On d=3 Yang-Mills-Chern-Simons theories with “fractional branes” and their gravity duals

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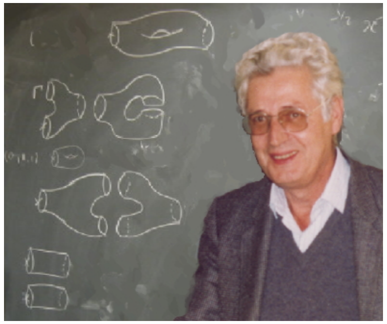

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Based on: O.A., Bergman and Jafferis, arXiv:0807.4924

O.A., Hashimoto, Hirano and Ouyang, arXiv:0906.2390 and work in progress

Outline

1) Motivations, review of $\mathcal{N}=3$ Yang-Mills-Chern-Simons (YM-CS) theories in general, $U(N)_k \times U(N)_{-k}$ (ABJM) in particular.
2) Adding fractional branes, possibility of duality cascades. Gravity dual ?
3) D-brane charges in the presence of Chern-Simons terms in the bulk.
4) The precise dual for $U(N)_k \times U(N+M)_{-k}$.
5) Conclusions and open questions.

Motivations

- Use SUSY, AdS/CFT to understand strong coupling dynamics in d=3 – relation to d=4 ? Condensed matter ?
- AdS_4 backgrounds are a large part (half of ?) the d=4 string landscape. Can they be understood / classified by studying the dual CFT_3's ?
- The Chern-Simons term is useful for writing explicit actions for CFTs (especially with SUSY)

$$\frac{1}{4g_{YM}^2} tr\left(F_{\mu\nu}^2\right) + \frac{k}{4\pi} tr\left(A \wedge dA + \frac{2}{3} A^3\right) + \cdots$$

Makes gauge field massive without breaking.

Review of $\mathcal{N}$=3 YM-CS theories

- The CS term gives masses of opposite sign to the two spin components of the massive gauge field, so the most SUSY a YM-CS theory can have is $\mathcal{N}$=3. In IR, when gauge fields are massive and decouple, can in some cases get more SUSY.
- $\mathcal{N}$=3 YM-CS theories arise by starting with any $\mathcal{N}$=4 SUSY gauge theory (any spectrum of hypermultiplets), and adding a SUSY CS term that gives a mass to the full $\mathcal{N}$=4 vector multiplet.

Review of $\mathcal{N}$=3 YM-CS theories

- In d=3 $\mathcal{N}$=2 superspace we have schematically

$$W = \tilde{Q}_i \Phi Q_i - \frac{k}{8\pi} tr\left(\Phi^2\right),$$

so the effective low-energy superpotential takes the form

$$W \sim \frac{1}{k} tr\left(\sum_i Q_i \tilde{Q}_i\right)^2 .$$

In the IR we get a CS-matter theory with this (marginal) W (and other interactions related by SUSY). In many cases this can be argued to be an exact SCFT (Gaiotto-Yin), with an $SO(3)_R \sim SU(2)_R$ symmetry acting on $\left(Q_i, \tilde{Q}_i^*\right)$.

The U(N)xU(N) case - basics

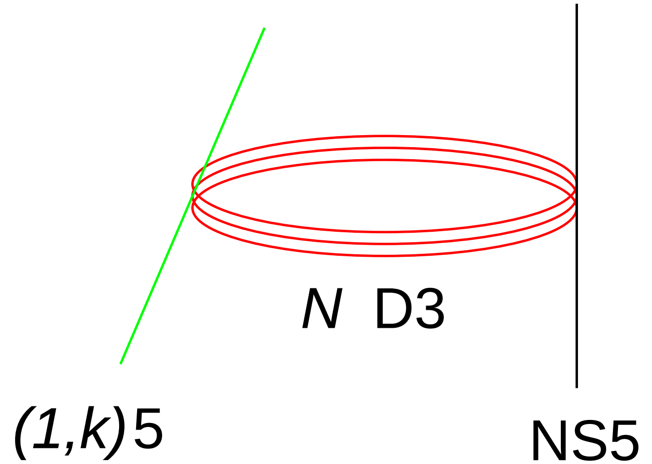
- An especially interesting case is the $U(N)_k \times U(N)_{-k}$ quiver theory (ABJM), with two bi-fundamental hypermultiplets. In this case the IR superpotential is

$$W = \frac{4\pi}{k} tr(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1)$$

(just like for Klebanov-Witten in d=4). This has an extra $SU(2)_A \times SU(2)_B$ flavor symmetry, which does not commute with $SU(2)_R$; together they give an $SU(4)_R = SO(6)_R$ symmetry, implying $\mathcal{N}=6$ superconformal symmetry in the IR.

The U(N)xU(N) case - duality

- The moduli space of this theory can be shown to be $\left(C^4/Z_k\right)^N/S_N$, as for N M2-branes at a C^4/Z_k singularity, so natural to conjecture theories are the same (same SUSY). This can be confirmed by dualities acting on the brane configuration that gives this theory :
- This gives at low energies precisely the $\mathcal{N}=3$ YM-CS theory discussed above.



The U(N)xU(N) case - duality

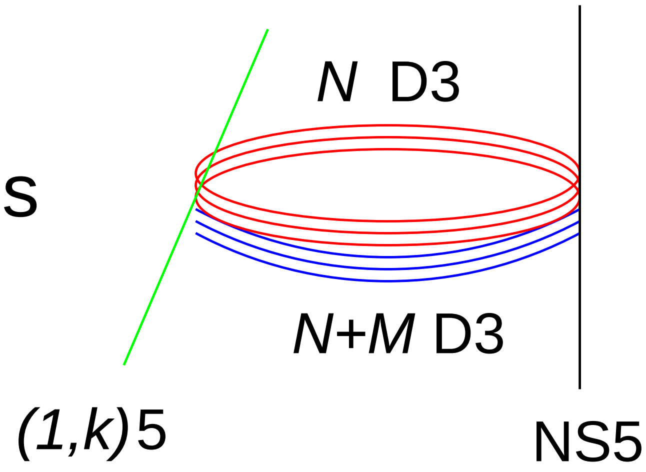
- T-duality relates this to type IIA with N D2-branes, 2 KK monopoles and k D6-branes. Lifting to M theory gives N M2-branes in a known (LWY) geometry, preserving 3/16 of SUSY. This geometry is non-singular except at the origin, where it has a C^4/Z_k singularity, leading to the relation above between the low-energy theories.
- Clearly, the $\mathcal{N}=6$ SCFTs we discussed are then dual to M theory on $AdS_4 x S^7/Z_k$. This description is valid (weakly curved) for large N with $k << N^{1/5}$, otherwise the M theory circle becomes small.

The U(N)xU(N) case - duality

- When k is larger we need to reduce to type IIA. We obtain type IIA string theory on $AdS_4 \times CP^3$, with N units of 6-form flux on CP^3 and k units of 2-form flux on the CP^1 in CP^3. This is weakly curved for $k \ll N$; for $k \gg N$ the field theory becomes weakly coupled (the 't Hooft coupling is $\lambda \sim N/k$).
- This gives an interesting example of AdS_4/CFT_3, with weak and strong coupling limits that can be compared; there are interesting integrable structures on both sides. I will not discuss any of the applications here…

Adding fractional branes

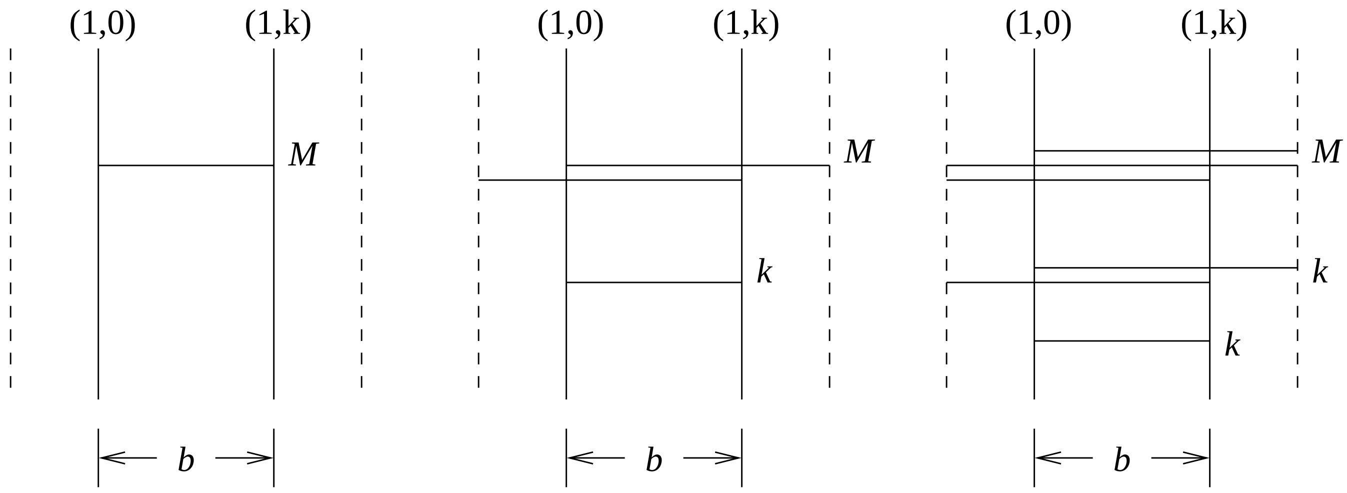
- Easy to generalize brane construction to add fractional branes :



- The s-rule suggests that perhaps for M>k this will break SUSY. For N=0 this is believed to be true. For higher N, naively true since can go on moduli space and obtain the N=0 (pure $\mathcal{N}=3$ SYM) case. For M=0,…,k have SUSY, seem to flow to $U(N)_k \times U(N+M)_{-k}$ $\mathcal{N}=6$ SCFT similar to above (still have same global symmetry).

Adding fractional branes

- The distance b between the branes maps to the relative Yang-Mills coupling between the two gauge groups; this is expected to decouple in IR.
- As usual in brane constructions, can try to move the branes around the circle to obtain new theories that are the same at low energies :



Adding fractional branes

- This suggests a possible IR equivalence between the $\mathcal{N}=3$ YM-CS theories with parameters (N,M), (N+M, M+k), (N+2M+k, M+2k), … Perhaps all flow to the same $U(N)_k \times U(N+M)_{-k}$ $\mathcal{N}=6$ SCFT.
- This requires a “modified s-rule” allowing D3-branes to stretch to different images of the NS5-brane (or to wind a different number of times) – this actually follows (Dasgupta,Mukhi) from the derivation of the s-rule.
- It also requires a modification in the moduli space.
- Note that some theories still do break SUSY.

Duality cascades ?

- The equivalence above is related to $d=3$ Seiberg duality in the same way as in the Klebanov-Strassler cascade. This suggests the possibility of a duality cascade here, where we start from the $U(N+nM+n(n-1)k/2)_k \times U(N+(n+1)M+n(n+1)k/2)_{-k}$ theory in the UV, gradually flow close to smaller values of n, and end up with $U(N)_k \times U(N+M)_{-k}$ in the IR (or with SUSY breaking if N is too small).
- On the field theory side, the evidence for this is similar to the KS cascade, except that the modified moduli space is less understood. Can we find evidence from gravity description ?

Duality cascades ?

- There are also some differences between the putative d=3 duality cascade and the KS cascade:

❑ For d=3 the theories involved are asymptotically free, so one can end the cascade in the UV at a finite value of n without more degrees of freedom.

❑ For d=3 there is no dimensionless parameter, and no limit of the cascade where one of the groups is weakly coupled.

❑ In the IR, we find either an $\mathcal{N}=6$ SCFT or SUSY breaking, rather than an (almost) massive SUSY theory as in KS.

Gravity dual of fractional branes ?

- Can start with IR limit of $M <= k$, which should be a $U(N)_k x U(N+M)_{-k}$ $\mathcal{N}=6$ SCFT. Following the duality chain, the fractional brane maps to a D4-brane wrapping CP^1 in CP^3. Naively, this creates a RR 4-form flux F_4 on CP^2 in CP^3. However, there are no known solutions with such a flux (certainly not with $\mathcal{N}=6$ SUSY).
- Another mystery – naively can turn on B_2 field on CP^1 in CP^3, without breaking SUSY – but $\mathcal{N}=6$ SCFTs have no exactly marginal deformations.
- Need to understand fluxes / charges better…

Charges in presence of CS terms

- The definition of brane charges turns out to be subtle in the presence of Chern-Simons-type terms in the action, like the B_2^F_4^F_4 term of type IIA supergravity. Naively, one expects charges to satisfy :

a) Gauge-invariance,

b) Dirac quantization,

c) Locality of sources,

d) Conservation.

- However, in the presence of Chern-Simons terms, no single charge satisfies all this.

Charges in presence of CS terms

- Recall that the gauge-invariant 4-form in type IIA SUGRA is $\tilde{F}_4 = dA_3 + dB_2 \wedge A_1.$
- The naïve D4-brane charge is thus the integral of this flux – we will call this (following Marolf) the Maxwell charge, $d\tilde{F}_4 = -*j_5^{Maxwell}.$
- This charge is gauge-invariant and conserved, but since in the vacuum

$$d\tilde{F}_4 + F_2 \wedge H_3 = 0,$$

its sources are not localized, and it is not quantized (it varies continuously when F_2, H_3 are non-zero).

Charges in presence of CS terms

- Another natural charge is the brane charge, defined by

$$d\tilde{F}_4 + F_2 \wedge H_3 = - * j_5^{brane}.$$

- This only gets contributions from localized sources, and it is gauge-invariant. But, it is not conserved or quantized. In particular, it gets contributions from the B_2^A_5 term on D6-branes, proportional to B_2.

Charges in presence of CS terms

- We want a quantized charge that just measures the integer number of D4-branes; can cancel all other sources by defining

$$\hat{F}_4 = \tilde{F}_4 + F_2 \wedge B_2 = d(A_3 + B_2 \wedge A_1), \qquad d\hat{F}_4 = -* j_5^{Page}.$$

  and this Page charge is then quantized and conserved, and only gets contributions from localized sources (D4-branes, or D4-branes inside D6-branes).

- However, this charge is not gauge-invariant under the gauge-transformations of the B_2 field.

Charges in presence of CS terms

- Naively this means that the Page charge is meaningless, but the ambiguity of the charge just comes from large gauge transformations of B_2, and shifts it by a multiple of F_2; so the charge modulo this transformation is still physically meaningful.
- This plays a role in cascades like KS : the D3-brane Maxwell charge (from F_5) varies, as does B_2. The D3-brane Page charge is fixed (quantized), but well-defined only mod M. So, have same gauge-invariant quantized charges for U(N)xU(N+M), U(N+M)xU(N+2M), etc.

Back to gravity dual of fractional branes

- This can be used to resolve both of our puzzles !
- We argued that the number of fractional branes should be the 4-form flux; but it should really be related to the quantized D4-brane Page charge,

$$Q_4^{Page} = -\int_{CP^2} \hat{F}_4 = -\int_{CP^2} (\tilde{F}_4 + F_2 \wedge B_2).$$

- This means that (a) With no 4-form, the B_2 field is actually quantized, $B_2 = -Q_4^{Page} / k,$

  (b) the solutions for the $U(N)_k x U(N+M)_{-k}$ SCFTs are precisely the solutions with this B_2 field, which are supersymmetric and do not involve any non-zero F_4. (k different solutions)

Back to gravity dual of fractional branes

- This fits nicely with the possibility of getting duality cascades in the gravity duals of the $\mathcal{N}=3$ YM-CS theories. We could start in the UV with some $U(N')_k \times U(N'+M')_{-k}$ theory, and with some relative gauge couplings determining b_{infinity}. We could then have a KS-like flow, in which B_2 is gradually reduced. Every time B_2 decreases by one, we can bring it back to [0,1] by a large gauge transformation, changing $Q_4^{Page} \to Q_4^{Page} - k.$

  This corresponds to a cascade step of the type we discussed above. In the IR we can end with some $AdS_4 \times CP^3$ with a B_2 field, as above.

What is Q_4^{Page} ?

- Naively we expect to have $Q_2^{Page}=N$, $Q_4^{Page}=M$, so that in the IR we would have $B_2=-M/k$. However, in the brane construction it is possible that the D6-branes also carry some charge, so one might expect $Q_2^{Page}=N+a_2k$, $Q_4^{Page}=M+a_4k$.
- To test this, let us go back to 2 facts :

➢ Moving the branes around, shifting $b_{infinity}$ by one, takes $N \to N+M, M \to M+k.$

➢ The Page and Maxwell charges are related by

$$Q_4^{Page} = Q_4^{Maxwell} - kb_\infty,$$

$$Q_2^{Page} = Q_2^{Maxwell} - Q_4^{Page} b_\infty - \frac{1}{2} kb_\infty^2.$$

and the Maxwell charges should not change by the large gauge transformation.

What is Q_4^{Page} ?

- To reproduce the correct transformation implied by the brane configuration, we must have precisely $Q_4^{Page}=M-k/2$ (the shift in Q_2^{Page} is not constrained). This implies that the correct B_2 field for describing the $U(N)_k \mathrm{x} U(N+M)_{-k}$ SCFT is actually $B_2 = \frac{1}{2} - \frac{M}{k}.$
- Such half-integer charges seem strange. To test this, consider a D6-brane wrapped on CP^2. This is a domain wall changing k by one, and we claim it also needs to shift Q_4^{Page} by a half. In fact, this follows from an argument of Freed+Witten. The path integral for a D6-brane wrapped on a non-spin manifold like CP_2 is only consistent if it has a non-integer gauge flux, $\int_{CP^1} F^{D6} = n - \frac{1}{2}, n \in Z.$

Additional consistency checks

- We can check that this shift in $Q_4^{Page}=M-k/2$ is also consistent with :
- The D4-brane wrapped on CP^2, identified in ABJM as a di-baryon, should have precisely M strings ending on it.
- The D6-brane wrapped on CP^3, identified in ABJM as a baryon vertex, should have precisely N strings ending on it.
- Parity : $U(N)_k x U(N+M)_{-k}$ is dual by a "cascade step" to $U(N+k-M)_k x U(N)_{-k}$, related by parity to $U(N)_k x U(N+k-M)_{-k}$, consistent with the parity transformation in type IIA taking B_2 to $(-B_2)$.

Conclusions

- The gravity dual for the $U(N)_k \times U(N+M)_{-k}$ $\mathcal{N}=6$ SCFT is type IIA on $AdS_4 \times CP^3$ with $B_2=1/2-M/k$ on CP^1. The gravity duals for the $\mathcal{N}=3$ YM-CS theories are still under construction; expect them to exhibit duality cascades and SUSY breaking.
- Understanding these systems requires a careful analysis of charge/flux quantization. In cascades, this analysis is closely related to the brane creation process in brane configurations. The same relation is useful also in other cases, in particular in the analogous $\mathcal{N}=4$ quiver whose gravity solution is explicitly known (so we can verify the qualitative picture of the RG above).

Open questions

- We discussed the gravity dual of “fractional branes” just in type IIA; it would be nice to understand better the charge quantizations and shifts from the point of view of the lift to M theory.
- Construct the precise gravity solutions for the $\mathcal{N}$=3 YM-CS theory (work in progress). Need to construct self-dual 4-forms on the LWY space.
- Find better arguments on the field theory side for duality cascades.
- Many possible generalizations with less supersymmetry.