Institut de Physique Théorique

Quantum measurement theory

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Complementary notes and bibliography

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Contents

1 Outline of the lectures and their references

The course will be based on the following references:

- [1] *An ensemble theory of ideal quantum measurement processes.* Armen E. Allahverdyan, Roger Balian, Theo M. Nieuwenhuizen. This text gathers the main ideas which will be presented in this course. It is the draft of a manuscript not yet in final form.
- [2] *Understanding quantum measurement from the solution of dynamical models.* Armen E. Allahverdyan, Roger Balian. This text contains a review of the literature about measurement models (beyond the scope of this course), and the detailed solution of a specific model which serves to illustrate the ideas of [1]. These ideas were already present in this review, but they appear in a more elaborate and more explicit form in [1].
- [3] *Lectures on dynamical models for quantum measurements.* Theo M. Nieuwenhuizen, Marti Perarnau-Llobet, Roger Balian. This is a course containing details about the derivations of [2].
- Among the references included in [1] and [2], many can be studied for deepening this course, for instance F. David's presentation of the principles of quantum mechanics [4], and F. Laloë's book [5](also published in French) about its interpretations.

Lecture 1

1. The measurement problem : [1], sec 1 ; [2], sec 1 ; Laloë's book [5]
2. Statement of the problem in the language of quantum statistical mechanics, for tested system + apparatus : [1], sec 1
3. Minimalist formulation of quantum mechanics : [1], sec 3 ; [2], sec 10 ; David's course [4].
4. Digression on Liouville representations : *Incomplete descriptions and relevant entropies*, RB, Am. J. Phys. 67, 1078-1090 (1999), Appendix A (available on the IPhT site, t99/043).
5. Digression on impossibility to identifying q-probabilities as probabilities : see Section 2 below.

Lecture 2

1. The measurement problem, recall and 3-step strategy: [1], secs 1-2; [2], sec 1.

2. Form of the Hamiltonian for system + apparatus: [1], sec 3; [2], sec 3.

3. Dynamical equations: [1], sec 5; [2], sec 4.

4. Thermodynamic equilibrium states for system + apparatus: [1], sec 4.

5. Relaxation for the full ensemble of runs; (i) Truncation (decay of $< s_x >$ and $< s_y >$, possible recurrences and their elimination): [1], sec 5; [2], secs 5.1.1-2, 5.3.1, 6.1, 9.6.1.

Lecture 3

1. Truncation (continuation): the cascade of correlations, source of irreversibility: [2] sec 5.1.

2. Equations of motion of a system weakly coupled with a thermal bath: [2] appendix A

3. Relaxation for the full ensemble; (ii) Registration: [2] sec 7.1; its possible failure [2] sec. 7.3.

4. Second step: relaxation of the subensembles: [1] sec. 7; [2] sec. 11.2 and appendices H and I.

Lecture 4

1. Classical behaviour of the pointer observable in the final state: [1] sec. 7.

2. Three interpretative principles: [1] sec. 8.

3. Uniquiness of the outcome, emergence and meaning of Born's rule and von Neumann's reduction: [1] sec. 9.

4. Information transfers; Contextuality: [1] sec. 10.

2 Digression on impossibility to identifying q-probabilities as probabilities

1. **Two observables** $\hat{A}$, $\hat{B}$: their q-variances $\langle \hat{A}^2 \rangle - \langle \hat{A} \rangle^2 = \Delta \hat{A}^2$ satisfy Heisemberg's inequality

$$\Delta \hat{A}^2 \, \Delta \hat{B}^2 \geq \frac{1}{4} \langle \hat{C}^2 \rangle, \tag{1}$$

where $[\hat{A}, \hat{B}] = \mathrm{i} \, \hat{C}$, so that q-variances differ from variances of probability theory, which are not bounded.

2. **Four observables**: $\hat{A}$, $\hat{A}'$, $\hat{B}$, $\hat{B}'$, taken as $\hat{A} = \mathbf{a}\cdot\hat{\boldsymbol{\sigma}}^{(1)}$, $\hat{A}' = \mathbf{a}'\cdot\hat{\boldsymbol{\sigma}}^{(1)}$, $\hat{B} = \mathbf{b}\cdot\hat{\boldsymbol{\sigma}}^{(2)}$, $\hat{B}' = \mathbf{b}'\cdot\hat{\boldsymbol{\sigma}}^{(2)}$, where $\mathbf{a}$, $\mathbf{a}'$, $\mathbf{b}$, $\mathbf{b}'$, are unit vectors. Bell's inequality in the CHSH (Clauser, Horne, Shimony, Holt) form: The number

$$X \equiv \big\langle \hat{A}\hat{B} + \hat{A}\hat{B}' + \hat{A}'\hat{B} - \hat{A}'\hat{B}' \big\rangle = \big\langle \hat{A}\big(\hat{B}+\hat{B}'\big) + \hat{A}'\big(\hat{B}-\hat{B}'\big)\big\rangle$$

would be less than 2 if A, A', B, B' were ordinary random variables taking values ± 1 (since we would have either $[B = B', B + B' = \pm 2]$ or $[B = -B', B - B' = \pm 2]$). But quantum mechanically, in the singlet state $|\psi\rangle$ where

$$\big[\hat{\boldsymbol{\sigma}}^{(1)} + \hat{\boldsymbol{\sigma}}^{(2)}\big]|\psi\rangle = 0, \qquad \big\langle \hat{A}\hat{B}\big\rangle = -\mathbf{a}\cdot\mathbf{b},$$

we can have $X = 2\sqrt{2}$ if $\mathbf{a}$, $\mathbf{a}'$, $\mathbf{b}$, $\mathbf{b}'$, have polar angles 0, $-\frac{\pi}{2}$, $\frac{3\pi}{4}$, $-\frac{3\pi}{4}$, respectively. Hence q-correlations *are not* correlations.

3. **Five observables**, realized with 3 spins $\hat{A}_1$, $\hat{A}_2$, $\hat{A}_3$, $\hat{B}_1$, $\hat{B}_2$, and $\hat{B}_3 \equiv \hat{B}_1\hat{B}_2$:

$$\hat{A}_1 = \hat{\sigma}_z^{(1)},\ \hat{A}_2 = \hat{\sigma}_z^{(2)},\ \hat{A}_3 = \hat{\sigma}_z^{(3)},\ \hat{B}_1 = \hat{\sigma}_x^{(2)}\hat{\sigma}_x^{(3)},\ \hat{B}_2 = \hat{\sigma}_x^{(3)}\hat{\sigma}_x^{(1)},\ \hat{B}_3 = \hat{\sigma}_x^{(1)}\hat{\sigma}_x^{(2)}.$$

The operators $\hat{A}_i\hat{B}_i$ and $\hat{A}_j\hat{B}_j$ commute, and we can take $|\psi\rangle$ as their common eigenstate with eigenvalue $+1$. Then,

$$\big\langle \hat{A}_1\hat{B}_1\hat{A}_2\hat{B}_2\hat{A}_3\hat{B}_3\big\rangle = 1 \quad \text{(without fluctuation)}.$$

However,

$$\big\langle \hat{A}_1\hat{B}_1\hat{A}_2\hat{B}_2\hat{A}_3\hat{B}_3\big\rangle = -\big\langle \hat{A}_1\hat{A}_2\hat{A}_3\hat{B}_1\hat{B}_2\hat{B}_3\big\rangle$$

(anticommutation $\hat{A}_i\hat{B}_j + \hat{B}_j\hat{A}_i = 0$ for $i \neq j$). The relation $(\hat{\sigma}_x^{(i)})^2 = 1$ leads to $\hat{B}_1\hat{B}_2\hat{B}_3 \equiv \hat{1}$. Hence, in this state

$$\big\langle \hat{A}_1\hat{A}_2\hat{A}_3\big\rangle = -1 \quad \text{(without fluctuation)}.$$

For ordinary random variables, we would have $A_iB_i = 1$, that is, $A_i = B_i = \pm 1$ (full correlation) and $B_1B_2B_3 = 1$ would imply $A_1A_2A_3 = 1$.

Even when they are complete and yield exact statements, q-correlations are not correlations of probability theory.

References

[1] *An ensemble theory of ideal quantum measurement processes.* Armen E. Allahverdyan, Roger Balian, Theo M. Nieuwenhuizen. To be submitted soon. (A draft copy is available on IPhT website. Disregard the underlined sentences, which are still discussed between the authors so as to reach a final form.)

[2] *Understanding quantum measurement from the solution of dynamical models.* Armen E. Allahverdyan, Roger Balian, Theo M. Nieuwenhuizen Phys. Rep. 525 (2013) 1-166 [[arXiv:1107.2138, t11/166, 2 paper copies available for borrowing on the books display shelf near the librarian office, at IPhT]]

[3] *Lectures on dynamical models for quantum measurements.* Theo M. Nieuwenhuizen, Marti Perarnau-Llobet, Roger Balian [[arXiv:1406.5178, t14/005]].

[4] *A short introduction to the quantum formalism[s].* David F. [[arXiv:1211.5627 t12/042

[5] *Do We Really Understand Quantum Mechanics?* by Franck Laloë, Cambridge University Press (2012)

[6] *Incomplete descriptions and relevant entropies.* Roger Balian, Am. J. Phys. 67, 1078-1090 (1999), Appendix A (available on the IPhT site, t99/043).