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CHIRAL CONDENSATE WITH WILSON FERMIONS

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We show that for Wilson fermions on the lattice the continuum contribution to $\langle\bar{\psi}\psi\rangle$ can be unambiguously separated form the perturbative tail which arises because of the explicit chiral symmetry breaking. This phenomenon is shown to happen in the Gross-Neveu model to order $1/N$, and we give general arguments for the occurrence of the same situation in four-dimensional gauge theories.

1. Introduction

It is well known that lattice gauge theories with Wilson fermions present difficulties when one tries to evaluate chiral condensates because of the explicit chiral symmetry breaking [1]. In general the chiral condensate has a perturbative part which needs to be subtracted in order to evaluate the continuum renormalization group invariant contribution. Since the perturbative part is given by a divergent series in the gauge coupling constant, it is not clear whether an unambiguous prescription for the resummation of the series can be given, leading to a uniquely defined value for the condensate [2]. In fact it has been argued by one of us that this is not true for the so-called gluon condensate $\langle F_{\mu\nu}^2\rangle$, the reason for this being that the Borel transform of the perturbative part has a renormalon singularity on the positive real axis that leads to an ambiguity which is of the same order as the continuum part [3]. This can be seen to follow from the nonperturbative mixing of the operator $F_{\mu\nu}^2$ with the operator $\mathbb{1}$, which carries the same quantum numbers.

In the case of the chiral condensate chiral symmetry modifies the situation. On the lattice it is possible to write down regularizations of the fermion action which do or do not respect some of the chiral symmetries of the continuum lagrangian (for a review see [4]). In the first case (naive or Kogut-Susskind fermions) the condensate vanishes to all orders in perturbation theory for massless quarks. For Wilson fermions one has to adjust the hopping parameter κ (or equivalently the bare quark

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mass) in order to obtain a massless pion which represents a signal of the restoration of chiral symmetry. However there is no guarantee yet that the theory has all the properties of a chirally symmetric theory at that point. One additional quantity that can be studied would be the continuum chiral condensate which can be estimated only after the perturbative tail has been subtracted. Therefore it is a fundamental as well as practical question whether the condensate is calculable and its value universal (in the sense that it will be the same for Wilson and Kogut-Susskind fermions in the continuum).

In this paper we will argue that this is indeed the case. We will first look at the Gross-Neveu model [5] in two dimensions in the $1/N$ expansion and show explicitly the full restoration of chiral symmetry at the critical value of the Wilson hopping parameter. We will compute the chiral condensate up to order $1/N$ and show how the perturbative and continuum contributions can be separated unambiguously. For the later part the correct weak coupling scaling properties are verified and universality is shown to hold. We will then give a simple argument for the occurrence of an analogous phenomenon and argue that a similar separation can be done in four-dimensional gauge theories. We also comment at the end about the practical feasibility of the subtraction of the perturbative part.

2. The two-dimensional Gross-Neveu model

We start from the continuum action density for the 2-dimensional model, which in euclidean space is

$$S = \bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - \tfrac{1}{2}g^2(\bar{\psi}\psi)^2, \tag{2.1}$$

where $\psi = (\psi_i)$ is an N-component massless Dirac field. This action is invariant under the discrete chiral transformation

$$\psi \to \gamma_5\psi. \tag{2.2}$$

We discretize the action on a square lattice with spacing $a = 1$ by using Wilson's method. Introducing a composite field σ_n we get

$$S = \tfrac{1}{2}\sum_{n,i}\sum_{\mu}\left[\bar{\psi}^i_n\gamma_{\mu}\left(\psi^i_{n+\mu} - \psi^i_{n-\mu}\right) + r\bar{\psi}^i_n\left(\psi^i_{n+\mu} - 2\psi^i_n + \psi^i_{n-\mu}\right)\right]$$

$$+ \sum_{n,i}(m + \sigma_n)\bar{\psi}^i_n\psi^i_n + \frac{1}{2g^2}\sum_n \sigma_n^2. \tag{2.3}$$

For $m = 0$ and $r \neq 0$ the action of eq. (2.3) is not invariant under the symmetry (2.2), and renormalization of the bare mass m has to be performed in order to recover that symmetry in the continuum limit. m is related to the usual Wilson hopping parameter by $\kappa = 1/2(m-2r)$.

Integrating over the field ψ we get the effective action for the σ-field, which generates the $1/N$ expansion

$$S_{\text{eff}} = N\left[-\operatorname{tr}\ln\left(i\not{S} + M + m + \sigma\right) + \frac{1}{2f}\sum_n \sigma_n^2\right], \tag{2.4}$$

where we have set

$$f = g^2 N, \tag{2.5}$$

which is kept fixed as N goes to infinity. In momentum space the kernels $\not{S}$ and M become

$$i\not{S}(k) = \sum_\mu i\gamma_\mu \sin(k_\mu), \tag{2.6a}$$

$$M(k) = r\sum_\mu \left[1 - \cos(k_\mu)\right]. \tag{2.6b}$$

From the action (2.4) we get the effective potential $\Gamma(\sigma)$ as a function of the external background field σ in the $1/N$ expansion. For a spatially constant σ we obtain

$$\Gamma(\sigma) = N\Gamma_0(\sigma) = \Gamma_1(\sigma) + \mathrm{O}\left(\frac{1}{N}\right). \tag{2.7}$$

Introducing the new variable Ω,

$$\Omega = \sigma + m, \tag{2.8}$$

we get

$$\Gamma_0(\sigma) = \frac{1}{2f}(\Omega - m)^2 + F_0(\Omega) \tag{2.9}$$

with

$$F_0(\Omega) = -\int_k \operatorname{tr}\ln\left[i\not{S}(k) + M(k) + \Omega\right], \tag{2.10}$$

and

$$\Gamma_1(\Omega) = F_1(\Omega) = \tfrac{1}{2}\int_k \ln\left[\frac{1}{f} + \Pi(k)\right] \tag{2.11}$$

with

$$\Pi(p)=\int_k \mathrm{tr}\frac{1}{\left[i\not{S}(k)+M(k)+\Omega\right]\left[i\not{S}(p+k)+M(p+k)+\Omega\right]}\,. \tag{2.12}$$

Finally, the chiral condensate $\langle\bar{\psi}\psi\rangle$ is related to the minimum σ_c of the effective potential $\Gamma(\sigma)$ by

$$\langle\bar{\psi}^i\psi^i\rangle = N\sigma_c\,. \tag{2.13}$$

2.1. RESTORATION OF CHIRAL SYMMETRY

In the continuum theory the chiral symmetry will be dynamically broken. In order to ensure its existence in the continuum with the Wilson action we will renormalize the bare mass m so that the effective potential $\Gamma(\sigma)$ has two degenerate minima. In the weak coupling region ($f \ll 1$) this is possible for three values of m. Indeed the effective potential of eq. (2.7) is invariant under the transformation

$$\sigma\to-\sigma\,,\qquad m\to-4r-m\,. \tag{2.14}$$

If $m=-2r$, the effective action (2.4) has in fact an exact chiral invariance, the potential (2.7) two symmetric degenerate minima $+\sigma_c$ and $-\sigma_c$, and the fermion propagator two poles close to $(\pi,0)$ and $(0,\pi)$. Thus in this case there is a Z_2 chiral symmetry which exchanges the two corresponding particles. Therefore the continuum theory will describe two species of fermions with a mass of order σ_c and the chiral condensate (2.13) will have no perturbative tail, in striking similarity with Kogut-Susskind fermions.

The other possibility is for $m=m_c$ close to 0 (or $-4r$). The fermion propagator has only a pole close to $(0,0)$ (respectively (π,π)), and the continuum limit will describe only one species of fermion. In that case we have to find m_c, σ_+ and σ_- such that

$$\frac{\partial\Gamma}{\partial\sigma}(\sigma_+,m_c)=\frac{\partial\Gamma}{\partial\sigma}(\sigma_-,m_c)=0\,, \tag{2.15}$$

$$\Gamma(\sigma_+,m_c)=\Gamma(\sigma_-,m_c)\,. \tag{2.16}$$

Expanding the solutions in $1/N$

$$m_c=m^0+\frac{1}{N}m^1+\mathrm{O}\left(\frac{1}{N^2}\right), \tag{2.17}$$

$$\Omega_\pm=\sigma_\pm+m_c=\Omega^0_\pm+\frac{1}{N}\Omega^1_\pm+\mathrm{O}\left(\frac{1}{N^2}\right), \tag{2.18}$$

we get a set of equations which can be solved recursively in $1/N$. The way m approaches m_c (from above or below) will select one of the two degenerate ground states (σ_+ or σ_-) at $m = m_c$.

2.2. THE CHIRAL CONDENSATE AT $N = \infty$

Using eqs. (2.7) and (2.9) we get at leading order

$$\frac{1}{f}\left(\Omega^0_\pm - m^0\right) + F_0'\left(\Omega^0_\pm\right) = 0, \tag{2.19a}$$

$$\frac{1}{2f}\left(\Omega^0_+ - m^0\right)^2 + F_0\left(\Omega^0_+\right) = \frac{1}{2f}\left(\Omega^0_- - m^0\right)^2 + F_0\left(\Omega^0_-\right). \tag{2.19b}$$

In order to solve these equations in a small-f expansion we need the small-Ω expansion of the function F_0 given by the integral (2.10). This expansion is of the form

$$F_0(\Omega) \underset{\Omega\to 0}{\equiv} a_0 + a_1\Omega + \sum_{n=2}^{\infty}\left(a_n + b_n \ln|\Omega|\right)\Omega^n. \tag{2.20}$$

The constants a_n and b_n are numerical factors which depend on r. In particular we have

$$b_2 = \frac{1}{2\pi}, \qquad b_3 = \frac{r}{2\pi}, \tag{2.21}$$

and $a_1 \neq 0$ for $r \neq 0$. After some algebra one finds for $\Omega^0_\pm$ and m^0

$$m^0 = a_1 f = \mathrm{O}(\mathrm{e}^{-2\pi/f}), \tag{2.22a}$$

$$\Omega^0_\pm = \pm C\mathrm{e}^{-\pi/f} + \mathrm{O}(\mathrm{e}^{-2\pi/f}), \tag{2.22b}$$

with

$$C = \mathrm{e}^{-1/2 + 2\pi a_2}. \tag{2.23}$$

Therefore the chiral condensate is

$$\langle\bar{\psi}\psi\rangle = N\left(a_1 f \pm C\mathrm{e}^{-\pi/f}\right). \tag{2.24}$$

At leading order of the $1/N$ the chiral condensate gets a perturbative tail which vanishes if $r = 0$ (naive fermions). The continuum part is nevertheless unambiguously defined by

$$\langle\bar{\psi}\psi\rangle_{\mathrm{cont.}} = \pm NC\mathrm{e}^{-\pi/f}. \tag{2.25}$$

It scales accordingly to the renormalization group prediction and coincides with the value for the condensate obtained by other formulations. In particular let us note that Ω_0 is for $N=\infty$ the dynamically generated mass m_{f} of the fermion, and that we get the renormalization group invariant ratio

$$\frac{\langle\bar{\psi}\psi\rangle_{\text{cont.}}}{m_{\mathrm{f}}}=N. \tag{2.26}$$

The above results are not surprising; at leading order of the $1/N$ expansion it is known that the perturbative tails which are present in the definition of composite operators are always convergent series in the coupling constant and may be unambiguously subtracted from the continuum part. We must look at least at the first nontrivial term of the $1/N$ expansion to see if the continuum part of $\langle\bar{\psi}\psi\rangle$ is well defined. Before considering this issue, let us note that the subleading terms of the weak coupling expansion of $\Omega^0_{\pm}$ may be computed in the same way. We shall need the first subleading term in the next section, which is of the form

$$\Omega^0_{\pm}=\pm C\mathrm{e}^{-\pi/f}+\frac{Af+B}{Df+E}\mathrm{e}^{-2\pi/f}+\mathrm{O}(\mathrm{e}^{-3\pi/f}), \tag{2.27}$$

where A, B, D and E involve the coefficients a_n and b_n for $n=2,3,4$. The important point is that the term of order $\mathrm{e}^{-2\pi/f}$ is the same in both chiral phases $\pm$.

2.3 THE CHIRAL CONDENSATE AT ORDER $1/N$

As previously discussed, we expect to see an ambiguity in the perturbative tail, if it exists, at order $1/N$. Therefore we must compute the terms m^1 and $\Omega^1_{\pm}$ in (2.17)–(2.18). Using (2.7) we get two linear equations:

$$m^1=-\frac{F_1(\Omega^0_+)-F_1(\Omega^0_-)}{F_0(\Omega^0_+)-F_0(\Omega^0_-)}, \tag{2.28a}$$

$$\Omega^1_{\pm}=m^1-\left[\frac{1}{f}+F_0''(\Omega^0_{\pm})\right]^{-1}\left[m^1F_0''(\Omega^0_{\pm})+F_1'(\Omega^0_{\pm})\right], \tag{2.28b}$$

which reduce, using (2.19), to the explicit formula for the chiral condensate

$$\sigma^1_{\pm}=\left[\frac{1}{f}+F_0''(\Omega^0_{\pm})\right]^{-1}\left[fF_0''(\Omega^0_{\pm})\frac{F_1(\Omega^0_+)-F_1(\Omega^0_-)}{\Omega^0_+-\Omega^0_-}-F_1'(\Omega^0_{\pm})\right]. \tag{2.29}$$

In order to determine the properties of the perturbative part of $\sigma^1_{\pm}$, we need the small-Ω expansion of the function F_1 given by (2.11). This expansion follows from the small-Ω and small-p expansion of the "bubble" integral $\Pi(p,\Omega)$ given by (2.12).

In fact we have first to take the small-Ω limit, which is of the form

$$\Pi(p,\Omega) \underset{\Omega\to 0}{\equiv} \Pi_0(p) + \sum_{k=1}^{\infty} \Pi_k(p,\Omega), \tag{2.30}$$

where

$$\Pi_k(p,\Omega) = \Omega^k\left[A_k(p) + \ln|\Omega| B_k(p)\right]. \tag{2.31}$$

The functions Π_0, A_k and B_k depend only on the momentum p and are analytic in the Brillouin zone away from $(0,0)$. Possible singularities or ambiguities in the small-f expansion of F_1 can arise only from their small-p behavior, which we now investigate.

The integral $\Pi_0(p)$ corresponds to the 1-loop integral (2.12) taken at zero mass Ω. Its small-p behavior is universal:

$$\Pi_0(p) = \frac{1}{\pi}\ln|p| + \text{const} + \mathrm{O}(|p|). \tag{2.32}$$

The small-p behavior of A_1 and B_1 is very important, and is found to be

$$A_1(p) = \frac{r}{\pi}\ln|p| + \text{cst} + \mathrm{O}(p), \tag{2.33}$$

$$B_1(p) = -\frac{r}{\pi} + \mathrm{O}(p). \tag{2.34}$$

Finally, let us mention that A_2 and B_2 behave as

$$A_2(p) \propto \frac{1}{p^2}\ln|p|, \qquad B^2(p) \propto \frac{1}{p^2}. \tag{2.35}$$

We can now compute the perturbative part of $\sigma^1_{\pm}$ by retaining only the terms of order $\mathrm{O}(\Omega)$ in (2.29), using (2.23) and expanding in f. We get

$$\sigma^1_{\pm} \equiv -\frac{f}{2}\int_k \frac{A_1(k) + 2\left(\pi a_2 - \frac{1}{2}\right)B_1(k)}{1/f + \Pi_0(k)} + \mathrm{O}(\mathrm{e}^{-\pi/f}), \tag{2.36}$$

where $\equiv$ means that (2.36) has to be understood in the sense of a formal power series in f.

In general, in dealing with asymptotic series in the coupling constant

$$A(g^2) = \sum_{k=0}^{\infty} A_k (g^2)^{k+1} \tag{2.37}$$

it is useful to consider the Borel transform of the formal power series

$$B(b)=\sum_{k=0}^{\infty}\frac{A_k}{k!}b^k. \tag{2.38}$$

The original function $A(g^2)$ can then be recovered through the integral transform

$$A(g^2)=\int_0^{\infty}\mathrm{d}b\,\mathrm{e}^{-b/g^2}B(b). \tag{2.39}$$

In order to study the summability of that series, we have to look at its Borel transform $B^1_{\pm}(b)$ defined from the series (2.27) by

$$f^n\to\frac{b^{k-1}}{(k-1)!}. \tag{2.40}$$

We get

$$B^1_{\pm}=-\tfrac{1}{2}\int_k\left[A_1(k)+\left(2\pi a_2-\frac{1}{2}\right)\right]\left[\frac{1-\mathrm{e}^{-b\Pi_0(k)}}{\Pi_0(k)}\right]. \tag{2.41}$$

From the IR behavior of Π_0, A_1 and B_1 given by (2.32)–(2.34), we see that this integral is IR convergent as long as $\mathrm{Re}\,b>2\pi$, and has an IR renormalon singularity at

$$b=2\pi. \tag{2.42}$$

Therefore the perturbative part of $\sigma^1_{\pm}$, which is (formally) defined through eq. (2.40), is not Borel summable. However, the ambiguity corresponding to the singularity at $b=2\pi$ is of order

$$\mathrm{Im}\left(\sigma^1_{\pm\,\mathrm{pert.}}\right)\approx\mathrm{e}^{-2\pi/f}, \tag{2.43}$$

and is therefore negligible in the continuum limit, since it corresponds to irrelevant terms of the order of the lattice spacing a.

Since the perturbative tail of the chiral condensate has no ambiguity of order $\mathrm{e}^{-\pi/f}$, the continuum part of the chiral condensate is well defined and can be unambiguously separated from the perturbative tail (up to irrelevant terms). It is possible, using (2.30), to evaluate explicitly that continuum part. This involves not only the terms of order Ω^2 in $F_1(\Omega)$, namely A_2 and B_2, but also those in $\Omega^0_{\pm}$ given in (2.28). The final result is rather complicated and we shall give only its general form. We have

$$\sigma^1_{\pm}(f)_{\mathrm{cont.}}=\pm\mathrm{e}^{-\pi/f}\left(s_{-1}\ln f+\sum_{n=0}^{\infty}s_nf^n\right), \tag{2.44}$$

where the s_n's are numerical factors. The presence of a $\ln f$ is not surprising; it is induced by the two loops coefficients of the β-function and of the anomalous dimension of $\bar{\psi}\psi$ which appear at order $1/N$. The series in f is not Borel summable but its ambiguity is of order $e^{-\pi/f}$ and is therefore irrelevant in the continuum limit. Let us emphasize that the continuum part has opposite sign in the two chiral phases $\pm$. This is a strong indication of the restoration of the chiral symmetry in the continuum limit.

In conclusion, we have checked in this section, up to order $1/N$, that first the continuum part of the chiral condensate may be unambiguously separated from its perturbative tail (that is, up to terms irrelevant in the continuum limit), and second that the continuum part isolated in such a way has the expected scaling and symmetry properties. Of course a complete analysis should require a check of universality by computing the ratio condensate/fermion mass at order $1/N$. This is possible but very tedious. We will prefer to give in the next section a simpler but more heuristic argument for the separability of the continuum part which has the advantage of being applicable to four-dimensional gauge theories.

3. The four-dimensional case

In the Gross-Neveu model as well as in four-dimensional gauge theories the chiral condensate for Wilson fermions can be written as

$$\langle \bar{\psi}\psi \rangle = \int_k \mathrm{tr}\left[\frac{1}{i\not{k}(k) + \tilde{M}(k) + \Sigma(k, g^2)}\right], \tag{3.1}$$

where $i\not{k}(k) + \tilde{M}(k)$ is the propagator for free Wilson fermions and $\Sigma(k, g^2)$ is the fermion self-energy. The requirement for restoration of chiral symmetry is that the bare fermion mass is renormalized in such a way that $\Sigma(k, g^2)$ has a zero at $k = 0$ in perturbation theory, namely that

$$\Sigma(k, g^2) \underset{k \to 0}{\sim} \not{k}\left[(\text{series in } g^2 \text{ and } \ln k^2) + \mathrm{O}(|k|)\right]. \tag{3.2}$$

The dependence on $\ln k$ can be made more precise by using the renormalization group. Adapting an argument originally due to G. Parisi, we can show that if we take the Borel transform of $\Sigma(k, g^2)$, its small-k behavior is determined by the first term in the β-function $\beta(g) = \partial g / \partial \ln \Lambda = -\beta_0 g^3 + \cdots$ and is given by

$$\tilde{\Sigma}(k, b) \underset{k \to 0}{\sim} \not{k}[k^2]^{-b\beta_0}. \tag{3.3}$$

Inserting (3.2) in (3.1) we can estimate the small-momentum behavior of the Borel transform of the perturbative part of $\langle\bar{\psi}\psi\rangle$. We get

$$\text{Borel transform of } [\langle\bar{\psi}\psi\rangle] \approx \int_{k\approx 0} \mathrm{d}^2 k \left[\frac{|k|^{-2b\beta_0}}{i\not{k} + rk^2}\right]. \tag{3.4}$$

In the case of the Gross-Neveu model in $d=2$ the integral is IR divergent at $b=1/\beta_0$ and not at $b=1/2\beta_0$ as suggested by naive power counting arguments, since $\operatorname{tr} 1/\not{k}=0$. The cancellation of the IR renormalon at $b=1/2\beta_0$ occurs obviously because of chiral symmetry. The perturbative tail is ambiguous only at order $\exp(-1/\beta_0 f)$ and the continuum part, of order $\exp(-1/2\beta_0 f)$, can be unambiguously separated.

The advantage of this heuristic argument lies in the fact that it can be immediately extended to gauge theories, where the formula of eq. (3.4) holds with $d=4$. Here one again has

$$\begin{aligned}\langle\bar{\psi}\psi\rangle &= \int_k \operatorname{tr}\left[\frac{1}{i\not{S}(k) + \tilde{M}(k) + \Sigma(k, g^2)}\right] \\ &\approx \sum_{n=0}^{\infty} \int_{k\approx 0} \operatorname{tr}\frac{1}{i\not{k} + \frac{1}{2}rk^2}\left(\frac{1}{i\not{k}}\right)^n \left(-\beta_0 g^2 i\not{k} \ln k^2\right)^n,\end{aligned} \tag{3.5}$$

where β_0 is the coefficient in the first term in the beta function of QCD: $\beta(g) = -\beta_0 g^3 + \mathrm{O}(g^5)$. After taking the Borel transform one obtains

$$\langle\bar{\psi}\psi\rangle \approx g^{-2}\int_{k\approx 0} \operatorname{tr}\frac{1}{i\not{k} + \frac{1}{2}rk^2}[k^2]^{-b\beta_0}. \tag{3.6}$$

Here the perturbative part has a renormalon at $b=2/\beta_0$, while the continuum part of the quark condensate is of order $\exp(-3/2\beta_0 g^2)$ and may be unambiguously separated.

We have shown in the previous section that the continuum value of $\langle\bar{\psi}\psi\rangle$ is related to the discontinuity of the full $\langle\bar{\psi}\psi\rangle$ as κ_c is approached from above ($\kappa > \kappa_c$) and below ($\kappa < \kappa_c$). This follows from the fact that the perturbative tail is an even function of $\kappa - \kappa_c$ whereas the continuum contribution is odd. This suggests a possible practical way of separating the perturbative tail for $\langle\bar{\psi}\psi\rangle$ from its continuum contribution. Let us define

$$\langle\bar{\psi}\psi\rangle_{\text{cont.}} = \tfrac{1}{2}\left[\lim_{m_R\to 0^+} - \lim_{m_R\to 0^-}\right]\langle\bar{\psi}\psi\rangle_{m_R}, \tag{3.7}$$

where m_R is the "renormalized" quark mass as defined through the pion mass. The perturbative tail is then given by

$$\langle\bar{\psi}\psi\rangle_{\text{pert.}} = \tfrac{1}{2}\left[\lim_{m_R\to 0^+} + \lim_{m_R\to 0^-}\right]\langle\bar{\psi}\psi\rangle_{m_R}. \tag{3.8}$$

Let us finally consider the case of the topological charge density $F_{\mu\nu}\tilde{F}_{\mu\nu}$. For the lattice regularization of $F_{\mu\nu}\tilde{F}_{\mu\nu}$ one has two choices: either one defines a simple local operator on the lattice which reduces to the continuum counterpart for weak enough coupling and has no topological significance on the lattice [6], or one defines a more complicated, not necessarily local, operator which maintains the topological properties of the continuum expression [7]. In the first case one expects in general that the lattice definition has a perturbative tail. However here the topological density is of the same order as the ambiguity in the perturbative tail which comes from the mixing with the operator $F_{\mu\nu}^2$, and leads to a renormalon singularity at $2/\beta_0$. Therefore one does not expect that in this case a meaningful continuum quantity can be extracted.

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