Torsional heterotic geometries

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Introduction and motivation

Constructing of flux backgrounds for heterotic strings

$$g_{MN},\ H_{MNP},\ \phi,\ F_{MN}$$

The background space is a torsional heterotic geometry.

Why are these geometries important?

1) Generality

2) Phenomenology: moduli stabilization, warp factors, susy breaking,...

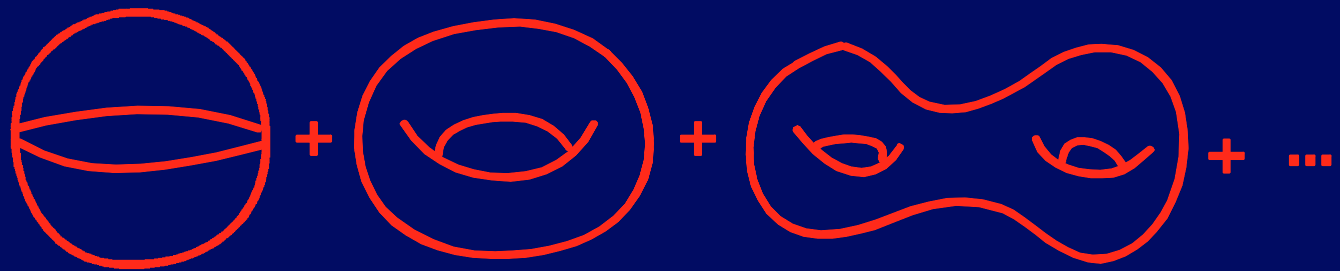
The heterotic string is a natural setting for building realistic models of particle phenomenology. The approach taken in the past is to specify a compact Kaehler six-dimensional space together with a holomorphic gauge field. For appropriate choices of gauge bundle it has been possible to generate space time GUT gauge groups like E_6, SO(10) and SU(5).

Candelas, Horowitz, Strominger,Witten, '85
Braun,He, Ovrut, Pantev, 0501070
Bouchard, Donagi, 0512149

It is natural to expect that many of the interesting physics and consequences for string phenomenology which arise from type II fluxes (and their F-theory cousins) will also be found in generic heterotic compactifications and that moduli can also be stabilized for heterotic strings.

3) <u>World-sheet description</u>: flux backgrounds for heterotic strings are more likely to admit a world-sheet description as opposed to the type II or F-theory counterparts.

Type II string theory or F-theory: Fluxes stabilize moduli but most of the time the size is not determined and remains a tunable parameter. This is very useful since we can take the large volume limit describe the background in a supergravity approximation. However, these are RR backgrounds and its not known how to quantize them. Moreover, the string coupling is typically O(1). It is unlikely that a perturbative expansion exists.



Heterotic strings: the string coupling constant is a tunable parameter which can be chosen to be small. There is a much better chance of being able to find a 2d world-sheet description in terms of a CFT, to control quantum corrections.

Based on:

K. Becker, S. Sethi,
Torsional heterotic geometries, 0903.3769.

K. Becker, C. Bertinato, Y-C. Chung, G. Guo,
Supersymmetry breaking, heterotic strings and fluxes, 0904.2932 .

Overview:

1) Motivation and introduction.

2) Heterotic low-energy effective action in 10d.

3) Method of construction.

4) Torsional type I background.

5) The Bianchi identity.

6) K3 base.

7) Conclusion.

Heterotic low energy effective action in 10d

$$S = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\,e^{-2\varphi}\ \Big[R + 4(\partial\varphi)^2 - \frac{1}{2}|H_{het}|^2 - \frac{\alpha'}{4}\left(\mathrm{tr}\,|F|^2 - \mathrm{tr}\,|R_+|^2\right) + O(\alpha'^3)\Big]$$

Bergshoeff, de Roo, NPB328 (1989) 439

The curvature is defined using the connection

$$\Omega_{\pm\ \mathrm{M}}^{\ AB} = \Omega^{AB}_{\ \ \mathrm{M}} \pm \frac{1}{2} H^{AB}_{\ \ M}$$

H includes a correction to $O(\alpha')$

$$H_{het} = dB_2 + \frac{\alpha'}{4}\left[Cs(\Omega_+) - Cs(A)\right]$$

$$dH_{het} = \frac{\alpha'}{4}\left[\mathrm{tr}(R_+ \wedge R_+) - \mathrm{tr}(F \wedge F)\right]$$

$$\delta\Psi_M = (\partial_M + \frac{1}{4}\Omega_{-\ M}^{AB}\Gamma_{AB})\varepsilon + O(\alpha'^2)$$

$$\delta\lambda = (\partial\!\!\!/\varphi - \frac{1}{2}H\!\!\!/)\varepsilon + O(\alpha'^2)$$

$$\delta\chi = F\!\!\!/\varepsilon + O(\alpha'^2)$$

We will solve:

$$\delta(fermi) = 0$$

$$dH_{het} = \frac{\alpha'}{4}[\mathrm{tr}(R_+ \wedge R_+) - \mathrm{tr}(F \wedge F)]$$

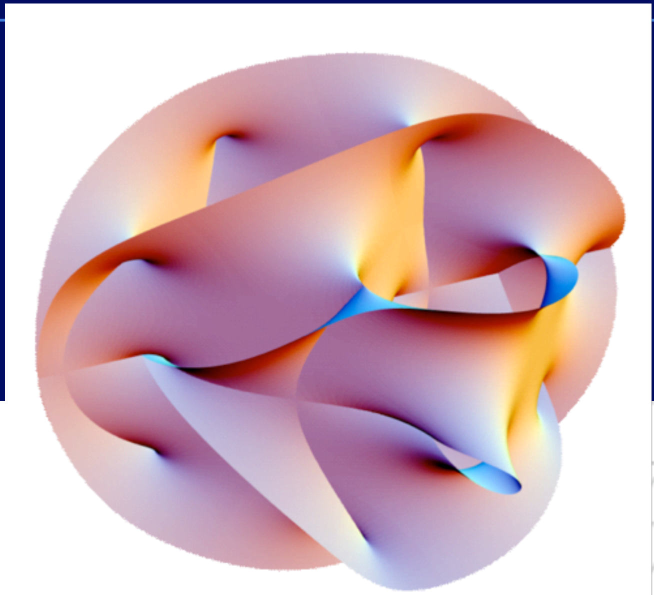
1) If H=0 the solutions are CY_3 or $K3xT^2$

2) If H does not vanish the spaces are complex but no longer Kaehler. There is still a globally defined two-form J but it is not closed. The deviation from `Kaehlerity' is measured by the flux

$$H = i(\partial - \bar{\partial})J$$

There are no algebraic geometry methods available to describe spaces with non-vanishing H. So we need a new way to describe the background geometry. Our approach will be to describe them explicitly with a metric.

Method of construction



$R^3 \times \mathrm{CY}_4$

M-theory

$$J \wedge G_{(2,2)} = 0$$

Kaehler form of CY_4

ds

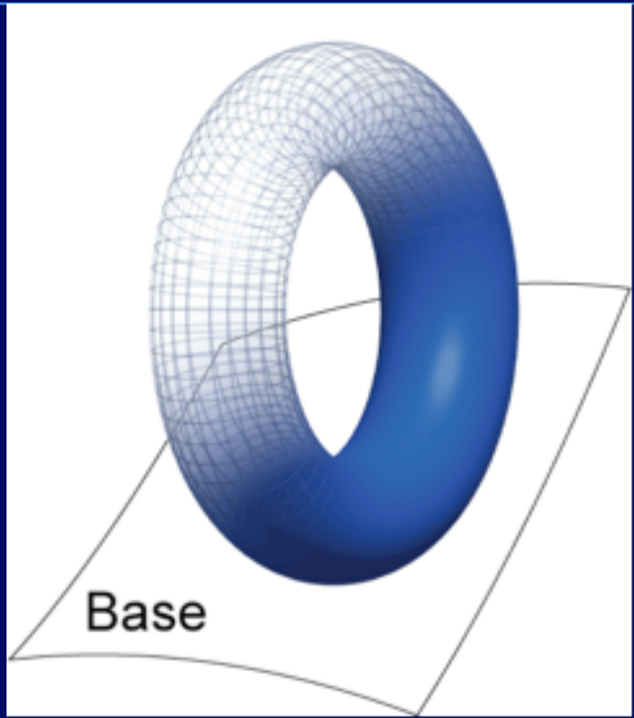
In the presence of flux the space-time metric is no longer a direct product

$$ds^2 = \Delta(y)^{-1}\, ds^2_{\mathrm{R}^3} + \Delta(y)^{1/2}\, ds^2_{\mathrm{CY}_4}$$

warp factor

coordinates of the CY_4

The CY_4 is elliptically fibred so that we can lift the 3d M-theory background to 4d...



$$\tau_B = C_0 + ie^{-\varphi_B} = \tau$$

The G_4 flux lifts to

$$G_3 = F_3 + ie^{-\varphi_B} H_3$$

together with gauge bundles on 7-branes. Susy requires

$$*G_3 = iG_3 \qquad J \wedge G_3 = 0$$

The orientifold locus

At the orientifold locus the background is type IIB on B_6 with constant coupling τ_B. To describe B_6 start with an elliptic CY_3

$$y^2 = x^3 + f(\vec{u})x + g(\vec{u})$$

(A. Sen 9702165, 9605150)

base coordinates

Symmetry $I : y \to -y, \quad B_6 = CY_3 / I$

$$Z_2 = (-1)^{F_L}\,\Omega\, I \quad \xrightarrow{R \to 1/R} \quad Z_2' = \Omega$$

The metric for an elliptic CY space

In the semi-flat approximation:

$$ds^2_{CY} = \underbrace{g_{ij}\, dy^i\, dy^j}_{4d\ \ base} + \frac{1}{\tau_2(y)} \,|\, dw_1 - \tau(y)\, dw_2\,|^2$$

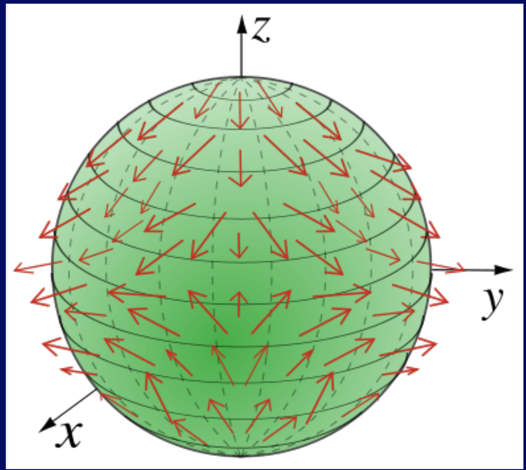
$$I : w_i \rightarrow -w_i$$

$$\bar{\partial}_{\bar{i}}\tau(y) = 0$$

$$\partial\bar{\partial}(\log \det g_{i\bar{j}} - \log \tau_2) = 0$$

Away from the singular fibers there is a U(1)xU(1) isometry...

Flux



In type II theories the fluxes are...

$$H_{NS-NS},\ F_{R-R},\ \varphi$$

....and should be invariant under

$$Z_2 = (-1)^{F_L}\,\Omega\, I$$

Since the NS-NS and R-R 3-forms are odd under $(-1)^{F_L}\,\Omega$

$$H_3 = H_{w_1} dw_1 + H_{w_2} dw_2$$

$$F_3 = F_{w_1} dw_1 + F_{w_2} dw_2$$

Type IIB background

- $$ds^2 = \Delta^{-1}\eta_{\mu\nu}\,dx^\mu\,dx^\nu + \Delta\,ds^2_{CY_3}$$

$$ds^2_{CY_3} = g_{ij}\,dy^i\,dy^j + \frac{1}{\tau_2(y)}\,|\,dw_1 - \tau(y)\,dw_2\,|^2$$

- $$\tau_B = \text{const.}$$

- 3-form flux $\quad *G_3 = iG_3 \qquad J \wedge G_3 = 0$

- 5-form flux $\quad C_4 = \Delta^{-2}dV_4$

$$\delta(fermi) = 0 \qquad dF_5 = F_3 \wedge H_3$$

Torsional type I background

In order to construct the type I solution we have to perform two T-dualitites in the fiber directions. To apply the dualitites rigorously we will assume that up to jumps by SL(2,Z), the complex structure parameter of of the elliptic fiber of the CY_3 can be chosen to be constant. With other words we assume that the CY_3 itself has a non-singular orientifold locus. In this case the base of the elliptic fibration is either K3 or one of the Hirzebruch surfaces F_n (n=0,1,2,4).

For these cases the duality can be done rigorously. But solutions also exists if the complex structure parameter is not constant....

Type I background

$$ds^2 = \Delta^{-1}\eta_{\mu\nu}\, dx^\mu\, dx^\nu + \Delta\, ds^2_{\rm tor}$$

$$ds^2 = \Delta g_{ij}\, dy^i\, dy^j + \frac{1}{\Delta \tau_2(y)} \left| dw_2 + \tau(y)\, dw_1 + A_H \right|^2$$

$$A_H = B_{w_2} + \tau B_{w_1}$$

$$F_3' = \sqrt{\frac{\Delta}{\tau_2(y)}}\, \mathrm{Im}[(F_{w_2} + \tau(y) F_{w_1}) E^{\bar{w}}] + *_b\, d\Delta^2$$

$$\varphi_I = \varphi_{IIB} - \log \Delta$$

one-bein in
the fiber direction

Comments:

1) The spinor equations are solved after imposing appropriate conditions on the flux
The SUSY conditions (spinor equations) are a repackaging of the spinor conditions in type IIB.

2) The spinor equations are solved if $\tau \neq const.$ and the base is not K3 or one of the Hirzebruch surfaces.

3) The scalar function Δ is not determined by susy.
It is the Bianchi identity which gives rise to a differential equation for Δ

The Bianchi identity

The Bianchi identity in SUGRA

In type II SUGRA $\quad F^{(n)} = dC^{(n-1)} + H_3 \wedge C^{(n-3)}$

After two T-dualities in the fiber directions the NS-NS field vanishes as expected from a type I background

$$F_3^{'} = dC_2^{'} \Rightarrow dF_3^{'} = 0$$

This is a differential equation for the scalar function Δ which turns out to be the warp factor equation of type IIB which schematically takes the form

$$\nabla^2 \Delta(y) + |H|^2 = 0$$

In type IIB backgrounds the warp factor equation arises from

$$dF_5 = F_3 \wedge H_3$$

$$F_5 = *F_5$$

$$iG_3 = *G_3$$

$$C_4 = \Delta^2 dV_4$$

one obtains

$$\nabla^2 \Delta(y) + |H|^2 = 0$$

Add D7/O7 sources

$$\sim \int C_4 \,\mathrm{tr}\left(R \wedge R\right)$$

Example: K3xT2

$$[\nabla^2 \Delta(y) + |H|^2] * 1 = \alpha'^2 \,\mathrm{tr}\left(r \wedge r\right) + \ldots$$

In general

$$S = \mu_p \int C \wedge e^{F-B} \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}}$$

$$ds^2 = g_{ij} dy^i dy^j + e^{\phi} (d\theta + A)^2$$

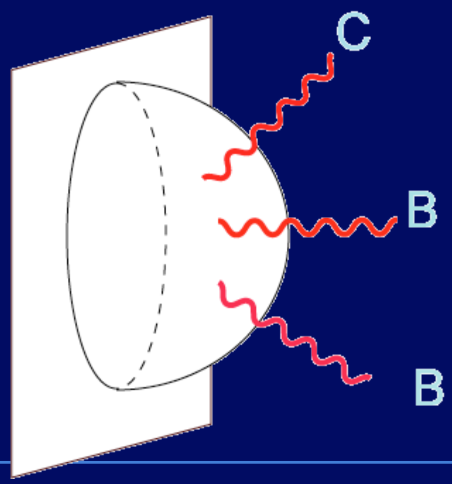
$$H_{NS-NS} = 0$$

$$d\tilde{s}^2 = g_{ij} dy^i dy^j + e^{-\phi} d\theta^2$$

$$H_{NS-NS} = dA \wedge d\theta$$

$$\int C_4 \mathrm{tr}\left(R \wedge R\right)$$

$$\sim \int C_3 [\mathrm{tr}\left(R \wedge R\right) + dH\, dH]$$



Since the anomalous couplings are not compatible with T-duality we need to solve the Bianchi identity directly on the type I/heterotic side

The Bianchi identity in a perturbative expansion

$$dF_3' = \frac{\alpha'}{4}\mathrm{tr}[R(\Omega_+)\wedge R(\Omega_+)]$$

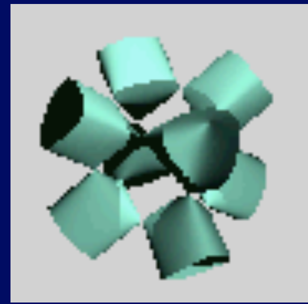
One can always solve the Bianchi identity in a perturbative expansion in 1/L, where L is the parameter which is mirror to the size in type IIB. On the type I side it corresponds to a large base and small fiber.

$$[\nabla^2\Delta(y)+|H|^2]*_b 1 = \frac{\alpha'}{4}\mathrm{tr}[r\wedge r]+O(L^{-1})$$

curvature 2-form of the base

K3 base

Torsional heterotic background with K3 base



The heterotic background is obtained from the type I background by S-duality

$$ds^2_{het} = e^{-\varphi_I}\, ds^2_I \qquad\qquad e^{\varphi_{het}} = e^{-\varphi_I}$$

The resulting metric is

$$ds^2_{het} = \eta_{\mu\nu}\, dx^\mu\, dx^\nu + \underbrace{e^{-4A(y)}}_{\Delta^2}\, \underbrace{g_{ij}\, dy^i\, dy^j}_{\text{K3}} + \underbrace{E_{w_1} E_{w_1} + E_{w_2} E_{w_2}}$$

$$E_{w_k} = dw_k + B_{y_i w_k}\, dy^i$$

$$H_{w_k} = dE_{w_k}$$

Topology change

When the fiber is twisted the topology of the space changes.
A caricature is a 3-torus with NS-flux

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2$$
$$x_i \sim x_i + 1, \quad i = 1,2,3$$
$$B_{NS} = N x_1\, dx_2 \wedge dx_3$$

$$ds^2 = dx_1^2 + dx_2^2 + (dx_3 - N x_1\, dx_2)^2$$
$$x_i \sim x_i + 1, \quad i = 2,3$$
$$x_1 \sim x_1 + 1, \quad x_3 \sim x_3 + N x_2$$
$$\omega = d(dx_3 - N x_1\, dx_2) = -N\, dx_1 \wedge dx_2$$

$$\cong S^1 \times S^2$$

The other fields are...

$$\varphi = -2A(y)$$

$$H_{het} = *_b de^{-4A(y)} - *_b H_{w_1} \wedge E_{w_1} - *_b H_{w_2} \wedge E_{w_2}$$

$$H_w = \frac{1}{2}(H_{w_1} - iH_{w_2}) \in \underbrace{H_-^{(1,1)}}_{N=2},\ \underbrace{H_+^{(2,0)},\ H_+^{(0,2)}}_{N=1},\ \underbrace{H_+^{(1,1)}}_{N=0}$$

Next we want to solve...

$$dH_{het} = \frac{\alpha'}{4} tr[R(\Omega_+) \wedge R(\Omega_+)]$$

To leading order

$$dH_{het} = d *_b de^{-4A} - H_{w_i} \wedge *_b H_{w_i} + O(\alpha')$$

For solutions with N=2 susy in 4d

$$\frac{\alpha'}{4} tr[R(\Omega_+) \wedge R(\Omega_+)] = \frac{\alpha'}{4} \{ tr(r_{K3} \wedge r_{K3}) + 4d *_b d(\nabla^2 A) +$$

$$+ d *_b d[(\nabla^2 e^{-4A} + |H|^2) e^{4A}]$$

$$+2d[(\nabla^2 e^{-4A} + |H|^2) *_b de^{4A}]\}$$

$O(\alpha'^2)$ equations of motion

$$ds^2_{het} = \eta_{\mu\nu} dx^\mu dx^\nu + e^{-4A'(y)} g_{ij} dy^i dy^j + E_{w_1} E_{w_1} + E_{w_2} E_{w_2}$$

K3

$$E_{w_k} = dw_k + B_{y_i w_k} dy^i$$

$$H_{w_k} = dE_{w_k}$$

$$\varphi = -2A'(y)$$

$$H_{het} = *_b de^{-4A'(y)} - *_b H_{w_1} \wedge E_{w_1} - *_b H_{w_2} \wedge E_{w_2}$$

$$A(y) \rightarrow A'(y)$$

$$e^{-4A'} = e^{-4A} + \alpha' \nabla^2 A$$

Conclusion

- We have seen how to construct torsional spaces given the data specifying an elliptic CY space.
- There are many more torsional spaces than CY's.
- Many properties which are generic for compactifications of heterotic strings on Kaehler spaces may get modified once generic backgrounds are considered.

The End