Quantum bosons for holographic superconductors

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Plan of talk

Motivation – unconventional phases at finite density

1. Low temperature and finite density (bosons and fermions)
2. Two uses of magnetic fields
3. Application to High - T_c superconductors

Holographic superconductors

1. Ingredients for a holographic superconductor
2. Black hole instabilities

Quantum bosons and magnetic fields

1. Quantum bosons and fermions in free field theories
2. Quantum bosons in strongly coupled field theories
3. The free energy and quasinormal modes

Motivation – unconventional phases at finite density

1. Low temperature and finite density (bosons and fermions)
2. Two uses of magnetic fields
3. Application to High - T_c superconductors
4. Quantum oscillations in High - T_c superconductors
5. Quantum criticality under the dome in High - T_c superconductors

Low temperature and finite density

- Effective field theories in condensed matter physics often have a finite charge density.
- Finite density: huge effect on the zero temperature ground state.
- Most commonly encountered states:
  - Charged fermions: Fermi surface is built up.
  - Charged bosons: condensation instabilities (e.g. superconductivity).
- The low energy excitations about a condensate or Fermi surface are very well characterised. It is a weak coupling description.
- There seem to be materials where these descriptions do not work.
- Perspective of this talk: AdS/CFT gives a tractable theory with an exotic finite density ground state.

Two uses of magnetic fields

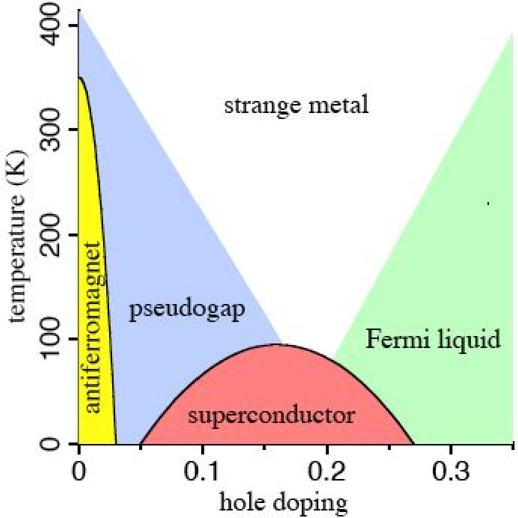
- de Haas - van Alphen effect: a Fermi surface leads to oscillations in the magnetic susceptibility as a function of $1/B$.
  - In a magnetic field

$$[P_x, P_y] \sim iB \quad \Rightarrow \quad \oint P_x dP_y \sim 2\pi(\ell + \tfrac{1}{2})B \,.$$

  - When the area of the orbit is a cross section of the Fermi surface there is a sharp response. I.e. at $1/B \sim \ell/A_F \sim \ell/k_F^2 \sim \ell/\mu^2$.

- Large magnetic field will suppress superconducting instabilities.
  - Energy cost of expelling magnetic field becomes too large, or
  - Vortices sufficiently dense to prevent superconductivity.

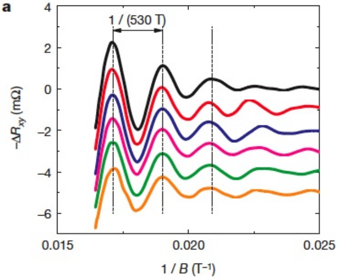
Application to High - T_c superconductors

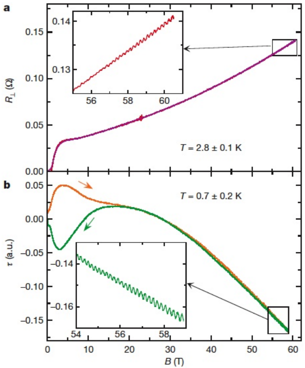


Quantum oscillations in High - T_c superconductors

Doiron-Leyraud et al. 2007 (Nature), Vignolle et al. 2008 (Nature).

- de Haas - van Alphen oscillations in underdoped and overdoped cuprates.



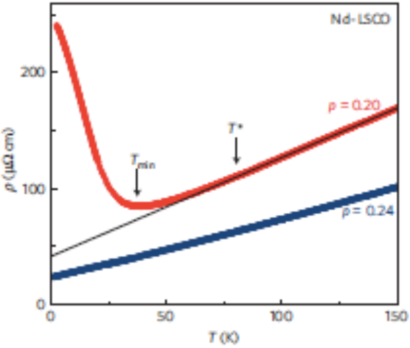


- In underdoped region, carrier density much lower than naïve expectation: "small Fermi surface".

Criticality under the dome in High - T_c superconductors

Daou et al. 2008 (Nature Physics)

- Resistivity in 'normal phase' linear in temperature (anomalous).
- Applying a large magnetic field shows persistance down to $T = 0$ at critical doping.



Holographic superconductors

1. Ingredients for a holographic superconductor
2. Black hole instabilities
3. Hairy black holes

Minimal ingredients for a holographic superconductor

- Minimal ingredients
  - Continuum theory ⇒ have $T^{\mu\nu}$ ⇒ need bulk g_{ab}.
  - Conserved charge ⇒ have J^{μ} ⇒ need bulk A_a.
  - 'Cooper pair' operator ⇒ have $\mathcal{O}$ ⇒ need bulk ϕ.
- Write a minimal 'phenomenological' bulk Lagrangian

$$\mathcal{L}_{1+3} = \frac{1}{2\kappa^2}R + \frac{3}{L^2\kappa^2} - \frac{1}{4g^2}F_{ab}F^{ab} - |\nabla\phi - iqA\phi|^2 - m^2|\phi|^2 \,.$$

There are four dimensionless quantities in this action.

- The central charge of the CFT is $c = 192L^2/\kappa^2$.
- DC conductivity $\sigma_{xx} = \frac{1}{g^2}$.
- $\Delta(\Delta - 3) = (mL)^2$. Either root admissible if $\Delta \geq \frac{1}{2}$.
- The charge q is the charge of the dual operator $\mathcal{O}$.

Two instabilities of a charged AdS black hole

- By dimensional analysis $T_c \propto \mu$.
- The dual geometry is therefore Reissner-Nordstrom-AdS.
- RN-AdS can be unstable against a (charged) scalar for two reasons.
- Reason 1 [Gubser '08]: Background charge shifts mass:

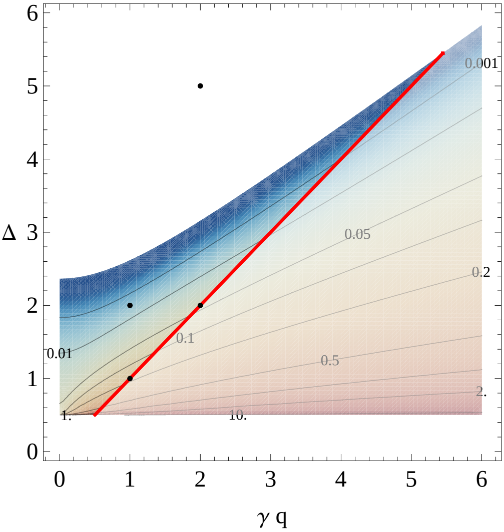
$$m^2_{\text{eff.}} \sim m^2 - q^2 A_t^2 \,.$$

- Reason 2 [SAH-Herzog-Horowitz '08]: Near extremality AdS_2 throat with

$$m^2_{\text{BF-2}} = -\frac{1}{4L_2^2} = -\frac{3}{2L^2} > -\frac{9}{4L^2} = m^2_{BF-4} \,.$$

- Precise criterion for instability at $T = 0$ [Denef-SAH '09, Gubser '08]

$$q^2\gamma^2 \geq 3 + 2\Delta(\Delta - 3) \,, \qquad \gamma^2 = \frac{2g^2L^2}{\kappa^2} \,.$$



[Denef-SAH '09]

Endpoint – hairy black holes

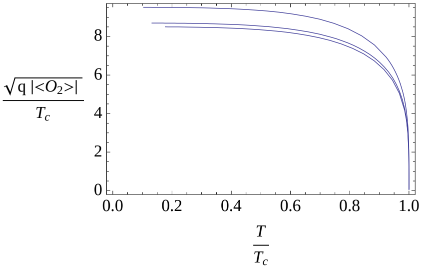
SAH-Herzog-Horowitz 2008

- Endpoint of instability is a hairy black hole:

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + \frac{L^2}{r^2}\left(dx^2 + dy^2\right) ,$$

$$A = A_t(r)dt\,, \qquad \phi = \phi(r)\,.$$

- Solve numerically (take $m^2 = -2/L^2$). Can obtain $\langle \mathcal{O} \rangle$:



- Compare 8 to ~ 3.5 for BCS and $\sim 5-8$ for High-T_C.

Quantum bosons and magnetic fields

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2. Quantum bosons in strongly coupled field theories
3. The free energy and quasinormal modes
4. Charged quasinormal modes of dyonic black holes
5. Towards quantum oscillations from bosons

Quantum bosons and fermions in free field theories

- Free bosons or fermions in magnetic fields have Landau levels

$$\varepsilon_\ell = \sqrt{2|qB|(\ell + \tfrac{1}{2})} \,.$$

- Free energy for fermions

$$\Omega = -\frac{|qB|AT}{4\pi} \sum_\ell \sum_\pm \log\left(1 + e^{(-q\mu \pm \varepsilon_\ell)/T}\right) \,.$$

- Zero temperature limit

$$\lim_{T\to 0} \Omega = -\frac{|qB|A}{4\pi} \sum_\ell (q\mu - \varepsilon_\ell)\theta(q\mu - \varepsilon_\ell) \,.$$

- Free energy for bosons – unstable at low temperatures

$$\Omega = -\frac{|qB|A}{4\pi} \sum_\ell \left[\log\left(e^{(\varepsilon_\ell - q\mu)/T} - 1\right) + \log\left(e^{(\varepsilon_\ell + q\mu)/T} - 1\right)\right] \,.$$

Quantum bosons in strongly coupled field theories

- AdS/CFT: classical description for large N strongly coupled theories.
- Magnetic field and charge density $\Rightarrow$ dyonic black hole

  $\Rightarrow$ unexciting free energy:

$$\Omega = -\frac{L^2}{2\kappa^2 r_+^3}\left(1 + \frac{r_+^2\mu^2}{\gamma^2} - \frac{3r_+^4 B^2}{\gamma^2}\right) \quad \text{with} \quad r_+(T,B,\mu)\,.$$

- Nontrivial Landau-level structure subleading in $1/N$?

  $\Rightarrow$ Quantum contribution from a charged scalar:

$$\Omega_{\text{1-loop}} = \frac{T}{2}\mathrm{tr}\log\left[-\hat{\nabla}^2 + m^2\right] + \cdots$$

with $\hat{\nabla} = \nabla - iqA$.

The free energy and quasinormal modes

- We derived a (new to my knowledge) formula for the determinant as a sum over quasinormal modes $z_\star(\ell)$ of the black hole

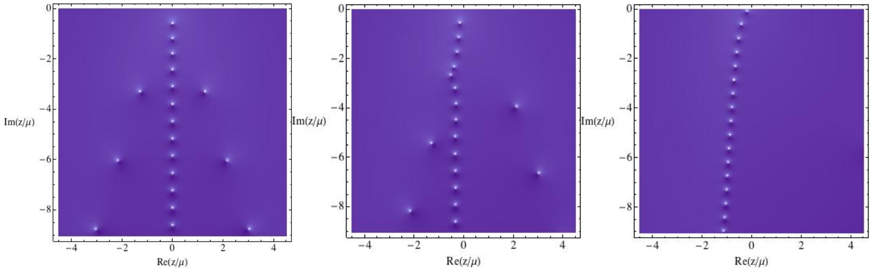
$$\Omega_{\text{1-loop}} = -\frac{|qB|AT}{4\pi} \sum_{\ell} \sum_{z_\star(\ell)} \log \left(\frac{|z_\star(\ell)|}{2\pi T} \left| \Gamma\left(\frac{i z_\star(\ell)}{2\pi T}\right) \right|^2 \right) .$$

- For the BTZ black hole we did the sum explicitly and checked agreement with the known result.

- Objective: (numerically) compute quasinormal modes for charged scalar in dyonic AdS black hole and do this sum!

Charged quasinormal modes of dyonic black holes

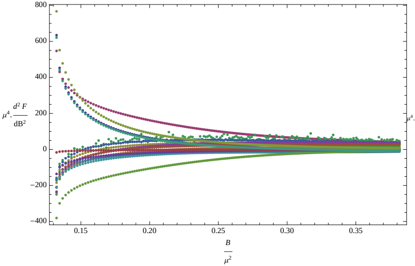
- The most common methods for computing quasinormal modes don't work for low/zero temperature Reissner-Nordstrom-AdS black holes.
- We used a matrix method proposed by Leaver in 1990.
- Some typical results – modes as a function of scalar field charge

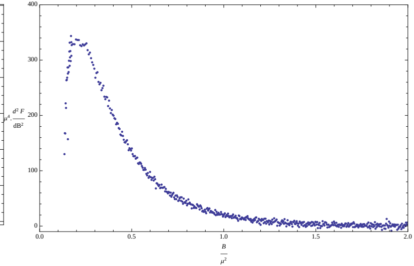


Towards quantum oscillations from bosons

- The magnetic susceptibility $d^2\Omega/dB^2$ has better convergence properties than Ω.

- Take the lowest 18 poles for a given ℓ and compute their contribution





- A quantum oscillation? (above plot is preliminary)

Conclusions

- There exist systems with finite charge density that are described as neither conventional Fermi liquids or superfluids.
- AdS/CFT provides model exotic stable finite density systems.
- Magnetic fields are an essential experimental and theoretical tool for probing such systems.
- There may be interesting structure at $1/N$ in AdS/CFT related to Landau levels for fermions and bosons.
- Found a method for computing determinants about black holes using quasinormal modes.
- Initial studies of RN-ADS quasinormal modes may suggest an analogue of quantum oscillations for strongly coupled bosons at finite chemical potential.