BPS SATURATED AMPLITUDES AND NON-PERTURBATIVE STRING THEORY.

Cargèse 1997, CPTH.PC571.1297, hep-th/9712079

P. VANHOVE
Centre de Physique Théorique École Polytechnique
91128 Palaiseau, France
vanhove@pth.polytechnique.fr

Abstract. The study of the special F^4 and R^4 in the effective action for the $\mathrm{Spin}(32)/\mathbf{Z}_2$ and Type II strings sheds some light on D-brane calculus and on instanton contribution counting. The D-instanton case is discussed separately.

Computations of non-perturbative contributions is still a difficult task. In (supersymmetric) Field theory a direct computation requires the construction of the measure of integration over vacua containing n-instantons, which is not yet available for every n. The use of dualities is still the main tool for getting the answer. In String theory the D-instantons contributions for the R^4 were conjectured thanks to the $Sl(2,\mathbf{Z})$ invariance of the Type IIb theory [1], and for the $\mathrm{Spin}(32)/\mathbf{Z}_2$ some *ad hoc* rules for the contributions for the R^4 and F^4 were derived from the S-duality relation between the Heterotic and Type I strings [2]. In this proceeding I will explain how these non-perturbative contributions can be computed exactly, concentrating on the eight-dimensional case. Details and generalizations will be found in [3].

On the Heterotic side the F^4 and R^4 terms are extracted from the fourth point amplitude on the torus involving four vertex operators. In order to get a non zero contribution, eight fermionic zero modes have to be soaked up. So all the fermionic zero modes from the supersymmetric side of the vertex operators have to be saturated, leaving a complete freedom on the non-supersymmetrical current algebra side. These contributions involve only states belonging to the short multiplets of the $N_{4d}=4$ supersymmetry algebra [4]. The non-perturbative contributions in eight-dimensions read

$$\mathcal{L}_{\mathrm{non-pert}} = -\frac{2V^{(8)}}{2^{10}\pi^6}\sum_{N>0} e^{-2i\pi N T_s \mathcal{T}_2}\frac{1}{N}\sum_{\substack{N=mn\\ 0\leq j<n}}[\mathcal{D}\mathcal{A}]\left(\frac{m\mathcal{U}+j}{n}\right) \ . \qquad (1)$$

In this expression $\mathcal{T}$ and $\mathcal{U}$ are the moduli of the two-torus of compactification, T_s is the tension of the soliton and the $\mathcal{A}(q, F, R)$ is a modular invariant partition function computed in the Ramond sector expanded at the eight order with respect to the space-time fermionic zero modes [5]. For Type II string for the R^4 terms we don't have anymore freedom for a current algebra, thus $\mathcal{A} = 1$. $\mathcal{D}$ is a differential operator whose appearance is related to non-holomorphicities of the elliptic genus [2]. Hereafter only the leading part is needed $\mathcal{D} = 1 + \dots$. These fluctuations around the wrapped D1-brane are described by a SO($2N$) Super-Yang-Mills (SYM) theory (N can be an half-integer). Thanks to the holomorphicity of $\mathcal{A}$, (1) can be computed at the infra-red (IR) fixed point of this SYM theory: $g_{\rm SYM} \to \infty$ with $\alpha' g_{\rm SYM}^2 g_s^2 = 1$ [6]. This limit is a (8,0) supersymmetrical σ-model with target space the symmetric orbifold

$$\mathcal{M} = (\mathbf{R}^8)^N / \mathcal{S}_N \ . \tag{2}$$

The Euclidean action in the light-cone gauge

$$\begin{aligned} S_{\rm E} = \int dt \int_0^{2\pi} \frac{d\sigma}{2\pi} {\rm Tr} \Big[&\partial \mathbf{X}^i \left(\bar\partial \delta_{ij} + i \mathcal{R}_{ij}/2\pi \right) \mathbf{X}^j + \mathbf{S}_a \partial \mathbf{S}_a + \\ + {}^{\rm T}\lambda_A^{(P)} \left(\bar\partial \delta^{AB} + i \mathcal{F}^{AB}/2\pi \right) \lambda_B^{(P)} &+ {}^{\rm T}\lambda_A^{(A)} \left(\bar\partial \delta^{AB} + i \mathcal{F}^{AB}/2\pi \right) \lambda_B^{(A)} \Big] \end{aligned} \tag{3}$$

describes the coupling between the fluctuations, $\mathbf{X}^i$, $\mathbf{S}_a$ living in the adjoint representation of SO($2N$) and λ_A in vectorial representation of SO($2N$), with the background fields

$$\begin{aligned} (\mathcal{R}_{ij})_{MN} &= -\frac{1}{8} \mathcal{R}_{ijkl} (\mathbf{S}_0^a)_{MP} \gamma_{ab}^{kl} (\mathbf{S}_0^b)_{PN} \\ (\mathcal{F}^{AB})_{MN} &= -\frac{1}{8} \mathcal{F}_{kl}^{AB} (\mathbf{S}_0^a)_{MP} \gamma_{ab}^{kl} (\mathbf{S}_0^b)_{PN} \end{aligned} \qquad M, N, P \in \{1, \cdots, 2N\} \ . \tag{4}$$

It is important to remark that the space-time fermionics zero modes $\mathbf{S}_0^a$ enter only through these tensors. The various twisted sectors of the Hilbert space constructed on $\mathcal{M}$ are labeled by the action of the permutations $\sigma \in \mathcal{S}_N$ decomposed in disjoint cycles as $\sigma = [1^{N_1}, \dots, N^{N_N}]$ with $N = \sum_i i N_i$ [6]. A sector with boundary conditions twisted by σ will contain $8 \times \sum_i N_i$ fermionic zero modes. As only terms with eight fermionic zero modes are needed, we must have $\sigma = [N^1]$. According to the rules of orbifold computation the determinant is given by the partition function on a torus computed over the invariant states [7]:

$$Z = \frac{1}{N!} \sum_{\omega = [N^1]} \sum_{\substack{\eta \\ \eta\omega = \omega\eta}} {}_{\eta}\square_{\omega} \ , \tag{5}$$

${}^{\eta}\square_{\omega}$ represents the functional integral

$${}^{\eta}\underset{\omega}{\square} = \int_{\text{Torus}} \mathcal{D}X\mathcal{D}S^a\mathcal{D}\lambda\ e^{-S_{\text{E}}}$$

calculated for the boundary conditions specified by η and ω. Yielding exactly the determinant of (1)

$$Z = \frac{1}{N}\left[\mathcal{A}(N\mathcal{U}) + \sum_{\substack{n_1 n_2 = N\\ n_2\neq 1}} \sum_{j=0}^{n_2-1} \mathcal{A}\left(\frac{n_1\mathcal{U}+j}{n_2}\right)\right] . \tag{6}$$

The Type II case, is very similar in spirit, the fluctuations around a vacua containing wrapped D-branes are described by a U(N) SYM model with an IR limit described by a (8,8) supersymmetrical σ-model (coupled to background for terms involving less zero modes). The big difference here is the existence of a U(1) gauge field freedom allowing 't Hooft fluxes [8]. Theses fluxes constraint the twisted states to appear as bound states of (p,q)-strings, $T_s = |p+q\tau|$ in (1). This gauge field is a remnant of the invariance under volume preserving diffeomorphism of the D-brane's world-volume theory. For the Spin(32)/$\mathbf{Z}_2$ case it was projected out by the Ω parity projection. As briefly explained in [9], seeing the global topology of the solitonic configuration, the gauge configuration contributions to the determinant of fluctuations are responsible for the appearance of the Bessel function $K_1(\cdots)$. In a T-dual language, this is the origin of the fractionalization of the D-instanton charge $N = nm$ [1, 10]. The D-instanton case is more tricky because the associated topological SYM model does not have a well defined IR limit. Nevertheless, [11] shown that these contributions stem from a one-loop computation involving the super-graviton compactified on a two-torus with complex structure $\tau = C^{(0)} + ie^{-\phi}$

$$\begin{aligned}\mathcal{L}^{FT} &= \int_0^\infty \frac{dt}{t}\,\mathrm{Str}\left((\mathcal{R}_{ijkl}M^{ij}\tilde{M}^{kl})^4 e^{-tS}\right)\\ &= t_{(8)}t_{(8)}\mathcal{R}^4\frac{\pi^{3/2}}{\Gamma(3/2)}\int_0^\infty \frac{dt}{t}\ t^{3/2}\sum_{(l_1,l_2)\neq(0,0)} e^{-\pi t\frac{|l_1+l_2\tau|^2}{\tau_2}} .\end{aligned} \tag{7}$$

M^{ij} ($\tilde{M}^{kl}$) is a Lorentz generator in the adjoint representation of SO(8) containing two left (right) fermionic zero modes [12], $\mathcal{R}_{ijkl}$ is the curvature. The super-trace being computed over 1/2-BPS multiplet of the IIb supergravity algebra. In lower dimensions the super-trace computed over the 1/2-BPS momentum multiplets of the U-duality group [9], yields the weight-3/2 Poincaré series, of course 1/2-BPS states from fluxes multiplets can contribute too [13]. In ten dimension this expression can be rewritten as

$$\begin{aligned}\mathcal{L}^{FT} &= 2\zeta(3)\tau_2^{3/2} + \frac{2\pi^2}{3}\tau_2^{-1/2} + \\ & 8\pi\sum_{N\geq 1}\left(e^{2i\pi N\tau}+c.c.\right)\left[\sum_{N=mn}\frac{1}{m^2}\right]\left\{\sqrt{N}\left(1+\sum_{k=1}^{\infty}\frac{(-1)^k}{(4\pi N\tau_2)^k}\frac{\Gamma\left(k-\frac{1}{2}\right)\Gamma\left(k+\frac{3}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2}\right)k!}\right)\right\}.\end{aligned} \tag{8}$$

The contribution in square bracket $[\cdots]$ should come from the same counting of zero modes as before [10]. The contribution in braces $\{\cdots\}$ is the signature of higher loops corrections around the D-instantons.

Acknowledgements

I want to thank the (*gentils*) organizers of the Cargèse summer school for financial support, and allowing me to give a talk. These works were done in collaboration with C. Bachas, C. Fabre, M.B. Green, M. Gutperle, E. Kiritsis, N.A. Obers. Moreover it is a pleasure to thanks K. Förger, P. Windey, F. Laudenbach and N. Berline for interesting discussions.

References

1. M.B. Green and M. Gutperle, Nucl. Phys. **B498** (1997) 195.
2. C. Bachas, C. Fabre, E. Kiritsis, N.A. Obers and P. Vanhove Nucl. Phys. **B509** (1998) 33.
3. C. Bachas, *Heterotic versus Type I*, hep-th/9710102;
   C. Bachas and P. Vanhove, *The Rules of D-brane Calculus*, CPTH-S587.1297.
4. C. Bachas and E. Kiritsis, Nucl. Phys. Proc. Suppl. **55B** (1997) 194.
5. W. Lerche, B. Nilsson and A. Schellekens, Nucl. Phys. **B289** (1987) 609;
   W. Lerche, B. Nilson, A. Schellekens and N. Warner, Nucl. Phys. **B299** (1988) 91;
   W. Lerche, Nucl. Phys. **B308** (1988) 102
6. W. Taylor, Phys. Lett. **394B** (1997) 283;
   R. Dijkgraaf, E. Verlinde and H. Verlinde, Nucl. Phys. **B500** (1997) 43 and *Notes on Matrix and Micro Strings*, hep-th/9709107;
   S.J. Rey, Nucl. Phys. **B502** (1997) 170;
   D.A. Lowe, Phys.Lett. **403B** (1997) 243.
7. P. Ginsparg, *Applied Conformal Field Theory* Les Houches session XLIX, ed. E. Brézin et J. Zinn-Justin Elsevier Science Publisher B.V. 1989.
8. G. 't Hooft, Nucl. Phys. **B153** (1979) 141;
   O.J. Ganor, S. Ramgoolam and W. Taylor IV, Nucl. Phys. **B492** (1997) 191.
9. M.B. Green and P. Vanhove, Phys. Lett. **408B** (1997) 122.
10. M.B. Green and M. Gutperle, *D-particle bound states and the D-instanton measure*, hep-th/9711107.
11. M.B. Green, M. Gutperle and P. Vanhove, Phys. Lett. **409B** (1997) 177.
12. M.B. Green, J.H. Schwarz and E. Witten *Superstring theory*, Appendix 9.A, Cambridge Univ. Press. (1987).
13. S. Elitzur, A. Giveon, D. Kutasov and E. Rabinovici, *Algebraic Aspects of Matrix Theory on* T^d, hep-th/9707217.