The centrality dependence of $v_2/\varepsilon$: the ideal hydro limit and $\eta/s$

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Abstract

The large elliptic flow observed at RHIC is considered to be evidence for almost perfect liquid behavior of the strongly coupled quark-gluon plasma produced in the collisions. In these proceedings we present a two parameter fit for the centrality dependence of the elliptic flow $v_2$ scaled by the spatial eccentricity $\varepsilon$. We show by comparing to viscous hydrodynamical calculations that these two parameters are in good approximation proportional to the shear viscosity over entropy ratio $\eta/s$ and the ideal hydro limit of the ratio $v_2/\varepsilon$.

1. Introduction

The goal of the ultra-relativistic nuclear collision program is the creation and study of a new state of matter, the quark-gluon plasma. The azimuthal anisotropy of the transverse momentum distribution in non-central heavy-ion collisions is thought to be sensitive to the properties of this state of matter. The second Fourier coefficient of this anisotropy, $v_2$, is called elliptic flow. For a recent review see [1].

In ideal hydrodynamics $v_2$ is proportional to the spatial eccentricity with a magnitude which depends on the Equation of State $EoS$. This spatial eccentricity is defined by

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

where $x$ and $y$ are the spatial coordinates of the colliding nucleons in the plane perpendicular to the collision axis and where the brackets denote an average. In practice $\varepsilon$ is not a measured quantity but obtained from model calculations, using Glauber or Color Glass Condensate (CGC) models, for instance.

The ratio $v_2/\varepsilon$ versus particle density is a sensitive gauge to test if the system approaches ideal hydrodynamic behavior [2]. It was observed that this ratio reaches the expected ideal hydrodynamic values only for the more central collisions at the highest RHIC center of mass energy [3,4] which indicates that certainly for non-central collisions, as well as at lower energies, and away from mid-rapidity the elliptic flow contains significant non-ideal hydro contributions.

Much of this discrepancy can be explained by incorporating the viscous contribution from the hadronic phase [5,6,7]. However, we expect that also the hot and dense phase must deviate from an ideal hydrodynamic description. Kovtun, Son and Starinets (KSS) [8], showed that conformal field theories with gravity duals have a ratio of shear viscosity $\eta$ to entropy density $s$ of, in natural units, $\eta/s = 1/4 \pi$. They conjectured that this value is a lower bound for any relativistic thermal field theory. In addition, Teaney [9] pointed out that very small shear viscosities, of the magnitude of the bound, would already lead to a significant reduction in the predicted elliptic flow.
Based on the centrality dependence of $v_2/\varepsilon$, the magnitude of $\eta/s$ for the created system has been estimated recently from a transport theory motivated calculation [10, 11] and from viscous hydrodynamical calculations [12, 13]. Both approaches have their merits and drawbacks.

In these proceedings we explore how well a parameterization can be used to estimate $\eta/s$ as well as the ideal hydrodynamical limit of $v_2/\varepsilon$ which is closely related to the EoS.

2. Simple Parameterization

We use the parameterization from [2, 10] which is defined by

$$\frac{v_2}{\varepsilon} = \frac{h}{1 + B/(1/S dN/dy)},$$

(1)

where $S$ is the transverse area of the collision region and $h$ and $B$ are the two free parameters in the fit. The parameter $h$ corresponds to the ideal hydro limit of $v_2/\varepsilon$ and $B$ is proportional to $\eta/s$.

Figure 1 shows how the parameterization behaves for two different values of the ideal hydro limit (the dashed line represents the harder EoS) and two different values of $\eta/s$ (the full line represents the smaller $\eta/s$). The effect of the EoS is clearly seen in the magnitude of $v_2/\varepsilon$ in Fig. 1 and the value of $\eta/s$ is reflected by the change in this magnitude versus $1/S dN/dy$ (for $\eta/s = 0$ the magnitude will be constant). The magnitude of $\eta/s$ is easier to quantify if one plots $v_2/h\varepsilon$, as is done in Fig. 2. A larger deviation from unity at fixed value of $1/S dN/dy$ then indicates a larger $\eta/s$.

To test if this simple parameterization does describe a state of the art viscous hydrodynamical calculation we fit the calculations from Luzum and Romatschke [14]. Figure 3 shows that Eq. 1 well describes results from viscous hydrodynamical calculations, done with three different values of $\eta/s$ and two different parameterizations of the spatial eccentricity (Glauber and CGC). As expected, $v_2$ is to good approximation proportional to the initial spatial eccentricity. In addition, it is seen that the deviation of $v_2/\varepsilon$ from unity at a given $1/S dN/dy$ increases for larger values of $\eta/s$.

Figure 4 shows $v_2/h\varepsilon$ from viscous hydrodynamical calculations [12, 13, 14] done by different groups using the same set of values of $\eta/s$ but different parameterization of the EoS and $\varepsilon$. The value of $\varepsilon$ is that used in the hydrodynamical calculations while the value of $h$ is obtained.

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Figure 1: The dependence of $v_2/\varepsilon$ versus transverse density of equation 1 for two values of $h$ and two values of $\eta/s$.

Figure 2: The dependence of $v_2/h\varepsilon$ versus transverse density of equation 1 for the same parameters as Fig. 1.
Figure 3: A fit of viscous hydrodynamical model results using CGC and Glauber initial eccentricities with Eq. 1.

Figure 4: Comparing viscous hydrodynamical calculations of different groups with the fit.

from the fit. We conclude that our parameterization yields curves that depend on the value of \( \eta/s \) but are roughly independent of the EoS and \( \varepsilon \). However it turns out that if the EoS is very different (e.g. not incorporating a phase transition) this scaling does break down (not shown).

Using Eq. 1 we can now compare the various viscous hydrodynamical results with data and estimate the value of \( \eta/s \). Since the value of \( \varepsilon \) is not known we take the eccentricity calculated assuming CGC [15] or Glauber (wounded nucleon) initial conditions as two extremes. It is seen from Fig. 5 that, assuming the CGC initial conditions, the STAR data is well described with twice the KSS bound, \( \eta/s \approx 2/4\pi \). Using the Glauber initial conditions, however, the STAR data is not described within the range of \( \eta/s \) currently used by the viscous hydrodynamical calculations. From the deviation from unity one can estimate that the corresponding value of \( \eta/s \) would be approximately four times the KSS bound. Using the CGC or Glauber initial conditions we find for the ideal hydro limit of \( v_2/\varepsilon \) the value \( 0.2 \pm 0.01 \) and \( 0.36 \pm 0.07 \), respectively.

For the CGC initial conditions the value of \( \eta/s \) approximately matches the EoS used by Luzum and Romatschke [13]). This is illustrated in Fig. 6 where the centrality dependence of \( v_2 \) [13] is well described by CGC initial conditions, a value of \( \eta/s = 2/4\pi \). Using viscous hydrodynamics with these CGC initial conditions, EoS, and magnitude of

Figure 5: Comparing viscous hydrodynamical calculations with STAR data.

Figure 6: A direct comparison of viscous hydro calculations with PHOBOS data (from [14]).
$\eta/s$, the transverse momentum dependence of $v_2$ is also well described, as shown in Fig. 7. The figure illustrates that the $p_t$ dependence is very sensitive to the viscous correction such that larger corrections decrease the magnitude of $v_2$ and shift its maximum to lower $p_t$. Figure 8 shows the centrality dependence of $v_2(p_t)$ where one clearly observes that the deviation with $\eta/s = 0$ increases from central to peripheral collisions and that the peak position shifts to lower $p_t$, consistent with larger viscous effects.

3. Conclusions

We have shown that a simple parameterization can describe the centrality dependence of $v_2/\varepsilon$. When compared to viscous hydrodynamical calculations such a parameterization yields an estimate of $\eta/s$. We find that the current RHIC data is described well by a spatial eccentricity based on CGC initial conditions, a soft EoS with $v_2/\varepsilon \approx 0.2$ and $\eta/s$ twice the KSS bound.

References


Figure 7: $v_2$ from STAR (approximately corrected for nonflow) compared to viscous hydrodynamical calculations (from [13]).

Figure 8: $v_2$ from STAR as function of transverse momentum and centrality.