

Multi-Point Propagators for Non-Gaussian initial conditions

Francis Bernardeau¹, Martín Crocce² & Emiliano Sefusatti¹

¹*Institut de Physique Théorique, CEA/DSM/IPhT,*

Unité de recherche associée au CNRS, CEA/Saclay 91191 Gif-sur-Yvette cedex and

²*Institut de Ciències de l'Espai, IEEC-CSIC, Campus UAB,
Facultat de Ciències, Torre C5 par-2, Barcelona 08193, Spain*

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We show here how Renormalized Perturbation Theory (RPT) calculations applied to the quasi-linear growth of the large scale structure can be carried on in presence of primordial non-Gaussian (PNG) initial conditions. It is explicitly demonstrated that the series reordering scheme proposed in Bernardeau, Crocce and Scoccimarro (2008) is preserved for non-Gaussian initial conditions. This scheme applies to the power spectrum and higher order spectra and is based on a reorganization of the contributing terms into sum of products of multi-point propagators. In case of PNG new contributing terms appear, the importance of which is discussed in the context of current PNG models. The properties of the building blocks of such resummation schemes, the multi-point propagators, are then investigated. It is first remarked that their expressions are left unchanged at one-loop order irrespectively of statistical properties of the initial field. We furthermore show that the high-momentum limit of each of these propagators can be explicitly computed even for arbitrary initial conditions. They are found to be damped by an exponential cutoff whose expression is directly related to the moment generating function of the one-dimensional displacement field. This extends what had been established for multi-point propagators for Gaussian initial conditions. Numerical forms of the cut-off are shown for the so-called local model of PNG.

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I. INTRODUCTION

With the advent of precision measurements of the large-scale structure properties of the universe (e.g. [1–3]), it became important to develop semi-analytic tools that allow to accurately compute the large scale statistical properties of the cosmic fields for any cosmological model. Indeed, it has been realized that the simple linear theory is too crude for the precision one wishes to attain. This is the case for instance for the precise determination of the power spectrum at the scales of the baryonic acoustic oscillations (see for instance [4–6]).

The Standard Perturbation Theory approach (see [7]) is however not very efficient in producing well behaved next-to-leading order terms to the spectra or bispectra. It leads to perturbative series that have poor convergence properties (see [8] for more insights). Various resummation schemes have been proposed over the last few years to overcome those difficulties while aiming at providing tools for performing such calculations in a systematic way [8–13]. We will be interested here more particularly in the so called Renormalized Perturbation Theory (RPT) approach developed initially in [8] and [13] where rewritings of the resummation series are proposed. Whereas RPT as presented in [8] makes use of the one-point propagator only, in [13] it was introduced the concept of multi-point propagators that allow an alternative scheme for the computation of spectra and bispectra.

Such developments however, have been so far limited to cases corresponding to Gaussian initial conditions (but see [14] for a scheme that numerically solves the truncated integro-differential equations governing the dynamics in presence of PNG) and it should be stressed that this is not only for convenience. In a diagrammatic representation of the dynamics, as presented for instance in [7] or in [8] the assumptions on the statistical properties of the initial field are indeed crucial. At the same time, new ideas to constrain the non-Gaussian properties of the initial metric fluctuations from observations of the large-scale structure of the universe have emerged (see for instance the recent reviews [15, 16] and [17]). They suggest that future redshift surveys could be our best opportunity to probe a departure from Gaussian initial conditions. This is in fact our main motivation for the investigations presented in this paper, where we extend the schemes and results first introduced in [13] to the case of primordial non-Gaussianities (PNG). The findings presented here are rather general and, except for some numerical illustrations, are valid for any types of non-Gaussian initial conditions.

The paper is organized as follows. In Sec. II we recall the equations of motion for gravitational instabilities of a cosmic fluid. In Sec. III we extend the expansion of power spectrum and bispectrum in terms of multi-point propagators to the case of arbitrary initial statistics. In Sec IV we compute explicitly the multi-point propagators in the large- k limit for non Gaussian initial conditions. Lastly, in Sec. V we study the concrete example of primordial non Gaussianities of the local type. Our concluding remarks are presented in Sec. VI.

II. THE EQUATIONS OF MOTION

We are interested here in the development of cosmological instabilities in a cosmological dust fluid. In general the dynamical evolution of such a fluid can be described with the Vlasov equation. As usual we restrict our investigations to the regime where multi-flow regions play a negligible role. In the one flow limit, the equations of motion then take the form of a set of three coupled equations relating the local density contrast, the peculiar velocity field and the gravitational potential (see [7]).

At *linear order* these equations can easily be solved for an arbitrary background cosmology. One generically finds a growing solution and a decaying solution in the density contrast and peculiar velocity fields satisfying,

$$\frac{\partial}{\partial \tau} \delta(\mathbf{k}, \tau) + \theta(\mathbf{k}, \tau) = 0, \quad (1)$$

$$\frac{1}{\mathcal{H}} \frac{\partial}{\partial \tau} \frac{\theta(\mathbf{k}, \tau)}{\mathcal{H}} + \left(1 + \frac{1}{\mathcal{H}^2} \frac{\partial \mathcal{H}}{\partial \tau}\right) \frac{\theta(\mathbf{k}, \tau)}{\mathcal{H}} + \frac{3}{2} \Omega_m(\tau) \delta(\mathbf{k}, \tau) = 0, \quad (2)$$

with $\delta(\mathbf{k}, \tau)$ and $\theta(\mathbf{k}, \tau)$ being respectively the Fourier transforms of the density contrast, $\delta(\mathbf{x}, \tau) = \rho(\mathbf{x})/\bar{\rho} - 1$, and of the peculiar velocity divergence, $\theta \equiv \nabla \cdot \mathbf{v}$. In Eqs. (1,2), $\Omega_m(\tau)$ is the matter density and $\mathcal{H} \equiv d \ln a / d\tau$ is the conformal expansion rate with $a(\tau)$ the cosmological scale factor and τ the conformal time. If one denotes $D_+(\tau)$ the growing mode solution of this system and $f_+(\tau)$ its logarithmic derivative with respect to the expansion then,

$$\delta(\mathbf{k}, \eta) = D_+(\eta) \delta_0(\mathbf{k}), \quad \theta(\mathbf{k}, \eta) / \mathcal{H} = -f_+(\eta) D_+(\eta) \delta_0(\mathbf{k}), \quad (3)$$

is the solution for the growing mode and

$$\delta(\mathbf{k}, \eta) = D_-(\eta) \delta_0(\mathbf{k}), \quad \theta(\mathbf{k}, \eta) / \mathcal{H} = -f_-(\eta) D_-(\eta) \delta_0(\mathbf{k}), \quad (4)$$

for the decaying. We can note that in the Einstein-de Sitter limit we have $f_+ = 1$ and $f_- = -3/2$, leading to $D_+ = a$ and $D_- = a^{-3/2}$. However, in the more generic case of arbitrary matter and dark-energy content f_+ and f_- do not admit an analytic solution [7].

Following [18], the equations of motion can be written in a compact form with the use of the two component quantity $\Psi_i(\mathbf{k}, \tau)$, defined as

$$\Psi_i(\mathbf{k}, \tau) \equiv \left(\delta(\mathbf{k}, \tau), -\frac{1}{f_+(\tau)\mathcal{H}} \theta(\mathbf{k}, \tau) \right), \quad (5)$$

where the index $i = 1, 2$ selects the density or velocity components and which makes explicit use of the growing solution. It is then convenient to re-express the time-dependence in terms of the growing solution and in the following we will use the time variable η defined as

$$\eta = \log D_+(\eta), \quad (6)$$

assuming the growing factor set to unity at initial time. Then the *fully nonlinear* equations in Fourier space read [7] (we henceforth use the convention that repeated Fourier arguments are integrated over),

$$\frac{\partial}{\partial \eta} \Psi_i(\mathbf{k}, \eta) + \Omega_{ij}(\eta) \Psi_j(\mathbf{k}, \eta) = \gamma_{ijk}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) \Psi_j(\mathbf{k}_1, \eta) \Psi_k(\mathbf{k}_2, \eta), \quad (7)$$

where

$$\Omega_{ij}(\eta) \equiv \begin{bmatrix} 0 & -1 \\ -\frac{3}{2} \frac{\Omega_m}{f_+^2} & \frac{3}{2} \frac{\Omega_m}{f_+^2} - 1 \end{bmatrix}, \quad (8)$$

and the *symmetrized vertex* matrix γ_{ijk} describes the non linear interactions between different Fourier modes. Its components are given by

$$\begin{aligned} \gamma_{222}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) &= \delta_D(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2) \frac{|\mathbf{k}_1 + \mathbf{k}_2|^2 (\mathbf{k}_1 \cdot \mathbf{k}_2)}{2k_1^2 k_2^2}, \\ \gamma_{121}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) &= \delta_D(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2) \frac{(\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{k}_1}{2k_1^2}, \end{aligned} \quad (9)$$

with $\gamma_{ijk}(\mathbf{k}, \mathbf{k}_a, \mathbf{k}_b) = \gamma_{ijk}(\mathbf{k}, \mathbf{k}_b, \mathbf{k}_a)$, and $\gamma = 0$ otherwise, where δ_D denotes the Dirac delta distribution. The matrix γ_{ijk} is independent on time (and on the background evolution) and encodes all the non-linear couplings of the system. The formal integral solution to Eq. (7) is given by (see [8, 18, 19] for a detailed derivation)

$$\Psi_i(\mathbf{k}, \eta) = g_{ij}(\eta) \phi_j(\mathbf{k}) + \int_0^\eta d\eta' g_{ij}(\eta, \eta') \gamma_{jkl}^{(s)}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) \Psi_k(\mathbf{k}_1, \eta') \Psi_l(\mathbf{k}_2, \eta'), \quad (10)$$

where $\phi_i(\mathbf{k}) \equiv \Psi_i(\mathbf{k}, \eta = 0)$ denotes the initial conditions, set when the growth factor $D_+ = 1$ and where $g_{ij}(\eta)$ is the *linear propagator*, that is the Green's function of the linearized version of Eq. (7) and describes the standard linear evolution of the density and velocity fields from their initial state. It is such that $g_{ab}(\eta, \eta') = 0$ for $\eta < \eta'$ due to causality, and $g_{ab}(\eta, \eta') \rightarrow \delta_{ab}$ as $\eta - \eta' \rightarrow 0^+$. It naturally encapsulates the linear solutions growth and reads

$$g_{ij}(\eta, 0) = \frac{D_+(\eta)}{5} \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix} - \frac{D_-(\eta)}{5} \begin{bmatrix} -2 & 2 \\ -2\frac{f_-(\eta)}{f_+(\eta)} & 2\frac{f_-(\eta)}{f_+(\eta)} \end{bmatrix}, \quad (11)$$

for $\eta \geq 0$ assuming the initial conditions (for $a = 1$) are set at a time when the universe is very close to Einstein-de Sitter. We note that growing mode initial conditions correspond to setting $\phi_1(\mathbf{k}) = \phi_2(\mathbf{k})$, in such a way that the second term in Eq. (11) does not intervene. The general expression (see [11]) of the propagator $g_{ij}(\eta, \eta')$ is obtained from the property

$$g_{ij}(\eta, \eta') g_{jk}(\eta', 0) = g_{ik}(\eta, 0), \quad (12)$$

so that

$$g_{ij}(\eta, \eta') = g_{ik}(\eta, 0) g_{kj}^{-1}(\eta', 0). \quad (13)$$

We can remark that for a Einstein-de Sitter background $g_{ij}(\eta, \eta') = g_{ij}(\eta - \eta', 0)$.

III. STATISTICS WITH NON-GAUSSIAN INITIAL CONDITIONS

A. The Γ -expansion

We are interested in the statistical properties of the density and velocity divergence fields, in particular in the construction of their power spectra and higher-order correlators. The (equal time) power spectra P_{ij} are defined as

$$\langle \Psi_i(\mathbf{k}) \Psi_j(\mathbf{k}') \rangle = \delta_D(\mathbf{k} + \mathbf{k}') P_{ij}(k), \quad (14)$$

and we want to reconstruct them from the expression of $\Psi_i(\mathbf{k})$ in terms of the initial density field $\phi_j(\mathbf{k})$, the correlation properties of which are assumed to be known. The aim of this paper is precisely to explore the effects of dropping the assumption that the initial conditions are Gaussian distributed.

A perturbative solution to Eq. (10) can be obtained by expanding the fields in terms of the initial ones,

$$\Psi_i(\mathbf{k}, \eta) = \sum_{n=1}^{\infty} \Psi_i^{(n)}(\mathbf{k}, \eta), \quad (15)$$

such that,

$$\Psi_i^{(n)}(\mathbf{q}, \eta) = \int d^3\mathbf{q}_1 \dots d^3\mathbf{q}_n \delta_D(\mathbf{q} - \mathbf{q}_{1\dots n}) \mathcal{F}_{ij_1 j_2 \dots j_n}^{(n)}(\mathbf{q}_1, \dots, \mathbf{q}_n; \eta) \phi_{j_1}(\mathbf{q}_1) \dots \phi_{j_n}(\mathbf{q}_n), \quad (16)$$

where we adopt the notation $\mathbf{q}_{1\dots n} \equiv \mathbf{q}_1 + \dots + \mathbf{q}_n$ for vectors sums and where $\mathcal{F}^{(n)}$ are fully symmetric functions of the wave-vectors that can be obtained recursively in terms of g_{ij} and γ_{ijk} [7]. Note that these functions have a non-trivial time-dependence because they also include sub-leading terms in η . Their fastest growing contribution is of course given by the well known $\{F_n, G_n\}$ kernels in PT (assuming growing mode initial conditions) whose dependence on time we recall here for an Einstein-de Sitter background,

$$\mathcal{F}_i^{(n)} = \exp(n\eta) \{F_n(\mathbf{q}_1, \dots, \mathbf{q}_n), G_n(\mathbf{q}_1, \dots, \mathbf{q}_n)\}, \quad (17)$$

for $i = 1, 2$ (density or velocity divergence fields respectively).

A formal expression for $P_{ij}(k)$ can be written using the expansion in Eq. (15) as,

$$\delta_D(\mathbf{k}_1 + \mathbf{k}_2) P_{ij}(k_1, \eta) = \sum_{n_1, n_2} \langle \Psi_i^{(n_1)}(\mathbf{k}_1, \eta) \Psi_j^{(n_2)}(\mathbf{k}_2, \eta) \rangle. \quad (18)$$

For simplicity in what follows we will simply drop the component indices i and j but they are implicit. Then, for a given choice of indices n_1 and n_2 one has to compute the ensemble average of $n_1 + n_2$ factors $\phi(\mathbf{q}_i)$, following the field expansion in Eq. (16). The ensemble average of $\phi(\mathbf{q}_1) \dots \phi(\mathbf{q}_{n_1}) \phi(\mathbf{q}_{n_1+1}) \dots \phi(\mathbf{q}_{n_1+n_2})$ is given by the sum of product of cumulants of all set of subsets of $\{\phi(\mathbf{q}_1) \dots \phi(\mathbf{q}_{n_1}) \phi(\mathbf{q}_{n_1+1}) \dots \phi(\mathbf{q}_{n_1+n_2})\}$ that form a partition. Let us be more precise. Let us define \mathcal{P}_n the set of partitions of a set of n variables¹. Its elements \mathcal{S}_i are lists of subsets, s_i , and each element of s_i is an index in the $1 \dots n$ range. As a result,

$$\langle \phi(\mathbf{q}_1) \dots \phi(\mathbf{q}_{n_1}) \phi(\mathbf{q}_{n_1+1}) \dots \phi(\mathbf{q}_{n_1+n_2}) \rangle = \sum_{\mathcal{S}_i \in \mathcal{P}_{n_1+n_2}} \prod_{s_i \in \mathcal{S}_i} \langle \prod_{i \in s_i} \phi(\mathbf{q}_i) \rangle_c, \quad (19)$$

where $\langle \dots \rangle_c$ are the cumulants. Contrary to the case of Gaussian initial conditions, there exist cumulants that involve more than two variables. We can still assume though that there are no singleton although this hypothesis does not change the end of the calculation.

The idea is now to sort the elements of $\mathcal{P}_{n_1+n_2}$. Each element \mathcal{S}_i of $\mathcal{P}_{n_1+n_2}$ defines four cardinal numbers; the numbers q_1 and q_2 of points in subsets that are entirely within the first n_1 points or the last n_2 points and the numbers $p_1 = n_1 - q_1$ and $p_2 = n_2 - q_2$ of points within the first n_1 or last n_2 that are in parts that are neither a subset of $\{1, \dots, n_1\}$ nor of $\{n_1 + 1, \dots, n_2\}$. The resulting value of the partition contribution to the moment does not depend on which of the p_1 or p_2 points is thus chosen because the \mathcal{F}^n functions are fully symmetric in their arguments. Up to symmetry factors $\binom{n_1}{p_1} \binom{n_2}{p_2}$, it is then possible to assume that the p_1 points correspond to the first ones of n_1 and similarly for p_2 .

Let us now define $\mathcal{P}_{q_1, p_1, q_2, p_2}$ as a subset of $\mathcal{P}_{n_1+n_2}$, e.g. the set of partitions in $\mathcal{P}_{n_1+n_2}$ with fixed values of q_1, p_1, q_2 and p_2 that are formed from the union of an element of \mathcal{P}_{q_1} , an element of \mathcal{P}_{q_2} and an element of \mathcal{P}_{p_1, p_2}^X . The latter is defined as the set of all partitions of $\{1, \dots, p_1 + p_2\}$ that do not contain subsets contained entirely within the first n_1 points or within the last n_2 . In Fig. 1 we give an explicit example of this partitioning scheme where an example of two different partitions having the same value are presented.

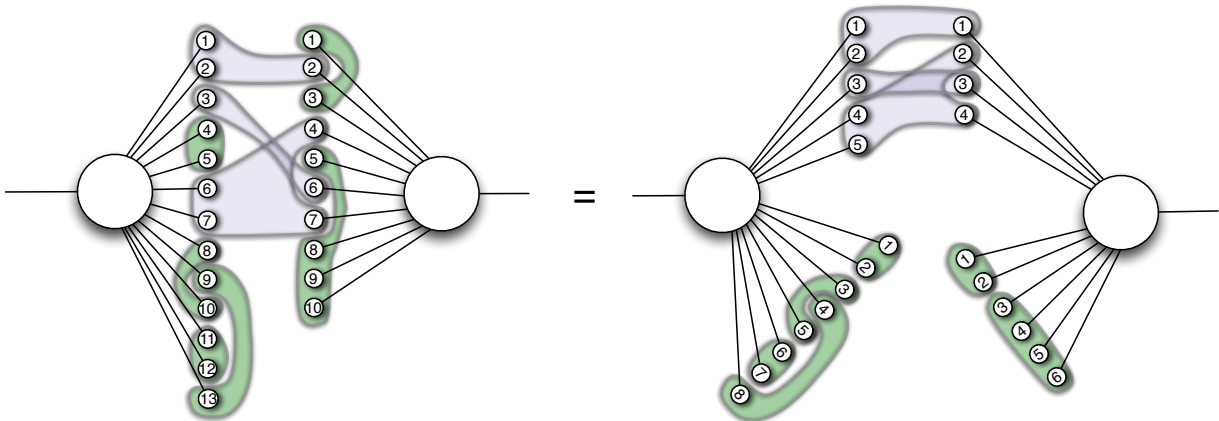


FIG. 1: Example of partitions of $n_1 + n_2$ points (here $n_1 = 13$ and $n_2 = 10$). On each ensemble the elements have been numbered. For this choice of partitions there are 5 elements out of 13 that are in subsets that span both column and 4 out of 10; therefore $p_1 = 5$, $q_1 = 8$ and $p_2 = 4$, $q_2 = 6$. The left hand side shows a generic partition of that kind. The right panel shows a partition which is identical to the left one up to a renumbering of the indices. These two contributing diagrams take the same values. When the order of the indices in each subset is preserved, the number of such contributing terms (e.g. the number of diagrams that lead to the same right hand side figure) is $\binom{p_1+q_1}{p_1} \binom{p_2+q_2}{p_2}$. The summation over all possible partitions can then be restricted to those of the type of the left hand side with a weight given by the aforementioned symmetry factor. Such terms are defined by three new partitions that are elements respectively of \mathcal{P}_{q_1} , \mathcal{P}_{q_2} and \mathcal{P}_{p_1, p_2}^X (see text for definitions).

¹ There is no simple expression for the number of such partitions. Its number is called the Bell number, B_n , after E.T. Bell, [20].

As a result, the power spectrum in Eq. (18) reads

$$\delta_D(\mathbf{k}_{12})P_{ij}(k_1, \eta) = \sum_{n_1, n_2} \int d^3 \mathbf{q}_1 \dots d^3 \mathbf{q}_{n_1+n_2} \delta_D(\mathbf{k}_1 - \mathbf{q}_{1\dots n_1}) \delta_D(\mathbf{k}_2 - \mathbf{q}_{n_1+1\dots n_1+n_2}) \times \mathcal{F}^{(n_1)}(\mathbf{q}_1, \dots, \mathbf{q}_{n_1}) \mathcal{F}^{(n_2)}(\mathbf{q}_{n_1+1}, \dots, \mathbf{q}_{n_1+n_2}) \sum_{\mathcal{S}_i \in \mathcal{P}_{n_1+n_2}} \prod_{s_i \in \mathcal{S}_i} \langle \prod_{i \in s_i} \phi(\mathbf{q}_i) \rangle_c, \quad (20)$$

with

$$\begin{aligned} \sum_{\mathcal{S}_i \in \mathcal{P}_{n_1+n_2}} \prod_{s_i \in \mathcal{S}_i} \langle \prod_{i \in s_i} \phi(\mathbf{q}_i) \rangle_c &= \sum_{q_1, p_1, q_2, p_2} \binom{p_1+q_1}{p_1} \binom{p_2+q_2}{p_2} \sum_{\mathcal{S}_i \in \mathcal{P}_{q_1, p_1, q_2, p_2}} \prod_{s_i \in \mathcal{S}_i} \langle \prod_{i \in s_i} \phi(\mathbf{q}_i) \rangle_c \\ &= \sum_{q_1, p_1, q_2, p_2} \binom{p_1+q_1}{p_1} \binom{p_2+q_2}{p_2} \left(\sum_{\mathcal{R}_i \in \mathcal{P}_{q_1}} \prod_{r_i \in \mathcal{R}_i} \langle \prod_{i \in r_i} \phi(\mathbf{q}_{p_1+i}) \rangle_c \right) \times \\ &\quad \left(\sum_{\mathcal{T}_i \in \mathcal{P}_{q_2}} \prod_{t_i \in \mathcal{T}_i} \langle \prod_{i \in t_i} \phi(\mathbf{q}_{n_1+p_2+i}) \rangle_c \right) \left(\sum_{\mathcal{U}_i \in \mathcal{P}_{p_1, p_2}^X} \prod_{u_i \in \mathcal{U}_i} \langle \prod_{i \in u_i} \phi(\mathbf{r}_i) \rangle_c \right), \end{aligned} \quad (21)$$

where we have introduced the set of wave numbers $\{\mathbf{r}_i\}$ that corresponds to reindexation of some of the \mathbf{q}_i ,

$$\{\mathbf{r}_1, \dots, \mathbf{r}_{q_1+q_2}\} \equiv \{\mathbf{q}_1, \dots, \mathbf{q}_{q_1}, \mathbf{q}_{n_1+1}, \dots, \mathbf{q}_{n_1+q_2}\}, \quad (22)$$

and where $\sum_{q_1, p_1, q_2, p_2}$ stands for $\sum_{q_1=0}^{n_1} \sum_{p_1=0}^{n_1-q_1} \sum_{q_2=0}^{n_2} \sum_{p_2=0}^{n_2-q_2}$. The crucial property is then that the first two parenthesis that appear in the last line of the previous expression depend only on q_1 or q_2 respectively. The sum over partitions can then be reorganized by summing for fixed values of p_1 and p_2 first (see Fig. 1, right panel). That is, by doing

$$\sum_{n \geq 0} \mathcal{F}^{(n)} \sum_{q=0}^n \sum_{p=0}^{n-q} \binom{n}{p} \rightarrow \sum_{p \geq 0} \sum_{q \geq 0} \binom{p+q}{q} \mathcal{F}^{(p+q)}$$

in each index 1 and 2 (after using that $n = p + q$) one can identify the function $\Gamma^{(p)}$ defined as,

$$\begin{aligned} \Gamma^{(p)}(\mathbf{q}_1, \dots, \mathbf{q}_p, \eta) &= \sum_{q=0}^{\infty} \binom{p+q}{q} \int d^3 \mathbf{q}_1 \dots d^3 \mathbf{q}_q \mathcal{F}^{(p+q)}(\mathbf{q}_1, \dots, \mathbf{q}_p, \mathbf{q}_{p+1}, \dots, \mathbf{q}_{p+q}; \eta) \\ &\quad \times \left(\sum_{\mathcal{R}_i \in \mathcal{P}_q} \prod_{r_i \in \mathcal{R}_i} \langle \prod_{i \in r_i} \phi(\mathbf{q}_{p+i}) \rangle_c \right), \end{aligned} \quad (23)$$

(here $i \in [1, q]$) which naturally extends to the case of arbitrary initial statistics the results for $\Gamma^{(p)}$ studied in [9, 13] for Gaussian initial conditions. In such case q takes only even values and the two point initial spectrum determines the partition \mathcal{P}_q , see for instance Eq. (15) in [9] and Eq. (21) in [13] for $\Gamma^{(1)}$ and $\Gamma^{(2)}$ respectively.

After inserting Eq. (23) back into Eq. (20) we arrive at,

$$\begin{aligned} \delta_D(\mathbf{k}_{12})P(k_1, \eta) &= \sum_{p_1, p_2} \int d^3 \mathbf{q}_1 \dots d^3 \mathbf{q}_{p_1} \int d^3 \mathbf{q}_{p_1+1} \dots d^3 \mathbf{q}_{p_1+p_2} \delta_D(\mathbf{k}_1 - \mathbf{q}_{1\dots p_1}) \delta_D(\mathbf{k}_2 - \mathbf{q}_{p_1+1\dots p_1+p_2}) \\ &\quad \Gamma^{(p_1)}(\mathbf{q}_1, \dots, \mathbf{q}_{p_1}, \eta) \Gamma^{(p_2)}(\mathbf{q}_{p_1+1}, \dots, \mathbf{q}_{p_1+p_2}, \eta) \sum_{\mathcal{U}_i \in \mathcal{P}_{p_1, p_2}^X} \prod_{u_i \in \mathcal{U}_i} \langle \prod_{i \in u_i} \phi(\mathbf{q}_i) \rangle_c, \end{aligned} \quad (24)$$

which is the final expression for the Γ -expansion of the power spectrum. It naturally extends the Γ -expansion obtained in [13] for Gaussian initial conditions to an arbitrary initial statistics. Note that contrary to that case the sum is not restricted to $p_1 = p_2$. As a consequence there is in general no guarantee that all terms of this sum are positive. In addition, note that from Eq. (20) to Eq. (24) $\delta_D(\mathbf{k} - \mathbf{q}_{1\dots p})$ could automatically be factorized out since $\langle \prod_{i \in r_i} \phi(\mathbf{q}_{p+i}) \rangle_c$ are all proportional to $\delta_D(\mathbf{q}_{p+1\dots p+q})$.

In Fig. 2 we show this expansion diagrammatically up to one-loop terms. Explicitly, these diagrams corresponds to

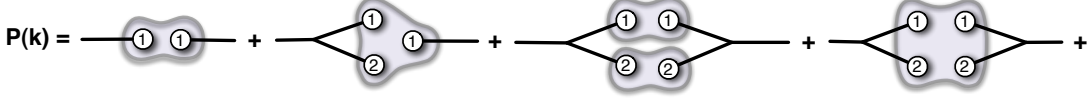


FIG. 2: Γ -expansion for the power spectrum up to one-loop. The shaded regions indicate the initial (or “primordial”) statistics corresponding to each diagram, that is: one initial power spectra (2-points) in the left most diagram, one initial bispectrum in the center-left one, two power spectra in the center-right and one initial trispectrum in the right most figure.

$$\begin{aligned}
P(k) = & [\Gamma^{(1)}(k)]^2 P_0(k) + 2\Gamma^{(1)}(k) \int d^3q \Gamma^{(2)}(\mathbf{q}, \mathbf{k} - \mathbf{q}) B_0(\mathbf{q}, \mathbf{k} - \mathbf{q}, -\mathbf{k}) + \\
& + 2 \int d^3q [\Gamma^{(2)}(\mathbf{q}, \mathbf{k} - \mathbf{q})]^2 P_0(\mathbf{k} - \mathbf{q}) P_0(\mathbf{q}) + \\
& + \int d^3q_1 d^3q_2 \Gamma^{(2)}(\mathbf{q}_1, \mathbf{k} - \mathbf{q}_1) \Gamma^{(2)}(\mathbf{q}_2, -\mathbf{k} - \mathbf{q}_2) T_0(\mathbf{q}_1, \mathbf{k} - \mathbf{q}_1, \mathbf{q}_2, -\mathbf{k} - \mathbf{q}_2). \quad (25)
\end{aligned}$$

We stress that each term in the Γ -expansion above involves an infinite number of perturbative contributions in δ as the $\Gamma^{(p)}$ functions correspond to a full resummation of the propagator *and* the vertex in the language of [4, 10].

The resummation leading to the Γ -expansion for $P(k)$ can obviously be extended for higher order correlators: higher order multi-point spectra can be obtained by gluing together $\Gamma^{(p)}$ functions multiplied by the proper cumulant. In the case of 3-point statistics (bispectrum) there are 3 “external” legs and the equivalent of Eq. (18) for the bispectrum $B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ now runs over indices n_1, n_2 and n_3 . Following the same line of reasoning as above, the total set of partitions $\mathcal{P}_{n_1+n_2+n_3}$ is broken up into three subsets \mathcal{P}_{q_i} contained fully with n_i and one with “cross” elements $\mathcal{P}_{p_1, p_2, p_3}^X$. The bispectrum is then a sum over the product of elements of $\mathcal{P}_{p_1, p_2, p_3}^X$ times $\Gamma^{(p_1)} \Gamma^{(p_2)} \Gamma^{(p_3)}$, as in Eq. (24). Figure 3 shows all the contributions in the multi-point propagator expansion of $B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ up to one-loop diagrams. For PNG models satisfying the hierarchical scaling $B_0 \sim P_0^2, T_0 \sim P_0^3$ and so on these are all the terms up to $\mathcal{O}(P_0^3)$. These diagrams yield the following explicit expressions,

$$\begin{aligned}
B(k_1, k_2, k_3) = & 2\Gamma^{(2)}(\mathbf{k}_1, \mathbf{k}_2) \Gamma^{(1)}(k_1) \Gamma^{(1)}(k_2) P_0(k_1) P_0(k_2) + 2 \text{ perm.} + \\
& \Gamma^{(1)}(k_1) \Gamma^{(1)}(k_2) \Gamma^{(1)}(k_3) B_0(k_1, k_2, k_3) + \\
& 8 \int d^3q \Gamma^{(2)}(-\mathbf{q}, \mathbf{q} + \mathbf{k}_1) \Gamma^{(2)}(-\mathbf{q} - \mathbf{k}_1, \mathbf{q} - \mathbf{k}_2) \Gamma^{(2)}(\mathbf{k}_2 - \mathbf{q}, \mathbf{q}) P_0(q) P_0(|\mathbf{k}_1 + \mathbf{q}|) P_0(|\mathbf{k}_2 - \mathbf{q}|) + \\
& 6 P_0(k_1) \Gamma^{(1)}(k_1) \int d^3q \Gamma^{(3)}(\mathbf{k}_1, \mathbf{k}_2 - \mathbf{q}, \mathbf{q}) \Gamma^{(2)}(\mathbf{k}_2 - \mathbf{q}, \mathbf{q}) P_0(q) P_0(|\mathbf{k}_2 - \mathbf{q}|) + 5 \text{ perm.} \\
& 2 P_0(k_1) \Gamma^{(1)}(k_1) \left[\Gamma^{(2)}(\mathbf{k}_1, \mathbf{k}_2) \int d^3q \Gamma^{(2)}(\mathbf{q}, \mathbf{k}_2 - \mathbf{q}) B_0(k_2, q, |\mathbf{k}_2 - \mathbf{q}|) + (k_2 \leftrightarrow k_3) \right] + 2 \text{ perm.} \\
& 4 \Gamma^{(1)}(k_1) \int d^3q \Gamma^{(2)}(\mathbf{q}, \mathbf{k}_2 - \mathbf{q}) \Gamma^{(2)}(\mathbf{k}_1 + \mathbf{q}, \mathbf{k}_2 - \mathbf{q}) P_0(|\mathbf{k}_2 - \mathbf{q}|) B_0(k_1, q, |\mathbf{k}_1 + \mathbf{q}|) + 2 \text{ perm.} + \\
& 3 \Gamma^{(1)}(k_1) \Gamma^{(1)}(k_2) P_0(k_1) \int d^3q \Gamma^{(3)}(\mathbf{k}_1, \mathbf{q}, \mathbf{k}_2 - \mathbf{q}) B_0(k_2, q, |\mathbf{k}_2 - \mathbf{q}|) + (k_1 \leftrightarrow k_2) + 2 \text{ perm.} + \\
& \Gamma^{(1)}(k_1) \Gamma^{(1)}(k_2) \int d^3q \Gamma^{(2)}(\mathbf{q}, \mathbf{k}_3 - \mathbf{q}) T_0(\mathbf{k}_1, \mathbf{k}_2, \mathbf{q}, \mathbf{k}_3 - \mathbf{q}) + 2 \text{ perm.}, \quad (26)
\end{aligned}$$

where in the r.h.s. we implicitly assumed $\mathbf{k}_3 = -\mathbf{k}_1 - \mathbf{k}_2$ (and so the l.h.s only depends on the magnitudes of the wave-vectors). The first two contributions are “tree level”. The next two correspond to the gravitationally induced non-Gaussianity, while the last four arise from PNG.

Comparing with the standard perturbative approach up to one-loop given in [21] we see that his terms $B_{112}^I, B_{111}, B_{222}^I, B_{123}^I, B_{122}^I, B_{122}^{II}, B_{113}^I, B_{112}^{II}$ correspond to those in Eq. (26) when the $\Gamma^{(p)}$ functions are taken at their lowest or “bare” order in Eq. (23) (i.e. prior to any resummation). In turn, his B_{123}^I (B_{113}^I) correspond to the next-to-leading order term in the resummation of $\Gamma^{(1)}$ in the first (second) line of Eq. (26). Lastly, his B_{114}^I resums into $\Gamma^{(2)}$ of the first line.

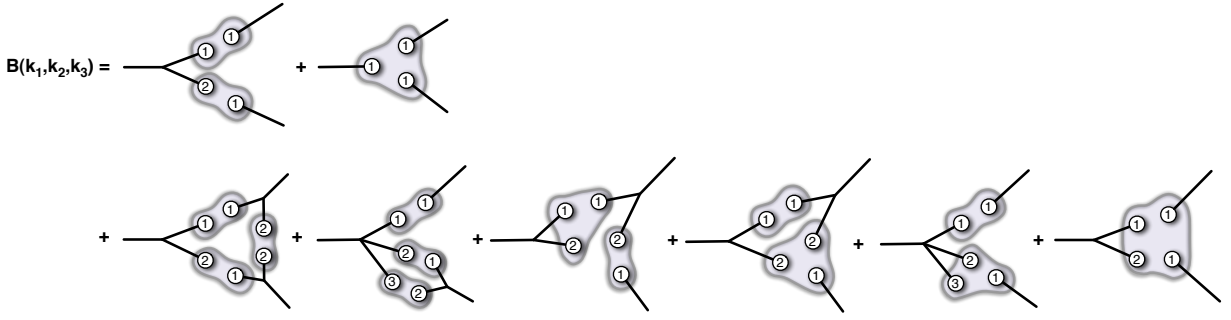


FIG. 3: Γ -expansion for the bispectrum up to one-loop. The first two contributions are proportional to P_0^2 (“tree level”), the remaining ones to P_0^3 , assuming that the bispectrum B_0 and the trispectrum T_0 scale respectively as P_0^2 and P_0^3 . The meaning of shaded regions is described in the caption to Fig. 2 (see text for details).

B. Multi-point propagators

In the previous section we showed how the $\Gamma^{(p)}$ functions in Eq. (23) serve as basic building blocks out of which one can construct series expressions for the polyspectra. In [13] these functions were further identified as n -point propagators since they can be obtained by functional differentiation of the final fields with respect to the initial ones. For PNG the $\Gamma^{(p)}$ functions are still equal to the ensemble average of the p -order functional derivative of Ψ with respect to ϕ , that is

$$\frac{1}{p!} \left\langle \frac{\partial^p \Psi_i(\mathbf{k}, \eta)}{\partial \phi_{j_1}(\mathbf{q}_1) \dots \partial \phi_{j_p}(\mathbf{q}_p)} \right\rangle \equiv \delta_{\mathbb{D}}(\mathbf{k} - \mathbf{q}_{1\dots p}) \Gamma_{ij_1\dots j_p}^{(p)}(\mathbf{q}_1, \dots, \mathbf{q}_p, \eta), \quad (27)$$

where we have re-introduced the doublet indices for clarity. This can be easily checked starting from Eq. (16) and getting to Eq. (23). Notice that, contrary to the standard PT kernels in Eq. (17), the multi-point propagators depend on the statistics of the initial fields.

C. Multi-point propagators and correlation functions

Within the framework developed so far, it is clear that a precise description of the large scale clustering can be achieved provided with a good understanding of the $\Gamma^{(p)}$ functions. For Gaussian initial condition it has been observed that the Γ -functions can be expressed in terms of correlation functions between initial and final fields. For instance, the nonlinear propagator $\Gamma^{(1)}$ satisfies [9],

$$\langle \Psi_i(\mathbf{k}, \eta) \phi_j(\mathbf{k}') \rangle = \delta_{\mathbb{D}}(\mathbf{k} + \mathbf{k}') \Gamma_{il}^{(1)}(k, \eta) P_{0,lj}(k), \quad (28)$$

where P_0 is the power spectrum of the initial density field and $\Gamma^{(1)}$ is defined formally through functional differentiation, as in Eq. (27). An analogous expression is given in [13] for $\Gamma^{(2)}$. These relations played a key role for the Gaussian case since they allowed the measurement of the fully nonlinear $\Gamma^{(1)}$ and $\Gamma^{(2)}$ function in N-body simulations [9, 13].

These simple expressions however, are no longer valid for non-Gaussian initial conditions. In this case the cross-correlation function between Ψ and ϕ can be expanded as follows,

$$\langle \Psi(\mathbf{k}, \eta) \phi(\mathbf{k}') \rangle = \sum_p \int d^3 \mathbf{q}_1 \dots d^3 \mathbf{q}_p \delta_{\mathbb{D}}(\mathbf{k} - \mathbf{q}_{1\dots p}) \Gamma^{(p)}(\mathbf{q}_1, \dots, \mathbf{q}_p, \eta) \langle \phi(\mathbf{q}_1) \dots \phi(\mathbf{q}_p) \phi(\mathbf{k}') \rangle_c. \quad (29)$$

This is actually a peculiar case of the general expression as obtained in Eq. (24) where the sum is now restricted to $p_2 = 1$. This expression can then be written in terms of the spectra $P_0(k)$, bispectra $B_0(\mathbf{k}_1, \mathbf{k}_2)$ and so on, of the initial fields times the corresponding $\Gamma^{(p)}$ function from Eq. (23). It leads to,

$$\langle \Psi_i(\mathbf{k}, \eta) \phi_j(\mathbf{k}') \rangle = \delta_{\mathbb{D}}(\mathbf{k} - \mathbf{k}') \left[\Gamma_{il}^{(1)}(k, \eta) P_{0,lj}(k) + \int d^3 \mathbf{q}_1 d^3 \mathbf{q}_2 \delta_{\mathbb{D}}(\mathbf{k} - \mathbf{q}_{12}) \Gamma_{ilm}^{(2)}(\mathbf{q}_1, \mathbf{q}_2, \eta) B_{0,lmj}(\mathbf{q}_1, \mathbf{q}_2) + \dots \right] \quad (30)$$

The terms appearing in the previous equation are ordered in terms of importance for weakly non-Gaussian models such as the local [22–25] and equilateral [26] ones. Typically, the initial curvature bispectrum in these models satisfies the hierarchical² scaling $B_{\Phi}(k, k, k) \sim f_{NL} P_{\Phi}^2(k)$, where $P_{\Phi}(k)$ is the curvature power spectrum. The second term in Eq. (30) is thus sub-dominant by a factor $\sim f_{NL} D(z)$. Alternatively, notice that Eq. (30) resembles the expansion for $P(k, \eta)$ in Eq. (25). This expansion have been studied using standard perturbation theory to one-loop and the leading order induced by primordial non Gaussianity (i.e. the second term in the previous equation) introduces corrections of $\sim 1\% - 2\%$ for both local and equilateral models with $|f_{NL}| < 300$ [27].

Similarly the computation of $\langle \Psi_i(\mathbf{k}, \eta) \phi_j(\mathbf{k}_1) \phi_l(\mathbf{k}_2) \rangle$ will make intervene the whole range of initial spectra. Its leading contribution is

$$\begin{aligned} \langle \Psi_i(\mathbf{k}, \eta) \phi_j(\mathbf{k}_1) \phi_l(\mathbf{k}_2) \rangle = \delta_D(\mathbf{k} - \mathbf{k}_{12}) \left[\Gamma_{im}^{(1)}(k, \eta) B_{0,mjl}(\mathbf{k}_1, \mathbf{k}_2) \right. \\ \left. + 2 \Gamma_{imn}^{(2)}(\mathbf{k}_1, \mathbf{k}_2, \eta) P_{0,mj}(k_1) P_{0,nl}(0, k_2) + \dots \right]. \end{aligned} \quad (31)$$

The relative importance of the terms in this series can be studied for particular models but in general, for weakly non-Gaussian initial conditions, the first two terms are dominant over the remaining series (not shown). Again, this expansion resembles the Γ -expansion for $B(k, k_1, k_2)$ in Eq. (26). In turn, the bispectrum has been studied using one-loop perturbation theory by [21] for local and equilateral models, and additionally using N-body measurements in [28] for local models with $f_{NL} = \pm 100$. In these cases, the next-to-leading terms become increasingly important for $k \gtrsim 0.1 h^{-1} \text{Mpc}$ depending on the value of f_{NL} and redshift.

Therefore, if we neglect sub-leading terms it is still possible to estimate the Γ -functions from correlators. This is well justified for $\Gamma^{(1)}$, obtained from Eq. (30) as,

$$\delta_D(\mathbf{k} - \mathbf{k}_1) \Gamma^{(1)}(k, \eta) = \langle \Psi(\mathbf{k}, \eta) \phi(\mathbf{k}_1) \rangle / P_0(k). \quad (32)$$

Inserting this into Eq. (31) one recovers $\Gamma^{(2)}$ as,

$$\delta_D(\mathbf{k} - \mathbf{k}_{12}) \Gamma^{(2)}(\mathbf{k}_1, \mathbf{k}_2, \eta) \simeq \frac{\langle \Psi(\mathbf{k}, \eta) \phi(\mathbf{k}_1) \phi(\mathbf{k}_2) \rangle}{2 P_0(k_1) P_0(k_2)} - \frac{B_0(\mathbf{k}_1, \mathbf{k}_2) \langle \Psi(\mathbf{k}, \eta) \phi(\mathbf{k}_1 + \mathbf{k}_2) \rangle}{2 P_0(k) P_0(k_1) P_0(k_2)}. \quad (33)$$

As expected, the first term is the same as for Gaussian initial statistics. One can now insert this back into Eq. (30) and recover the correction introduced by the initial higher-order correlators such as B_0 ,

$$\delta_D(\mathbf{k} - \mathbf{k}_1) \Gamma^{(1)}(k, \eta) = \frac{\langle \Psi(\mathbf{k}, \eta) \phi(\mathbf{k}_1) \rangle}{P_0(k)} + \frac{1}{2P_0(k)} \int d^3\mathbf{k}_1 d^3\mathbf{k}_2 \frac{\langle \Psi(\mathbf{k}, \eta) \phi(\mathbf{k}_1) \phi(\mathbf{k}_2) \rangle}{P_0(k_1) P_0(k_2)} B_0(\mathbf{k}_1, \mathbf{k}_2) + \mathcal{O}(B_0^2). \quad (34)$$

In most commonly studied models of primordial non Gaussianity [22, 26], $B_0 \sim f_{NL}$. The expression above is thus neglecting terms of order f_{NL}^2 . In addition, as we discuss in the next section, the cross-correlations such as that in Eq. (34) will drop to zero towards high momenta. This is an important property to take into account when it comes to compute the momentum integration that cannot be recovered using the known tree-level analytical expression for these correlators.

Expressions such as Eq. (33) and (34) give a concrete path to the actual measurements of the Γ functions that can certainly shed light into their full description. This is however beyond the scope of this paper and will be left for further work.

IV. PROPAGATORS IN THE LARGE- k LIMIT

A. The α -method

The multi-point propagators or Γ functions play an essential role in our formalism (as they did in [9, 13] for Gaussian initial conditions) and setting the ground for their adequate analytical description is thus one of the main goals of this paper. This task is however extremely difficult as it involves summing over infinite terms in Eq. (23) or adding up

² The extension of this scaling to the density spectra is however not straightforward because of the shape of the transfer function that relates the density and the potential at linear order.

an infinite number of diagrams, if we were to work with diagrammatic techniques as in [8, 13] (but see also [10, 11]). Nonetheless well established physical arguments were put forward in [9] to show that indeed, the leading order set of contributions can be resummed in the large- k regime where the coupling of Fourier modes simplifies (see below). The low- k regime, that allows perturbative calculations, could then be matched to the high- k asymptotic using simple physical arguments [9]. This resulted in a good agreement with propagator measurements in N-body simulations and allowed accurate descriptions of power spectrum [4] and bispectrum [13] at large scales.

In what follows we are interested in extending these results to the case of non-Gaussian initial conditions, computing of the Γ functions (defined as the ensemble averages of functional derivatives of ψ_i with respect to the initial field values ϕ_j) in the large- k limit. As shown in [9, 13] this is a regime where the other intervening modes \mathbf{q} in the momentum (or “loop”) integrals are such that $q \ll k_i$ for all “external” \mathbf{k}_i . The intervening modes are also assumed to be in the linear regime with initial condition set in the growing mode. This later condition amounts to resum only those diagrams that are maximally connected with the initial conditions (that is, with all mode-mode interactions happening along the “principal path” as introduced in [9, 13]) what gives the leading contribution after resummation.

Following these assumptions we then compute $\partial\Psi_i(\mathbf{k}, \eta)/\partial\phi_j(\mathbf{k}')$ from Eq. (10) with the prescription that in its r.h.s. one replaces one factor Ψ_k by its linear solution $g_{kl}(\eta', 0)\phi_l$. Furthermore ϕ_l is set to be the growing mode, that is equal to $\delta_0(\mathbf{q})u_l$ with $u_l = (1, 1)$. From Eq. (10), one then has

$$\frac{\partial\Psi_i(\mathbf{k}, \eta)}{\partial\phi_j(\mathbf{k}')} = g_{ij}(\eta, 0)\delta_{\mathbf{D}}(\mathbf{k} - \mathbf{k}') + 2 \int_0^\eta d\eta' D_+(\eta') g_{ik}(\eta, \eta') \gamma_{klm}^{(s)}(\mathbf{k}, \mathbf{q}, \mathbf{k}_1) u_l \delta_0(\mathbf{q}) \frac{\partial\Psi_m(\mathbf{k}_1, \eta')}{\partial\phi_j(\mathbf{k}')} , \quad (35)$$

where the vertex function should be further taken in the $q \ll k_1$ limit (and recall that repeated Fourier arguments are assumed to be integrated over). As noticed in [9] this considerably simplifies its expression and we are left with

$$\frac{\partial\Psi_i(\mathbf{k}, \eta)}{\partial\phi_j(\mathbf{k}')} = g_{ij}(\eta, 0)\delta_{\mathbf{D}}(\mathbf{k} - \mathbf{k}') + i\alpha(\mathbf{k}) \int_0^\eta d\eta' D_+(\eta') g_{ik}(\eta, \eta') \frac{\partial\Psi_k(\mathbf{k}, \eta')}{\partial\phi_j(\mathbf{k}')} , \quad (36)$$

where the intervening modes were all collected in a *single* quantity,

$$\alpha(\mathbf{k}) = -i \int d^3\mathbf{q} \frac{\mathbf{k}\cdot\mathbf{q}}{q^2} \delta_0(\mathbf{q}) , \quad (37)$$

where $\delta_0(\mathbf{q})$ is the initial density contrast.

This possibility of collecting all the intervening modes in a single random variable (or more generally a finite number of variables) is at the heart of what we call here the α -method. This method was described and extensively used in [29] (and was already sketched in [30]). This method is much more powerful than standard diagram evaluations and countings. It eventually allows to compute the resulting propagator shapes either when the initial statistics is non trivial as in the present work and/or when the dependence of the propagators is complicated (as in [29]).

Let us go back to this specific calculation. We note that for a given value of \mathbf{k} , $\alpha(\mathbf{k})$ is a real random variable. It is Gaussian distributed if the initial conditions are Gaussian distributed but we will not make this hypothesis here. In the previous system we have introduced the i factor to make $\alpha(\mathbf{k})$ real and of positive variance. Note that in all cases,

$$\langle\alpha^2(\mathbf{k})\rangle = \int d^3\mathbf{q} P_0(q) \frac{(\mathbf{q}\cdot\mathbf{k})^2}{q^4} = k^2\sigma_{\text{displ.}}^2 , \quad (38)$$

where $\sigma_{\text{displ.}}$ is the one-component *displacement* dispersion in linear theory.

We are then left with an equation evolution for $\partial\Psi_i(\mathbf{k}, \eta)/\partial\phi_j(\eta')$ that can be solved explicitly. If we indeed write

$$\frac{\partial\Psi_i(\mathbf{k}, \eta)}{\partial\phi_j(\mathbf{k}')} = \xi(\mathbf{k}, \eta) g_{ij}(\tau, 0) \delta_{\mathbf{D}}(\mathbf{k} - \mathbf{k}') , \quad (39)$$

taking advantage of Eq. (12), we are left with the ODE

$$\frac{\partial}{\partial\eta} \xi(\mathbf{k}, \eta) = i\alpha(\mathbf{k}) D_+(\tau) \xi(\mathbf{k}, \tau) , \quad \xi(\mathbf{k}, \eta = 0) = 1 , \quad (40)$$

which can be solved explicitly to give,

$$\frac{\partial\Psi_i(\mathbf{k}, \eta)}{\partial\phi_j(\mathbf{k}')} = \exp[i\alpha(\mathbf{k})(D_+(\eta) - 1)] g_{ij}(\eta, 0) \delta_{\mathbf{D}}(\mathbf{k} - \mathbf{k}') . \quad (41)$$

This is the first result of this section. We will see later its implication for the form of the propagators.

We can then proceed to the computation of higher order Γ functions from successive differentiations of Eq. (10) with respect to ϕ . The second order partial derivative induces two terms.

$$\begin{aligned} \frac{\partial^2 \Psi_i(\mathbf{k}, \eta)}{\partial \phi_j(\mathbf{k}_1) \partial \phi_k(\mathbf{k}_2)} &= \int_0^\eta d\eta' g_{il}(\eta, \eta') \gamma_{lmn}^{(s)}(\mathbf{k}, \mathbf{k}'_1, \mathbf{k}'_2) \frac{\partial \Psi_m(\mathbf{k}'_1, \eta')}{\partial \phi_j(\mathbf{k}_1)} \frac{\partial \Psi_n(\mathbf{k}'_2, \eta')}{\partial \phi_k(\mathbf{k}_2)} \\ &+ 2 \int_0^\eta d\eta' D_+(\eta') g_{il}(\eta, \eta') \gamma_{lmn}^{(s)}(\mathbf{k}, \mathbf{q}, \mathbf{k}') \phi_m(\mathbf{q}) \frac{\partial^2 \Psi_n(\mathbf{k}', \eta')}{\partial \phi_j(\mathbf{k}_1) \partial \phi_k(\mathbf{k}_2)}, \end{aligned} \quad (42)$$

which can once again be simplified in the high- k limit into,

$$\begin{aligned} \frac{\partial^2 \Psi_i(\mathbf{k}, \eta)}{\partial \phi_j(\mathbf{k}_1) \partial \phi_k(\mathbf{k}_2)} &= \int_0^\eta d\eta' g_{il}(\eta, \eta') \gamma_{lmn}^{(s)}(\mathbf{k}, \mathbf{k}'_1, \mathbf{k}'_2) \frac{\partial \Psi_m(\mathbf{k}'_1, \eta')}{\partial \phi_j(\mathbf{k}_1)} \frac{\partial \Psi_n(\mathbf{k}'_2, \eta')}{\partial \phi_k(\mathbf{k}_2)} \\ &+ i\alpha(\mathbf{k}) \int_0^\eta d\eta' D_+(\eta') g_{il}(\eta, \eta') \frac{\partial^2 \Psi_l(\mathbf{k}', \eta')}{\partial \phi_j(\mathbf{k}_1) \partial \phi_k(\mathbf{k}_2)}. \end{aligned} \quad (43)$$

It can easily be checked that the solution of Eq. (43) is

$$\frac{\partial^2 \Psi_i(\mathbf{k}, \eta)}{\partial \phi_j(\mathbf{k}_1) \partial \phi_k(\mathbf{k}_2)} = \exp[i\alpha(\mathbf{k})(D_+(\eta) - 1)] \int_0^\eta d\eta' g_{il}(\eta, \eta') \gamma_{lmn}^{(s)}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) g_{mj}(\eta', 0) g_{nk}(\eta', 0), \quad (44)$$

where the integral in the left hand part of this equation is the expression of $\partial^2 \Psi_i(\mathbf{k}, a) / \partial \phi_j(\mathbf{k}_1) \partial \phi_k(\mathbf{k}_2)$ for a vanishing value of α that is its tree order expression. Indeed, inserting this expression in the second term of the right hand side of Eq. (43) and using the result of Eq. (41), we are left with

$$\begin{aligned} \frac{\partial^2 \Psi_i(\mathbf{k}, \eta)}{\partial \phi_j(\mathbf{k}_1) \partial \phi_k(\mathbf{k}_2)} &= \int_0^\eta d\eta' g_{il}(\eta, \eta') \gamma_{lmn}^{(s)}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) \exp[i\alpha(\mathbf{k}_1 + \mathbf{k}_2)(D_+(\eta') - 1)] g_{mj}(\eta', 0) g_{nk}(\eta', 0) + \\ &i\alpha(\mathbf{k}) \int_{\eta''}^\eta d\eta' D_+(\eta') g_{il}(\eta, \eta') \exp[i\alpha(\mathbf{k})(D_+(\eta') - 1)] \int_0^{\eta''} d\eta'' g_{lq}(\eta', \eta'') \gamma_{qmn}^{(s)}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) g_{mj}(\eta'', 0) g_{nk}(\eta'', 0). \end{aligned} \quad (45)$$

Then remarking that

$$\begin{aligned} i\alpha(\mathbf{k}) \int_{\eta''}^\eta d\eta' D_+(\eta') g_{il}(\eta, \eta') \exp[i\alpha(\mathbf{k})(D_+(\eta') - 1)] g_{lq}(\eta', \eta'') = \\ g_{iq}(\eta, \eta'') \left\{ \exp[i\alpha(\mathbf{k})(D_+(\eta) - 1)] - \exp[i\alpha(\mathbf{k})(D_+(\eta') - 1)] \right\}, \end{aligned} \quad (46)$$

and that $\alpha(\mathbf{k})$ is a linear function of its argument, the two terms of the right hand side of Eq. (45) simplify and recombine into the expression of Eq. (44).

This result can naturally be extended to any order³ as it can be established recursively by successive use of the relation (46). More precisely, we have

$$\frac{\partial^p \Psi_i(\mathbf{k}, \eta)}{\partial \phi_{j_1}(\mathbf{k}_1) \dots \partial \phi_{j_p}(\mathbf{k}_p)} = \exp[i\alpha(\mathbf{k})(D_+ - 1)] \frac{\partial^p \Psi_i(\mathbf{k}, \eta)}{\partial \phi_{j_1}(\mathbf{k}_1) \dots \partial \phi_{j_p}(\mathbf{k}_p)} \Big|_{\alpha(\mathbf{k})=0}, \quad (47)$$

where $\mathbf{k} = \sum \mathbf{k}_j$. After taking the ensemble average of this expression we finally obtain,

$$\Gamma^{(p)}(\mathbf{k}_1, \dots, \mathbf{k}_p, \eta) = \langle \exp[i\alpha(\mathbf{k})(D_+(\eta) - 1)] \rangle \Gamma_{\text{tree}}^{(p)}(\mathbf{k}_1, \dots, \mathbf{k}_p, \eta), \quad (48)$$

where we have used that the derivatives on the r.h.s of Eq. (47) are evaluated at $\alpha = 0$ (i.e. at tree level) and thus are independent of initial random fields. This is a remarkable result that extends in a concrete and simple way the findings of [9, 13] for Gaussian initial conditions. Hence, the statistical properties of “random variable” $\alpha(\mathbf{k})$ determine a transition function that relates the fully nonlinear multi-point propagator with their tree level expression (which, up to sub-leading terms, are nothing else than the standard PT kernels in Eq. 17), in the high- k regime. In what follows, we will use this result to explicitly compute the Γ -functions in this regime.

³ An alternative way of obtaining this result is to remark that in presence of low q perturbing modes Eq. (7) can be rewritten – for an Einstein-de Sitter background – as $a \frac{\partial}{\partial a} \Psi_i(\mathbf{k}, a) + \Omega_{ij} \Psi_j(\mathbf{k}, a) - i\alpha(\mathbf{k}) a \Psi_i(\mathbf{k}, a) = \gamma_{ijk}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) \Psi_j(\mathbf{k}_1, a) \Psi_k(\mathbf{k}_2, a)$. The linear propagator is $\tilde{g}_{ij}(\mathbf{k}, a_1, a_2) = \exp(i\alpha(\mathbf{k})(a_1 - a_2)) g_{ij}(a_1/a_2)$. The quantities we derive in this paper are the tree order Γ -functions of this theory, whose α dependence can be easily computed from the fact that $\tilde{g}_{ij}(\mathbf{k}_1 + \mathbf{k}_2, a_1, a_2) \tilde{g}_{kl}(\mathbf{k}_1, a_2, a_3) \tilde{g}_{mn}(\mathbf{k}_2, a_2, a_3) = \exp(i\alpha(\mathbf{k}_1 + \mathbf{k}_2)(a_1 - a_3)) g_{ij}(a_1/a_2) g_{kl}(a_2/a_3) g_{mn}(a_2/a_3)$.

B. The transition function

Irrespectively of their order, the Γ -functions are all obtained, in the large- k regime, from the computation of the ensemble average in

$$f(k) = \langle \exp [i\alpha(\mathbf{k})(D_+ - 1)] \rangle, \quad (49)$$

that will depend only on k due to homogeneity and isotropy. From the definition of $\alpha(\mathbf{k})$ in Eq. (37) we observe that the function $f(k)$ can in fact be easily related to the statistical properties of the primordial field: it is nothing but the moment generating function \mathcal{M} (see Eq. (133) of [7]) at point $t = \mathbf{k}(D_+ - 1)$ of the single point one component displacement field \mathbf{d} ,

$$\mathbf{d} = \int d^3\mathbf{q} \frac{\mathbf{q}}{q^2} \delta_0(\mathbf{q}). \quad (50)$$

As a consequence of Eq. (135) in [7], $\log f(k)$ is nothing but the cumulant generating function of that same variable at the same point defined through,

$$\log f(k) = \sum_{p=2}^{\infty} \frac{\langle (\mathbf{d} \cdot \mathbf{k})^p \rangle_c}{p!} (D_+ - 1)^p. \quad (51)$$

For Gaussian initial conditions only $p = 2$ is non-zero and one recovers the well known result $f(k) = \exp(-k^2 \sigma_{\text{displ.}}^2 (D_+ - 1)^2 / 2)$ [9, 13]. Note also that only even values of p contribute to this sum because the cumulants cannot depend on the direction of the wave-vector \mathbf{k} . This is at variance with the result derived in [31] following the prescription for propagator resummation put forward in [10] for Gaussian initial statistics. It is however a very important result because it shows that, for any given model of primordial non-Gaussianity, the first ‘‘non-Gaussian’’ correction to the propagator decay (in the high- k limit) is given by the four point connected function. For weakly non-Gaussian initial conditions this will represent a minor contribution.

V. PREDICTIONS FOR THE LOCAL MODEL OF PNG

We now turn into the evaluation of the transition function $f(k)$ defined in Eq. (51) for a specific model of primordial non-Gaussianity. We will focus in the local model [22–25], which is perhaps the most studied model of primordial non Gaussianity within the context of large scale structure (see for instance the recent reviews [15, 16] and references therein). This model is build upon the nonlinear relation [22]

$$\Phi(\mathbf{x}) = \phi(\mathbf{x}) + f_{NL}(\phi^2(\mathbf{x}) - \langle \phi^2(\mathbf{x}) \rangle) + g_{NL}\phi^3(\mathbf{x}), \quad (52)$$

between the Gaussian field ϕ and Bardeen’s gauge-invariant potential Φ , where up to cubic terms are considered. The local model corresponds to f_{NL} and g_{NL} constants independent of space. The connection to the matter overdensity is through Poisson’s equation $\delta_{\mathbf{k}}(z) = M(k, z)\Phi_{\mathbf{k}}$, with

$$M(k, z) = \frac{2}{3} \frac{k^2 T(k) D(z)}{\Omega_m H_0^2}, \quad (53)$$

where $T(k)$ is the matter transfer function, and $D(z)$ the growth factor. In what follows we will assume the same cosmological model as in [28] (and the CMB convention for f_{NL} and g_{NL}). This can be summarized as $h = 0.7$, $\Omega_m = 0.279$, $\Omega_b = 0.0462$, $n_s = 0.96$ and $\sigma_8 \simeq 0.81$ (see Sec. 3 in [28] for more detail)

We start by writing the first terms of the sum defining $f(k)$,

$$\log f(k) = \frac{\langle (\mathbf{d} \cdot \mathbf{k})^2 \rangle_c}{2} (D_+ - 1)^2 + \frac{\langle (\mathbf{d} \cdot \mathbf{k})^4 \rangle_c}{24} (D_+ - 1)^4 + \dots, \quad (54)$$

with the displacement field \mathbf{d} given by Eq. (50). As noticed before, the first contribution reduces to $\langle (\mathbf{d} \cdot \mathbf{k})^2 \rangle_c = -\sigma_{\text{disp}}^2 k^2$ and $\sigma_{\text{disp}} = 6.01 h^{-1} \text{Mpc}$ (at $z = 0$) for our cosmology. For Gaussian initial conditions this is the only non-zero contribution. For non-Gaussian initial conditions we need to evaluate, in addition

$$\begin{aligned} \langle (\mathbf{d} \cdot \mathbf{k})^4 \rangle_c &= \int d^3q_1 \int d^3q_2 \int d^3q_3 \int d^3q_4 \frac{\mathbf{k} \cdot \mathbf{q}_1}{q_1^2} \frac{\mathbf{k} \cdot \mathbf{q}_2}{q_2^2} \frac{\mathbf{k} \cdot \mathbf{q}_3}{q_3^2} \frac{\mathbf{k} \cdot \mathbf{q}_4}{q_4^2} \langle \delta_0(\mathbf{q}_1) \delta_0(\mathbf{q}_2) \delta_0(\mathbf{q}_3) \delta_0(\mathbf{q}_4) \rangle_c \\ &= \int d^3q_1 \int d^3q_2 \int d^3q_3 \frac{\mathbf{k} \cdot \mathbf{q}_1}{q_1^2} \frac{\mathbf{k} \cdot \mathbf{q}_2}{q_2^2} \frac{\mathbf{k} \cdot \mathbf{q}_3}{q_3^2} \frac{\mathbf{k} \cdot \mathbf{q}_4}{q_4^2} T_0(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4), \end{aligned} \quad (55)$$

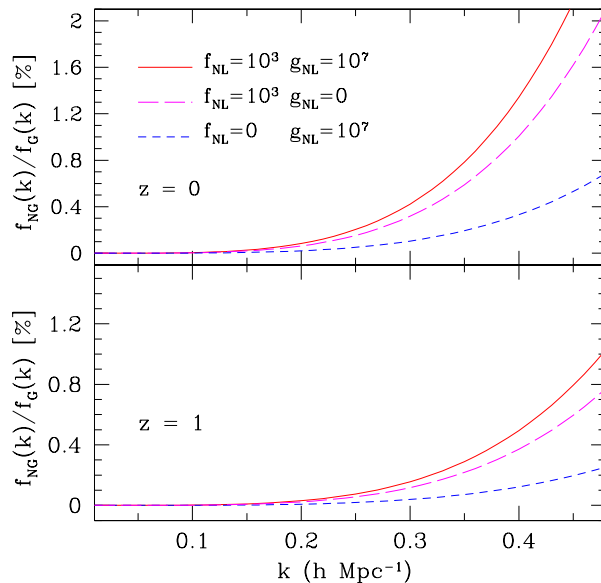


FIG. 4: The non Gaussian transition function $f(k)$ in the large- k limit in ratio to its expression for Gaussian statistics (i.e. setting $f_{NL} = g_{NL} = 0$). For currently accepted values of non Gaussian parameters (i.e. close to those shown) the correction to the damping is remarkably sub-dominant, even more so at higher redshift (e.g. bottom panel).

with $\mathbf{q}_4 = -\mathbf{q}_{123} \equiv -(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3)$ in the last equality. The matter trispectrum T_0 is given by

$$T_0(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = M(k_1, z)M(k_2, z)M(k_3, z)M(k_4, z) T_\Phi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4), \quad (56)$$

where T_Φ is the curvature trispectrum. For local non-Gaussianity there are two distinct contributions,

$$T_\Phi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = 4f_{NL}^2 P_\Phi(k_1)P_\Phi(k_2) [P_\Phi(k_{13}) + P_\Phi(k_{14})] + 5 \text{ perm.} \\ + 6g_{NL} P_\Phi(k_1)P_\Phi(k_2)P_\Phi(k_3) + 3 \text{ perm.} \quad (57)$$

Hence, in what follows we will distinguish the f_{NL}^2 component from the g_{NL} one. In appendix A we include a detailed account of the evaluation of the integral in Eq. (55). The final result for our cosmology is $\langle (\mathbf{d} \cdot \mathbf{k})^4 \rangle_c = f_{NL}^2 k^4 s_{4,f_{NL}}^4 + g_{NL} k^4 s_{4,g_{NL}}^4$ with $s_{4,f_{NL}} = 0.05537 h^{-1} \text{ Mpc}$ and $s_{4,g_{NL}} = 0.02359 h^{-1} \text{ Mpc}$ (at $z = 0$). After putting together this result and the one for the variance into Eq. (54), and normalizing the growth to $D(z = 0) = 1$, we obtain the prediction for the transition function in the local model at arbitrary redshift.

Figure 4 shows the ratio of the $f(k)$ function so obtained, assuming $f_{NL} = 10^3$ and/or $g_{NL} = 10^7$, to its value for Gaussian initial conditions (i.e. with the kurtosis set to zero). We see that the modification to the Gaussian case for $f(k)$ is minor for this choice of the PNG parameters, with up to 2% weaker decay at $z = 0$ (top panel) and 1% at $z = 0.5$ (bottom panel). In turn, Fig. 5 shows $\log f(k)$ as a function of k^2 for Gaussian and non-Gaussian initial conditions, with $f_{NL} = 5 \times 10^3$ and, separately, $g_{NL} = 10^9$. As expected, the correction becomes relevant for values of f_{NL} that make the ϕ^2 correction the same order of the Gaussian component.

Provided with the damping function $f(k)$ we are in place to explicitly compute the Γ -expansion for the bispectrum, as given in Eq. (26), but assuming the multi-point propagators in their large- k limit (a well justified approximation, at least for Gaussian initial conditions [13]). Figure 6 shows the terms contributing to this expansion assuming a local model for primordial non-Gaussianity with $f_{NL} = 300$ and $g_{NL} = 10^5$, and equilateral configurations. In solid blue line we show the equivalent to the tree-level bispectrum induced by gravity, i.e. the first term in Eq. (26). The contribution from the primordial bispectrum is depicted by solid red line, it dominates the total signal at very large scales but it is exponentially suppressed at high- k by $\Gamma^{(1)}$. At one-loop there are two terms also present for Gaussian initial conditions, see discussion after Eq. (26), which are $\mathcal{O}(P_0^3)$. These are shown by the dashed red line. The purely PNG induced ones at this order are given by the solid green line, they comprise terms $\mathcal{O}(P_0 B_0) \sim f_{NL} P_0^3$ and $\mathcal{O}(T_0) \sim (f_{NL}^2, g_{NL}) P_0^3$. In dashed line we show the result of standard perturbation theory at one-loop from [28]. Contrary to that case, here the number of terms is reduced (due to the resummation of several contributions). In addition, each term dominates over a narrow range of scales.

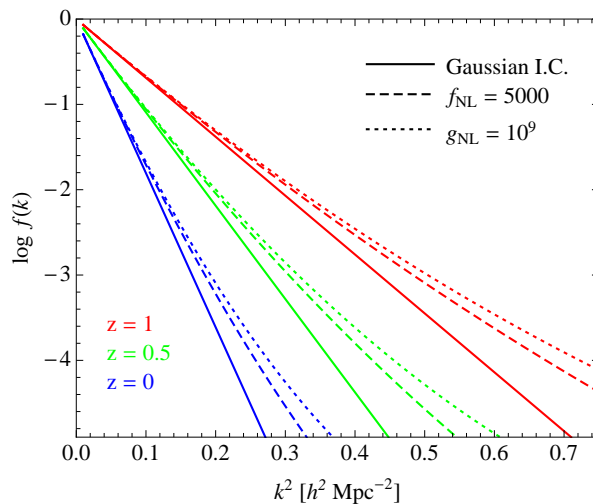


FIG. 5: The damping function $\log f(k)$ as a function of k for Gaussian and non-Gaussian initial conditions, with $f_{NL} = 5 \times 10^3$ and $g_{NL} = 10^9$ respectively

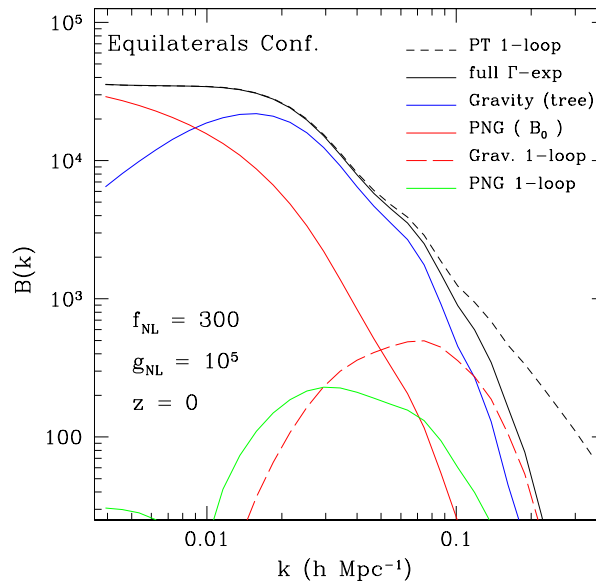


FIG. 6: The Γ expansion for the equilateral bispectrum in the local model of PNG, up to one-loop. The “gravitational” tree level bispectrum is shaped by $\Gamma^{(2)}$ while the PNG one arises from the primordial bispectrum and is exponentially suppressed to high- k by $\Gamma^{(1)}$. At one-loop there are terms $\sim P_0^3$ (grav.), $\sim P_0 \times B_0$ (PNG, f_{NL}) and $\sim T_0$ (PNG, f_{NL}^2 and g_{NL}).

VI. CONCLUSIONS

The impact of primordial non-Gaussian (PNG) initial conditions on the statistical properties of the cosmic density field is a priori not easy to infer. This is because most of the diagrammatic expansions that have been lately used to study the large scale clustering are rooted in the assumption of primordial Gaussian fluctuations, that considerably simplifies the structure of multi-loop diagrams. We show here explicitly that PNG can indeed be accounted for in the approaches developed in previous papers along the so-called RPT method [8]. In particular the Γ -expansion approach of [13] is preserved for such initial conditions.

Within this context we generalize the definition of multi-point propagator to the case of arbitrary initial statistics. We find that they can still be regarded as the basic building blocks out of which one can construct n -point spectra. The

Γ -expansions so obtained correspond to a resummation of infinite sub-sets of diagrams in the approach of [8, 10, 12], concretely those corresponding to propagator and vertex renormalization, in line with the results in [13] for initial Gaussian fields. For concreteness we give explicit expressions for the series expansion up to one-loop of the power spectrum and the bispectrum. In this way, the leading corrections to the power-spectrum are then those that make intervene the primordial bispectrum in the contraction of the $\Gamma^{(1)} - \Gamma^{(2)}$ product.

We then turned into the description of the $\Gamma^{(n)}$ functions themselves (a.k.a. the multi-point propagators). We show how the simple one-to-one correspondence between n -point propagators and n -point correlators (among initial and final fields) found for Gaussian initial conditions can be extended to the case of PNG. In the latter case, however, we find that this relations involves a infinite hierarchy among $\Gamma^{(n)}$ and correlators. Nonetheless for weakly non-Gaussian models, as those allowed by current data, well justified approximations can be put forward to close the hierarchy permitting the actual measurement of $\Gamma^{(n)}$ in N-body simulations. In this way one can extend the methodology carried out for Gaussian initial statistics [9, 13] to shed light in the full description of the multi-point propagators.

Finally, the large- k damping behavior of the multi-point propagators is derived for arbitrary initial conditions. In this regime, we find that the multi-point propagators are proportional to their tree-level expressions. Moreover we explicitly show that the rate of decay is the same irrespectively of their order, and that is given by the cumulant generating function of the displacement field. Remarkably, the first corrections to the ‘‘Gaussian’’ damping in this regime is due to the initial trispectrum, which for small departure from Gaussian initial conditions constitutes a sub-dominant contribution.

We leave for future work a more detailed analysis involving quantitative predictions for specific models of PNG. But we hope the results already presented here might serve as a basis to meet the accuracy requirements demanded by the analysis of future datasets tailored at deciphering the primordial statistics and the physics of inflation.

Acknowledgments

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Appendix A: Evaluation of the kurtosis cumulant

In this appendix we discuss the evaluation of the 4-order cumulant defining the first non-Gaussian contribution to the transition function $f(k)$ in Eq. (51). As discussed in Sec. V, and particularly in Eqs. (56,57), we must evaluate two separate contributions to the trispectrum defining the kurtosis cumulant. We have, in the first place

$$\langle (\mathbf{d} \cdot \mathbf{k})^4 \rangle_{c,f_{NL}} = \int d^3 q_1 \int d^3 q_2 \int d^3 q_3 \frac{\mathbf{k} \cdot \mathbf{q}_1}{q_1^2} \frac{\mathbf{k} \cdot \mathbf{q}_2}{q_2^2} \frac{\mathbf{k} \cdot \mathbf{q}_3}{q_3^2} \frac{\mathbf{k} \cdot \mathbf{q}_4}{q_4^2} \tilde{T}_{f_{NL}}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4), \quad (\text{A1})$$

with $\mathbf{q}_4 = -\mathbf{q}_{123}$ and

$$\tilde{T}_{f_{NL}}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4) = 48 f_{NL}^2 M(q_1) M(q_2) M(q_3) M(q_4) P_\Phi(q_1) P_\Phi(q_3) P_\Phi(q_{12}), \quad (\text{A2})$$

giving the f_{NL} term, and a similar expression with the trispectrum replaced by

$$\tilde{T}_{g_{NL}}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4) = 24 g_{NL} M(q_1) M(q_2) M(q_3) M(q_4) P_\Phi(q_1) P_\Phi(q_2) P_\Phi(q_3), \quad (\text{A3})$$

corresponding to the g_{NL} contribution. Notice that in Eqs. (A2,A3) we took advantage of the fact that we are integrating over all orientations so all permutations give the same result. Since both integrals depend only on the magnitude k , we can integrate over the orientation of \mathbf{k} . Hence, for the f_{NL} contribution we have

$$\langle (\mathbf{d} \cdot \mathbf{k})^4 \rangle_{c,f_{NL}} = \int d^3 q_1 \int d^3 q_2 \int d^3 q_3 \tilde{T}_{f_{NL}}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4) \frac{1}{4\pi} \int d\Omega_k \frac{\mathbf{k} \cdot \mathbf{q}_1}{q_1^2} \frac{\mathbf{k} \cdot \mathbf{q}_2}{q_2^2} \frac{\mathbf{k} \cdot \mathbf{q}_3}{q_3^2} \frac{\mathbf{k} \cdot \mathbf{q}_4}{q_4^2}, \quad (\text{A4})$$

and we are then allowed to put \mathbf{q}_1 along the z -axis and set the azimuthal angle of \mathbf{q}_2 equal to zero, obtaining the 6-dimensional integral

$$\begin{aligned} \langle (\mathbf{d} \cdot \mathbf{k})^4 \rangle_{c,f_{NL}} &= 2\pi \int dq_1 q_1^2 \int dq_2 q_2^2 \int d \cos \theta_2 \int d^3 q_3 \tilde{T}_{f_{NL}}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4) \int d\Omega_k \frac{\mathbf{k} \cdot \mathbf{q}_1}{q_1^2} \frac{\mathbf{k} \cdot \mathbf{q}_2}{q_2^2} \frac{\mathbf{k} \cdot \mathbf{q}_3}{q_3^2} \frac{\mathbf{k} \cdot \mathbf{q}_4}{q_4^2}, \\ &= 2\pi \int dq_1 \int dq_2 \int d \cos \theta_2 \int d^3 q_3 \frac{\tilde{T}_{f_{NL}}(q_1, q_2, q_3, q_4, q_{12})}{q_3^2 q_4^2} \int d\Omega_k (\mathbf{k} \cdot \mathbf{q}_1)(\mathbf{k} \cdot \mathbf{q}_2)(\mathbf{k} \cdot \mathbf{q}_3)(\mathbf{k} \cdot \mathbf{q}_4). \end{aligned}$$

Introducing the vector $\mathbf{p} \equiv \mathbf{q}_{12} \equiv \mathbf{q}_1 + \mathbf{q}_2$ we can rewrite the integral as

$$\langle (\mathbf{d} \cdot \mathbf{k})^4 \rangle_{c,f_{NL}} = 2\pi \int dq_1 \int dq_2 \int_{|q_1 - q_2|}^{q_1 + q_2} dp \frac{p}{q_1 q_2} \int d^3 q_3 \frac{\tilde{T}_{f_{NL}}(q_1, q_2, q_3, q_4, p)}{q_3^2 q_4^2} \int d\Omega_k (\mathbf{k} \cdot \mathbf{q}_1)(\mathbf{k} \cdot \mathbf{q}_2)(\mathbf{k} \cdot \mathbf{q}_3)(\mathbf{k} \cdot \mathbf{q}_4),$$

and recall that $\mathbf{q}_4 = -\mathbf{q}_{123} = -\mathbf{q}_{12} - \mathbf{q}_3 = -\mathbf{p} - \mathbf{q}_3$, so that the magnitude q_4 only depends on the magnitudes of \mathbf{p} and \mathbf{q}_3 and on the angle between them. Then, we can transform the integrand in the basis where $\mathbf{p} = \mathbf{q}_{12}$ is along the z -axis,

$$\begin{aligned} \langle (\mathbf{d} \cdot \mathbf{k})^4 \rangle_{c,f_{NL}} &= 2\pi \int dq_1 \int dq_2 \int_{|q_1 - q_2|}^{q_1 + q_2} dp \frac{p}{q_1 q_2} \int dq_3 q_3^2 \int d \cos \tilde{\theta}_3 \frac{\tilde{T}_{f_{NL}}(q_1, q_2, q_3, q_4, p)}{q_3^2 q_4^2} \\ &\quad \times \int d\tilde{\phi}_3 \int d\Omega_k (\mathbf{k} \cdot \mathbf{q}_1)(\mathbf{k} \cdot \mathbf{q}_2)(\mathbf{k} \cdot \mathbf{q}_3)(\mathbf{k} \cdot \mathbf{q}_4), \end{aligned} \quad (\text{A5})$$

with $\tilde{\theta}_3$ and $\tilde{\phi}_3$ define the orientation of \mathbf{q}_3 in the new basis and $\tilde{\theta}_3$ is now the angle between \mathbf{q}_3 and \mathbf{p} . Given the one-to-one correspondence between θ_3 and q_4 , as before we can change variable of integration to get

$$\begin{aligned} \langle (\mathbf{d} \cdot \mathbf{k})^4 \rangle_{c, f_{NL}} &= 2\pi \int dq_1 \int dq_2 \int_{|q_1 - q_2|}^{q_1 + q_2} dp \frac{p}{q_1 q_2} \int dq_3 \int_{|p - q_3|}^{p + q_3} dq_4 \frac{q_4}{p q_3} \frac{\tilde{T}_{f_{NL}}(q_1, q_2, q_3, q_4, p)}{q_4^2} \\ &\times \int d\tilde{\phi}_3 \int d\Omega_k (\mathbf{k} \cdot \mathbf{q}_1)(\mathbf{k} \cdot \mathbf{q}_2)(\mathbf{k} \cdot \mathbf{q}_3)(\mathbf{k} \cdot \mathbf{q}_4) \end{aligned} \quad (\text{A6})$$

$$\begin{aligned} &= 2\pi k^4 \int dq_1 \int dq_2 \int_{|q_1 - q_2|}^{q_1 + q_2} dp \int dq_3 \int_{|p - q_3|}^{p + q_3} dq_4 \frac{\tilde{T}_{f_{NL}}(q_1, q_2, q_3, q_4, p)}{q_1 q_2 q_3 q_4} \\ &\times F_{geom}(q_1, q_2, q_3, q_4, p), \end{aligned} \quad (\text{A7})$$

with the geometric factor given by,

$$\begin{aligned} F_{geom}(q_1, q_2, q_3, q_4, p) &\equiv \int d\tilde{\phi}_3 \int d\Omega_k (\hat{\mathbf{k}} \cdot \mathbf{q}_1)(\hat{\mathbf{k}} \cdot \mathbf{q}_2)(\hat{\mathbf{k}} \cdot \mathbf{q}_3)(\hat{\mathbf{k}} \cdot \mathbf{q}_4) \\ &= \frac{1}{120} [(q_1^2 - q_2^2)^2 - 7(q_1^2 + q_2^2)p^2 + 6p^4] \\ &\quad - \frac{1}{30} [(q_1^2 - q_2^2)^2 - 3(q_1^2 + q_2^2)p^2 + 2p^4] \frac{(q_3^2 + q_4^2)}{p^2} \\ &\quad + \frac{1}{120} [3(q_1^2 - q_2^2)^2 - 5(q_1^2 + q_2^2)p^2 + 2p^4] \frac{(q_3^2 - q_4^2)^2}{p^4}, \end{aligned} \quad (\text{A8})$$

being a function of the five variables. The g_{NL} integral follows from the same considerations, with $\tilde{T}_{g_{NL}}(q_1, q_2, q_3, q_4)$ replacing $\tilde{T}_{f_{NL}}(q_1, q_2, q_3, q_4, p)$. An explicit evaluation of these integrals for the cosmology assumed in this paper (see Sec. V) yields

$$\langle (\mathbf{d} \cdot \mathbf{k})^4 \rangle_c = k^4 f_{NL}^2 9.4 \times 10^{-6} + k^4 g_{NL} 3.1 \times 10^{-7} \quad (\text{A9})$$

assuming T_0 is evaluated at $z = 0$ and \mathbf{k} is in units of $h \text{ Mpc}^{-1}$.