

## INSTANTONS AND CONDENSATES IN TWO-DIMENSIONAL $CP^{n-1}$ MODELS

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We discuss the respective role of instantons and of vacuum expectation values of local operators in the structure of the Borel transform of  $CP^{n-1}$  models and suggest an IR finite definition for the contribution of instantons. The structure of the operator product expansion, the renormalization of composite operators, the role of the  $\theta$  parameter and the significance of semi-classical results are discussed from that point of view.

The exact role of instantons (topologically nontrivial minima of the action) [1] in the functional integral remains somewhat uncertain for massless asymptotically free theories. Indeed, the semi-classical methods suffer from important infra-red (IR) divergences when one integrates over the scale of the instantons. Related problems appear while comparing semi-classical arguments with those of the  $1/n$  expansion [2]. For those reasons the only rigorous semi-classical results have been obtained in a finite space-time volume [5–7], and eventually by taking the infinite volume limit after summation over all instantons configurations [3,4,7]. On the other hand a lot of phenomenological models of QCD vacuum based on instantons have been proposed [8–10], which in fact always involve some IR cut off on the instanton size. In this letter we shall discuss the precise relationship of these problems with the occurrence of the “non-perturbative effects” associated to vacuum expectation values of local operators (condensates) in such theories [11,12] and to the “IR renormalons” problem [13]. For that purpose we shall analyse the structure of the Borel transform of the two-dimensional  $CP^{n-1}$  models via their  $1/n$  expansion and the semi-classical results. This will lead us to conjecture a general, mathematically well-defined, and IR finite definition of the contribution of instantons to the observables of such theories.

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Let us first recall briefly some classical results about  $CP^{n-1}$  models (we use the notations of ref. [14]). The classical action of the euclidean  $CP^{n-1}$  model is

$$S = \frac{n}{2f} \int d^2x \overline{D}_\mu Z \overline{D}^\mu Z - \frac{i\theta}{2\pi} \int d^2x \epsilon^{\mu\nu} \partial_\mu A_\nu, \quad (1)$$

where  $Z(x)$  is an  $n$  component complex field with unit norm 1.

$$\overline{Z}_\alpha \cdot Z^\alpha = 1, \quad (2)$$

and  $D_\mu$  the covariant derivative

$$D_\mu = \partial_\mu + iA_\mu, \quad A_\mu = \frac{1}{2} i \overline{Z} \overleftrightarrow{\partial}_\mu Z. \quad (3)$$

We have already introduced the vacuum angle  $\theta$  via the term  $i\theta Q$  in the action. This model admits instanton (and anti-instanton) solutions of the form  $Z_\alpha(x) = w_\alpha(x)/|w(x)|$ , where the  $w_\alpha$  are rational functions of  $s = x_1 - ix_2$  ( $\bar{s} = x_1 + ix_2$ ), with classical action

$$S = (n\pi/f) |Q| - i\theta Q, \quad (4)$$

where the topological number  $Q$  is simply the number of poles (minus the number of zeros) of the  $w$ 's. The  $Q = 1$  solution (one instanton) is characterized by its position,  $SU(n)$  internal degrees of freedom, and an arbitrary scale  $\lambda$ . The contribution of the  $Q = 1$  solution in the semi-classical approximation (one-loop correction) is of the form:

$$\int_0^\infty d\lambda \lambda^{n-3} \exp(-n\pi/f_R). \quad (5)$$

( $f_R$  is the renormalized coupling constant) and is IR divergent for  $n \geq 2$  and UV divergent for  $n \leq 2$ .

The  $1/n$  expansion is obtained by introducing Lagrange multipliers  $\alpha(x)$  and  $\lambda_\mu(x)$  in order to take into account the constraints (2) and (3). Integrating over the  $Z$  and  $A^\mu$  fields we get an  $U(1)$  gauge invariant effective action.

$$S_{\text{eff}} = n \text{tr} \ln [-D_\mu \bar{D}^\mu - (i/\sqrt{n})\alpha] + \frac{i\sqrt{n}}{2f} \int d^2x \alpha(x) - \frac{i\theta}{2\pi\sqrt{n}} \int d^2x \epsilon_{\mu\nu} \partial^\mu \lambda^\nu(x), \quad (6)$$

with

$$D_\mu = \partial_\mu + (i/\sqrt{n})\lambda_\mu. \quad (7)$$

The  $1/n$  expansion corresponds at the level of Feynman diagrams to charged massive particles (the  $Z$  field) with propagator

$$G(p) = (p^2 + m^2)^{-1}, \quad m^2 = \mu^2 \exp(-2\pi/f_R), \quad (8)$$

interacting via the exchange of a scalar  $\alpha$  particle with propagator

$$D^{(\alpha)}(p) = \left( \int \frac{d^2k}{(2\pi)^2} G(k) G(p-k) \right)^{-1}, \quad (9)$$

and the exchange of a  $U(1)$  massless gauge boson  $\lambda^\mu$  with propagator (in Lorentz gauge)

$$D_{\mu\nu}^{(\lambda)}(p) = (\delta_{\mu\nu} - p_\mu p_\nu / p^2) [(p^2 + 4m^2)D^{(\alpha)}(p) - 1/\pi]^{-1} \quad (10)$$

The topologically nontrivial structure of the model emerges via the  $\theta$  term in  $S_{\text{eff}}$ , which corresponds to  $\lambda^\mu$  lines ending in the vacuum with a factor  $\theta$ .

*The Borel transform and the  $1/n$  expansion.* We are interested in the structure of the Borel transform (with respect to the coupling constant  $f_R$ ) of any observable of the model. In fact in the  $1/n$  expansion only locally  $U(1)$  gauge invariant observables are meaningful and free of the IR divergences coming from the massless propagator  $D^{(\lambda)}$  [14] (this corresponds to the fact that only globally  $SU(n)$  invariant observables are IR finite in the perturbative expansion [15]). If we consider such an observable  $G(x)$ , it is more interesting to look at the structure of the "modified" Borel transform [16]:

$$B(x; s) = \int_0^c df_R f_R^{-2} \exp(s/f_R) G(x; f_R). \quad (11)$$

This may be done by the methods that we have developed in refs. [17,18] for the  $O(n)$  nonlinear sigma model. The only difference between the  $O(n)$  and  $CP^{n-1}$  models lies in the additional massless propagator  $D^{(\lambda)}$ . We simply give the final result:

*Proposition 1.* At all orders of the  $1/n$  expansion, the Borel transform  $B$  of any invariant observable  $G$  has the following structure:

(a)  $B(s)$  has cuts at  $s = 2\pi k, k \geq 0$  integer.

(b) The discontinuity  $\Delta_k$  of  $B$  at  $s = 2\pi k$  is the Borel transform of the terms of the SVZ operator product expansion [11]

$$G(x; f_R) = \sum_i C_i(x; f_R) \langle 0|O_i|0 \rangle, \quad (12)$$

associated to operators  $O_i$  with canonical dimension  $2k$ . In (12) the sum runs over all gauge invariant and scalar local operators  $O_i$  constructed from the fields  $Z, \bar{Z}, \alpha$  and  $\lambda^\mu$ . The coefficients  $C_i$  are perturbative series in  $f_R$  and the "condensates"  $\langle 0|O_i|0 \rangle$  are non-perturbative terms of order

$$\langle 0|O_i|0 \rangle \approx [\exp(-\pi/f_R)]^{\dim O_i}. \quad (13)$$

(c) The terms of the RHS of (12) have the ambiguities found in ref. [18] for the  $O(n)$  model. Namely, each  $C_i(f_R)$  is a non-Borel summable series because of IR singularities (IR renormalons) and the condensates  $\langle 0|O_i|0 \rangle$  have  $(2)^{\dim O_i/2}$  different determinations (coming from UV ambiguities) depending on the resummation prescription chosen for the  $C_i$ 's.

(d) The only dependence of  $G$  in the vacuum angle  $\theta$  is contained in the condensates  $\langle 0|O_i|0 \rangle$  which are functions of  $\theta$ .

Points (a), (b) and (c) are simply the generalisation to  $CP^{n-1}$  models of the structure found in refs. [17, 18] for the  $O(n)$  model. The interesting point is (d), which shows that at any finite order of the  $1/n$  expansion, the topologically non-trivial structure of the model does not give new non-perturbative effects and is completely taken into account by the OPE (12).

*The IR structure of the semiclassical approximation.* Of course we expect that the above result does not hold at finite  $n$ . Indeed the one-instanton contribution is naively of order  $\exp(-n\pi/f_R)$ , so that the OPE (12) should be valid only for  $\dim O_i < n$ . In order to understand what new contributions instantons induce

at finite  $n$ , let us return to the results of the semi-classical calculations by putting an IR cutoff and looking at the structure of the IR divergences as the cutoff is removed.

For that purpose we shall simply use the results of ref. [3] where the one-loop contribution of  $k$ -instantons configurations is computed exactly for the  $CP^{n-1}$  models on a sphere with radius  $R$ . We use the conformal complex coordinates on the sphere where

$$z = R^{-1}(x_1 + ix_2), \quad g_{\mu\nu} = \delta_{\mu\nu} 4R^2/(1 + |z|^2)^2. \quad (14)$$

We shall restrict ourselves to the  $k = 1$  sector. The one-instanton may always be written, after an internal  $SU(n)$  rotation as

$$w_\alpha = \delta_{\alpha,1} \rho(1 + z\bar{z}_0) + \delta_{\alpha,n} (z - z_0), \quad 0 < \rho < 1, \quad (15)$$

$\rho$  is the scale of the instanton and  $z_0$  its position. The measure over the collective coordinates  $\rho$  and  $z_0$  is found at one loop <sup>+1</sup> [6] [after integrating over the  $SU(n)$  internal degrees of freedom which are irrelevant for any  $SU(n)$  invariant observable]:

$$d\mu = R^n C_n(f_R; \mu) 4d^2z_0 (1 + |z_0|^2)^{-2} \times d\rho \rho^{-3} (1 - \rho^2)^2 \rho^{n(1+\rho^2)/(1-\rho^2)}, \quad (16)$$

where the constant  $C_n$  is in the MS scheme

$$C_n(f_R; \mu) = (4\pi)^{-1} [\pi^n / \Gamma(n-1)] \times \exp\{-\frac{1}{2}(n+2) + \frac{1}{2}n[\ln 4\pi + \Gamma'(1)]\} \times [(2n/f)\mu \exp(\pi/f_R)]^n. \quad (17)$$

We are interested in the behavior as  $R \rightarrow \infty$  of invariant observables. Let us consider for instance the two-point function:

$$\Delta(\mathbf{x}, \mathbf{y}) = |w(\mathbf{x}) \cdot \bar{w}(\mathbf{y})|^2 / |w(\mathbf{x})|^2 |w(\mathbf{y})|^2. \quad (18)$$

Using (15) and (16) we get for the contribution of the  $k = 1$  sector to  $\Delta$  the integral

$$\Delta^{(k=1)}(\mathbf{x}, 0) = -R^{n-2} C_n |\mathbf{x}|^2 \int_0^1 d^2z_0 \int d\rho \times \frac{\rho^{-1+n(1+\rho^2)/(1-\rho^2)} (1-\rho^2)^2}{(\rho^2 + |z_0|^2)(\rho^2 |1 + x\bar{z}_0/R|^2 + |z_0 - x/R|^2)}, \quad (19)$$

The behavior of (19) as  $R \rightarrow \infty$  may be splitted in two

<sup>+1</sup> The parameter  $z$  of ref. [6] is  $z = \rho^2$ .

parts: first the divergences coming from the behavior of the integrand as  $R \rightarrow \infty$  (which give terms like  $R^{n-k}$  ( $k$  integer) and which correspond to the short distance behavior of  $\Delta$  in a constant background field, and second the divergences coming from the integration in the domain  $\rho \lesssim 1/R$ , which correspond to small size instantons, and which give terms like  $R^k (\ln R)^q$  ( $q \leq k$ ). The final result has a simple form for generic (non-rational)  $n$ , and may be extended for any observable  $G$ .

*Proposition 2.* Any observable  $G$  in the  $k = 1$  sector has the following large  $R$  expansion

$$G^{(k=1)}(\mathbf{x}) \equiv \sum_i R^{n-\dim O_i} C_i^{(k=0)}(\mathbf{x}) \langle 0 | O_i | 0 \rangle_{R=1}^{(k=1)} + C_0^{(k=1)}(\mathbf{x}, R), \quad (20)$$

where the sum runs over all local operators  $O_i$ .

$C_i^{(k=0)}(\mathbf{x})$  is the first term ( $f_R = 0$ ) of the coefficient  $C_i$  of the OPE (12) computed in perturbation theory (namely in the  $k = 0$  sector).

$R^{n-\dim O_i} \langle 0 | O_i | 0 \rangle_{R=1}^{(k=1)}$  is by homogeneity the UV finite part of the VEV of  $O_i$  computed in the  $k = 1$  sector (with the IR cutoff  $R$ ). The finite part prescription deals with the UV divergences of order

$$\int_0 d\rho \rho^{n-1-\dim O_i}, \quad (21)$$

which are present in  $\langle O_i \rangle_R^{k=1}$  if  $\dim O_i \geq n$ .

The remaining term  $C_0^{(k=1)}$  is the "finite part" of  $G$  in the  $k = 1$  sector and is of the form

$$C_0^{(k=1)} \underset{R \rightarrow \infty}{\equiv} a_0 + R^{-2} (a_1 \ln R + b_1) + R^{-4} [a_2 (\ln R)^2 + b_2 \ln R + c_2] + \dots. \quad (22)$$

The first term  $a_0$  is universal and independent of the IR regulator, and is obtained by computing naively  $G$  in the  $k = 1$  sector with  $R = \infty$  and dealing with the IR divergence at  $\rho = \infty$  by Hadamard's finite part prescription.

Thus, (20) shows that the IR singularities of the  $k = 1$  sector have the structure of an OPE exactly like the  $k = 0$  sector [15,19], but that there is a mixing between the  $k = 0$  and  $k = 1$  sectors in (20). There are no others  $C_i^{(k=1)}$  terms at leading order in  $f_R$  in (20)

because

$$\langle 0|O_i|0\rangle^{k=0} = O(f_R) \quad \text{if } \dim O_i > 0.$$

*The structure of the Borel transform for finite n.*  
 It is tempting to generalize that result to any  $k$  sector and to any order of perturbation theory and to expect that all IR singularities of the  $k$  sector may be recast into VEV of local operators computed in sectors  $0 \leq k' \leq k$ . (This is technically much more difficult to prove, in particular there is no simple way to take a generic  $n$  so that the different powers of  $R$  do not mix. We have only partial results on those points). Assuming, as done in ref. [19] for massless superrenormalizable theories, that in the limit  $R \rightarrow \infty$  those VEV have a finite limit, we are led to conjecture the following result.

*Conjecture.* For generic (non-rational)  $n$ , the Borel transform of any observable  $G$  has cuts as  $s = 2\pi(\delta + n \cdot k)$ ,  $\delta, k$  integer, corresponding to the OPE

$$G(x) = \sum_i C_i(x) \langle 0|O_i|0\rangle, \quad (23)$$

where the VEV of an operator  $O_i$  with dimension  $2\delta$  is of the form

$$\begin{aligned} \langle 0|O_i|0\rangle &= C(\theta) \exp\left(\int_{f_R} \frac{2\delta + \gamma(t)}{\beta(t)} dt\right) \\ &\simeq C(\theta) \exp[-(\pi/f_R) \dim O_i], \end{aligned} \quad (24)$$

and where each  $C_i$  is the sum over all topological sectors  $k$  of the IR finite part of the corresponding coefficient of the OPE in sector  $C_i^{(k)}$ :

$$C_i(x) = \sum_{k=-\infty}^{+\infty} \exp(i\theta k) C_i^{(k)}(x), \quad (25)$$

each  $C_i^{(k)}$  being of the form

$$\begin{aligned} C_i^{(k)}(x) &= [f^{-1} \exp(-\pi/f)]^{nk} \\ &\times [a_{i,0}^k(x) + f_R a_{i,1}^k(x) + \dots]. \end{aligned} \quad (26)$$

In (26) the bracket contains the whole perturbative expansion of  $C_i$  in the sector  $k$ , plus possible non-perturbative contributions coming from other finite action classical configurations.

*Comments.*

(a) This seems to us the simplest and most natural generalization of the partial results obtained above. It means that all IR divergences of the instantons can be recast in the condensates<sup>#2</sup> so that what remain are the series (26), which are in IR finite parts of the instanton contributions, and which are well defined for generic  $n$ . In particular all ‘‘paradoxes’’ encountered when comparing the large  $n$  limit and the semi-classical results are simply explained by (23)–(26).

(b) The  $f_R$  and  $\theta$  dependence of the condensates suggested in (24) is a consequence of the fact that the  $\langle O_i \rangle$ 's must obey standard renormalization group equations and that the  $\beta$  function and the anomalous dimensions  $\gamma$  are the same in all topological sectors [21]. Therefore, they do not depend on  $\theta$  and cannot contain non-perturbative contributions. As a consequence the  $\langle O_i \rangle$ 's depend on  $\theta$  only via the integration constants  $C(\theta)$  in (24).

(c) As for the  $O(n)$  model, those constants  $C(\theta)$  contain also the UV ambiguities related to the renormalons problem and therefore are determined only once a summation prescription has been chosen for the series (26) which contain IR renormalons.

(d) The above result is valid only for generic  $n$ . For integer values of  $n$ , some  $\langle O_i \rangle$  and  $C_j^{(k)}$  are of the same order and the associated cuts in the Borel plane should coalesce and have additional divergences. We shall discuss this point further in the case  $n = 2$  [the  $O(3)$  model].

(e) The first condensate  $O_1 = \langle 0|D_\mu Z \bar{D}^\mu \bar{Z}|0\rangle$  is well defined as long as  $n > 2$ , and its ambiguity [which is equal to the IR renormalon of  $C_0^{(0)}(f_R)$  at  $s = 1/2\pi$ ] is therefore independent of  $\theta$ . Thus  $O_1(\theta) - O_1(\theta = 0)$  is unambiguous for  $n > 2$  and the topological susceptibility  $\chi_T$ , which corresponds in the infinite volume limit to

$$\chi_T = -\frac{1}{2} [\beta(f_R)/f_R^2] d^2 O_1(\theta)/d\theta^2|_{\theta=0}, \quad (27)$$

is well defined as long as  $n \geq 2$ .

(f) Finally let us note that VEV of pseudoscalar operators such as the topological charge density ( $1/2\pi$ )

<sup>#2</sup> This fact was already suggested in ref. [12], however, our finite part prescription is unambiguous and differs from that proposed in ref. [12]. The fact that instantons must give in fact a contribution of order  $\exp(-2\pi/f_R)$  rather than  $\exp(-n\pi/f_R)$  was also pointed out by Munster [20].

$\times \epsilon_{\mu\nu} \partial^\mu A^\nu$  may appear in the OPE (23) via the terms  $C_i^{(k)}$ ,  $k$  odd, in (25). Such terms are zero for  $\theta = 0$  or  $\pi$  and cannot be seen in the  $1/n$  expansion.

*The case  $n = 2$ .* We now discuss the mixing of the condensates with the instantons which occurs for integer values of  $n$  on the example of the  $CP^1$  model [the  $O(3)$  non-linear  $\sigma$  model]. Then the first condensate  $O_1$  is from (24)

$$O_1 = C_1 f_R^{-4/n} \exp(-2\pi/f_R) [1 + O(f_R)], \quad (28)$$

and for  $n = 2$  is exactly of the same order than the contribution of the one-instanton given by (26)<sup>+3</sup>. Therefore the contribution of the one-instanton computed by (19) behaves like  $1/(n - 2)$  and has a pole at  $n = 2$ . Since  $C_0^{(1)}$  and  $O_1$  correspond to cuts in the Borel plane which coincide as  $n = 2$ ,  $O_1$  must have also a pole at  $n = 2$  (associated to divergences of small size instantons) so that the two poles cancel to give an additional logarithm of  $f_R^{-2} \exp(-2\pi/f_R)$ . This means that one can no longer separate the contribution of instantons to those of the condensates and that only their sum is meaningful. As a consequence, the topological susceptibility  $\chi_T$  given by (27) diverges at  $n = 2$ , because of small size fluctuations, a phenomenon already noticed by Lüscher [23].

This is in fact the consequence of a general phenomenon: For an integer (or rational) number of components  $n$ , an operator  $O$  with dimension  $m$  suffers from additional UV divergences, coming from small size instantons, which mix  $O$  with operators with dimension  $p < m$ , via different topological sectors [ $\Delta k = n^{-1}(m - p)$ ]. Therefore additional subtractions are needed in order to define the renormalized operator  $O$ .

This may be shown to occur in the so-called "semi-classical approximation" where one computes only the quadratic fluctuations around the  $k$ -instanton solutions and sums over all instantons (or anti-instantons) configurations. This approximation corresponds in fact (at least formally) to take the limit where the renormalized coupling constant  $f_R$  goes to zero and the  $\theta$  angle is taken complex<sup>+4</sup> and goes to  $-i\infty$  (or  $+i\infty$ )

<sup>+3</sup> The identity holds also for the power factor in  $f_R$ . We do not know if this is a coincidence or if there exists some deep reason for such an identity, like in some super-symmetric theories [22],

<sup>+4</sup> The idea of taking  $\theta$  complex has already been suggested by 't Hooft [24].

in such a way that the parameter

$$z = [f_R^{-1} \exp(-\pi/f_R)]^n \exp(\pm i\theta), \quad (29)$$

remains finite, and corresponds to the fugacity of the instantons (or anti-instantons). Such a limit likely exists in a finite volume but the existence of a thermodynamic limit is highly nontrivial and has been proved up to now only for the  $O(3)$  model [7]. If the above limit and the thermodynamic limit exist and commute, then the condensates (24) should also have a semi-classical limit, and the mixing of instantons and condensates remains valid, giving for an observable a small  $z$  expansion involving  $z^k (\ln z)^r$  ( $r \leq k$ ) terms. This is indeed the case for the  $O(3)$  model, which corresponds in the semi-classical approximation and the infinite volume limit to a neutral Coulomb gas at  $\beta_c = 1$ , i.e. to a free massive Dirac Field [3,4,7]. The two points function (18) has been computed exactly by Iwazaki and Yoshie [25] and is found to be a combination of Bessel functions of  $R = |x|m$  [where  $m$  is the mass gap and  $m^2$  is proportional to the fugacity  $z$  given by (29)] with a small  $R$  expansion of the form predicted by our arguments:

$$\langle \sigma(x) \sigma(0) \rangle = 1 + \sum_{k=1}^{\infty} R^{2k} \sum_{r=0}^k a_{k,r} (\ln R)^r. \quad (30)$$

This seems to us an additional argument for the correctness of our conjecture<sup>+5</sup>.

In conclusion, we think that the considerations developed here allow a better understanding of the significance of instantons, of semi-classical methods, and of the structure of the Borel transform in massless asymptotically free theories. It remains to understand whether instantons-anti-instantons configurations play a role and may be characterized in the same manner.

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<sup>+5</sup> However, the arguments of ref. [25] for some "topological symmetry breaking" seems irrelevant since the limit (29) explicitly breaks the instanton-anti-instanton (or parity) symmetry. One may also ask whether the semi-classical approximation ( $\theta = \pm i\infty$ ) is really close to the "true" model ( $\theta$  real)!

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### References

- [1] A.M. Polyakov, Phys. Lett. 59B (1975) 82;  
A.A. Belavin, A.M. Polyakov, A. Schwartz and Y. Tyupkin,  
Phys. Lett. 59B (1975) 85;  
G. 't Hooft, Phys. Rev. D14 (1976) 3435.
- [2] E. Witten, Nucl. Phys. B149 (1979) 285.
- [3] B. Berg and M. Lüscher, Commun. Math. Phys. 69  
(1979) 57.
- [4] V.A. Fateev, I.V. Frolov and A.S. Schwartz, Nucl. Phys.  
B154 (1979) 1.
- [5] M. Lüscher, Nucl. Phys. B205 (1982) 483.
- [6] P. Schwab, Phys. Lett. 118B (1982) 373.
- [7] A. Patrascioiu and A. Rouet, Nucl. Phys. B214 (1983)  
481;  
A. Rouet, Phys. Lett. 124B (1983) 379.
- [8] C. Callan, R. Dashen and D.J. Gross, Phys. Rev. D11  
(1978) 2717;  
R. Jackiw and C. Rebbi, Phys. Rev. Lett. 37 (1976) 172.
- [9] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl.  
Phys. B165 (1980) 45.
- [10] E.V. Shuryak, Nucl. Phys. B203 (1982) 93, 116, 140.
- [11] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl.  
Phys. B147 (1979) 385, 447, 519.
- [12] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I.  
Zakharov, Nucl. Phys. B174 (1980) 379.
- [13] G. Parisi, Nucl. Phys. B150 (1979) 163.
- [14] A. D'Adda, M. Lüscher and P. di Vecchia, Nucl. Phys.  
B146 (1978) 63.
- [15] S. Elitzur, Nucl. Phys. B212 (1983) 501;  
F. David, Commun. Math. Phys. 81 (1981) 149.
- [16] W.Y. Crutchfield, Phys. Rev. D19 (1979) 2370.
- [17] F. David, Nucl. Phys. B209 (1982) 433.
- [18] F. David, Nucl. Phys. B234 (1984) 237.
- [19] M. Bergere and F. David, Ann. Phys. (NY) 142 (1982)  
476.
- [20] G. Munster, Z. Phys. C12 (1982) 43.
- [21] D. Amati and A. Rouet, Phys. Lett. 73B (1978) 39;  
A. Rouet, Phys. Lett. 78B (1978) 608.
- [22] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I.  
Zakharov, Exact Gell-Mann–Low function of supersym-  
metric Yang–Mills theories from instanton calculus, ITEP  
preprint (1983).
- [23] M. Lüscher, Nucl. Phys. B200 (1982) 61.
- [24] G. 't Hooft, in: Proc. EPS Intern. Conf. on High energy  
physics (Lisbon, July 1981).
- [25] Y. Iwasaki, Phys. Lett. 104B (1981) 459;  
Y. Iwasaki and T. Yoshie, Phys. Lett. 112B (1982) 379.