Models of inflation with primordial non-Gaussianities

Francis Bernardeau*, Tristan Brunier* and Jean–Philippe Uzan†

*Service de Physique Théorique, CEA/DSM/SPhT, Unité de recherche associée au CNRS, CEA/Saclay 91191 Gif-sur-Yvette cédex
†Institut d'Astrophysique de Paris, UMR7095 CNRS, Université Pierre & Marie Curie - Paris, 98 bis bd Arago, 75014 Paris, France

Abstract. We present a class of models in which the primordial metric fluctuations do not necessarily obey Gaussian statistics. These models are realizations of mechanisms in which non-Gaussianity is first generated by a light scalar field and then transferred into curvature fluctuations during or at the end of inflation. For this class of models we present generic results for the probability distribution functions of the metric perturbation at the end of inflation. It is stressed that finite volume effects can induce non trivial effects that we sketch.

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1. INTRODUCTION

Cosmic Microwave Background (CMB) observations offer a precious window for the physics of the early Universe. And it is well known that inflation generically predicts Gaussian initial metric fluctuations with an almost scale invariant power spectrum [1]. With the advent of large scale structure and CMB surveys however it will be possible to test in details the statistical properties of the initial conditions. So far no non-Gaussian signal has been detected in CMB data [2] but the number of modes that can be probed is still small. In large-scale structure surveys the number of independent modes that are observed is large but the difficulty arises from the non-linear gravitational dynamics [3] which can shadow the primordial Non-Gaussianity (NG). Having at our disposal models of inflation in which non-Gaussian adiabatic metric fluctuations are generated can then serve as a guideline for designing strategies for detecting primordial non-Gaussianities, in particular in the context of the Planck mission.

A number of propositions have been made which circumvent the usual bounds on primordial NG generic inflation (e.g. slow-roll single field inflation) predicts (see below). This is the case in particular when primordial adiabatic fluctuations are subdominant compared to density fluctuations generated during the pre- or re-heating phases. Examples of such models are the curvaton model (which relies on assumptions on the reheating phase [5]) or recent propositions of multiple-field inflation which are based on relatively well understood phases of preheating [6].

2. GENERIC INFLATION

As mentioned in the introduction, we are interested in models that can produce sufficiently large non-Gaussianity. To be more precise, that should be the case for at least the range of modes that are observationally relevant, i.e. that corresponds to the large scale structure scales.

Let us start by considering a single field, φ, in slow-roll inflation. The Klein-Gordon equation for its perturbation, $\delta\varphi$, is of the form

$$\ddot{\delta\varphi} + 3H\dot{\delta\varphi} - \frac{\Delta}{a^2}\delta\varphi = -V''\delta\varphi - V'''\frac{\delta\varphi^2}{2} + \dots \quad (1)$$

During the slow-roll regime, $|\dot{H}| \ll H^2$ and $\ddot{\varphi} \ll 3H\dot{\varphi}$ so that[1]

$$3H^2M_4^2 \simeq V, \qquad H\dot{\varphi} \simeq -V' \quad (2)$$

[1] We have set $M_4^{-2} \equiv 8\pi G$.

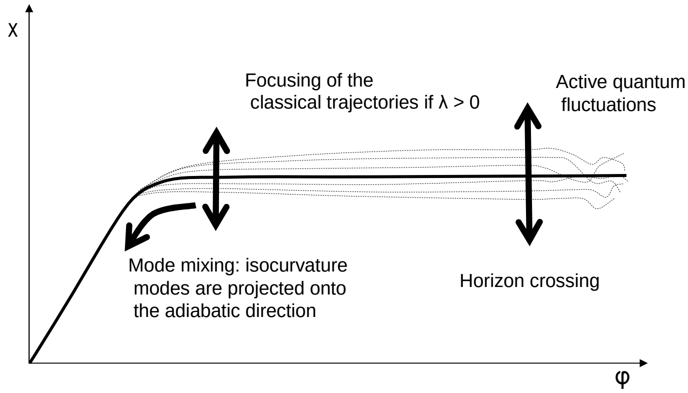

FIGURE 1. The trajectories of the fields in the plane (ϕ,χ), once smoothed on a scale R. Before horizon crossing ($R<H$), the trajectories behave quantumly because the quantum fluctuations are active up to scale H and are not smoothed out. After the horizon crossing ($R>H$), the trajectories can be treated as classical trajectories. Note that this transition happens at different time for different values of R. The bundle of classical trajectories then evolves in the two dimensional potential and its cross section evolves with time. In the case sketched here, because of the bending of the potential valley, the isocurvature modes induce metric fluctuations because of differences in length of the trajectories. The net result is a transfer of modes between isocurvature and adiabatic directions.

and the slow-roll conditions can be expressed as $\varepsilon \ll 1$ and $|\eta| \ll 1$ with

$$\varepsilon \equiv M_4 \frac{V'}{V}, \qquad \eta \equiv M_4^2 \frac{V''}{V}. \tag{3}$$

In order for the fluctuations of the scalar field to be large enough, one needs the mass of the field to be much smaller than H, in which case one gets $\langle\delta\varphi\rangle \sim H$. Furthermore to get the correct amplitude for the primordial fluctuations, one should have $V^{3/2}/(V'M_4^3) \sim 10^{-5}$ so that

$$H \sim 10^{-5}\varepsilon M_4. \tag{4}$$

The range of wavelengths that correspond to the observed large scale structure exits the Hubble radius over the number of e-foldings

$$N_\lambda = H\Delta t. \tag{5}$$

During that period, the slow-roll parameter η has varied over the range $\Delta\eta \sim (\xi - \eta\varepsilon)\Delta\varphi/M_4$, with $\xi \equiv M_4^3(V'''/V)$. From Eq. (4), one deduces that

$$\Delta\varphi/M_4 \sim -\varepsilon H\Delta t. \tag{6}$$

Now, if the quadratic term in the r.h.s. of Eq (1) is dominant over the linear term then $V''/V''' < \delta\varphi \sim H$ and, using Eq (4), one deduces that $\Delta\eta > 10^5\eta N_\lambda$. This will induce a rapid breakdown of the slow-roll inflation. This can be understood from the fact that the potential has to be both flat enough for the fluctuations to develop and steep enough for the non-linear terms not to be negligible[2]

3. MULTIPLE FIELD INFLATION

The previous analysis shows that it is not possible to get significant NG effects over the observable wavelengths with single field inflation. To obtain such effects, extra fields are necessary. A class of multiple field inflationary model corresponds to a Lagrangian where N scalar fields are minimally coupled to the metric, e.g.,

$$\mathscr{L} = -\frac{R}{16\pi G} + \frac{1}{2}\sum_{j=1}^{N}\partial_\mu\varphi_j\partial^\mu\varphi_j - V(\varphi_1,\ldots,\varphi_N), \tag{7}$$

[2] A way round to this argument is to consider potential with a sharp feature [4] but in that case the non-Gaussianity is associated with a departure from scale invariance and is located on a very small band of wavelengths.

where R is the Ricci scalar. It follows that the background Einstein equations take the form,

$$H^2 = \frac{4\pi G}{3}\left(\sum_{j=1}^{N}\dot{\varphi}_j^2 + 2V\right), \qquad \dot{H} = -4\pi G \sum_{j=1}^{N}\dot{\varphi}_j^2, \qquad \ddot{\varphi}_j + 3H\dot{\varphi}_j = -V_j, \tag{8}$$

where a dot refers to a derivation with respect to the cosmic time, t, $H \equiv \dot{a}/a$ and V_j refers to a derivation with respect to φ_j.

Then, let us consider the simplest case in which we have only two scalar fields φ_1 and φ_2. We decompose these two fields into the genuine inflationary direction $\delta\varphi$ and a transverse direction χ, taking advantage of the direction defined by the inflationary direction,

$$\begin{pmatrix}\dot{\delta\varphi}\\ \dot{\chi}\end{pmatrix} = \mathscr{M}(\theta)\begin{pmatrix}\dot{\varphi}_1\\ \varphi_2\end{pmatrix}, \qquad \mathscr{M}(\theta) \equiv \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}, \tag{9}$$

the angle θ being defined by

$$\cos\theta \equiv \frac{\dot{\varphi}_1}{\sqrt{\dot{\varphi}_1^2+\dot{\varphi}_2^2}}, \qquad \sin\theta \equiv \frac{\dot{\varphi}_2}{\sqrt{\dot{\varphi}_1^2+\dot{\varphi}_2^2}}. \tag{10}$$

From these definitions it follows that χ is stationary along the classical inflationary trajectory. To be more precise, the Klein-Gordon equations imply that,

$$\ddot{\delta\varphi} + 3H\dot{\delta\varphi} - \frac{\Delta\delta\varphi}{a^2} = -\frac{\partial^2 V}{\partial\delta\varphi^2}\delta\varphi - \frac{\partial^2 V}{\partial\delta\varphi\partial\chi}\chi, \tag{11}$$

$$\ddot{\chi} + 3H\dot{\chi} - \frac{\Delta\chi}{a^2} = \ldots - \frac{\partial^3 V}{\partial\chi^3}\frac{\chi^2}{2} - \frac{\partial^4 V}{\partial\chi^4}\frac{\chi^3}{3!}. \tag{12}$$

This set of equation clearly opens the way to new phenomena. Indeed, nothing prevents the fluctuations in the χ directions to develop significant non-Gaussian properties. This would be the case for instance for generic quartic potentials. Those field fluctuations can appear as metric fluctuations if, at some stage before the end of the inflationary period, the χ fluctuations can be transferred into the adiabatic fluctuation. According to Eq. (11), this is possible provided $\partial^2 V/(\partial\delta\varphi\partial\chi)$ is not zero. Geometrically speaking, it corresponds to a bent in the inflationary trajectory [3] [7, 8]. This idea is sketched on diagram 1.

3.1. A simple working example: extended hybrid inflation

Although generic predictions can be made for such a class of models, it is to be stressed that such a scenario does not require elaborate or exotic field theory constructions. Let's for instance consider the following multiple-field potential presented in [10],

$$V(\phi,\chi,\sigma) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\chi^4 + \frac{\mu}{2}\left(\sigma^2-\sigma_0^2\right)^2 + \frac{g}{2}\sigma^2\left(\phi\cos\alpha + \chi\sin\alpha\right)^2. \tag{13}$$

It is an extension of the hybrid model with one extra field. Here, one field, ϕ, is the inflaton; the second field is a light scalar, χ, with quartic coupling $0 < \lambda \leq 1$ and the third field, σ, is coupled to the two others so that the end of inflation is triggered when σ undergoes a phase transition and σ_0 is the final vev of σ. One extra parameter α describes the mixing angle between ϕ and χ in their coupling to σ. This parameter will determine the ratio between the initial isocurvature and adiabatic modes in the final metric fluctuations. Note that a quartic interaction has been introduced since this is the only one that does not require further fine tuned parameter (any other polynomial type interaction would require dimensioned parameter tuned to H, see [8]).

In this model, as for hybrid inflation, the inflationary stage ends when the effective mass of σ vanishes, that is when

$$g\left(\phi\cos\alpha + \chi\sin\alpha\right)^2 - 2\mu\sigma_0^2 = 0. \tag{14}$$

[3] Note that this bent can also be encoutered during the preheating phase. An example of such a model in presented in [9].

The value of ϕ at the end of inflation is therefore

$$\phi_{\rm end} \equiv \frac{\pm\sqrt{2\mu/g}\,\sigma_0 - \chi\sin\alpha}{\cos\alpha}. \tag{15}$$

For $\phi > \phi_{\rm end}$, $\sigma = 0$ and the two fields evolve independently: ϕ drives the inflation while χ develops non-Gaussianity. The amount of non-Gaussianity of χ then depends only on λ and on the total number of e-foldings between horizon crossing and the end of inflation.

3.2. Test quantum scalar field in de Sitter space

The non-Gaussian properties of the χ field during the inflationary phase are those developed by a self interacting scalar field in an expanding universe. The full resolution of this problem in quite involved. It can be addressed with a Perturbation Theory approach that can be done either at a quantum level or, once the horizon has been crossed, at a classical level (assuming the scalar field behaves like a classical stochastic field).

In [12] we present the result of the computation of the high order correlation function of a test scalar field in a de Sitter or quasi de Sitter background. The calculation is based on an expansion of the field evolution from the free field solution as summarized in the following. For a minimally coupled free quantum field, the solution can indeed be decomposed in plane waves as,

$$\widehat{v}_0(\mathbf{x},\eta) = \int {\rm d}^3\mathbf{k}\left[v_0(k,\eta)\widehat{b}_{\mathbf{k}}{\rm e}^{i\mathbf{k}\cdot\mathbf{x}} + v_0^*(k,\eta)\widehat{b}^\dagger_{\mathbf{k}}{\rm e}^{-i\mathbf{k}\cdot\mathbf{x}}\right], \tag{16}$$

where we have introduced $\widehat{v} \equiv a\,\widehat{\chi}$, a hat referring to a quantum operator and $b_{\mathbf{k}}$ and $b^\dagger_{\mathbf{k}}$ and the annihilation and creation operators for a particle of momentum $\mathbf{k}$ in a Bunch-Davies vacuum. In the massless limit, the free field solution is

$$v_0(k,\eta) = \left(1 - \frac{{\rm i}}{k\eta}\right)\frac{{\rm e}^{-{\rm i}k\eta}}{\sqrt{2k}}. \tag{17}$$

One can then express perturbatively the N-point correlation functions of the interacting field, χ, in terms of those of the free scalar field. The equal time correlators are expectation values of product of field operators for the current time vacuum state. Such computations can be performed following general principles of quantum field calculations [11]. The simplest formulation is to apply the evolution operator $U(\eta_0,\eta)$ backward in time to transform the interacting field vacuum into the free field vacuum at an arbitrarily early time η_0 so that,

$$\langle v_{\mathbf{k}_1}\ldots v_{\mathbf{k}_n}\rangle \equiv \langle 0|U^{-1}(\eta_0,\eta)\; v_{\mathbf{k}_1}\ldots v_{\mathbf{k}_n}\; U(\eta_0,\eta)|0\rangle \tag{18}$$

where $|0\rangle$ is here the *free field* vacuum [4].

The evolution operator U can be written in terms of the interaction Hamiltonian, H_I, as

$$U(\eta_0,\eta) = \mathscr{T}\exp\left(-{\rm i}\int_{\eta_0}^{\eta}{\rm d}\eta'\;H_I(\eta')\right) \tag{19}$$

where $\mathscr{T}$ is the time ordered product operator[5]. Eventually the connected part of the above ensemble average at a time η reads,

$$\langle v_{\mathbf{k}_1}\ldots v_{\mathbf{k}_n}\rangle_c = -{\rm i}\int_{\eta_0}^{\eta}{\rm d}\eta'\;\langle 0|\left[v_{\mathbf{k}_1}\ldots v_{\mathbf{k}_n}, H_I(\eta')\right]|0\rangle, \tag{20}$$

where the brackets stand for the commutator.

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[4] It is to be noted that these calculations do not correspond to those of diffusion amplitudes of some interaction processes in a de Sitter space (see [13] for a comprehensive presentation of those calculations.). When one tries to do these latter calculations with a path integral formulations, mathematical divergences are encountered as it has been stressed in [14, 8]. With that respect de Sitter space strongly differs from Minkowski space-time.

[5] Then the inverse operator for U reads, $U^{-1}(\eta_0,\eta) = \bar{\mathscr{T}}\exp\left(+{\rm i}\int_{\eta_0}^{\eta}{\rm d}\eta'\;H_I(\eta')\right)$ where $\bar{\mathscr{T}}$ is the inverse $\mathscr{T}$ product.

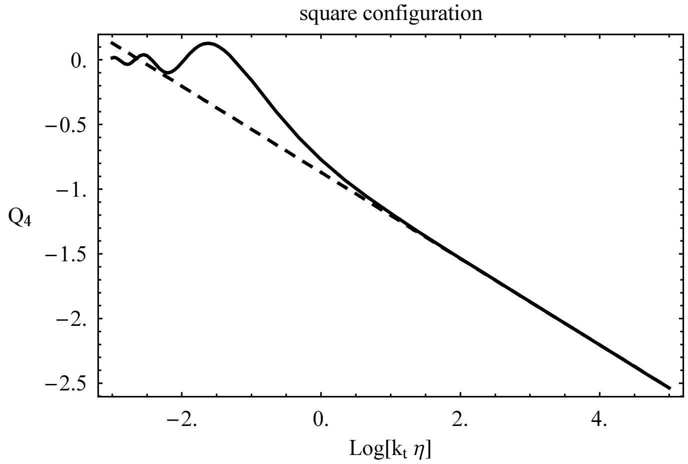

FIGURE 2. Behaviour of the function Q_4 as of function of time. The transition to the superhorizon behavior (dashed line) is shown. The function Q_4 is shown here for a "square" configuration ($k_1 = k_2 = k_3 = k_4$) as a function of $k_t\eta = \sum k_i\eta$.

The result for the four-point function in case of a quartic interaction (with parameter λ) can finally be written as,

$$\langle\chi_{\mathbf{k}_1}\dots\chi_{\mathbf{k}_4}\rangle_c = \delta_{\rm Dirac}\left(\sum \mathbf{k}_i\right)P_4(k_1,k_2,k_3,k_4), \qquad P_4(k_1,k_2,k_3,k_4) = \nu_3(\{k_i\})\sum_i\prod_{j\neq i}\frac{H^2}{2k_j^3}, \tag{21}$$

where the vertex value, ν_3, reads

$$\nu_3(\{k_i\}) = \frac{\lambda}{3H^2}\left[\zeta(\eta,k_i)+\log\left(-\eta\sum k_i\right)\right]. \tag{22}$$

In this expression $\zeta(\eta,k_i)$ can be expressed[6] in terms of the four wavelength k_i. It is finite for any values of η and k_i (in particular in the super-Hubble limit).

When the term $\log\left(-\eta\sum k_i\right)$ is large (and negative), that is when the number of e-foldings, N_e, between the time of horizon crossing for the modes we are interested in and the end of inflation is large, the vertex value is simply given by,

$$\nu_3(\{k_i\}) = -\lambda\, N_e/(3H^2),$$

which corresponds exactly to what a classical stochastic approach would give [8]. On Fig. 2 one can then appreciate the transition from a regime which is dominated by quantum fluctuations to a regime following a classical evolution. In this figure, what is plotted is the reduced four-point correlation function, Q_4, defined as,

$$Q_4(\{k_i\}) = \frac{P_4(k_1,k_2,k_3,k_4)}{P_2(k_1)P_2(k_2)P_2(k_3)+\text{sym.}} \tag{23}$$

where $P_2(k)$ is the free field power spectrum.

These results illustrate one quantitative aspects of the NG properties that self-coupled scalar fields can develop, namely high order correlation functions, whose computation is given here at tree order. These calculations are on solid ground. They do not give however a complete prescription for the description of the statistical properties of the field.

3.3. PDF and finite volume effects, a classical stochastic approach

The use of a classical perturbative approach can provide us with a complete description of the expected properties of the resulting field properties. In particular it is possible to infer the whole one point probability distribution function (PDF) of the field value.

[6] note that ζ differs from the one introduced in [12].

The approach detailed in [8] and [15] is based on the expansion the filtered field in terms of the coupling constant as

$$\chi_{\rm S}(\eta) = \chi_{\rm S}^{(0)}(\eta) + \chi_{\rm S}^{(1)}(\eta) + \ldots. \tag{24}$$

The subscript S stands for the fact that the field has been convolved with a given smoothing window function in such a way that only large enough scales are taken into account. Then $\chi_{\rm S}^{(0)}$ represents the value of the filtered field when the self-interacting term in the potential is dropped. In the slow-roll regime the field $\chi_{\rm S}$ follows the Klein-Gordon equation (12). Then $\chi_{\rm S}^{(0)}$ is simply constant during the slow roll evolution. It represents the initial conditions for the non-linear field evolution. At first order term in λ, $\chi_{\rm S} \equiv \chi_{\rm S}{}^{(1)}$, evolves according to

$$3H\dot{\chi}_{\rm S}{}^{(1)} = -\frac{\lambda}{3!}\left[\chi_{\rm S}^{(0)}\right]^3 \tag{25}$$

for a quartic potential. In this approach the treatment of the filtering of the r.h.s. of this equation is very crude. This is the price to pay for using this approach. The results we are going to find can anyway be checked against the more rigorous calculations based on the computation shape of the tri-spectrum presented in the previous subsection.

Resolving Eq. (25), leads to the expression of $\chi_{\rm S}{}^{(1)}$ as a function of $\chi_{\rm S}^{(0)}$. $\chi_{\rm S}$ is actually built out from modes whose wavelengths are larger than the smoothing scale. During inflation, $\chi_{\rm S}$ can be viewed as a classical random field as soon as the smoothing scale has crossed the comoving horizon. Today, $\chi_{\rm S}$ is made of a the superposition of modes that are super-horizon and of modes which are now within our horizon. It is important to understand that, although the ensemble average of $\chi_{\rm S}$ actually vanishes, its geometrical mean at the survey size, as given by observations, is expected to be non-zero (since at least super-horizon modes are out of reach).

The value of $\chi_{\rm S}$ at our horizon scale, say $\overline{\chi_{\rm S}^{(0)}}$, appears therefore as a new free parameter. Its value is determined by the peculiar realization of the inflationary path our Universe has followed. In our model case $\chi_{\rm S}^{(0)}$ is bounded ($\lambda > 0$) and its range can be estimated through a Fokker-Planck equation [16, 15]. Phenomenologically it means that one should decompose $\chi_{\rm S}^{(0)}$ as,

$$\chi_{\rm S}^{(0)} = \delta\chi_{\rm S}^{(0)} + \overline{\chi_{\rm S}^{(0)}}. \tag{26}$$

where $\delta\chi_{\rm S}^{(0)}$ is then Gaussian distributed with zero average and given variance σ_δ. Similarly,

$$\chi_{\rm S} = \delta\chi_{\rm S} + \bar{\chi} \tag{27}$$

where $\bar{\chi}$ corresponds to the averaged value of χ at our horizon size.

The equation of evolution (25) can be solved to get

$$\chi_{\rm S}^{(1)}(t) = -\lambda(t - t_{\rm H})\frac{\left(\chi_{\rm S}^{(0)}\right)^3}{18H}, \tag{28}$$

which also reads

$$\chi_{\rm S}^{(1)}(t) = -\frac{\lambda N_e}{18H^2}\left(\chi_{\rm S}^{(0)}\right)^3, \tag{29}$$

N_e being the number of e-foldings between t_H and the end of inflation.

These results imply that

$$\bar{\chi} \approx \overline{\chi_{\rm S}^{(0)}} - \frac{\lambda N_e}{18H^2}\left[\left(\overline{\chi_{\rm S}^{(0)}}\right)^3 + 3\,\overline{\chi_{\rm S}^{(0)}}\,\sigma_\delta^2\right] \tag{30}$$

which explicitly shows that $\bar{\chi}$ and $\overline{\chi_{\rm S}^{(0)}}$ are equal at leading order in λ. It also gives

$$\delta\chi_{\rm S} = \delta\chi_{\rm S}^{(0)} - \frac{\lambda N_e}{18H^2}\left\{\left(\delta\chi_{\rm S}^{(0)}\right)^3 + 3\left[\left(\delta\chi_{\rm S}^{(0)}\right)^2 - \sigma_\delta^2\right]\bar{\chi} + 3\delta\chi_{\rm S}^{(0)}\bar{\chi}^2\right\}. \tag{31}$$

It is straightforward to see that $\delta\chi_{\rm S}$ has acquired a non zero third order moment,

$$\overline{\delta\chi_{\rm S}^3} = -\frac{\lambda N_e}{H^2}\bar{\chi}\sigma_\delta^4 \tag{32}$$

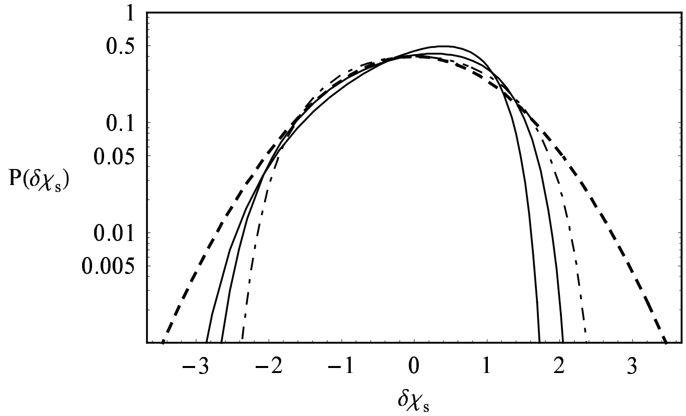

FIGURE 3. PDF of $\delta\chi_s = \delta\chi_s - \bar{\chi}$ for different values of $\bar{\chi}$. The dashed line corresponds to a Gaussian distribution; the dot-dashed line to the deformed distribution of $\delta\chi_s$ when $\lambda N_e/H^2 = 1$ and $\bar{\chi} = 0$ and the solid lines when the latter equals 0.5 and 1.

at leading order in λ. This is a finite volume effect in the sense that it exists for a fixed (not ensemble averaged) value of $\bar{\chi}$. This effect cannot a priori be neglected. It can be shown that $\bar{\chi}$ should be of the order of $H/\lambda^{1/4}$, which implies that the reduced skewness of $\delta\chi_s$, $\overline{\delta}\chi_s^3/(\overline{\delta}\chi_s^2)^{3/2} \sim \lambda^{3/4} N_e \sigma_\delta / H$ is significant as soon as $\lambda^{3/4} N_e$ approaches unity.

An attentive reader cannot but notice that the evolution equation for χ_s can in fact be solved in

$$\chi_s = \frac{\chi_s^{(0)}}{\sqrt{1 - \frac{\lambda N_e}{9H^2}\left(\chi_s^{(0)}\right)^2}} \tag{33}$$

and the distribution of χ_s can then be infered from that of $\chi_s^{(0)}$ assuming the latter is Gaussian distributed with a non-zero mean value [7].

In figure 3, we present the deformation of the PDF of $\delta\chi_s$ while $\bar{\chi}$ is varied. As expected, it shows that when $\bar{\chi}$ is not zero, the PDF gets skewed in a way that can be easily understood: when $\bar{\chi}$ is positive it gets more difficult to have excursion towards larger value of χ_s, but easier to roll down to smaller values. It is as if the field χ was actually evolving in the potential $\lambda(\chi + \bar{\chi})/4!$.

What should then be expected for observations? In this family of models, the surviving couplings in the metric, are expected to be, to a good approximation, equivalent to those induced by the superposition of two stochastically independent fields, a Gaussian one and one obtained by a non-linear transform of a Gaussian field with the same spectrum, e.g. Eq. (33). In other words the local (Bardeen) potential would read,

$$\Phi(\mathbf{x}) = \cos\alpha\ \Phi_1(\mathbf{x}) + \sin\alpha\ \mathscr{F}[\Phi_2(\mathbf{x})] \tag{34}$$

where ratio between the initial adiabatic and isocurvature fluctuations is described by a mixing angle α and where the function $\mathscr{F}$ takes into account the self coupling of the field χ that gave rise to this part of the metric fluctuations. This description is the starting point of investigations for the CMB properties presented in [17].

4. CONCLUSIONS

We have shown here that it is possible to build explicit models of inflation where primordial metric fluctuations can develop significant non-Gaussian features. The class of models we have identified induce NG properties that can

[7] As noted in Ref. [8], such a simple variable change implicitly incorporates "loop order" effects that, because of sub-Hubble physics, are not necessarily correctly estimated. In that paper we developed a more elaborated method which allows the reconstruction of the PDF from the only tree order contributions of each cumulant. As the two approaches eventually give the same qualitative results we restrict here our analysis to the simplest method.

be described by a few identified parameters. Two of them are of fundamental origin, the amplitude of the quartic coupling and the mixing angle; a third is due to finite volume effects and its value depends on the peculiar realization of the universe we live in, the transverse field value at Hubble size. Unlike most models proposed in the literature, they provide us with well defined properties of the metric fluctuations that can serve as a starting point for further investigations on its various observational aspects, both for CMB and large scale structure surveys [17]. In particular, the models do not require to deal with the more intricate second order dynamics.

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