Chapter 1

Introduction

1.1 Motivation

Quantum mechanics in its modern form is now more than eighty years old. It is probably the most successful and complete physical theory that was ever proposed in the whole history of science. The quantum theory is born with the twentieth century from the attempts to understand the structure and the properties of matter, of light and of their interactions at the atomic scale. In a quarter of a century a consistent formulation of the quantum theory was achieved, and it became the general conceptual framework to understand and formulate the laws of the physical world. This theory is quantum mechanics. It is valid from the microscopic distances (the presently accessible high energy scales 10 TeV$\simeq 10^{-19}$m, and possibly from the Planck scale (10^{-35} m), up to the macroscopic distances (from $\ell \sim 1$ nm up to $\ell \sim 10^5$ m depending of physical systems, from molecules to neutron stars, and experiments). Beyond these scales, for most scientists the "old" classical mechanics takes over as an effective theory, which becomes valid when quantum interferences and non-local correlations effects can be neglected.

Quantum mechanics has fully revolutionized and unified the fields of physics (as a whole, from particle and nuclear physics to atomic an molecular physics, optics, condensed matter physics and material science), and of chemistry (again as a whole). It had a fundamental impact in astronomy, cosmology and astrophysics, in biology. It had and still has a big influence on mathematics (with feedback). It had of course a huge impact on modern technology, communications, computers, energy, sensors, weaponry (unfortunately) etc. In all these fields of science, and despite the impressive experimental and technical progresses of the last decades quantum mechanics has never been found at fault or challenged by experiments, and its theoretical foundations are considered as very solid. These tremendous successes have overcome the doubts and the discussions about the foundational principles and the paradoxical and non-classical features of quantum mechanics and the possible interpretations of the formalism, that take place since its beginning.

Quantum information became a important and very active field (both theoretically and experimentally) in the last decades. It has enriched our points of view on the quantum theory, and on its applications, such as cryptography and quantum computing. Quantum information science, together with the experimental tests of quantum mechanics on microscopic quantum systems, the theoretical advances in quantum gravity and cosmology, the slow diffusion of the concepts of quantum theory in the general public, etc. have led to a revival of the discussions about the principles of quantum mechanics and its seemingly paradoxical aspects.

This development of the reflexions and of the discussions about quantum theory, and the concurrent expansion of its successes and of its applications (possibly together with the rise of sensationalism in science outreach and in the practice of science...) give sometimes the feeling that quantum mechanics is both: (i) the unchallenged and dominant paradigm of modern physical sciences, (ii) but at the same time a still mysterious and poorly understood theory, that awaits some (imminent) revolution.

The purpose of these lecture notes is to discuss more the first point: why is quantum mechanics so consistent and successful? They will present a brief and introductory (but hopefully coherent) view of the main formalizations of quantum mechanics (and of its version compatible with special relativity, namely quantum field theory), of their interrelations and of their theoretical foundations.

Two formulations are the standard tools used in most applications of quantum theory in physics and chemistry. These are: (i) the "standard" formulation of quantum mechanics (involving the Hilbert space of pure states, self-adjoint operators as physical observables, and the probabilistic interpretation given by the Born rule); and (ii) the path integral and functional integral representations of probabilities amplitudes. It is important to be aware that there are other formulations of quantum mechanics, i.e. other representations (in the mathematical sense) of quantum mechanics, which allow a better comprehension and justification of the quantum theory. This course will focus on two of them: (i) the algebraic quantum theory and quantum field theory, and (ii) the so called "quantum logic" approach. These are the formulations that I find the most interesting (besides the standard one) and that I think I managed to understand (somehow...).

In my opinion discussing and comparing these various formulations is useful in order to get a better understanding of the coherence and the strength of the quantum formalism. This is important when discussing which features of quantum mechanics are basic principles and which ones are just natural consequences of the former. Indeed what are the principles and what are the physical consequences depends on the precise formulations chosen. For instance, as we shall see, the Born rule or the projection postulate are postulates in the standard formulation, while in some other formulations, like quantum logic, they are mere consequences of other postulates.

There are many excellent books that present and discuss the principles of quantum mechanics. Nowadays most of them treat to some extend the foundational and interpretational issues. Nevertheless, usually only the standard formalism is presented in some details, with its physical applications, and it is in this framework that the questions of the principles and of the possible interpretations of the formalism are discussed: Bell-like inequalities, non-locality, determinism and chance, hidden variable models, Copenhagen interpretation versus the rest of the world, information-theoretic formulations, etc. Even in the excellent and highly advisable reviews by Auletta [Aul01] and by Laloë [Lal12] the algebraic formalism and quantum logic are discussed in a few pages. I think that discussing in more details these other formulations is quite useful and sheds lights on the meaning of the formalism and on its possible interpretations. It is also useful when considering the relations between quantum theory, information theory and quantum gravity. Thus I hope these notes will not be "just another review on quantum theory" (and possibly rather amateurish), but that they may fill some gap and be useful, at least to some readers.

1.2 Organization

After this introductory section, the second section of these notes is a reminder of the basic concepts of classical physics, of probabilities and of the standard (canonical) and path integral formulations of quantum physics. I tried to introduce in a consistent way the important classical concepts of states, observables and probabilities, which are of course crucial in the formulations of quantum mechanics. I discuss in particular the concept of probabilities in the quantum world and the issue of reversibility in quantum mechanics in the last subsection.

The third section is devoted to a presentation and a discussion of the algebraic formulation of quantum mechanics and of quantum field theory, based on operator algebras. Several aspects of the discussion are original. Firstly I justify the appearance of abstract C^*-algebras of observables using arguments based on causality and reversibility. In particular the existence of a *-involution (corresponding to conjugation) is argued to follow from the assumption of reversibility for the quantum probabilities. Secondly, the formulation is based on real algebras, not complex algebras as is usually done, and I explain why this is more natural. I give the mathematical references which justify that the GNS theorem, which ensures that complex abstract C^*-algebras are always representable as algebras of operators on a Hilbert space, is also valid for real algebras. The standard physical arguments for the use of complex algebras are only given after the general construction. The rest of the presentation is shorter and quite standard.

The fourth section is devoted to one of the formulations of the so-called quantum logic formalism. This formalism is much less popular outside the community interested in the foundational basis of quantum mechanics, and in mathematics, but deserves to be better known. Indeed, it provides a convincing justification of the algebraic structure of quantum mechanics, which for an important part is still postulated in the algebraic formalism. Again, if the global content is not original, I try to present the quantum logic formalism in a similar light than the algebraic formalism, pointing out which aspects are linked to causality, which ones to reversibility, and which ones to locality and separability. This way to present the quantum logic formalism is original, I think. Finally, I discuss in much more details than is usually done Gleason's theorem, a very important theorem of Hilbert space geometry and operator algebras, which justify the Born rule and is also very important when discussing the status of hidden variable theories.

The final section contains short, introductory and more standard discussions of some other questions about the quantum formalism. I present some recent approaches based on quantum information. I discuss some features of quantum correlations: entanglement, entropic inequalities, the Tisrelson bound for bipartite systems. The problems with hidden variables, contextuality, non-locality, are reviewed. Some very basic features of quantum measurements are recalled. Then I stress the difference between the various formalizations (representations) of quantum mechanics, the various possible interpretations of this formalism, and the alternative proposal for a quantum theory that are not equivalent to quantum mechanics. I finish this section with a few very standard remarks on the problem of quantum gravity.

1.3 What this course is not!

These notes are tentatively aimed at a non specialized but educated audience: graduate students and more advanced researchers in physics. The mathematical formalism is the main subject of the course, but it will be presented and discussed at a not too abstract, rigorous or advanced level. Let me stress that **these notes do not intend to be**:

— a real course of mathematics or of mathematical physics;

— a real physics course on high energy quantum physics, on atomic physics and quantum optics, of quantum condensed matter, discussing the physics of specific systems and their applications;

— a course on what is *not* quantum mechanics;

— a course on the history of quantum physics;

— a course on the present sociology of quantum physics;

— a course on the philosophical and epistemological aspects of quantum physics.

But I hope that it could be useful as an introduction to these topics. Please keep in mind that this is not a course made by a specialist, it is rather a course made by an amateur, but theoretical physicist, for amateurs!

1.4 Acknowledgements

These lecture notes started from: (i) a spin-off of yet-to-be-published lecture notes for a master course in statistical field theory that I am giving a Ecole normale supérieure since 2001, and of courses in quantum field theory that I gave in the recent years at the Perimeter Institute, (ii) a growing interest in quantum information science and the foundational discussions about quantum formalism (a standard syndrome for the physicist over fifty... but motivated also, more constructively I hope, by my work during five years as an evaluator for the European Research Council), (iii) a course for the Graduate School of Physics of the Paris Area (ED107) that I gave in 2012 at the Institut de Physique Théorique. A first version of the notes of this course is available online at [Dav12].

Most of the content of these lecture notes is review material (I hope properly assimilated). A few sections and aspects of the presentation contain some original material. In particular the emphasis on the role of reversibility in the formalism (especially in sections 3 and 4) has been published in a short form in [Dav11]. These notes can be considered partly as a very extended version of this letter.

I would like to thank for their interest, advices, critics and support at various stage of this work Roger Balian, Michel Bauer, Marie-Claude David, Michel Le Bellac, Kirone Mallick, Vincent Pasquier, Catherine Pépin.