

The Point of E_8 In F-theory GUTs

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Based on work with C. Vafa, as well as:

C. Beasley, V. Bouchard, S. Cecotti, M. Cheng, G.L. Kane, J. Marsano

N. Saulina, S. Schäfer-Nameki, J. Seo, J. Shao, A. Tavanfar

Outline

- Motivation: Bottom Up GUTs
- F-theory Ingredients
- Flavor and E_8
- ~~SUSY~~ and Cosmology

Motivation

Standard Model/MSSM \subset Strings?

What is possible in string constructions?

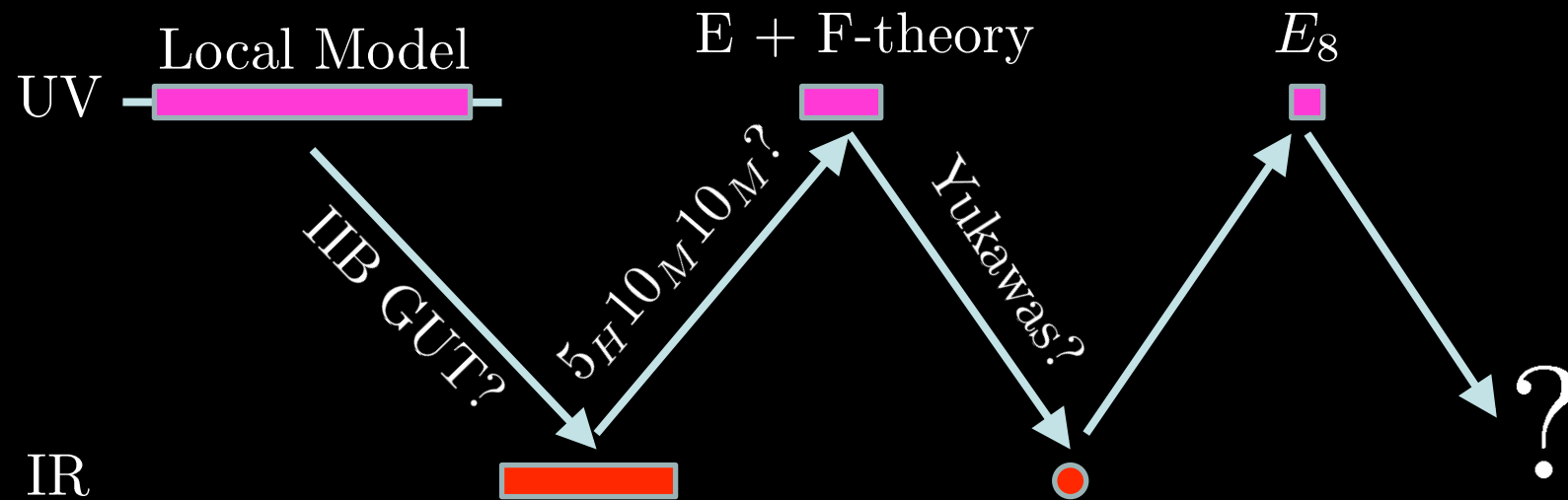
Hybrid Strategy:

Top Down: Specify All Details in UV (Global Models)

Where to look first?

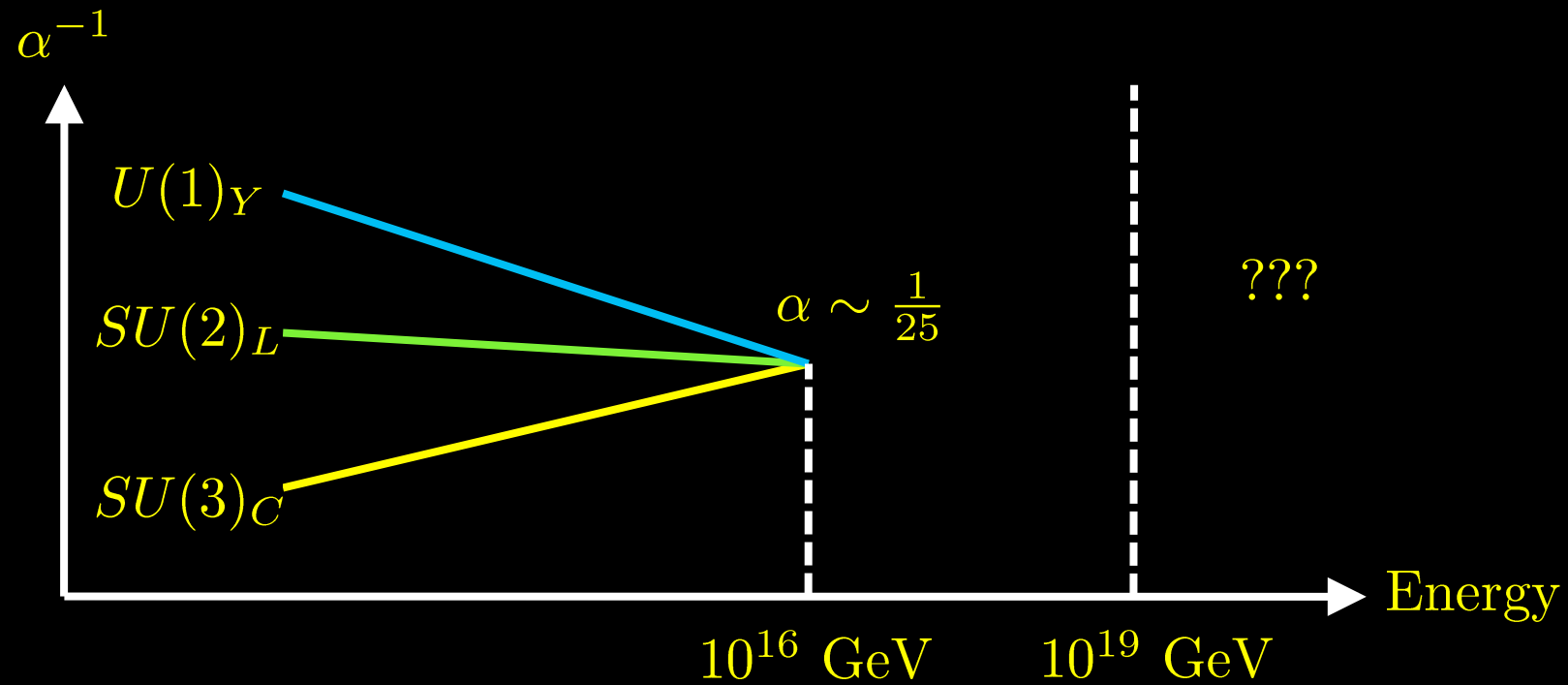
Bottom Up: Decouple some of UV (Local Models)

Too flexible?



Simplifying Assumptions:

1) Low energy supersymmetry & Unification:



2) $M_{GUT}/M_{pl} \ll 1$

Assumption 1: GUTs

$$SU(5)_{GUT} \supset SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$10_M = \begin{bmatrix} 0 & U & U & Q & Q \\ -U & 0 & U & Q & Q \\ -U & -U & 0 & Q & Q \\ -Q & -Q & -Q & 0 & E \\ -Q & -Q & -Q & -E & 0 \end{bmatrix} \quad 5_H = \begin{bmatrix} T_u \\ T_u \\ T_u \\ H_u \\ H_u \end{bmatrix}$$

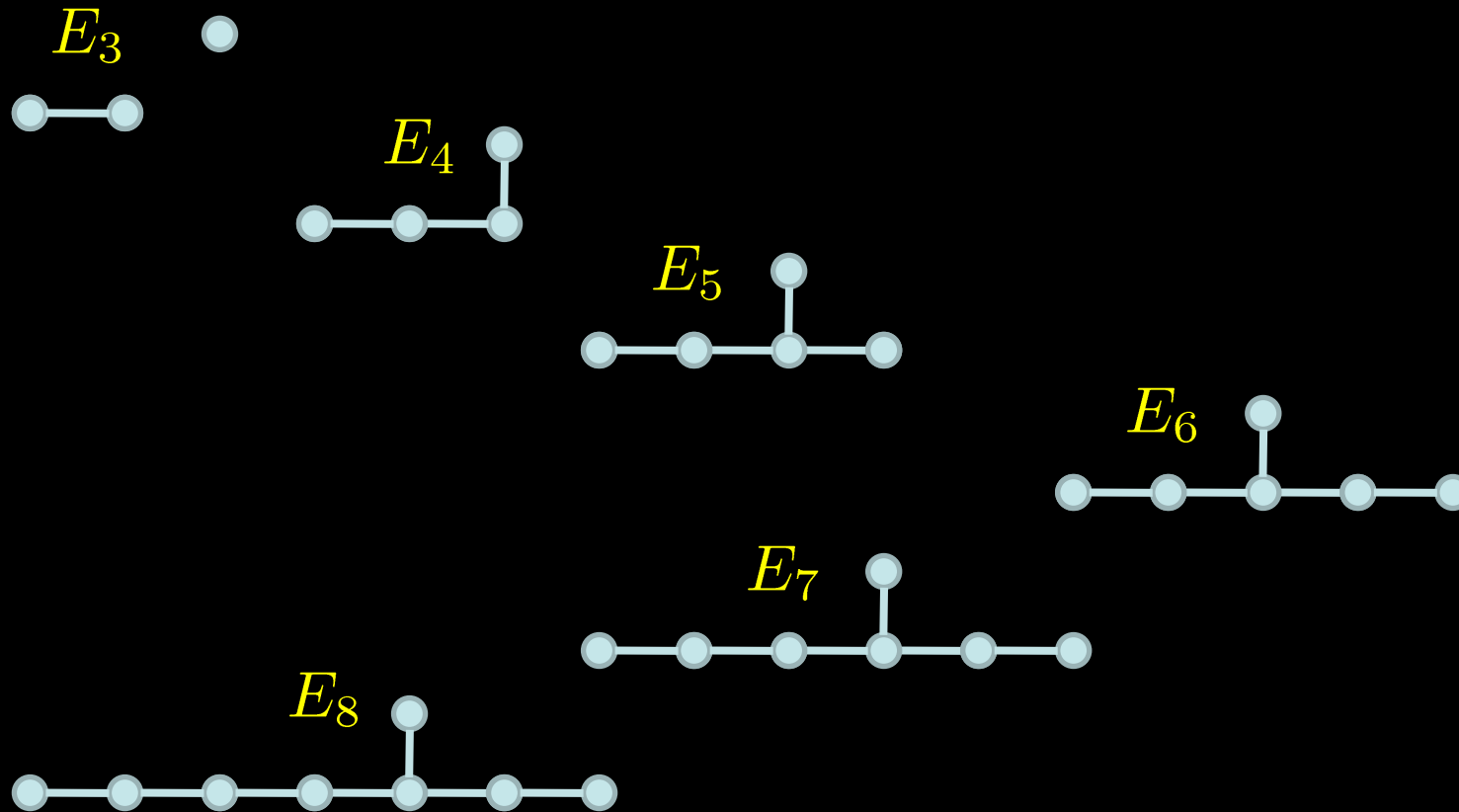
$$\bar{5}_M = [D \quad D \quad D \quad L \quad L]$$

$$\bar{5}_H = [T_d \quad T_d \quad T_d \quad H_d \quad H_d]$$

$$L_{GUT} \supset 5_H \times 10_M \times 10_M \Rightarrow t \text{ quark mass}$$

$$L_{GUT} \supset \bar{5}_H \times \bar{5}_M \times 10_M \Rightarrow b \text{ quark \& } \tau \text{ lepton mass}$$

GUTs and E_N



How much E is necessary? How much is aesthetics?

Assumption 2: $M_{GUT}/M_{pl} \ll 1$

$$10\text{D Gravity: } R^{3,1} \times \mathcal{M}_6 \Rightarrow G_{Newton} \sim \frac{1}{Vol(\mathcal{M}_6)}$$

Gravity decouples when $Vol(\mathcal{M}_6) \rightarrow \infty$

Gauge Theory on $R^{3,1} \times \mathcal{M}_k \subset R^{3,1} \times \mathcal{M}_6$:

$$\Rightarrow g_{YM}^2 \sim \frac{1}{Vol(\mathcal{M}_k)} \Rightarrow Vol(\mathcal{M}_k) \not\rightarrow \infty$$

Local Flexibility

Local Model suggests GUT from p - brane, $p = 3, 4, 5, 6, 7$

\Rightarrow Type II strings

E-type Structure: $g_s \rightarrow O(1)$

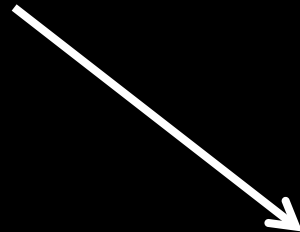
F-theory branes: 3-branes & 7-branes

E-type and 4d Chiral Matter \Rightarrow 7-branes

1) \exists a GUT



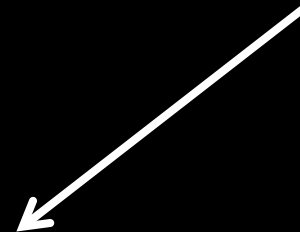
E-type Structures



2) \exists Decoupling Limit



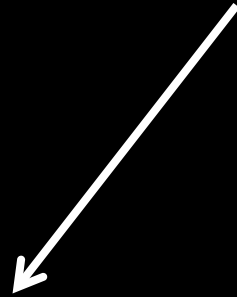
IIB 7-branes



F-theory?

Roadmap

- Motivation: Bottom Up GUTs



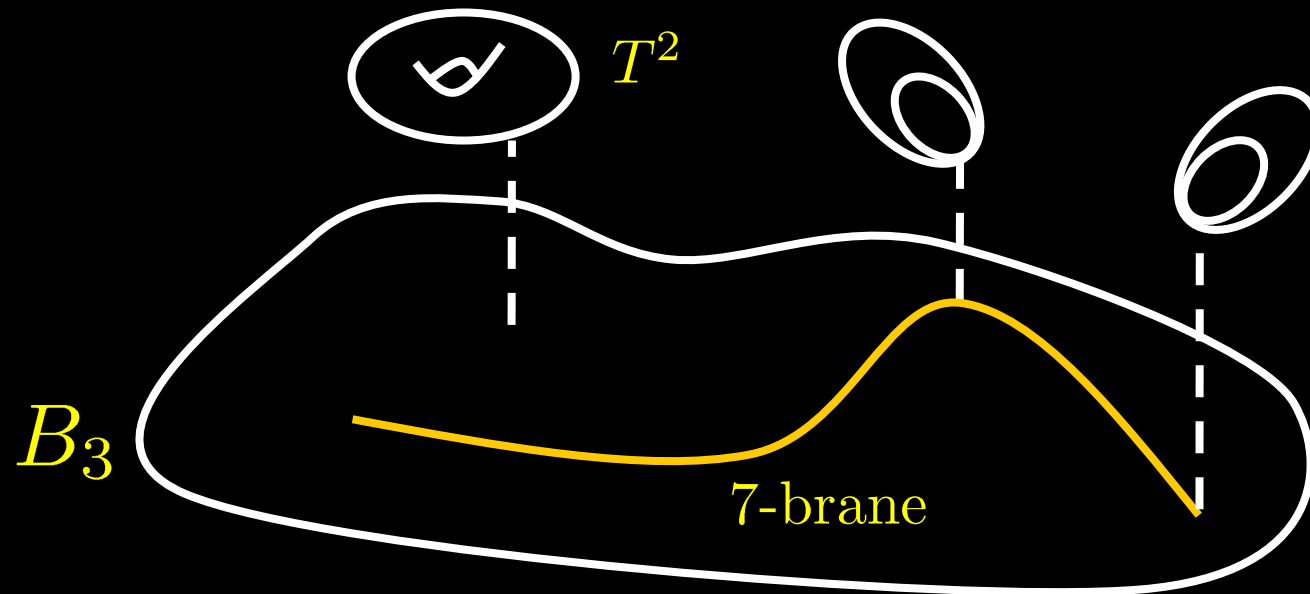
- F-theory Ingredients

F-theory Review I

F-theory = Strongly Coupled Formulation of IIB in 12d

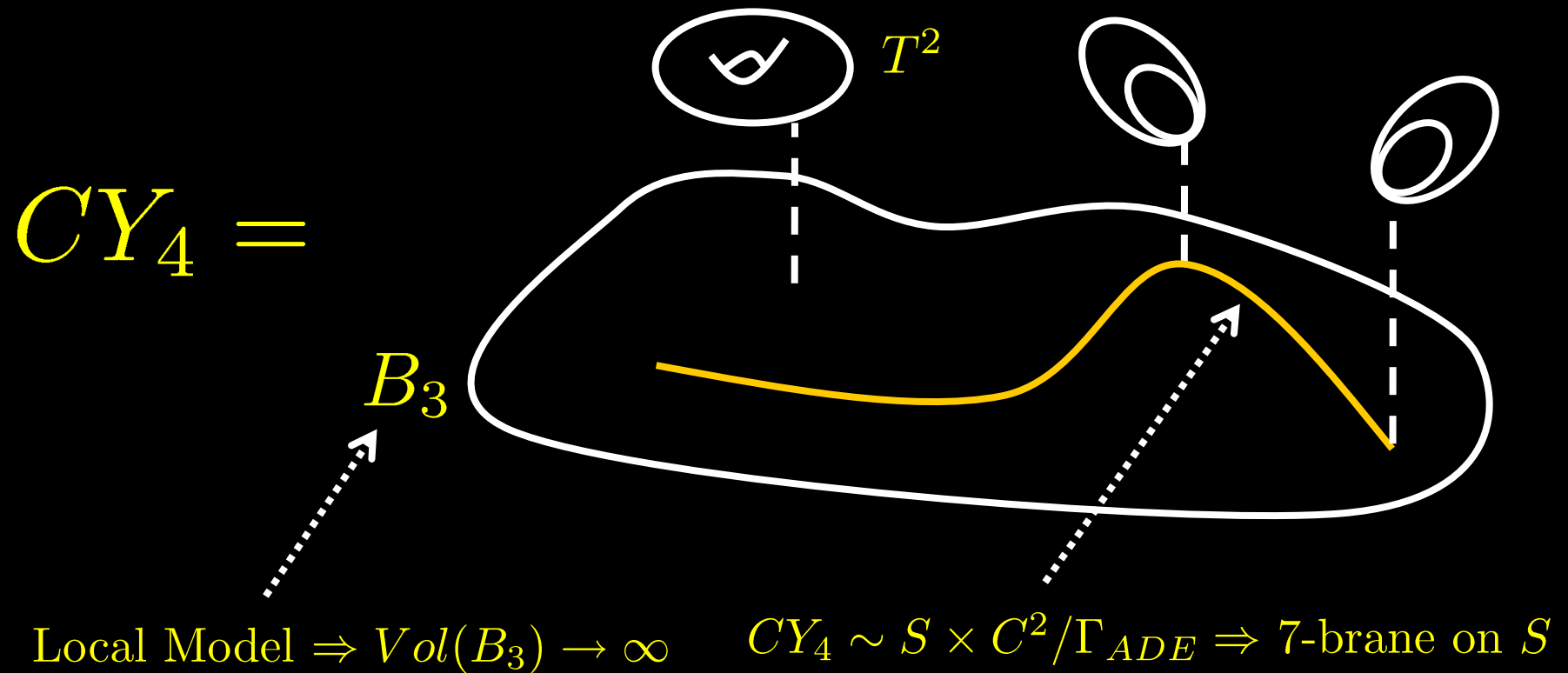
$\tau_{IIB} = C_0 + ie^{-\phi}$ is cplx str. of a T^2

This T^2 pinches off near 7-branes:



F-theory Review II

$$4d \mathcal{N} = 1 \Rightarrow F / R^{3,1} \times \text{Elliptic } CY_4$$



Geometry \Rightarrow Gauge Theory

$$F - th/R^{3,1} \times S \times C^2/\Gamma_{ADE} \Rightarrow 8d \text{ SYM w/gp } G_{ADE}$$

Example: 8d $SU(N)$ at $z = 0$ from $y^2 = x^2 + z^N$

10d \Rightarrow Gravity (decoupled)

8d : 7 \Rightarrow Gauge Group

6d : 7 \cap 7' \Rightarrow Matter

4d : 7 \cap 7' \cap 7'' \Rightarrow Yukawas

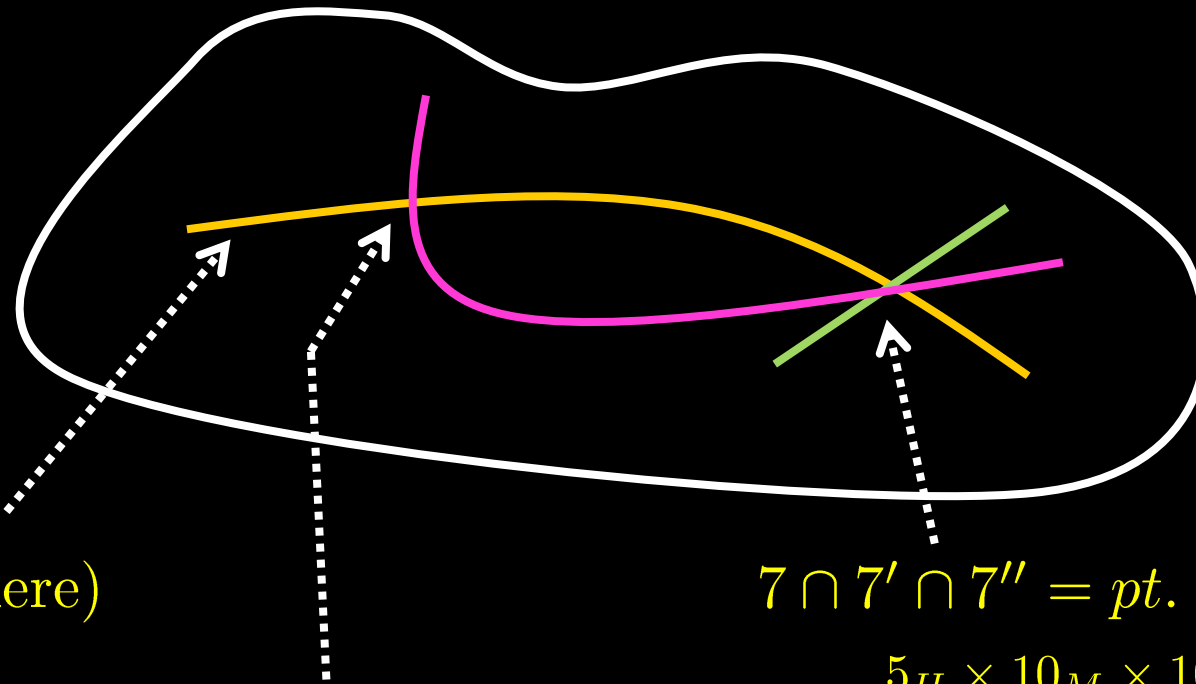
F-theory GUTs

Beasley JJH Vafa '08 (BHV I, II), Donagi Wijnholt I II '08
(see also Hayashi et al. '08 '09)

See also talk by Weigand

B_3 :

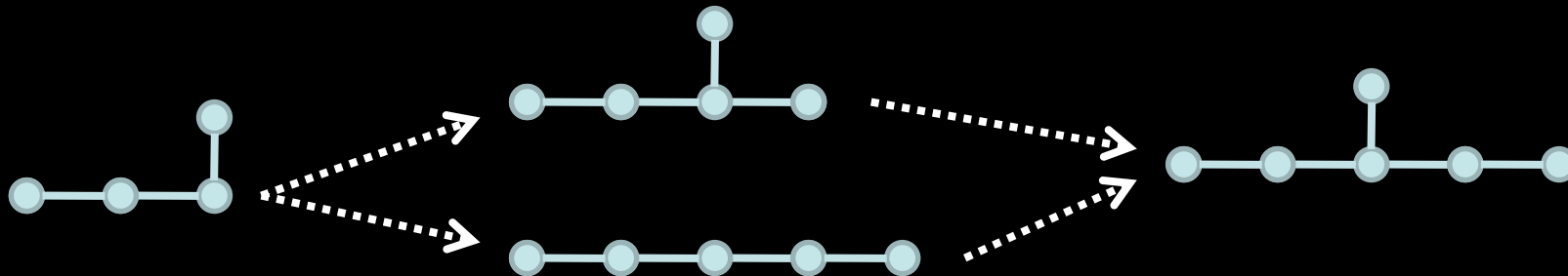
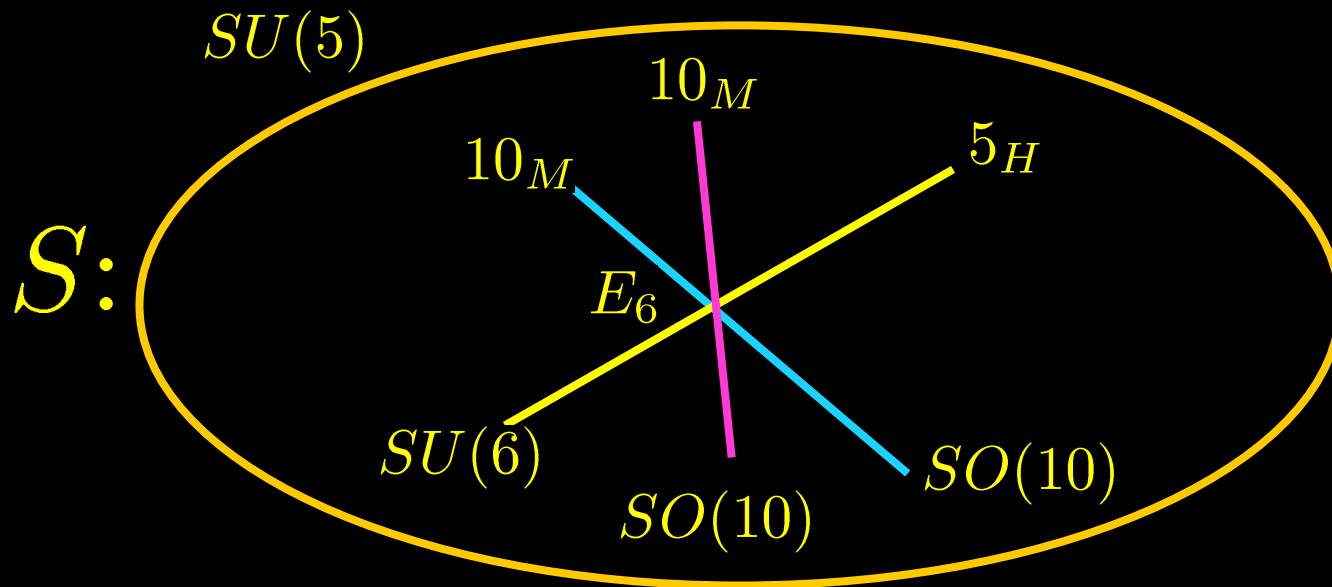
7-brane / S
(GUT lives here)



$7 \cap 7' = \text{curve} \subset S$
 $\bar{5}, 10 \in SU(5), 16 \in SO(10)...$

$7 \cap 7' \cap 7'' = pt. \subset S$
 $5_H \times 10_M \times 10_M...$

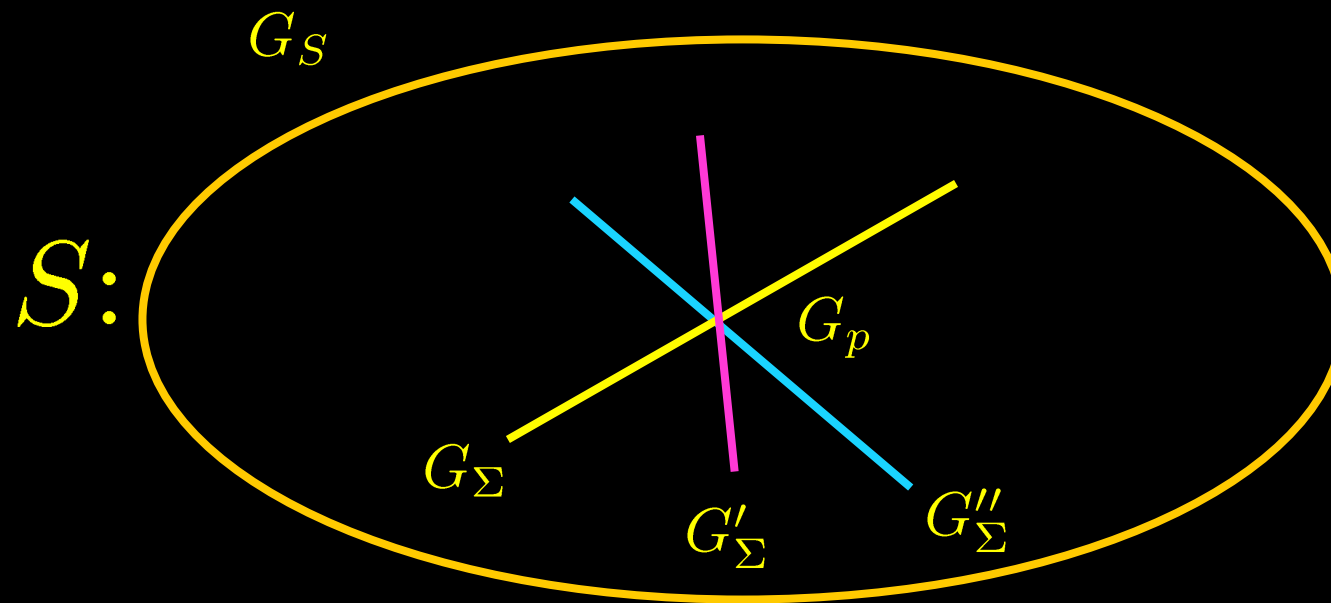
Higgsing By Geometry



$5_H 10_M 10_M$ needs E_N

Higgsing By Geometry

Local Higgsing: $G_p \rightarrow G_S \times G_{def}$



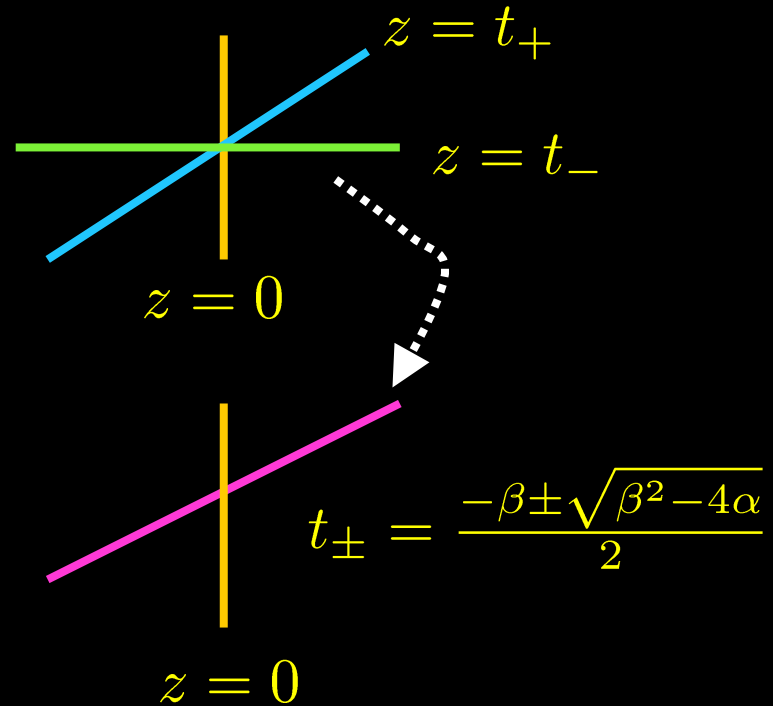
Parameterized by $\text{Cartan}(G_{def}) \text{ mod } \mathfrak{S} \subset \text{Weyl}(G_{def})$

“Monodromy Group” $\cdots \uparrow$

Monodromy Example

Location of 7-branes = roots of polynomials

$$y^2 = x^2 + z^5(z - t_+)(z - t_-)$$



$$y^2 = x^2 + z^5(z^2 + \alpha z + \beta)$$

$$(\beta^2 - 4\alpha) \rightarrow e^{2\pi i}(\beta^2 - 4\alpha) \Rightarrow t_+ \rightarrow t_-$$

More On Monodromy

Monodromy Important For Yukawas:

Hayashi et al. '09

$$\lambda_{ij} \cdot 5_H \times 10_M^{(i)} \times 10_M^{(j)}$$

$$\text{No Monodromy: } \lambda^{ij} = \begin{bmatrix} 0 & A & B \\ A' & 0 & C \\ B' & C' & 0 \end{bmatrix} \Rightarrow \geq 2 \text{ heavy gens}$$

$$\Sigma_{(i)} \overset{\mathfrak{S}_{mono}}{\longleftrightarrow} \Sigma_{(j)} \Rightarrow 1 \text{ heavy gen allowed}$$

4d Spectrum

$$G_S \xrightarrow{\text{instanton}} \Gamma_S \times H_S$$

4d matter \iff zero modes in instanton background

$$\left. \begin{array}{l} \text{S Modes: } \bar{\partial}_A \Psi = 0 \\ \text{\Sigma Modes: } \bar{\partial}_{A+A'} \sigma = 0 \end{array} \right\} \Rightarrow \text{Index Computation}$$
$$\int_M ch(V) Td(M)$$

Beasley JJH Vafa I '08
Donagi Wijnholt I '08

Minimal Spectrum

Beasley JJH Vafa II '08

$$G_S = SU(5) \xrightarrow{U(1)_Y \text{ flux}} SU(3) \times SU(2) \times U(1)_Y$$

No bulk exotics \Rightarrow unique internal flux

Higgs: $\int_{\Sigma_H} F_{U(1)_Y} \neq 0 :$

$\bar{5}_H =$	T_d	T_d	T_d	H_d	H_d
$5_H =$	T_u	T_u	T_u	H_u	H_u
	out			in	

Matter: $\int_{\Sigma_M} F_{U(1)_Y} = 0$

Matter: $\int_{\Sigma_M} F_{U(1)_\perp} = 3$

$3 \times \bar{5}_M$
$3 \times 10_M$
in

Roadmap

- F-theory Ingredients



- Flavor and E_8

Quark Wishlist

Two Light Generations

Hierarchical CKM Matrix:

$$|V_{CKM}| \sim \begin{bmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{bmatrix} \sim \begin{bmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.008 & 0.04 & 0.99 \end{bmatrix}$$

Minimal Ingredients

With Minimal Ingredients

What Yukawas Do We Get?

3 10_M 's on Σ_{10} curve

3 $\bar{5}_M$'s on $\Sigma_{\bar{5}}$ curve

1 $5_H \times 10_M \times 10_M$ point

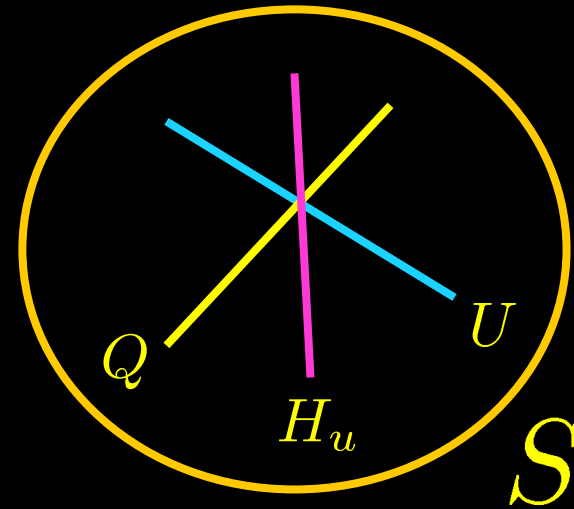
1 $\bar{5}_H \times \bar{5}_M \times 10_M$ point

+ Flux

Quark Yukawas:

$$R^{3,1}: W \supset \lambda_u^{ij} \cdot Q^i U^j H_u + \dots$$

$$\mathcal{M}_6: \bar{\partial}\Psi = 0: \Psi_Q^i, \Psi_U^i, \Psi_{H_u}, \dots$$



$$\lambda_u^{ij} = \underbrace{\Psi_Q^i(p) \Psi_U^j(p) \Psi_{H_u}(p)} + \dots$$

See Beasley JJH Vafa II '08
And Hayashi et al. '09

(outer product)

$$\begin{bmatrix} m_u & & \\ & m_c & \\ & & m_t \end{bmatrix} = \begin{bmatrix} 0 & & \\ & 0 & \\ & & m \end{bmatrix}$$

+ ... ?

$$\lambda_u^{ij} = \Psi_Q^i(p) \Psi_U^j(p) \Psi_{H_u}(p) + \dots$$

$$\lambda_d^{ij} = \Psi_Q^i(p) \Psi_D^j(p) \Psi_{H_d}(p) + \dots$$

Two light generations

$$\lambda_u^{ij} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & \lambda_t \end{bmatrix}$$

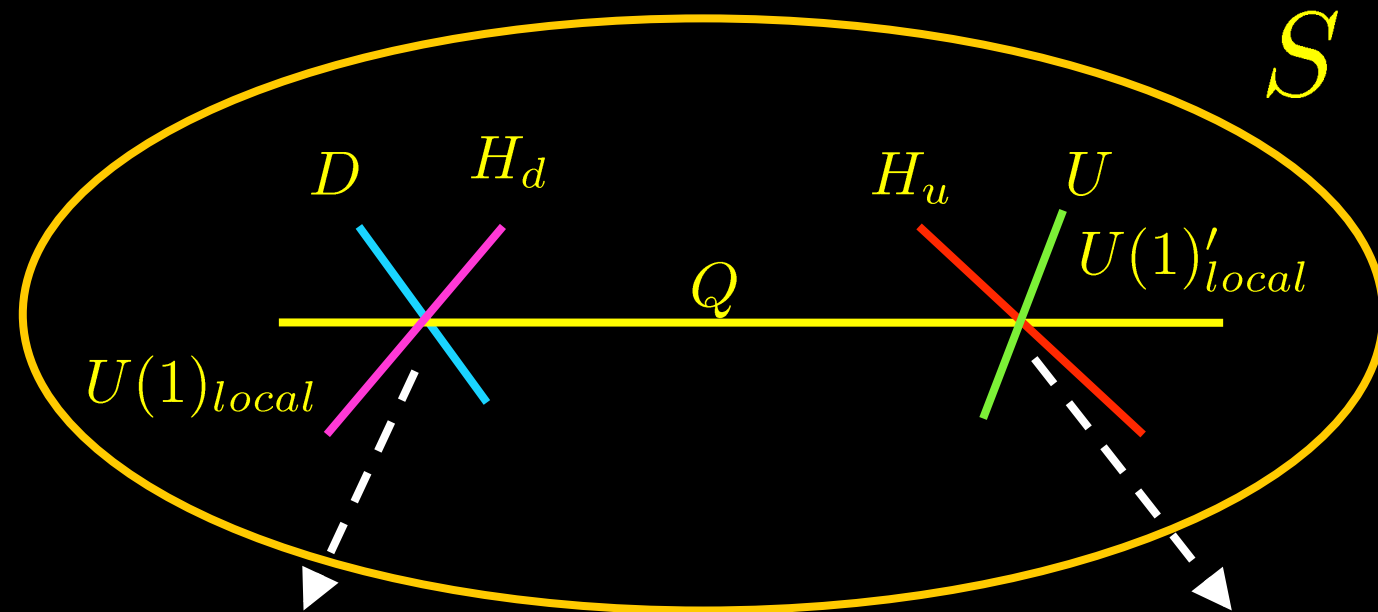
$$\lambda_d^{ij} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & \lambda_b \end{bmatrix}$$

What corrects this structure?

$$\partial B_{(0,2)} \neq 0$$

Cecotti, Cheng, JH, Vafa
In Progress

$U(1)$ Selection Rules



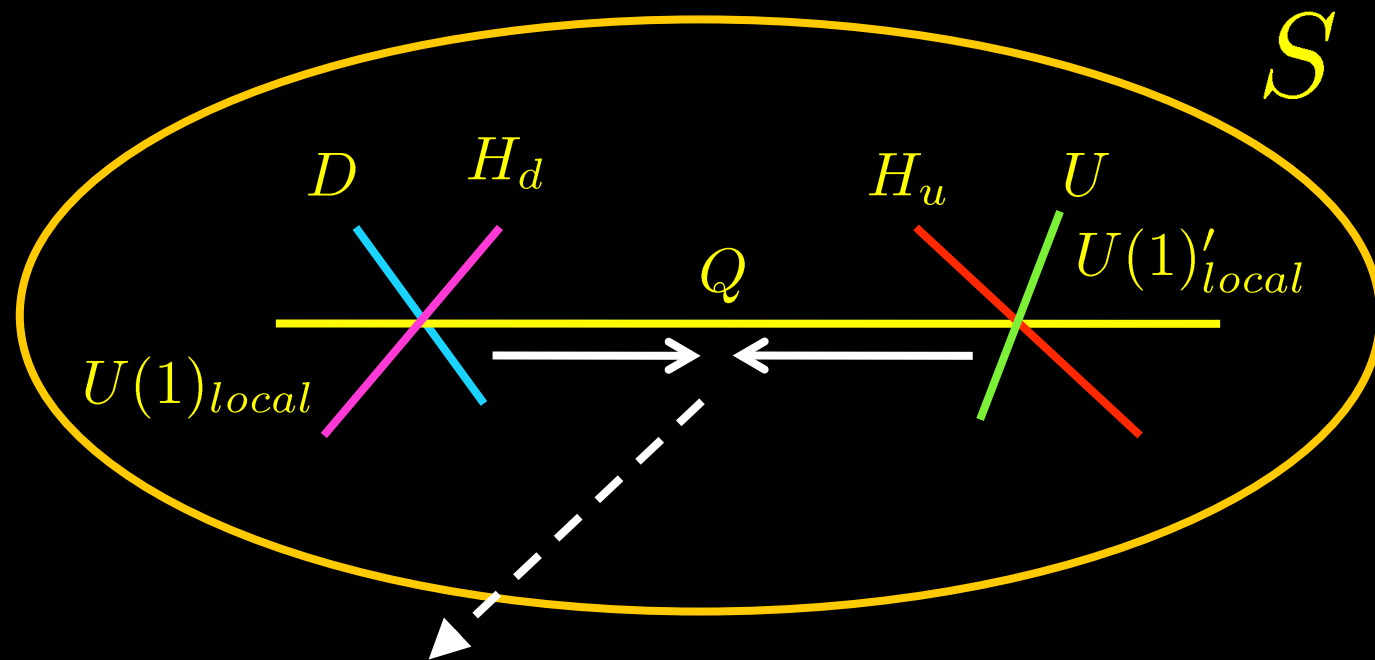
$$\lambda_d \sim \begin{bmatrix} \varepsilon_D^5 & \varepsilon_D^4 & \varepsilon_D^3 \\ \varepsilon_D^4 & \varepsilon_D^3 & \varepsilon_D^2 \\ \varepsilon_D^3 & \varepsilon_D^2 & 1 \end{bmatrix}$$

$$T_L \cdot \lambda_u \cdot T_R^\dagger \sim \begin{bmatrix} \varepsilon_U^8 & \varepsilon_U^6 & \varepsilon_U^4 \\ \varepsilon_U^6 & \varepsilon_U^4 & \varepsilon_U^2 \\ \varepsilon_U^4 & \varepsilon_U^2 & 1 \end{bmatrix}$$

IF $U(1)_{local} \neq U(1)'_{local}$: No CKM Hierarchy

$$\mathcal{P}_{down} \rightarrow \mathcal{P}_{up}$$

$$U(1)_{local} \rightarrow U(1)'_{local} \Rightarrow \text{CKM Hierarchy}$$



$$|V_{CKM}| \sim \begin{bmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{bmatrix}$$

$$E_6 : 5_H 10_M 10_M$$

$$SO(12) : \bar{5}_H \bar{5}_M 10_M$$

$$E_7$$

CKM Matrix

$$|V_{CKM}| \sim \begin{bmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{bmatrix} \quad \begin{aligned} \varepsilon^2 &\sim Flux^2 / M_*^4 \\ &\sim Vol(S)^{-1} / M_*^4 \sim \alpha_{GUT} \end{aligned}$$



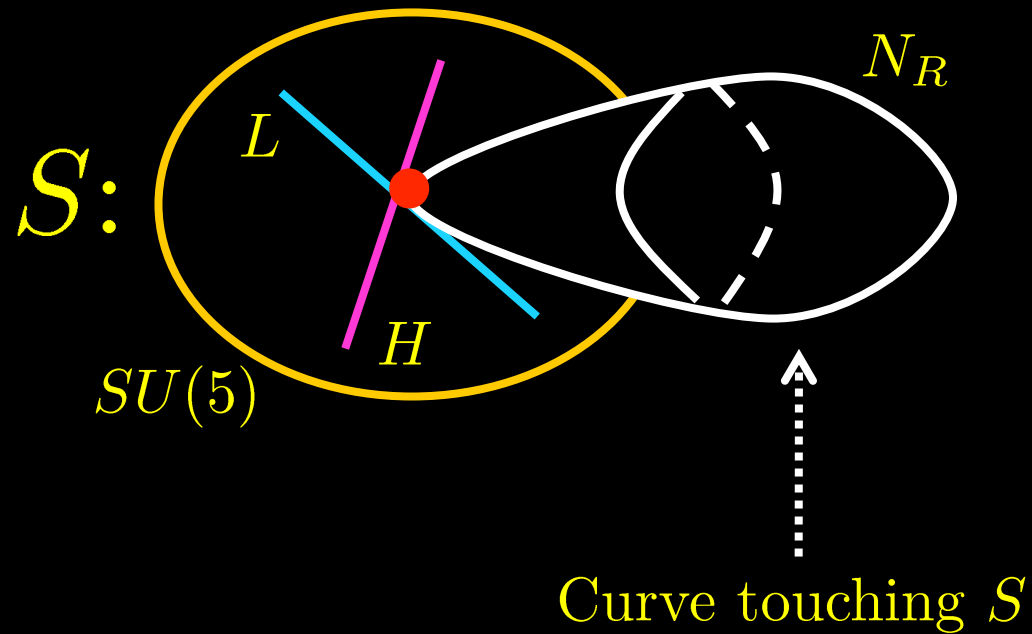
$$|V_{CKM}^{F-th}| \sim \begin{bmatrix} 1 & 0.2 & 0.008 \\ 0.2 & 1 & 0.04 \\ 0.008 & 0.04 & 1 \end{bmatrix}$$

$$|V_{CKM}^{obs}| \sim \begin{bmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.008 & 0.04 & 0.99 \end{bmatrix}$$

Neutrinos

Beasley, JJH, Vafa '08

Bouchard, JJH, Seo Vafa '09



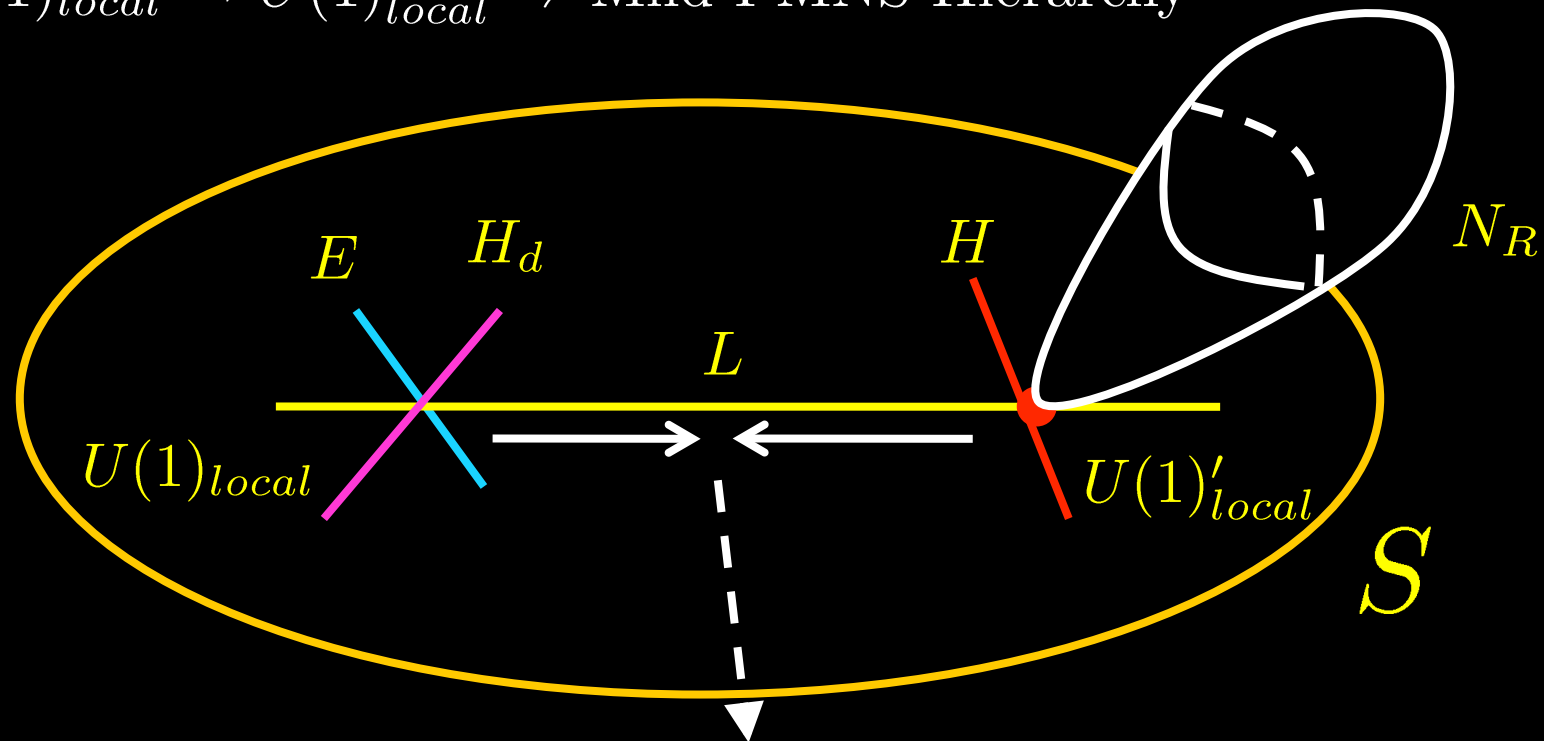
$$H = \gamma_{GUT} \cap \gamma'_{\perp}$$

$$L = \gamma_{GUT} \cap \gamma''_{\perp}$$

$$N_R = \gamma'_{\perp} \cap \gamma''_{\perp}$$

ν Mixing Hierarchy

$U(1)_{local} \rightarrow U(1)'_{local} \Rightarrow$ Mild PMNS Hierarchy



$$|V_{PMNS}| \sim \begin{bmatrix} U_{e1} & \varepsilon^{1/2} & \varepsilon \\ \varepsilon^{1/2} & U_{\mu 2} & \varepsilon^{1/2} \\ \varepsilon & \varepsilon^{1/2} & U_{\mu 3} \end{bmatrix}$$

ν Masses

$$m_{\nu_1} : m_{\nu_2} : m_{\nu_3} \sim \varepsilon_N^2 : \varepsilon_N : 1 \Rightarrow \begin{array}{c} \nu_3 \\ \hline \nu_2 \\ \hline \nu_1 \end{array} \quad \text{“Normal Hierarchy”}$$

$$\text{Predict: } \frac{m_{\nu_2}^2 - m_{\nu_1}^2}{m_{\nu_3}^2 - m_{\nu_2}^2} \sim \alpha_{GUT} \sim 0.04$$

Close!

$$\text{Observe: } \frac{m_{\nu_2}^2 - m_{\nu_1}^2}{m_{\nu_3}^2 - m_{\nu_2}^2} = \frac{m_{sol}^2}{m_{atm}^2} \sim 0.03$$

PMNS Matrix

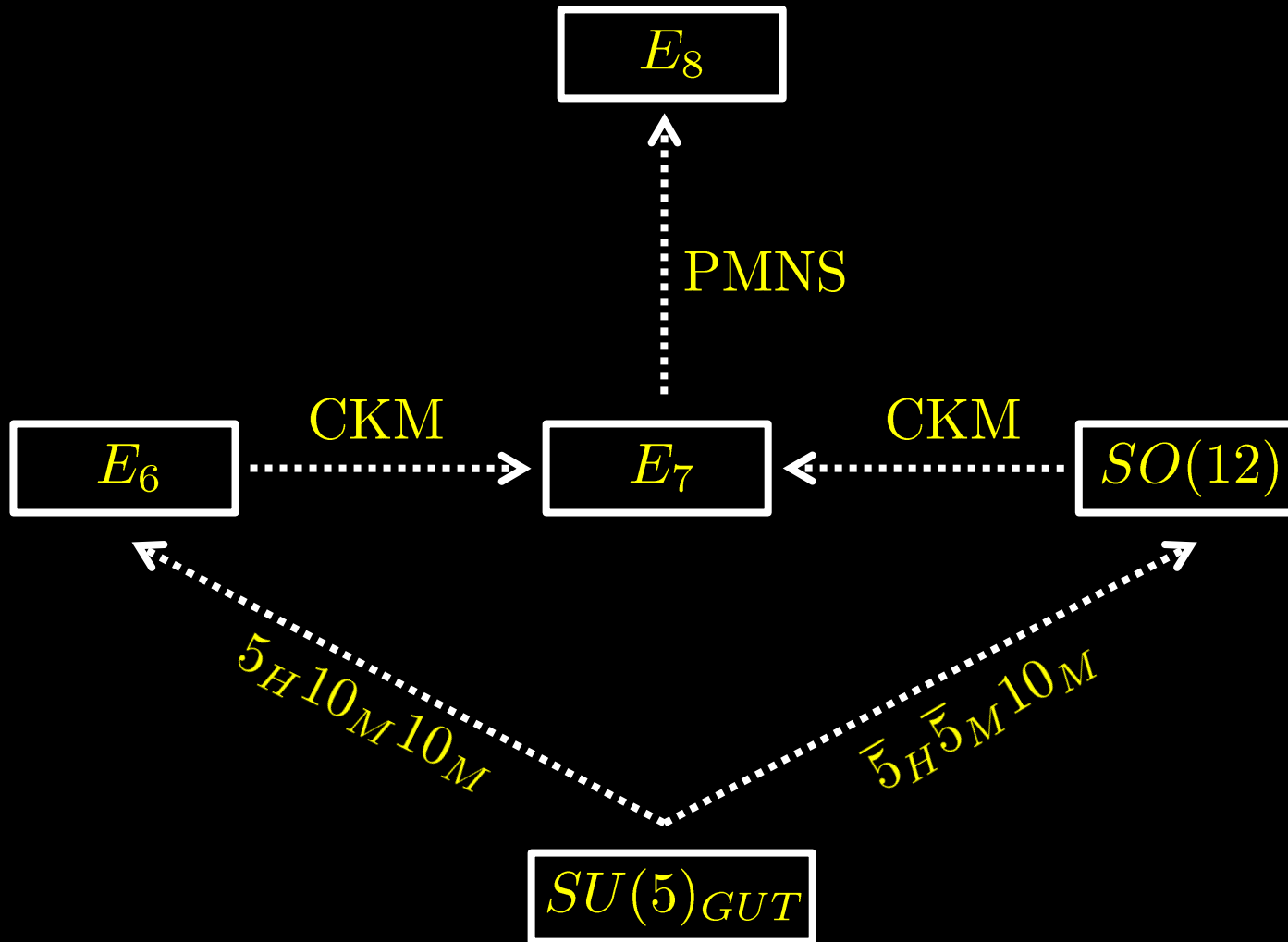
Bouchard, JJH, Seo Vafa '09

$$|V_{PMNS}^{F-th}| \sim \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \\ \left[\begin{array}{ccc} 0.87 & 0.45 & 0.2 \\ 0.45 & 0.77 & 0.45 \\ 0.2 & 0.45 & 0.87 \end{array} \right] & \begin{array}{l} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \end{array}$$

$$|V_{PMNS}^{obs(3\sigma)}| \sim \left[\begin{array}{ccc} 0.77 - 0.86 & 0.50 - 0.63 & 0.00 - 0.22 \\ 0.22 - 0.56 & 0.44 - 0.73 & 0.57 - 0.80 \\ 0.21 - 0.55 & 0.40 - 0.71 & 0.59 - 0.82 \end{array} \right]$$

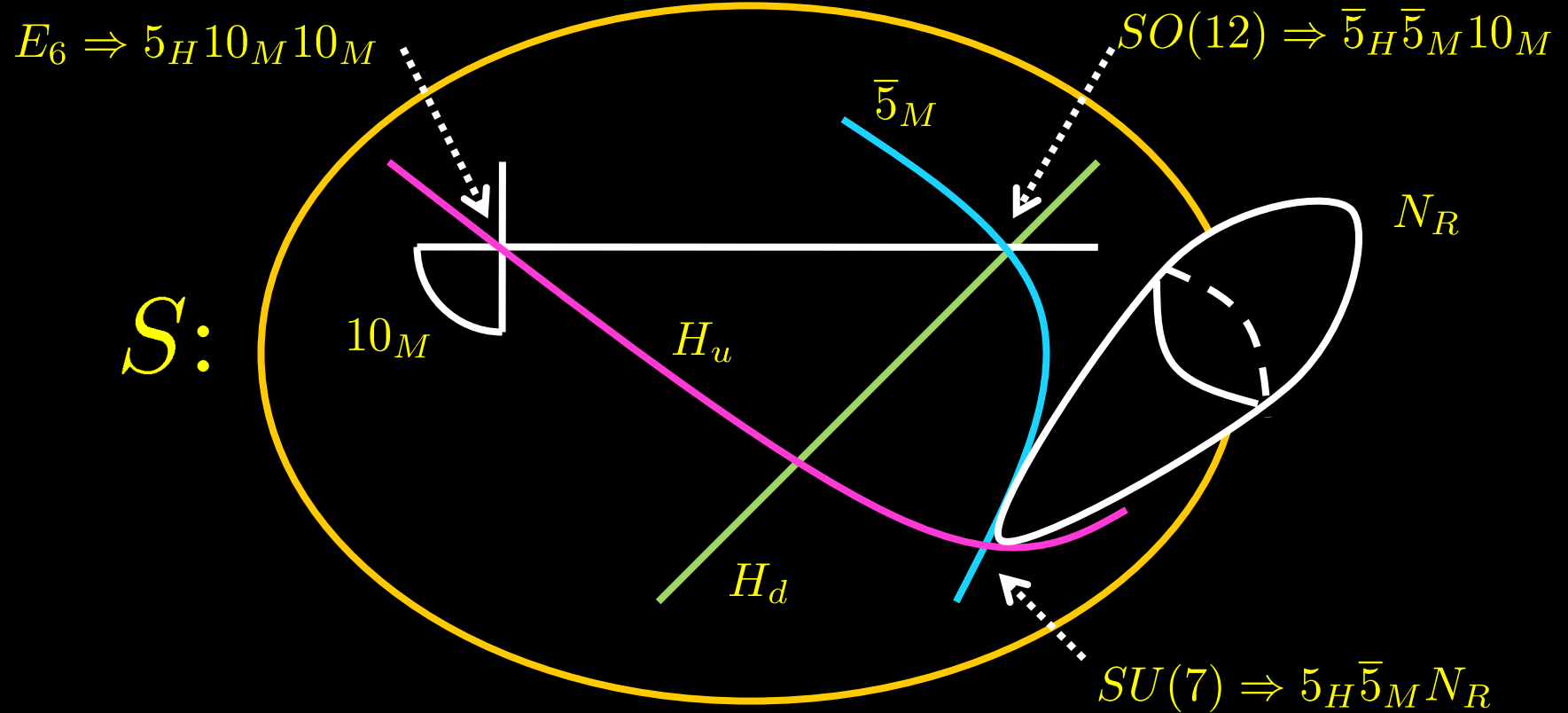
\Rightarrow Predict $V_{PMNS}^{1,3}$ close to current bound

$$\text{CKM} + \text{PMNS} \Rightarrow E_8$$



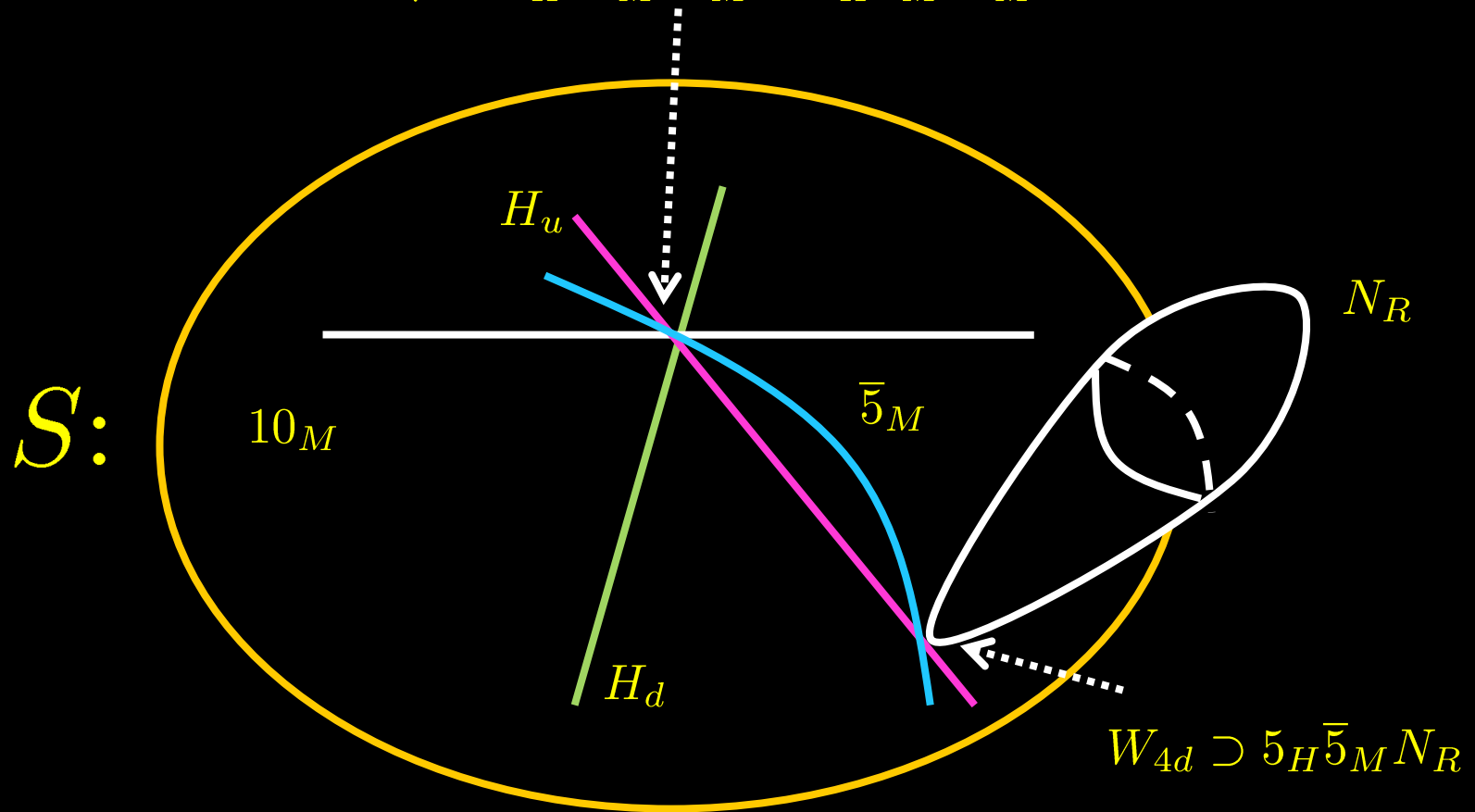
Point Unification

Beasley JJH Vafa II '08

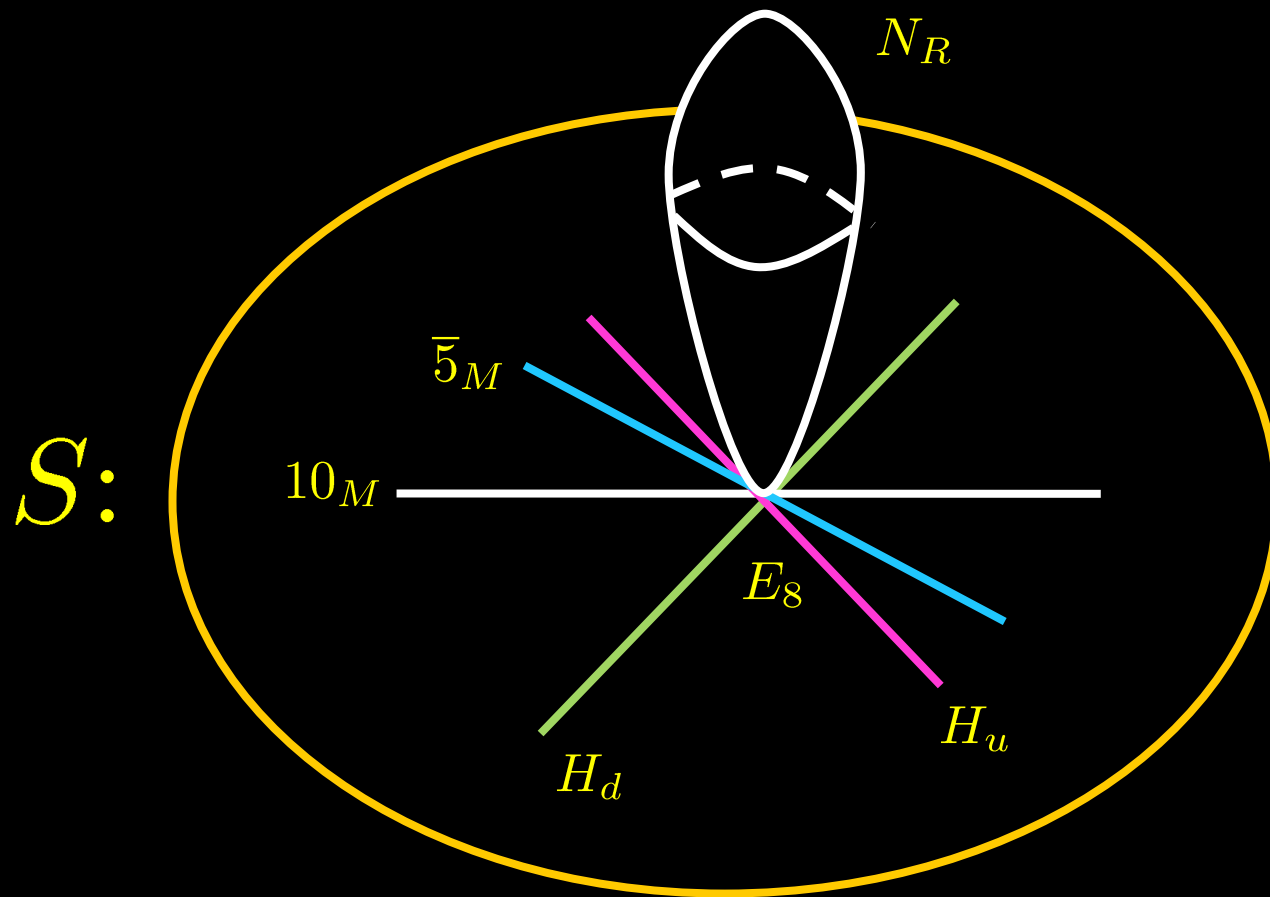


Point Unification

$$E_7 \Rightarrow 5_H 10_M 10_M + \bar{5}_H \bar{5}_M 10_M$$



Point Unification



Extra Matter?

Roadmap

- Flavor and E_8



- ~~SUSY~~ and Cosmology

Monodromy and E_8

$$E_8 \supset SU(5)_{GUT} \times SU(5)_{def}$$

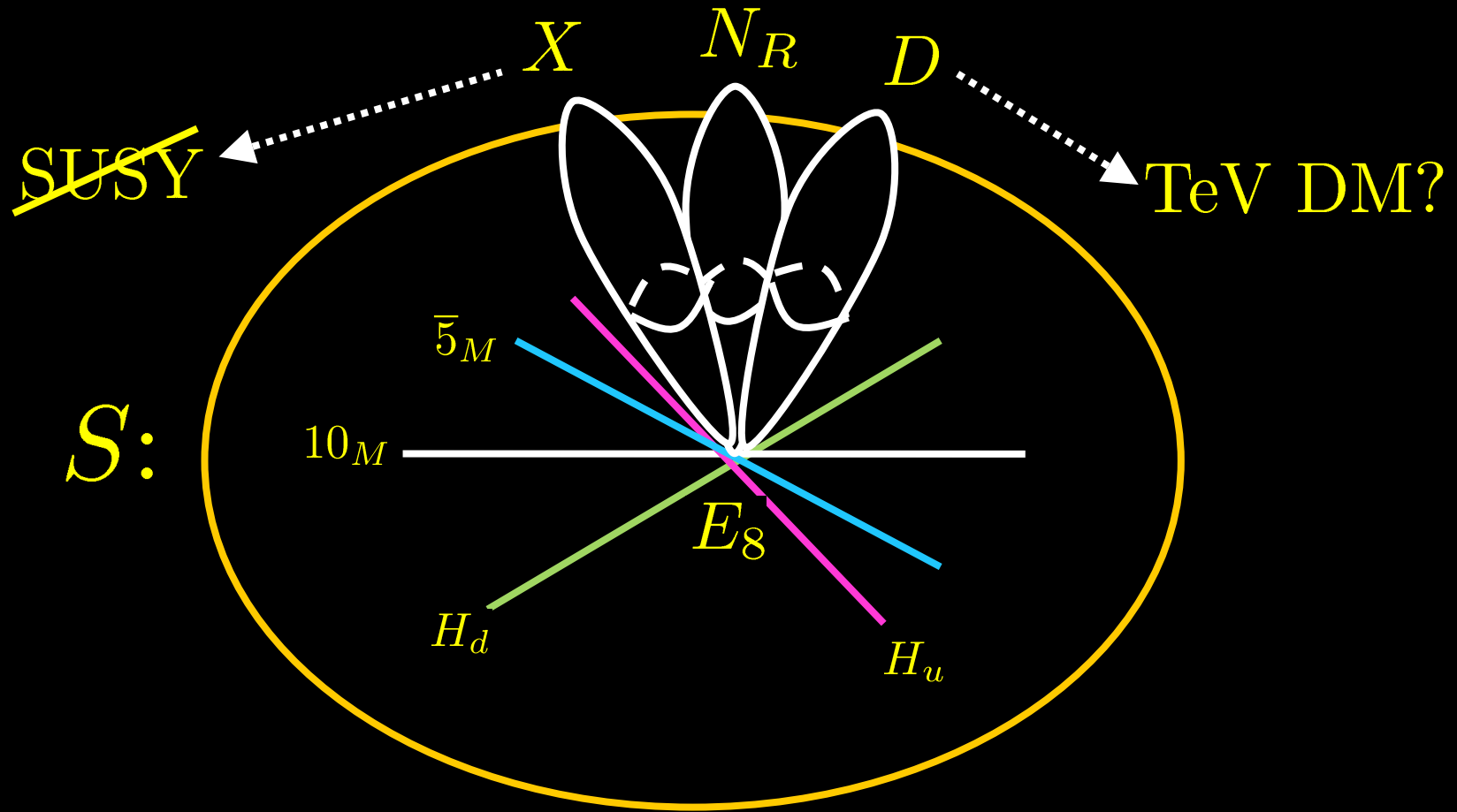
$$248 \rightarrow (5_G, 10_\perp) + (\bar{5}_G, \bar{10}_\perp) + (10_G, \bar{5}_\perp) + (\bar{10}_G, 5_\perp) + adj$$

$$\mathfrak{S}_{mono} \subset Weyl(SU(5)_{def}) = S_5$$

Flavor + Monodromy \Rightarrow Small List of Available Groups:

$$Z_2, Z_2 \times Z_2, Z_3, S_3, Dih_4$$

GUT Singlets



Left-Over?

$$E_8 \supset SU(5)_{GUT} \times SU(5)_{\perp} + \text{Monodromy} \Rightarrow$$

Almost no room left except:

JJH Tavanfar Vafa '08

~~SUSY~~: $U(1)_{PQ}$, $\int d^4\theta \frac{X^\dagger H_u H_d}{\Lambda_{UV}}$ and $\int d^2\theta X Y_R Y'_R$

$$\langle X \rangle = x + \theta^2 F \Rightarrow \mu\text{-term and min. Gauge Mediation}$$

In nearly all cases messengers in $10 \oplus \overline{10}$

Some Singlets: TeV Dark Matter?

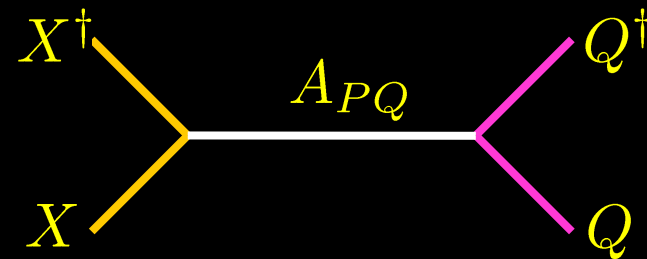
PQ Deformed GMSB

JJH, Vafa '08

String Theory $\Rightarrow U(1)_{PQ}$ gauge boson

Heavy $U(1)_{PQ}$ exchange \Rightarrow

(see Arkani-Hamed, Dine, Martin '97)



$$\text{@ UV: } m_{soft}^2 = m_{mGMSB}^2 - q\Delta_{PQ}^2$$

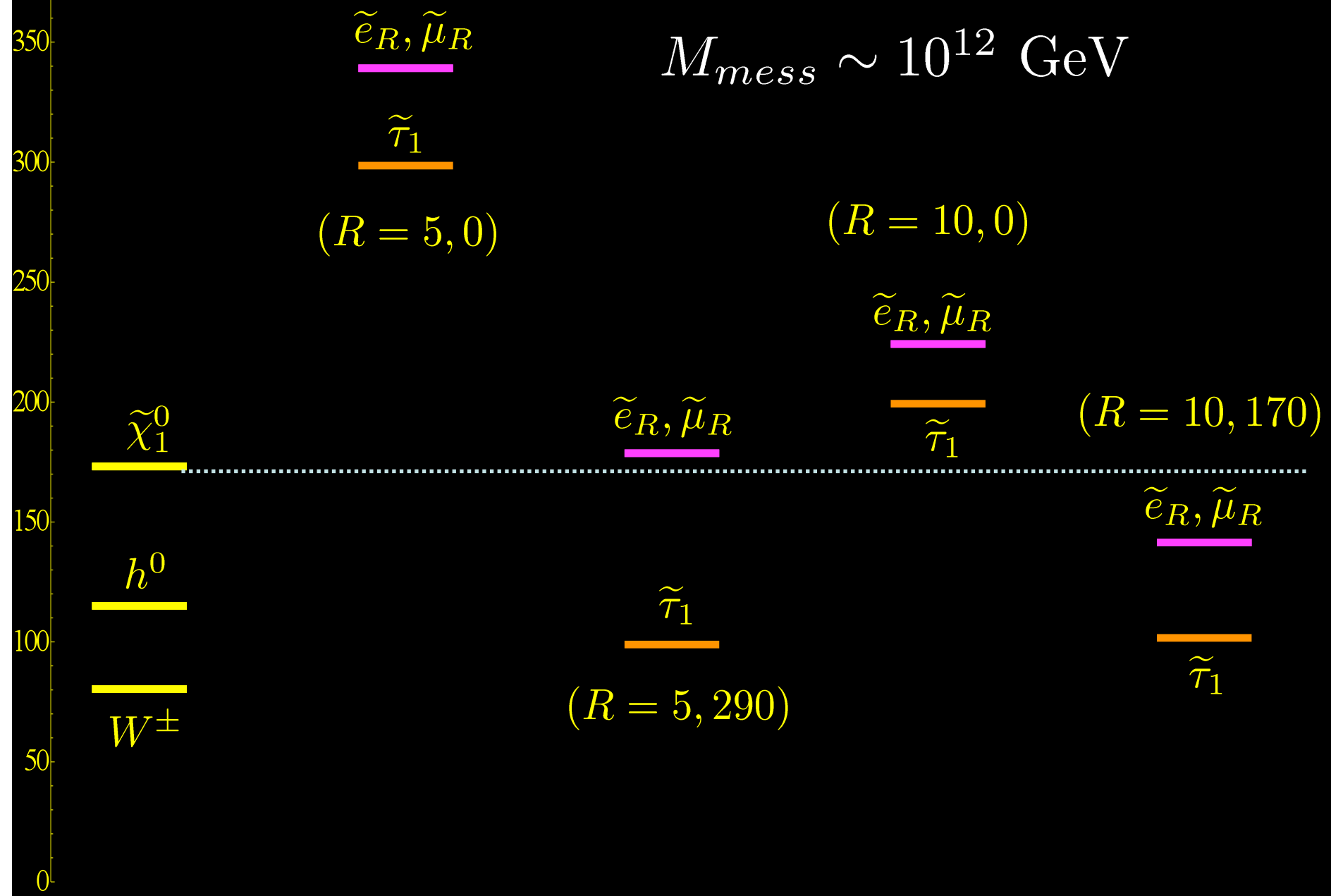
For most common $10 \oplus \overline{10}$ scenario

$\tilde{\tau}_1$ is typically the quasi-stable NLSP

GeV

$$R \oplus \bar{R}, \Delta_{PQ} \text{ (GeV)}$$

$$M_{mess} \sim 10^{12} \text{ GeV}$$



LHC?

JJH, Kane, Shao Vafa '09

NLSP is either quasi-stable $\tilde{\tau}_1$ or $\tilde{\chi}_1^0$

$\tilde{\tau}_1$ NLSP: $\tilde{\tau}_1$ track \Rightarrow mass reconstruction of Δ_{PQ}

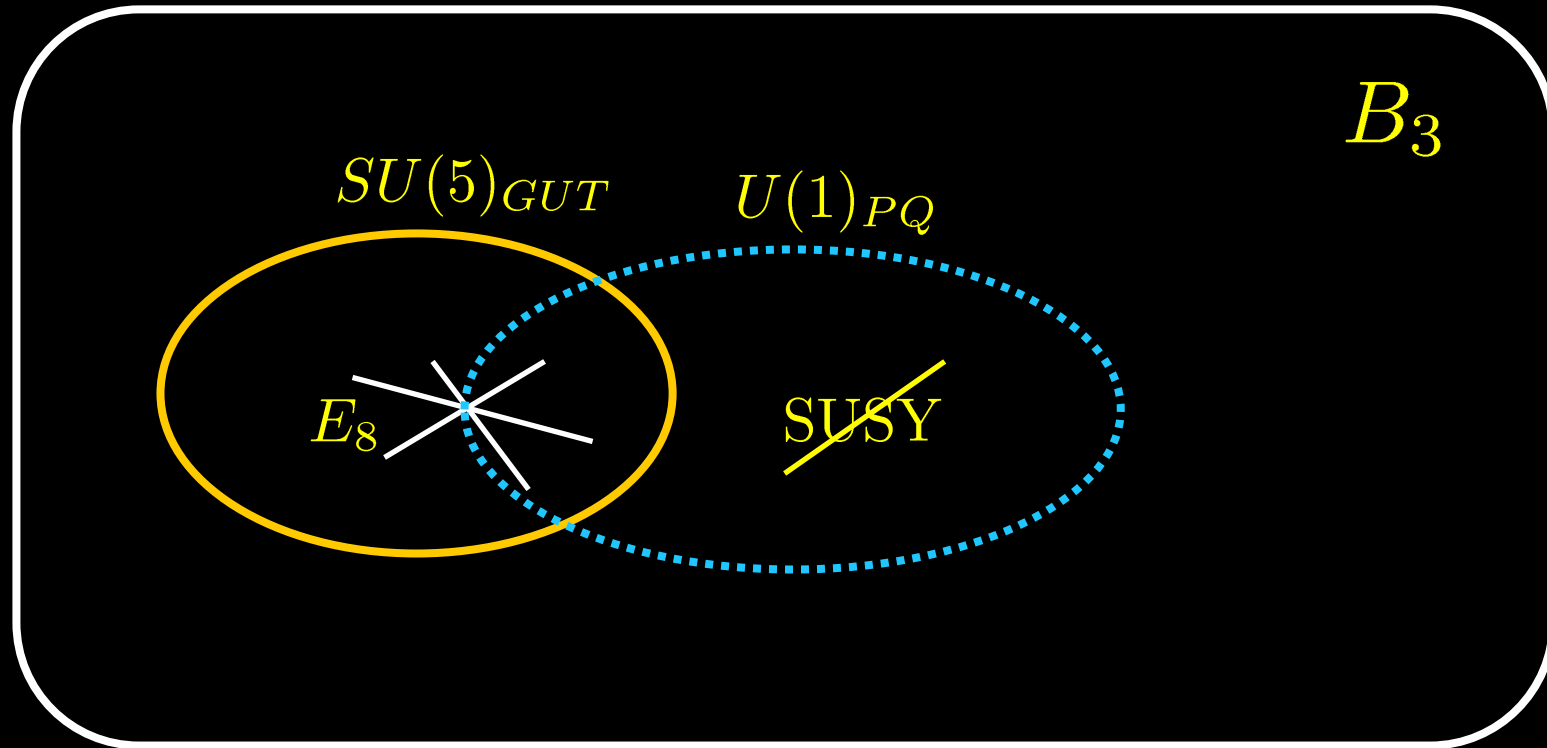
$\tilde{\chi}_1^0$ NLSP e.g. $5 \oplus \bar{5}$ case

can STILL distinguish F-th($\Delta_{PQ}^{(1)}$) and F-th($\Delta_{PQ}^{(2)}$)

5 fb^{-1} : $\pm 100 \text{ GeV}$

50 fb^{-1} : $\pm 10 \text{ GeV}$

TeV Dark Matter?



Must be nearby PQ brane to get TeV mass JH Tavanfar Vafa '09

Singlets from E_8 all decay too rapidly \Rightarrow no candidate

Any Dark Matter?

JJH Tavanfar Vafa '08

~~SUSY~~ \Rightarrow 10 – 100 MeV LSP Gravitino

“Gravitino Problem”: Predicted $\Omega_{3/2} h^2 \gg 1$?

Oscillation and decay of saxion dilutes thermal relics

$$\Omega_{3/2}^{F-th} h^2 \sim 0.1$$

\Rightarrow astrophysical origin for PAMELA, ATIC, FERMI, ...

JJH Tavanfar Vafa '09

Conclusions

- Bottom Up GUTs and F-theory
- Flavor $\Rightarrow E_8$
- $10 \oplus \overline{10}$ Messengers Most Common
- 10 – 100 MeV Gravitino = Dark Matter