PHYSICAL IMPLEMENTATION AND RESIDUAL DECOHERENCE

OF

PROTECTED QUBITS

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Principle of qubit protection I

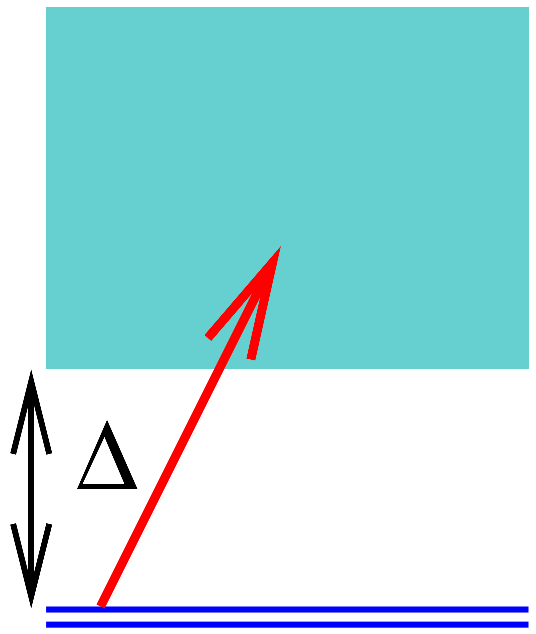
A. Kitaev, Ann. Phys. **303**, 2, (2003)

Coding space separated from non-coding ones by large gap Δ.

Absence of decoherence to first order:

$$PH_cP = P \otimes \tilde{H}^{(1)}_{\text{env}}$$

Single error that can be detected.



Principle of qubit protection II

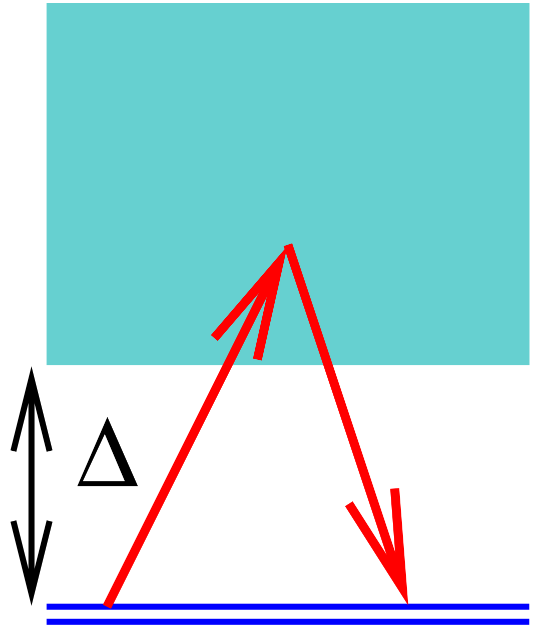
Absence of decoherence to second order:

$$PH_cQ\frac{1}{H_0}QH_cP = P \otimes \tilde{H}^{(2)}_{\text{env}}$$

Can be generalized to arbitrary order in H_c $\rightarrow$ notion of protected system at order N.

Can we achieve N large in a physical system ?

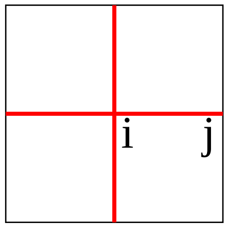
Potentially dangerous double error.



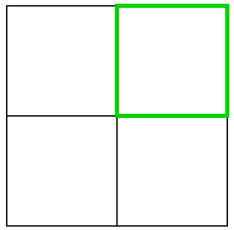
Lattice gauge theory in deconfined regime I

A. Kitaev, Ann. Phys. **303**, 2, (2003)

Z_2 charge $U_i = \prod_j^{(i)} \sigma_{ij}^x$



Z_2 flux $B_\square = \prod_{ij\in\square} \sigma_{ij}^z$



$$H_{\text{Kitaev}} = -\frac{\Delta_c}{2}\sum_i U_i - \frac{\Delta_f}{2}\sum_\square B_\square$$

Localized excitations with finite energy gap

Ground-state degeneracy depends on global topology of the lattice.

Lattice gauge theory in deconfined regime II

Two-fold degenerate ground-state on a cylinder

Degeneracy enforced by non-local symmetries:

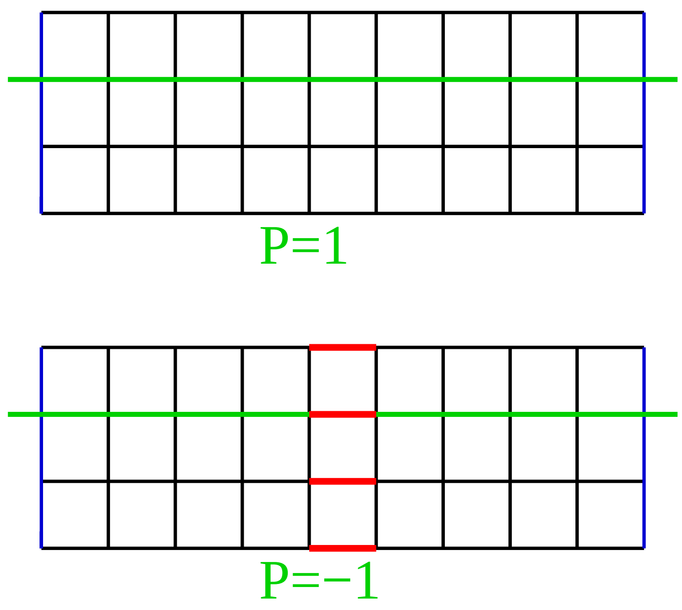
row operators:

$$P_i = \prod_j \sigma^z_{ij}$$

column operators:

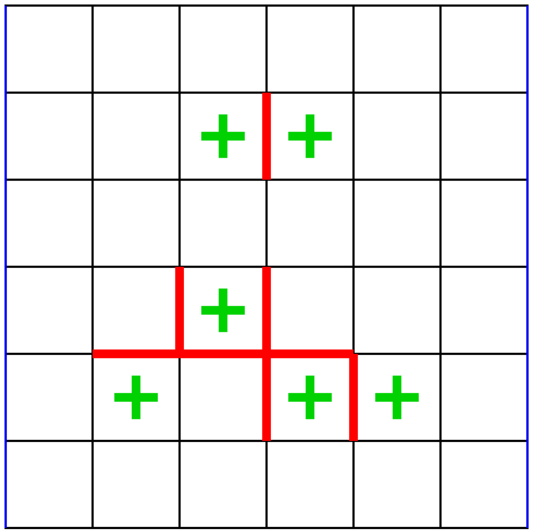
$$Q_j = \prod_i \sigma^x_{ij}$$

$$\{P_i, Q_j\} = 0$$



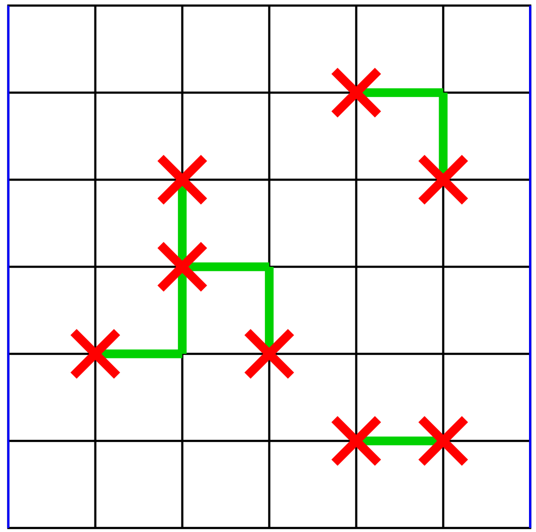
Local errors are harmless

Electric noise: $\prod_{ij \in \text{cluster}} \sigma_{ij}^{x}$



Creates localized Z_2 fluxes.

Magnetic noise: $\prod_{ij \in \text{cluster}} \sigma_{ij}^{z}$



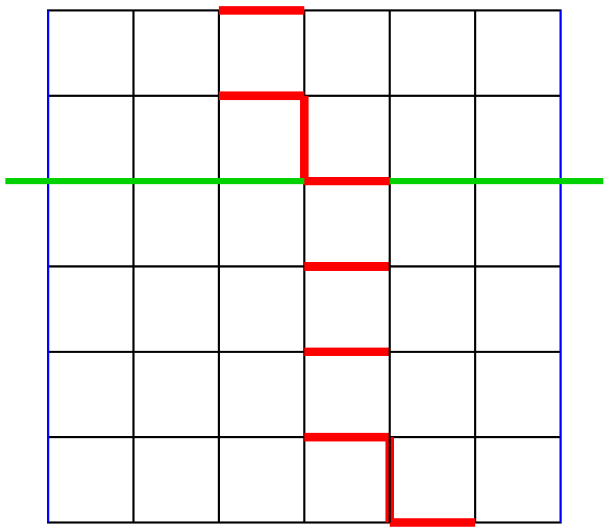
Creates local Z_2 charges.

These errors create only virtual states above finite energy gap.

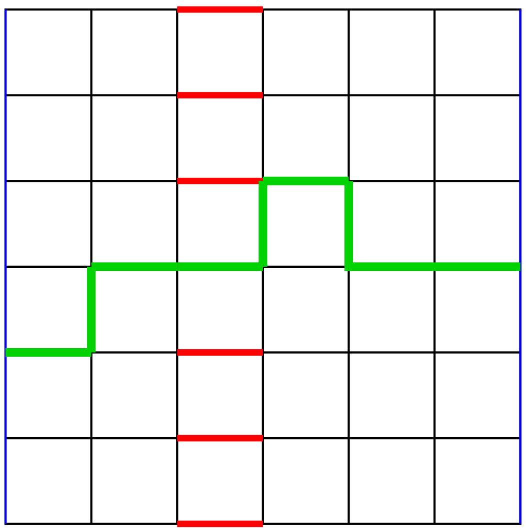
The only dangerous errors are non-local !

They are suppressed by a factor $(\mathrm{noise}/\Delta_{\mathrm{c,f}})^{\mathrm{L}}$

Electric noise transfers one Z_2 flux along v-path and flips P_i: Relaxation in *flux* basis or dephasing in *charge* basis.



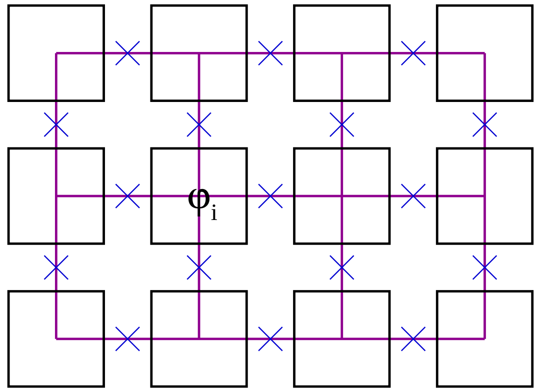
Magnetic noise transfers one Z_2 charge along h-path and flips Q_j: Relaxation in *charge* basis or dephasing in *flux* basis.



Basics of Josephson junction arrays

ϕ_j; local phase of Cooper pair condensate

$\hat{n}_j = \frac{\partial}{i\partial\phi_j}$: number of Cooper pairs on island j



$$\Delta\phi_j \Delta n_j \simeq 2\pi$$

$$A_{ij} = \frac{2\pi}{\Phi_0}\int_i^j \vec{A}_{ij}.d\vec{r}$$

$$H = -E_{\mathrm{J}} \sum_{\langle ij \rangle} \cos(\phi_i - \phi_j - A_{ij}) + \frac{E_{\mathrm{C}}}{2}\sum_{ij}(C_{ij}^{-1})\hat{n}_i\hat{n}_j$$

E_{J}: Josephson coupling energy

E_{C}: Charging energy

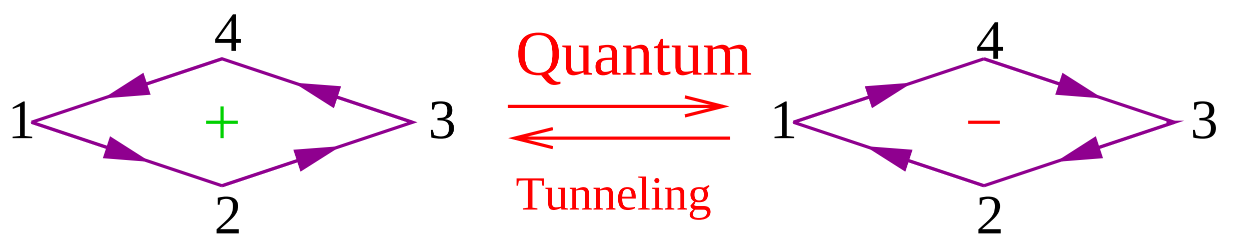
A rhombus with half a flux quantum

Define $\theta_{ij} = \phi_i - \phi_j - A_{ij}$, then:

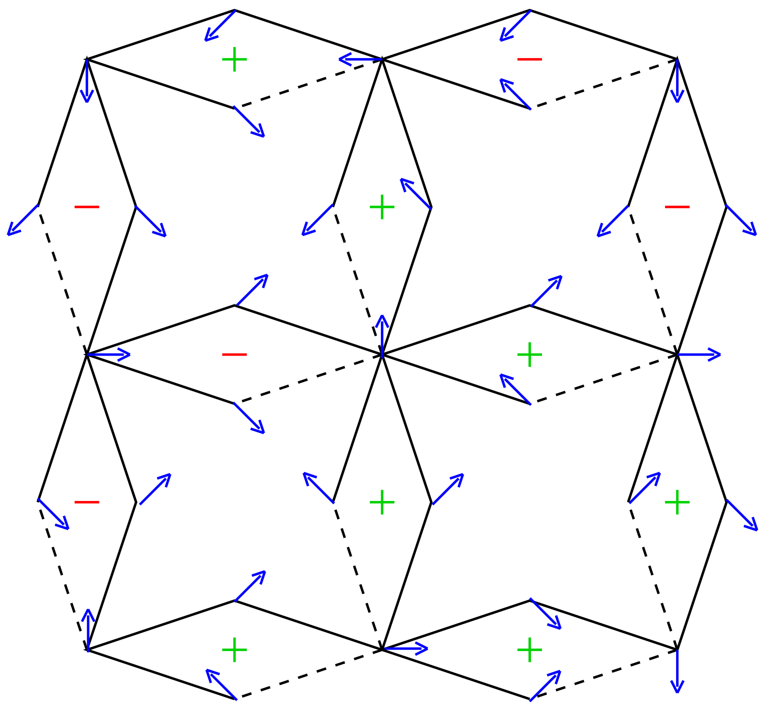
$$\theta_{12} + \theta_{23} + \theta_{34} + \theta_{41} \equiv \pi, \text{ mod } 2\pi$$

$\longrightarrow$ Get two-fold degenerate classical ground-state, with $\theta_{ij} = \pm\frac{\pi}{4}$

$\longrightarrow$ Quantum fluctuations ($E_C \neq 0$) of phases lift this degeneracy

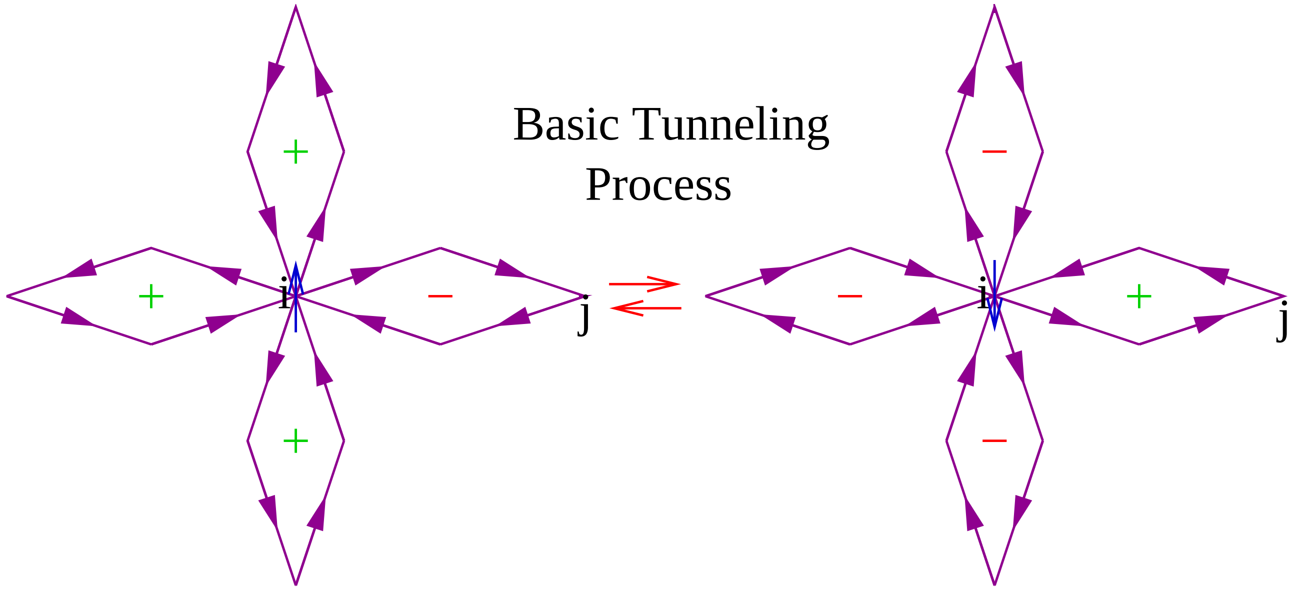


Local constraint on the classical ground-states



Enforces $B_\square = 1$ for classical ground-states.
Physical origin of Δ_f.

Effect of quantum fluctuations of phase variables



Basic tunneling process acts as: $\prod_j^{(i)} \sigma_{ij}^x$

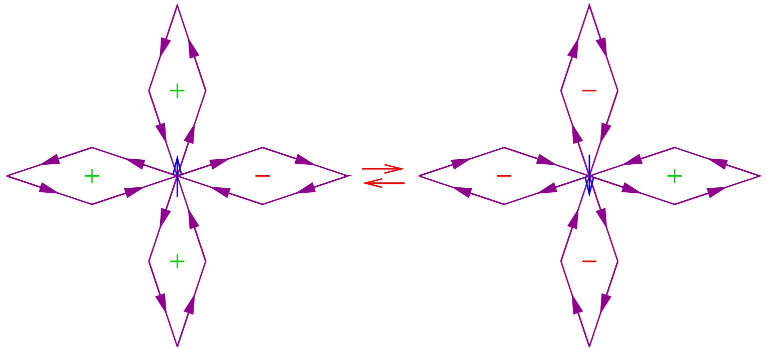
Tunnel rate: $\Delta_\mathrm{C} \simeq E_\mathrm{J}^{3/4} E_\mathrm{C}^{1/4} \exp(-4S_0)$

where: $S_0 = 1.61(E_\mathrm{J}/E_\mathrm{C})^{1/2}$, (Ioffe and Feigel'man, 2002)

Localization of Cooper pairs, and charge $4e$ condensate

Physical interpretation of local flip operator U_j:

$$U_j|\phi_j\rangle = |\phi_j + \pi\rangle$$



$$U_j|n_j\rangle = (-1)^{n_j}|n_j\rangle$$

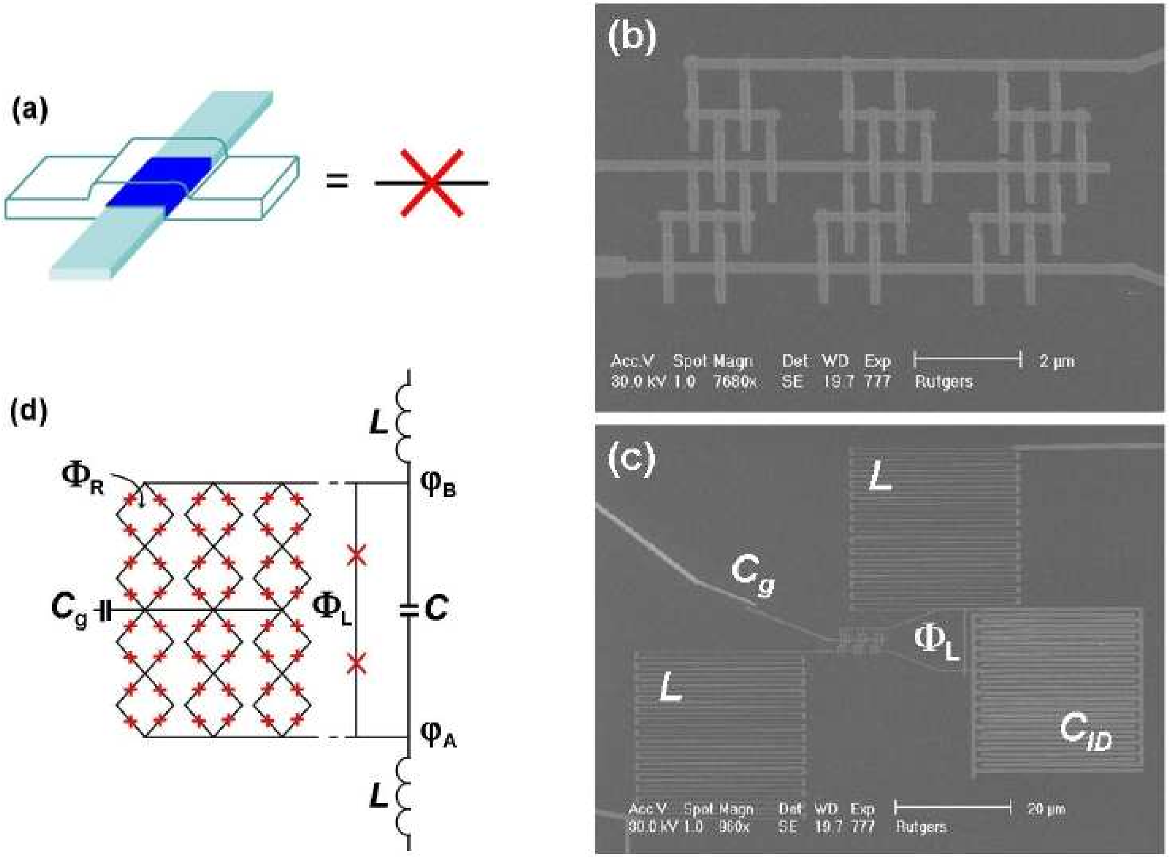
The parity U_j of n_j is conserved and single Cooper pairs are localized in Aharonov-Bohm cages.

$$\langle \exp(i\phi_j)\rangle = 0$$

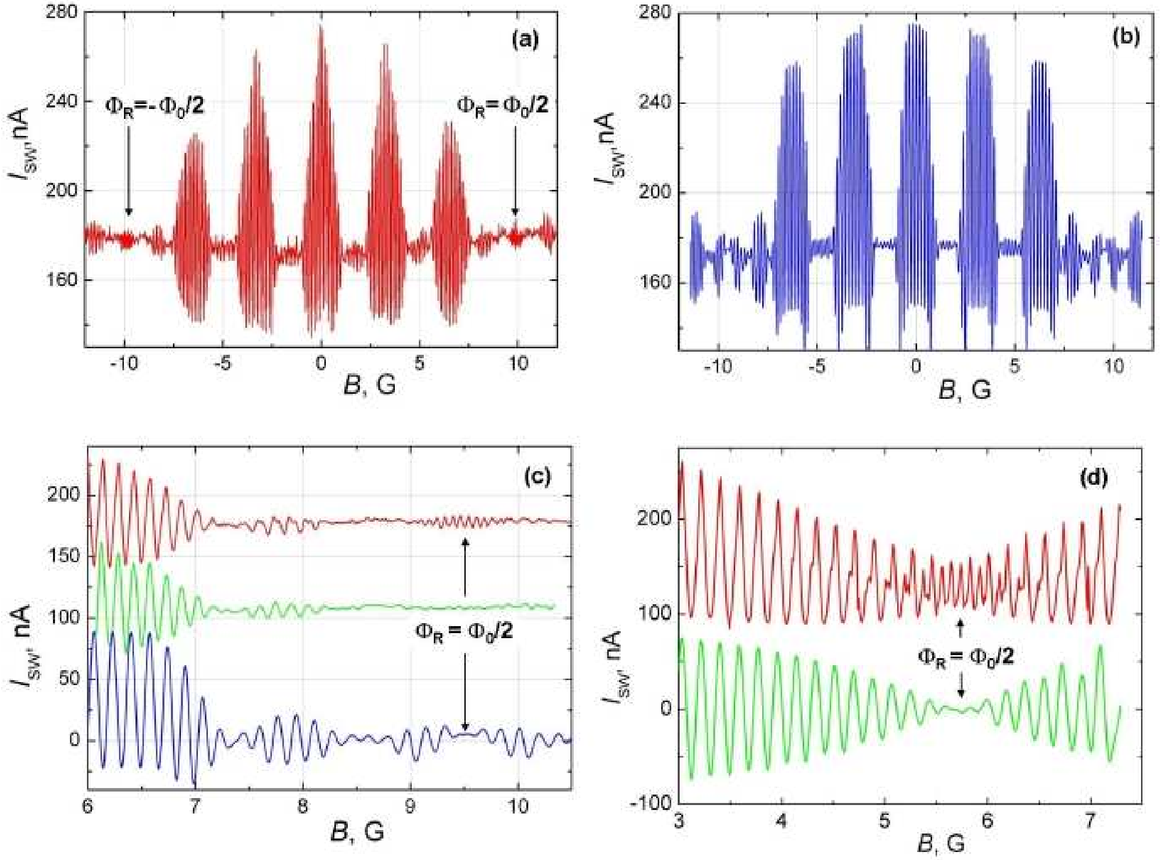
$$\langle \exp(i2\phi_j)\rangle \neq 0$$

No 2e condensate if $\Delta_c \neq 0$, but $4e$ condensate!

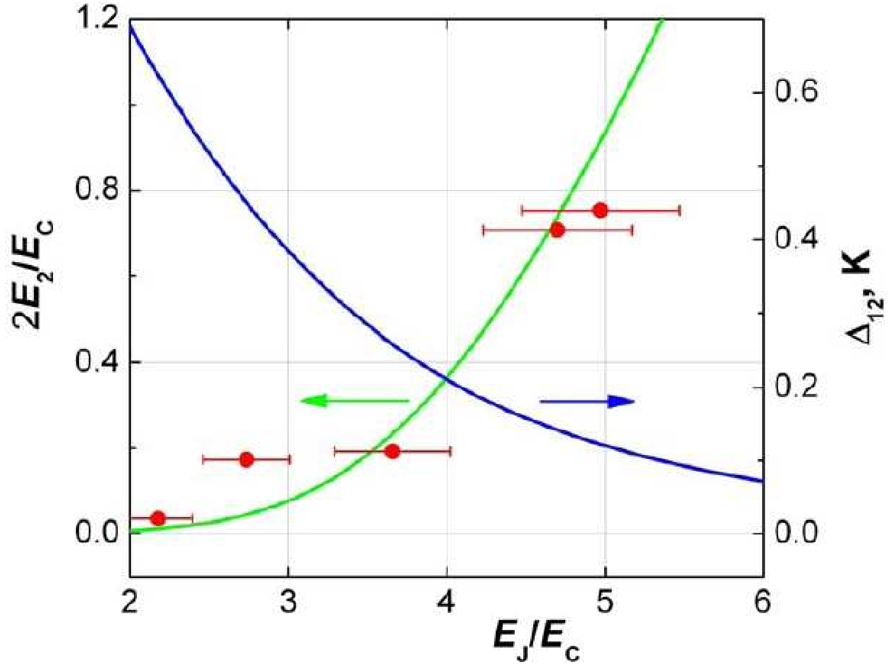
Experimental realization: M. Gershenson et al. (2007)



Evidence for finite Δ_C and charge $4e$ condensate



Phase stiffness of charge $4e$ condenstate



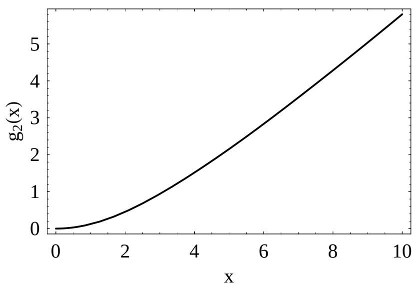
Computational issues I: hierarchical approximation

Series composition of Z_2 junctions

$$V(\phi) = -E_2 \cos(2\phi)$$

$$E'_2 = \left[1 - \frac{7}{256}\left(\frac{E_2}{E_C}\right)^2\right]\frac{1}{8}\frac{E_2^2}{E_C}$$

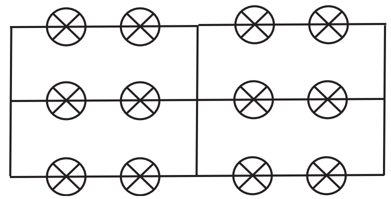
$$E'_C = \left[1 - \frac{1}{16}\left(\frac{E_2}{E_C}\right)^2\right] 2E_C$$

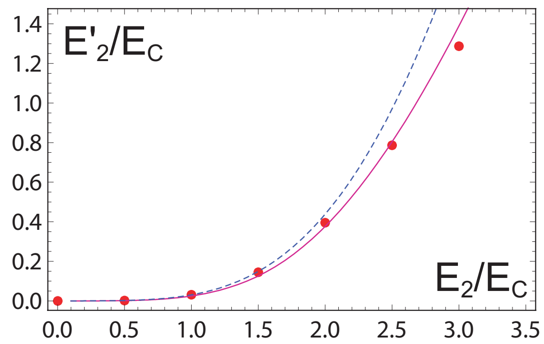


Parallel composition

$$E'_2 = K E_2$$

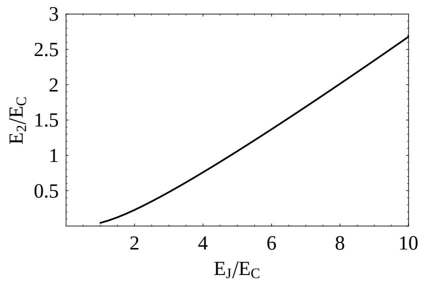
$$E'_C = K^{-1} E_C$$



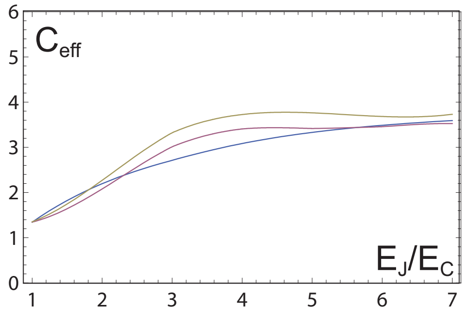


Computational issues II: single rhombus as Z_2 junction

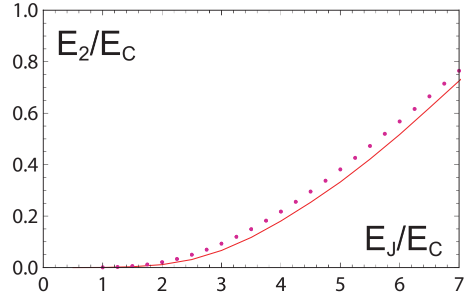
Effective E_2

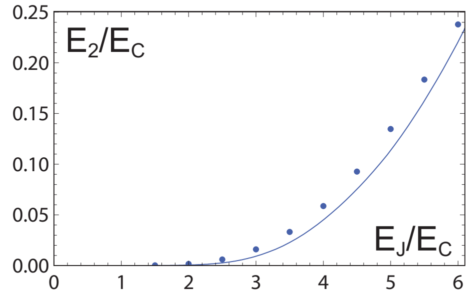


Effective capacitance



Test of coarse graining: 2 and 3 rhombi in series





Decoherence induced by finite frequency fluctuations

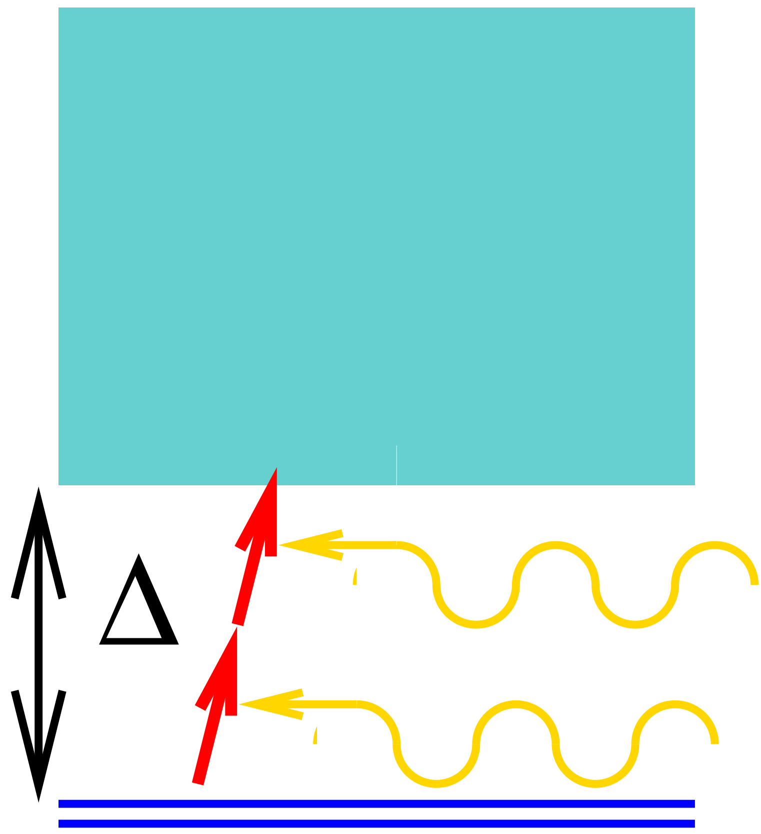
So far, we have considered only *virtual* transition to excited states.

But the bath may provide some energy: problem of *real* transitions.

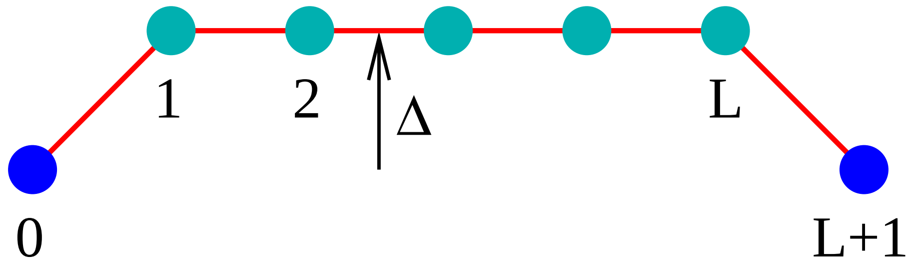
Spectral width of bath: D

$$D_{\text{eff}} = \text{Min}(k_{\text{B}}T, D)$$

$$n = \Delta / D_{\text{eff}}$$



Toy model



$$H = H_{\text{syst}} + H_{\text{bath}} + H_{\text{c}}$$

$$H_{\text{syst}} = \Delta \sum_{j=1}^{L} |j\rangle\langle j|$$

$$H_{\text{c}} = - \sum_{j=0}^{L} |j\rangle\langle j+1| \otimes X_{j+1/2}$$

Tree approximation

Density of states at generation p:

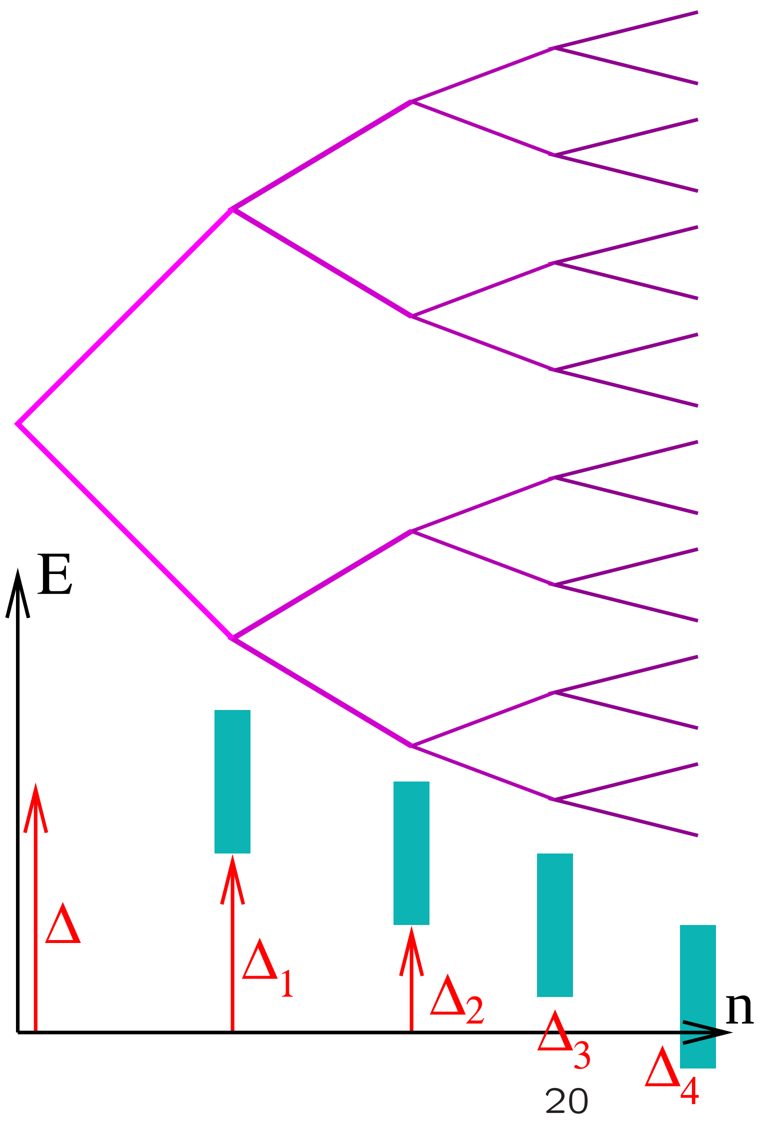
$$\rho_p(\omega) = \alpha_p(\omega - \Delta_p)^{r_p},$$

restricted to $\Delta_p < \omega < \Delta_p + D_p$.

$$R(z) = \langle \text{in}|(z - H)^{-1}|\text{in}\rangle$$

$$R(z) = \frac{1}{z - \Sigma_1(z)}$$

$$\Sigma_p(z) = \alpha_p W_p^2 \int \frac{\rho_p(\omega)\, d\omega}{z - \omega - \Sigma_{p+1}(z)}$$



Weak coupling analysis

Assume $z \simeq 0$, and

$$\begin{aligned} z - \Re\Sigma_1(z) &\leq \Delta_1 \\ \ldots &\leq \ldots \\ z - \Re\Sigma_{n-1}(z) &\leq \Delta_{n-1} \\ z - \Re\Sigma_n(z) &\geq \Delta_n \end{aligned}$$

Then, imaginary parts satisfy:

$$\begin{aligned} -\Im\Sigma_1(z) &\simeq c_1\alpha_1 W_1^2 D_1^{r_1-1}(-\Im\Sigma_2) \\ \ldots &\simeq \ldots \\ -\Im\Sigma_{n-1}(z) &\simeq c_{n-1}\alpha_{n-1} W_{n-1}^2 D_{n-1}^{r_{n-1}-1}(-\Im\Sigma_n) \\ -\Im\Sigma_n(z) &= \alpha_n W_n^2 (z + |\Delta_n|)^{r_n} \end{aligned}$$

$$-\Im\Sigma_1(z) \simeq \left(\frac{W_1 \ldots W_n}{D_1 \ldots D_{n-1}}\right)^2 \left(c_1\alpha_1 D_1^{r_1+1} \ldots c_{n-1}\alpha_{n-1} D_{n-1}^{r_{n-1}+1}\right) \alpha_n (z + |\Delta_n|)^{r_n}$$

$\longrightarrow$ Master equation, with rates appearing at order $2n$ in perturbative expansion.

$\longrightarrow$ No use to make systems of size L with $L > n$.

Conclusions

1) Kitaev's Z_2 lattice model implemented in the low energy sector of some Josephson junction arrays.
2) These arrays are composed of fully frustrated rhombi.
3) Topological protection arises in the phase where quantum phase fluctuations destroy the $2e$ condensate, while preserving the $4e$ condensate.
4) Experimental evidence for this phase: observation of enhanced immunity against static flux fluctuations, evidence of a finite Δ_{c}.
5) Protection still works in the presence of dynamical fluctuations, up to order $n = \Delta/D_{\text{eff}}$.