The Statistics of the large–scale Velocity Field

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Abstract. A lot of predictions for the statistical properties of the cosmic velocity field at large-scale have been obtained recently using perturbation theory. In this contribution I report the outcomes of a set of numerical tests that aim to check these results. Using Voronoi and Delaunay tessellations for defining the velocity field by interpolation between the particle velocities in numerical simulations, we have been able to get reliable estimates of the local velocity gradients. Thus, we have been able to show that the properties of the velocity divergence are in very good agreement with the analytical results. In particular we have confirmed the Ω dependence expected for the shape of its distribution function.

1. Introduction

The advent of reliable redshift-independent distance estimators lead to an enormous growth of activity in the field of measuring and interpreting the peculiar velocities of galaxies. The velocity field can in particular be fruitfully investigated by means of perturbation analysis. One important result is the, Ω-dependent, velocity-density relationship that follows from linear theory (see e.g. Peebles 1980). Moreover, taking advantage of the fact that Perturbation Theory predicts that the rotational part of the velocity field vanishes, Bertschinger & Dekel (1989) developed the non-parametric POTENT method in which the local cosmological velocity field is reconstructed from the measured line-of-sight velocities (Bertschinger et al. 1990).

Then, via the velocity-density relationship it is possible to estimate the value of $\Omega^{0.6}/b$, where it is assumed that the bias of the galaxies can be simply represented by a linear bias factor b (see the review paper of Dekel 1994 and references therein).

There are however other methods that have been proposed that uses *intrinsic* properties of the large–scale velocity field to estimate Ω. For example, Nusser & Dekel (1993) proposed to use a reconstruction method assuming Gaussian initial conditions to constrain Ω, while Dekel & Rees (1994) use voids to achieve the same goal. Another approach has been proposed by Bernardeau et al. (1995) and Bernardeau (1994a) based on the use of statistical properties of the divergence of the locally smoothed velocity field. Preliminary comparisons of the analytical predictions with numerical simulations (Bernardeau 1994b, Juszkiewicz et al. 1995, Łokas et al. 1995) yielded encouraging results. However, such a comparison is complicated due to the fact that the velocities are only known at, non-uniformly distributed, particle locations.

Figure 1. Sketch of the PDF of the velocity divergence as given by Eq. (1) for $\Omega = 0.3$ and $\Omega = 1$.

Here, I report recent results obtained by Bernardeau & van de Weygaert (1996) and Van de Weygaert et al. (1996) addressing specifically the issue of the discrete nature of the velocity sampling.

2. Theoretical results

To start with, let me remind a few analytical results obtained from Perturbation Theory applied to the large-scale velocity field. I consider the statistical properties of the one-point *volume* averaged velocity divergence, θ, (in units of the Hubble constant) when the average is made with a top-hat window function. Although the first analytical results that have been obtained dealt with the values of high order moments of its distribution function (Bernardeau et al. 1995, Bernardeau 1994a) it is more convenient to consider its global shape. Particularly interesting is the $n = -1$ case (where n is the index of the power spectrum) for which there is a simple analytical fit for the PDF (Bernardeau 1994b),

$$p(\theta)\mathrm{d}\theta = \frac{([2\kappa - 1]/\kappa^{1/2} + [\lambda - 1]/\lambda^{1/2})^{-3/2}}{\kappa^{3/4}(2\pi)^{1/2}\sigma_\theta} \exp\left[-\frac{\theta^2}{2\lambda\sigma_\theta^2}\right] \mathrm{d}\theta, \tag{1}$$

with

$$\kappa = 1 + \frac{\theta^2}{9\lambda\Omega^{1.2}}, \qquad \text{and} \qquad \lambda = 1 - \frac{2\theta}{3\Omega^{0.6}}, \tag{2}$$

where σ_θ is the variance of the distribution. The resulting shape of the PDF is shown in Fig. 1. It is worth noting that the Ω dependence shows up in the shape and position of the peak (given by $\theta_{\rm peak}$ indicated on the figure) and by the

Figure 2. Voronoi and Delaunay tessellations of a 2D set of particles (filled circles). The solid lines form the Voronoi tessellation, the dashed lines the Delaunay tessellation. I represented a normal vector $\vec{n}$ of the wall separating the points **A** and **B**.

position of the large θ cutoff ($\theta_{\rm max}$ on the figure). A similar property to the later one was also found by Dekel & Rees (1994) using the Zel'dovich approximation. It allows indeed to constraint Ω from the largest expanding void. In principle all these features can be used to constrain Ω.

3. The Delaunay and Voronoi methods

Before trying to apply these ideas to the data we have extensively checked these features in numerical simulations. In usual numerical analysis, the velocity field is defined with a momentum average of the closest particles on grid points. This method, however, yields very poor results when a subsequent *volume* average is required. The main reason is that the two smoothing scales, grid size and smoothing radius, cannot be very different from each other. The problem is in fact to define properly the velocity *field* (the velocity at any location in space) from the velocities of a given set of discrete and sparse points. This is what the methods proposed by Van de Weygaert and I are designed for.

3.1. The Voronoi method

In the first method we propose the local velocity to be given by the *velocity of the closest particle*. For defining the velocity in the whole space one has then to divide space in cells, each containing a particle (of a simulation for instance), and so that any point inside the cell is closer to it than to any other particle. This partition is called the *Voronoi* tessellation. In Fig 2. I present a 2D sketch of such a partition: the solid lines form the Voronoi tessellation of the filled circles representing the particles. Then, from the initial assumption that the velocity is constant in the Voronoi cells, the velocity gradients (in particular the divergence) are localized on the walls. They have actually a surface density given by ($\vec{u}$ is the peculiar velocity)

$$\vec{u}_{i,j}(\mathrm{wall}) = (\vec{\mathrm{u}}_{\mathrm{ext}} - \vec{\mathrm{u}}_{\mathrm{int}})_{\mathrm{i}} \cdot \vec{\mathrm{n}}_{\mathrm{j}}(\mathrm{wall}) \tag{3}$$

where $\vec{n}(\mathrm{wall})$ is the unit vector normal to the wall and going outward of the cell (see Fig. 2). The local smoothed velocity divergences are then just given by the sum of the fraction of all walls that are within a given sphere of radius R_0 (thick solid line in the Figure) multiplied by the value of the divergence on each wall,

$$\theta_{\mathrm{smoothed}} = \frac{3}{4\,\pi\,R_0^3} \sum_{\mathrm{walls}} \mathrm{Surface}(\mathrm{wall} \cap \mathrm{sphere})\;\theta(\mathrm{wall}). \tag{4}$$

3.2. The Delaunay method

In the Delaunay method the local velocity is supposed to be given by a *linear combination of the velocities of the four neighbors*. If in 1D it is easy to identify the closest neighbors, this is no longer the case in 2D or 3D. The solution is once again provided by the Voronoi tessellation or rather its dual, the *Delaunay* tessellation. This is the triangulation in which the particles are connected together when they share a wall in the Voronoi tessellation (dashed lines in Fig. 2). Because of the properties of the Voronoi tessellation, the Delaunay triangulation satisfies a criterion of compactness: tetrahedra that are defined in such a way (or triangles in 2D) are as less elongated as possible. This is a crucial property for doing a subsequent linear interpolation in the tetrahedra, since it ensures that the linear interpolation will be as good as possible.

So having identified the four neighbors (A, B, C, D) of a point M its velocity is assumed to be given by

$$\vec{u}(M) = \alpha_A\,\vec{u}(A) + \alpha_B\,\vec{u}(B) + \alpha_C\,\vec{u}(C) + \alpha_D\,\vec{u}(D), \tag{5}$$

where α_i are the barycentric weights of the points (A, B, C, D) at the position M,

$$\alpha_A\,\vec{u} + \alpha_B\,\vec{u} + \alpha_C\,\vec{u} + \alpha_D\,\vec{u} = 0, \qquad \sum_{i=1}^{4} \alpha_i = 1. \tag{6}$$

The velocity gradients are then uniform in the tetrahedra and the local smoothed divergence is given by a sum over all the tetrahedra that intersect a given sphere (gray area in Fig. 2).

3.3. Practical implementation, Validity of the methods

In practice to use these methods we have to select only a fraction of the points provided by the N-body codes. This is done in such a way that the largest voids retain as many particles as possible. For details see Bernardeau and Van de Weygaert (1995). The tessellations are then built using the codes developed by Van de Weygaert (1991).

So far we have applied these methods to two different numerical simulations. One kindly provided by H. Couchman (1991) with CDM initial condition for

Figure 3. Scatter of the local divergences measured by different methods

$\Omega = 1$ and a PM simulation with $\Omega \approx 0.3$ and a power law spectrum of index $n = -1$ (Van de Weygaert et al. 1996)

Here I just present a very significant figure showing the scatter plots of the local divergences measured in 8000 different locations (Fig. 3) with the various available methods. When the Delaunay method is compared to the previous grid method the correlation is very noisy and there is even a systematic error in the variance. When the Voronoi and the Delaunay methods are compared to each other, no such features are seen and there is a perfect correlation between the two estimations. This gives us a good confidence in our methods.

4. The measured Ω dependence of the velocity divergence distribution, conclusions

The resulting PDF-s are presented in Fig. 4. They show a remarkable agreement between the numerical estimations and the predictions of Eq. (1). Moreover the left panel demonstrates that the statistics of the velocity divergence is indeed sensitive to Ω (comparison of the dashed line with the solid line).

An interesting remark to make is that, in principle, it is not only possible to measure Ω but it must also be possible to test the gravitational instability scenario. The PDF given in (1) is indeed a two-parameter family of curves. If the actual distribution fails to reproduce one of these curves it is not compatible with the gravitational instability scenario with Gaussian initial conditions. That would be the case, for instance, if the observed distribution is skewed the other way around.

However, so far, the analysis have been done essentially in numerical simulations. The construction of a procedure to be used with observational data is still in progress.

Figure 4. The measured shapes of the velocity divergence PDF. The left panel is for $\Omega = 0.33$ and the right for $\Omega = 1$. The solid lines are the predictions from Eq. (1), and the dashed line for $\Omega = 1$ with the same variance as for $\Omega = 0.33$. The open squares are from the grid method, the filled triangles and squares are respectively from the Delaunay and Voronoi methods.

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