# Reconstructing azimuthal distributions in nucleus–nucleus collisions

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**Abstract**

Azimuthal distributions of particles produced in nucleus-nucleus collisions are measured with respect to an estimated reaction plane which, because of finite multiplicity fluctuations, differs in general from the true reaction plane. It follows that the measured distributions do not coincide with the true ones. I propose a general method of reconstructing the Fourier coefficients of the true azimuthal distributions from the measured ones. This analysis suggests that the Fourier coefficients are the best observables to characterize azimuthal anisotropies because, unlike other observables such as the in-plane anisotropy ratio or the squeeze-out ratio, they can be reconstructed accurately.

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A characteristic aspect of collective behavior in nucleus–nucleus collisions is that the directions of the outgoing particles are correlated to the orientation of the impact parameter [1]: azimuthal distributions measured from the reaction plane (which is the plane containing the impact parameter and the beam axis) are not uniform. At energies between 100 MeV and 2 GeV per nucleon, several experiments have measured azimuthal distributions of charged particles [2], identified protons and light nuclei [3, 4], charged [5] and neutral [6] pions, neutrons [7, 8] and $\Lambda$ baryons [9]. Similar results have recently become available from the ultrarelativistic heavy ion experiments performed at the Brookhaven AGS [10] and at the CERN SPS[11] where proton and charged pion azimuthal distribution have been measured. In all these analyses, the azimuthal angle is defined with respect to an estimated reaction plane which, because of finite multiplicity fluctuations, differs from the true reaction plane. It has been recently emphasized that a determination of nuclear equation of state from azimuthal anisotropies requires a good accuracy [12]. It is therefore important to correct the errors which are made in determining the reaction plane.

We propose a systematic procedure to reconstruct the true azimuthal distribution from the measured ones. Normalized azimuthal distributions can be expressed as Fourier series:

$$\frac{dN}{d\phi} = \frac{1}{2\pi}\left(1 + 2\sum_{n\geq 1} c_n \cos n\phi\right) \tag{1}$$

where $c_n = \langle \cos n\phi \rangle$, the brackets denoting average values, and we assume that the azimuthal distributions are symmetric with respect to the reaction plane (i.e. even in $\phi$), which holds for spherical nuclei. The knowledge of all the Fourier coefficients $\langle \cos n\phi \rangle$ allows to reconstruct the full distribution using Eq.(1).

In an actual experiment, the reaction plane is not known exactly. It is reconstructed event by event from the reaction products. The reconstructed plane differs in general from the true reaction plane by an error $\Delta\phi$, which varies from one event to the other. Thus, the measured azimuthal angle $\psi$ is related to the true azimuthal angle $\phi$ by $\psi = \phi - \Delta\phi$ (see Fig.1). Averaging over many events, assuming that $\phi$ and $\Delta\phi$ are statistically independent (this assumption will be discussed below), one obtains the following relation between the measured and true Fourier coefficients[10, 13, 14]:

$$\langle \cos n\psi \rangle = \langle \cos n\phi \rangle \langle \cos n\Delta\phi \rangle. \tag{2}$$

From Eq.(2), we can reconstruct the true distribution once the correction factor $\langle \cos n\Delta\phi \rangle$ is known.

Before we calculate $\langle \cos n\Delta\phi \rangle$, let us briefly comment Eq.(2). This equation shows that the measured anisotropies are always smaller than the true ones: they are smeared by the error $\Delta\phi$. More precisely, if the probability distribution of $\Delta\phi$ has a typical width $\delta$, i.e. if $\delta$ denotes the typical error made in determining the reaction plane, $\langle \cos n\Delta\phi \rangle$ will decrease with $n$ and become small for $n > 1/\delta$. This has two consequences. First, the higher order coefficients disappear in the measurement procedure. Indeed, all the distributions measured so far are well reproduced by keeping only the first two Fourier coefficients $n = 1, 2$ in Eq.(1) [2, 5, 6, 7, 12]. However, higher order components might be sizeable in the true distributions.

The second consequence is that observables involving higher order Fourier coefficients cannot be reconstructed accurately. This is unfortunately the case for widely used observables such as the in-plane anisotropy [7, 15]

$$R_{\text{in-plane}} = \frac{(dN/d\phi)_{\phi=0}}{(dN/d\phi)_{\phi=180^\circ}} \tag{3}$$

or the squeeze-out ratio [2, 5, 6, 8]

$$R_{\text{squeeze}} = \frac{(dN/d\phi)_{\phi=90^\circ} + (dN/d\phi)_{\phi=270^\circ}}{(dN/d\phi)_{\phi=0^\circ} + (dN/d\phi)_{\phi=180^\circ}} \tag{4}$$

which both involve an infinite number of Fourier coefficients (see Eq.(1)). Furthermore, Fourier coefficients, which are integrated quantities, are also easier to evaluate in theoretical models (especially in Monte-Carlo calculations) than observables using the value of the distribution at a specific point.

We now recall how the orientation of the reaction plane is estimated from the reaction products: in high energy collisions ($E/A > 100$ MeV), the projectile (target) fragments are deflected away from the target (projectile). Therefore, the vector obtained by summing all the transverse momenta of the particles produced in the projectile (target) rapidity region is parallel (antiparallel) to the impact parameter. More generally, one constructs a vector $\mathbf{Q}$ [16]:

$$\mathbf{Q} = \sum_{k=1}^{N} w_k \, \mathbf{u}_k \tag{5}$$

where the sum runs over all the detected particles in the event. $\mathbf{u}_k$ is the unit vector parallel to the transverse momentum of the particle, and $w_k$ is a weight which may depend on the type of particle, its rapidity and transverse momentum. The choice of $w_k$ is to a large extent arbitrary. Danielewicz and Odyniec [16] choose $w_k = p_T$ for $y > 0.3$, $w_k = -p_T$ if $y < -0.3$ and $w_k = 0$ if $|y| < 0.3$. Many alternative definitions have been used [17, 18], some of which do not require particle identification. This method of determining the reaction plane is commonly referred to as the transverse momentum method. In experiments which measure the energy deposited in a calorimeter, without counting individual particles, an equation similar to Eq.(5) must be used, with $w_k$ replaced by the energy deposited in each segment of the calorimeter [19]. We come back to this case at the end of this paper.

For an ideal system with infinite multiplicity $N$, $\mathbf{Q}$ lies in the true reaction plane, and azimuthal distributions can be measured from $\mathbf{Q}$. In an actual experiment, the multiplicity is finite, which has two effects. First, because of statistical fluctuations, there is a deviation $\Delta\phi$ between the true reaction plane and $\mathbf{Q}$. Another effect of the finite multiplicity is that when one measures the azimuthal angle of a particle with respect to $\mathbf{Q}$, there is a correlation if the particle is included in the sum in Eq.(5). This can be avoided by constructing a new vector $\mathbf{Q}$ obtained by summation over the $N-1$ remaining particles [16]. Then, it is reasonable to assume, as we have done, that $\phi$ and $\Delta\phi$ are statistically independent. However, this is not quite true for small nuclei where the constraint of global momentum conservation creates important correlations. A method to subtract these correlations is described in [20].

We now proceed to evaluate $\langle \cos n\Delta\phi \rangle$. We first show that the distribution of $\Delta\phi$ is a universal function of a single real parameter $\chi$, which measures the accuracy of the reaction plane determination, and scales with $N$ like $\sqrt{N}$. It is normalized in such a way that for large $\chi$, the standard angular deviation is $\langle \Delta\phi^2 \rangle^{1/2} = 1/(\chi\sqrt{2})$. Then we express $\langle \cos n\Delta\phi \rangle$ as a function of $\chi$. Finally, we explain how to extract $\chi$ from the data.

We consider a large sample of events having the same magnitude of impact parameter. Experimentally, this can be done approximately by selecting events having the same multiplicity, or the same transverse energy, or the same energy in a zero degree calorimeter. The number $N$ of particles entering the definition of the vector $\mathbf{Q}$ in Eq.(5) is usually much larger than unity. Then the central limit theorem shows that, for given magnitude and orientation of the true impact parameter, the fluctuations of $\mathbf{Q}$ around its average value, $\langle \mathbf{Q} \rangle$, are gaussian. Note that both the magnitude and angle of the vector $\mathbf{Q}$ fluctuate (see Fig.2). We choose the direction of impact parameter as the $x$-axis. Then $\mathbf{Q} = Q(\mathbf{e}_x \cos\Delta\phi + \mathbf{e}_y \sin\Delta\phi)$ and $\langle \mathbf{Q} \rangle = \bar{Q}\mathbf{e}_x$, and the two dimensional distribution of $\mathbf{Q}$ takes the form

$$\frac{dN}{QdQd\Delta\phi} = \frac{1}{\pi\sigma^2} \exp\left( -\frac{|\mathbf{Q} - \langle \mathbf{Q} \rangle|^2}{\sigma^2} \right) = \frac{1}{\pi\sigma^2} \exp\left( -\frac{Q^2 + \bar{Q}^2 - 2Q\bar{Q}\cos\Delta\phi}{\sigma^2} \right). \quad (6)$$

We have assumed that the fluctuations are isotropic. This will be justified later. Note that $\bar{Q}$ scales like $N$ while $\sigma$ scales like $\sqrt{N}$.

Eq.(6) can be easily integrated over $Q$ [21] to yield the distribution of $\Delta\phi$:

$$\frac{dN}{\Delta\phi} = \frac{1}{\pi} \exp(-\chi^2) \left\{ 1 + z\sqrt{\pi}\, [1 + \mathrm{erf}(z)] \exp(z^2) \right\}. \quad (7)$$

where $z = \chi \cos \Delta\phi$ and $\mathrm{erf}(x)$ is the error function. This distribution depends on $\bar{Q}$ and $\sigma$ only through the dimensionless parameter $\chi \equiv \bar{Q}/\sigma$. The Fourier coefficients are most easily calculated by integrating Eq.(6) first over $\Delta\phi$ and then over $Q$ [14]:

$$\langle \cos n\Delta\phi \rangle = \frac{\sqrt{\pi}}{2} \chi e^{-\chi^2/2} \left[ I_{\frac{n-1}{2}}\left( \frac{\chi^2}{2} \right) + I_{\frac{n+1}{2}}\left( \frac{\chi^2}{2} \right) \right] \quad (8)$$

where $I_k$ is the modified Bessel function of order $k$. The variations of the first coefficients with $\chi$ is displayed in Fig.3. As expected, $\cos n\Delta\phi$ decreases with increasing $n$, and becomes vanishingly small for $n \gg \chi$. Experiments report values of $\langle \cos\Delta\phi \rangle$ ranging from 0.35 [13] for light nuclei ($A \simeq 20$) to 0.94 [5] for the heaviest ones ($A \simeq 200$), corresponding to values of $\chi$ between 0.4 and 2.2 respectively. Fig.3 shows that the corrections are important in this range.

If $\chi \gg 1$, the distribution of $\Delta\phi$, Eq.(7), becomes approximately gaussian

$$\frac{dN}{d\Delta\phi} \simeq \frac{\chi}{\sqrt{\pi}} \exp\left( -\chi^2 \Delta\phi^2 \right) \quad (9)$$

and the Fourier coefficients are given by

$$\langle \cos n\Delta\phi \rangle \simeq \exp(-n^2/4\chi^2). \quad (10)$$

In the limit $\chi \ll 1$, the $n^{\text{th}}$ Fourier coefficient is of order $\chi^n$:

$$\langle \cos n\Delta\phi \rangle \simeq \frac{\sqrt{\pi}}{2^n \Gamma\left(\frac{n+1}{2}\right)} \chi^n \tag{11}$$

where $\Gamma$ is the Euler function.

We now turn to the determination of $\chi$. The most widely used method [5, 6, 7, 12] to estimate the accuracy of the reaction plane determination is to divide each event randomly into two subevents containing half of the particles each, and to construct $\mathbf{Q}$ for the two subevents [16]. One thus obtains two vectors $\mathbf{Q}_I$ and $\mathbf{Q}_{II}$. The distributions of $\mathbf{Q}_I$ and $\mathbf{Q}_{II}$ are given by an equation similar to Eq.(6). However, since each subevent contains only $N/2$ particles, the corresponding average value and fluctuations must be scaled: $\bar{Q}_I = \bar{Q}_{II} = \bar{Q}/2$, $\sigma_I = \sigma_{II} = \sigma/\sqrt{2}$, and therefore $\chi_I = \chi_{II} = \chi/\sqrt{2}$. The distribution of the relative angle $\Delta\phi_R \equiv |\Delta\phi_I - \Delta\phi_{II}|$ can be calculated analytically (see the Appendix of [21] and the note added in proof)

$$\frac{\mathrm{d}N}{\mathrm{d}\Delta\phi_R} = \frac{\mathrm{e}^{-\chi_I^2}}{2} \left\{ \frac{2}{\pi}(1+\chi_I^2) + z\left[I_0(z) + \mathbf{L}_0(z)\right] + \chi_I^2 \left[I_1(z) + \mathbf{L}_1(z)\right] \right\} \tag{12}$$

where $z = \chi_I^2 \cos\Delta\phi_R$ and $\mathbf{L}_0$ and $\mathbf{L}_1$ are modified Struve functions [22]. This distribution is normalized to unity between 0 and $\pi$.

The value of $\chi$ can be obtained by fitting Eq.(12) to the measured distribution. However, it can be calculated more simply from the fraction of events for which $\Delta\phi_R > 90°$, which is calculated by integrating Eq.(12) over $\Delta\phi_R$:

$$\frac{N(90° < \Delta\phi_R < 180°)}{N(0° < \Delta\phi_R < 180°)} = \frac{\exp(-\chi_I^2)}{2} = \frac{\exp(-\chi^2/2)}{2}. \tag{13}$$

Alternatively, one can obtain $\chi$ by measuring [19, 21]

$$\begin{aligned} \langle \cos\Delta\phi_R \rangle &= \langle \cos\Delta\phi_I \rangle \langle \cos\Delta\phi_{II} \rangle \\ &= \frac{\pi}{8}\chi^2 e^{-\chi^2/2} \left[ I_0\left(\chi^2/4\right) + I_1\left(\chi^2/4\right) \right]^2 \end{aligned} \tag{14}$$

where we have used Eq.(8) with $n = 1$ and $\chi$ replaced by $\chi_I = \chi/\sqrt{2}$. The variation of $\langle \cos\Delta\phi_R \rangle$ with $\chi$ is displayed in Fig.4. Eq.(14) is more reliable than Eq.(13) if $\chi$ is large, for in this case the ratio in Eq.(13) is very small and is therefore subject to relatively large statistical fluctuations. Other methods to measure $\chi$ are described in [21, 23].

We finally justify the hypothesis that was made in writing Eq.(6), namely that the fluctuations have the same magnitude in both $x$ and $y$ directions. This is not a consequence of the central limit theorem, which only ensures that the two dimensional distribution of $\mathbf{Q}$ is gaussian. The most general gaussian compatible with the symmetry $\Delta\phi \to -\Delta\phi$ can be written as

$$\frac{dN}{QdQd\Delta\phi} = \frac{1}{\pi\sigma_x\sigma_y} \exp\left[ -\frac{(Q\cos\Delta\phi - \bar{Q})^2}{\sigma_x^2} - \frac{Q^2\sin^2\Delta\phi}{\sigma_y^2} \right]. \tag{15}$$

The quantities $\sigma_x$ and $\sigma_y$ characterize the fluctuations along the $x$ and $y$ axis, which are *a priori* different (the circle in Fig.2 should then be replaced by an ellipse). $\bar{Q}$, $\sigma_x$ and $\sigma_y$ are related to the azimuthal distribution of particles in the following way. Assuming for simplicity that the multiplicity $N$ is the same for all events in the sample (it is at least approximately true since the impact parameter is fixed), we get from Eq.(5)

$$\bar{Q} = \langle \mathbf{Q} \cdot \mathbf{e}_x \rangle = N \langle w \cos\phi \rangle \tag{16}$$

where $\phi$ is the *true* azimuthal angle of particles, and the last average involves all the detected particles of all events. Similarly, the fluctuations in the $x$ and $y$ directions are given by

$$\begin{aligned} \sigma_x^2 &= 2\left[\langle (\mathbf{Q} \cdot \mathbf{e}_x)^2 \rangle - \bar{Q}^2\right] = 2N\left[\langle w^2 \cos^2\phi \rangle - \langle w \cos\phi \rangle^2\right] \\ \sigma_y^2 &= 2\langle (\mathbf{Q} \cdot \mathbf{e}_y)^2 \rangle = 2N \langle w^2 \sin^2\phi \rangle. \end{aligned} \tag{17}$$

We define the average fluctuation $\sigma$ by

$$\sigma^2 = \frac{1}{2}(\sigma_x^2 + \sigma_y^2) = \langle Q^2 \rangle - \bar{Q}^2 = N\left[\langle w^2 \rangle - \langle w \cos\phi \rangle^2\right] \tag{18}$$

and the anisotropy of the fluctuations by

$$\frac{1}{2}(\sigma_x^2 - \sigma_y^2) = N\left[\langle w^2 \cos 2\phi \rangle - \langle w \cos\phi \rangle^2\right]. \tag{19}$$

Three cases must be distinguished, depending on the relative magnitudes of $\bar{Q}$, $\sigma_x$ and $\sigma_y$.

- (A) Azimuthal anisotropies are small, i.e. the Fourier coefficients of the azimuthal distribution are much smaller than unity. Then $\langle w^2 \cos 2\phi \rangle$ and $\langle w \cos\phi \rangle^2$ are both small compared to $\langle w^2 \rangle$. We deduce from Eqs.(18) and (19) that $\sigma_x \simeq \sigma_y \simeq \sigma$: our assumption is justified in this case.

- (B) There are situations where $\langle w \cos\phi \rangle^2$ cannot be neglected compared to $\langle w^2 \rangle$. This is the situation when the flow is strong. Using Eqs.(16) and (17) and the fact that $N \gg 1$, we see that in this case $\bar{Q} \gg \sigma_x, \sigma_y$: fluctuations are small and $\Delta\phi \simeq \mathbf{Q} \cdot \mathbf{e}_y / \bar{Q} \ll 1$ (see Fig.2). Therefore, the distribution of $\Delta\phi$ involves only $\sigma_y$, not $\sigma_x$. Although $\sigma_x$ and $\sigma_y$ may differ in this case, one can replace $\sigma_x$ by $\sigma_y$ without altering the distribution of $\Delta\phi$. Note that in this case, $\chi = \bar{Q}/\sigma \gg 1$, hence the distribution of $\Delta\phi$ reduces to its asymptotic form, Eq.(9).

- (C) Finally, there is the case when $\langle w^2 \cos 2\phi \rangle$ is of order $\langle w^2 \rangle$ while $\langle w \cos\phi \rangle^2$ is much smaller. In this case, there is a strong anisotropy in the second Fourier component, which should be used to determine the reaction plane. Instead of $\mathbf{Q}$, one should construct the vector $\mathbf{Q}_2$ defined as

$$\mathbf{Q}_2 = \sum_{k=1}^{N} w'_k \left(\mathbf{e}_x \cos 2\phi_k + \mathbf{e}_y \sin 2\phi_k\right) \tag{20}$$

with the same notations as in Eq.(5), and $w'_k$ is an appropriate weight. The azimuthal angle of the reaction plane is estimated as half the azimuthal angle of $\mathbf{Q}_2$, hence it is defined modulo $\pi$, i.e. one cannot distinguish $\phi$ and $\phi+\pi$. This method is equivalent to the diagonalisation of the transverse sphericity tensor, which has been claimed to be more efficient than the transverse momentum method at intermediate energies ($E/A < 100$ MeV, see [24] and in particular Eq.(12) of [25]) and at ultrarelativistic energies where the flow angle is very small due to increasing nuclear transparency [21, 26]. Then, the procedure described in this paper can be applied to reconstruct the azimuthal distributions, replacing $\mathbf{Q}$ by $\mathbf{Q_2}$. However, since the azimuthal angle is defined modulo $\pi$, only the even Fourier components can be reconstructed.

Before we conclude, let us briefly comment on corrections which have been applied, in previous works, to the measured azimuthal distributions. The method proposed by Tsang *et al.* [25, 27] is based on an ansatz for the distributions of $\phi$ and $\Delta\phi$, which are assumed to be proportional to $\exp(-\omega^2 \sin^2\phi)$, where $\omega$ is a fitted parameter. Our analysis is more general in the sense that it does not make any a priori hypothesis on the shape of azimuthal distributions. The analysis of Demoulins *et al.* [13] is similar to ours, although limited to $n=1$ and $n=2$. The main difference is that they calculate $\langle\cos\Delta\phi\rangle$ and $\langle\cos 2\Delta\phi\rangle$ independently from measured quantities. The analysis done by the E877 collaboration [10] is similar, in the sense that the corrections $\langle\cos n\Delta\phi\rangle$ are extracted directly, for each value of $n$, from measured correlations between subevents. On the contrary, our method allows to calculate simply all the Fourier coefficients $\langle\cos n\Delta\phi\rangle$ from Eq.(14) or Fig.3, as soon as we know the first one, $\langle\cos\Delta\phi\rangle$.

In conclusion, we have described a simple procedure to reconstruct the true azimuthal distributions from the measured ones by means of analytical formulae. Let us summarize this procedure: given a large sample of events in a restricted centrality interval, one measures the distribution of the relative angle between subevents $\Delta\phi_R$, from which one extracts, using Eq.(13) or Eq.(14), the crucial parameter $\chi$ which measures the accuracy of the reaction plane determination. Then one uses Eqs.(2) and (8) to reconstruct the Fourier coefficients of the true azimuthal distribution from the measured ones.

A first assumption, on which our calculation relies crucially, is that the fluctuations of the momentum transfer $\mathbf{Q}$ are gaussian. This is reasonable only if the multiplicity is large enough. The method should not be applied if, for instance, a single big fragment of the projectile gives the largest contribution to $\mathbf{Q}$. In order to test the validity of the gaussian hypothesis, one may check that Eq.(12) provides a good fit to the measured distribution of the angle $\Delta\phi_R$. Alternative tests are proposed in [23]. Our second assumption was that the subevents used in determining the value of $\chi$ are equivalent. This holds strictly only if the subevents are defined by a random selection of $N/2$ particles. The subevents are not equivalent if they correspond to different pseudorapidity windows as in [10]. However, Eq.(14) can still be used to calculate the correction for arbitrary $n$, once the correction for $n=1$ is known.

We have introduced a parameter $\chi$ to measure the accuracy of the reaction plane determination. Unlike other quantities frequently used in the literature such as $\langle\cos\Delta\phi\rangle$ [4, 13], or $\langle\Delta\phi^2\rangle^{1/2}$ [5, 6, 7, 9], $\chi$ scales simply with the multiplicity like $\sqrt{N}$. It has a simple physical interpretation, being the ratio of the average value of the flow vector $\mathbf{Q}$ to the typical statistical fluctuation $\sigma$. And furthermore, it can be directly deduced from measured quantities by simple expressions such as Eq.(13).

Our analysis suggests that the Fourier coefficients of the azimuthal distribution, which can be reconstructed accurately and are also easy to estimate in theoretical models, are the best observables to characterize azimuthal anisotropies. Note that we have chosen to measure azimuthal angles around the beam axis. It was argued in Ref.[28] that azimuthal distributions should rather be measured around the flow axis, determined by a sphericity tensor analysis [29]. But the fluctuations of the sphericity tensor, which is a $3 \times 3$ matrix, are much more complex [30] than those of $\mathbf{Q}$, and cannot be described in terms of a single parameter $\chi$. There exists no simple procedure to subtract statistical errors in this case.

What accuracy can be attained with our method? If the azimuthal angles of $\mathcal{N}$ particles are measured (summing over all events), the corresponding statistical error on the measured Fourier coefficient is $1/\sqrt{2\mathcal{N}}$. Systematic errors can be removed, at least partly, using a mixed event technique. The error on the true Fourier coefficient will be larger by a factor $1/\langle\cos n\Delta\phi\rangle$, according to Eq.(2). However, with high enough statistics, accurate measurements are possible even if the reaction plane is poorly known, as is the case with light projectiles [13]. In experiments using heavy ions, it would be interesting to try to measure higher order Fourier coefficients with $n \geq 3$ which, although probably small, could be measured accurately.

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## References

[1] H. Gustafsson *et al.*, Phys. Rev. Lett. **52** (1984) 1590.

[2] H. H. Gutbrod *et al.*, Phys. Lett. **B216** (1989) 267.

[3] K. G. R. Doss *et al.*, Phys. Rev. Lett. **59** (1987) 2720;

[4] H. A. Gustafsson *et al.*, Mod. Phys. Lett. **A3** (1988) 1323.

[5] D. Brill *et al.*, Phys. Rev. Lett. **71** (1993) 336.

[6] L. B. Venema *et al.*, Phys. Rev. Lett. **71** (1993) 835.

[7] M. Elaasar *et al.*, Phys. Rev. **C49** (1994) R10.

[8] D. Lambrecht *et al.*, Z. Phys. **A350** (1994) 115.

[9] M. Justice *et al.*, Nucl. Phys. **A590** (1995) 549c; M. Justice *et al.*, preprint nucl-ex/9708005.

[10] J. Barrette *et al.*, Phys. Rev. **C55** (1997) 1420; nucl-ex/9707002, Phys. Rev. **C**, in print.

[11] NA49 collaboration, H. Appelshäuser *et al.*, preprint nucl-ex/9711001

[12] Ph. Crochet *et al.*, preprint nucl-ex/9709004.

[13] M. Demoulins *et al.*, Phys. Lett. **B241** (1990) 476; M. Demoulins, Thèse, Université de Paris-Sud (1989), unpublished.

[14] J.-Y. Ollitrault, "A method of reconstructing azimuthal distributions in heavy ion collisions", in *XXV International Symposium on Multiparticle Dynamics*, 11-16 Sept. 1995, Stara Lesna, Slovakia, Bruncko D., Sandor L., Urban J. eds., World Scientific (1996), p. 290.

[15] G. M. Welke, M. Prakash, T. T. S. Kuo, S. Das Gupta and C. Gale, Phys. Rev. **C38** (1988) 2101.

[16] P. Danielewicz and G. Odyniec, Phys. Lett. **B157** (1985) 146.

[17] G. Fai, W. M. Zhang and M. Gyulassy, Phys. Rev. **C36** (1987) 597.

[18] P. Beckmann *et al.*, Mod. Phys. Lett. **A2** (1987) 163.

[19] S. Voloshin and Y. Zhang, Z. Phys. **C70** (1996) 665; J. Barrette *et al.*, Phys. Rev. Lett. **73** (1994) 2532.

[20] P. Danielewicz *et al.*, Phys. Rev. **C38** (1988) 120.

[21] J.-Y. Ollitrault, Phys. Rev. **D48** (1993) 1132.

[22] H. Bateman, *Higher Transcendental Functions*, Vol. 2, MacGraw-Hill, 1953.

[23] J.-Y. Ollitrault, Nucl. Phys. **A590** (1995) 561c.

[24] W. K. Wilson, R. Lacey, C. A. Ogilvie and G. D. Westfall, Phys. Rev. **C45** (1992) 738.

[25] M. B. Tsang *et al.*, Phys. Rev. **C44** (1991) 2065.

[26] J.-Y. Ollitrault, Phys. Rev. **D46** (1992) 229.

[27] M. B. Tsang *et al.*, Phys. Lett. **B297** (1992) 243.

[28] H. H. Gutbrod *et al.*, Phys. Rev. **C42** (1990) 640.

[29] M. Gyulassy, K.A. Frankel and H. Stöcker, Phys. Lett. **B110** (1982) 185.

[30] P. Danielewicz and M. Gyulassy, Phys. Lett. **B129** (1983) 283.

# Figure captions

**Fig. 1:** Schematic picture of a semi-central nucleus-nucleus collision viewed in the transverse plane (the beam axis is orthogonal to the figure). $\mathbf{b}$ is the impact parameter oriented from the target to the projectile and $\mathbf{Q}$ is the vector defined by Eq.(5). A particle is emitted along the dashed arrow. Its azimuthal angle measured with respect to $\mathbf{Q}$ is $\psi$, while the "true" azimuthal angle is $\psi + \Delta\phi$.

**Fig. 2:** Schematic picture of the distribution of $\mathbf{Q}$, given by Eq.(6). The solid thick arrow indicates the average value $\langle\mathbf{Q}\rangle = \bar{Q}\mathbf{e}_x$, which lies along the direction of the true impact parameter. $\mathbf{Q}$ fluctuates around this average value with a standard deviation $\sigma$, so that a typical value of $\mathbf{Q}$ (dotted arrow) lies within the dotted circle with radius $\sigma$. It is obvious from this figure that the typical magnitude of $\Delta\phi$ is $\sigma/\bar{Q} = 1/\chi$.

**Fig. 3:** Solid lines: variation of $\langle\cos n\Delta\phi\rangle$ with the parameter $\chi$, calculated from Eq.(8). The curves are labeled by the value of $n$. The dotted curves and dash-dotted curves are the asymptotic forms given respectively by Eqs.(11) and (10).

**Fig. 4:** Variation of $\langle\cos\Delta\phi_R\rangle$ with $\chi$, given by Eq.(14).