ANISOTROPY : A NEW SIGNATURE OF TRANSVERSE COLLECTIVE FLOW ?

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ABSTRACT

We propose a method for identifying hydrodynamical flow of the matter produced in the central rapidity region of ultrarelativistic heavy ion collisions. Quantitative predictions are made on the basis of a hydrodynamical model. We estimate statistical fluctuations and conclude that collective behaviour, if any, should be seen unambiguously in Au or Pb collisions.

1. Introduction

In searching for evidence of the formation of quark–gluon plasma in ultrarelativistic nucleus–nucleus collisions, one is led to address the question whether the matter produced in such collisions can be considered as a thermalized gas. Since the system under consideration here is by no means static or homogeneous, thermal equilibrium, if it exists, is at most local. Local equilibrium may be achieved if the produced particles scatter among each other several times before they escape and fly to the detectors. It may then have observable consequences, such as the occurrence of collective flow.

Collective flow of nuclear matter was first observed at Bevalac in 1984 and has been thoroughly studied since then[1], for energies up to 1 GeV per nucleon. Observables which describe the flow are always constructed through a global event-by-event analysis. However, this has not proven successful at ultrarelativistic energies[2] so far. Nevertheless, the larger number of particles created at AGS and SPS energies in the central rapidity region offers an opportunity to study collective flow, not only of spectators and/or participant nucleons, but also among the produced particles (already at AGS energies, the number of charged pions in the central rapidity region is expected to be larger than the number of protons for central Au-Au collisions, and this is true *a fortiori* at SPS, RHIC and LHC energies). Effects of such a flow on inclusive variables, such as the shape of p_T-spectra[3] or the impact parameter dependence[4] of $\langle p_T \rangle$, have already been studied, but none is unambiguously characteristic of collective behaviour. Here, I would like to show that global analysis is still relevant at ultrarelativistic energies if applied to the particles produced in the central rapidity region.

2. An observable for collective flow

2.1 Flow analysis from intermediate to ultrarelativistic energies

The first analyses of collective behaviour in nucleus–nucleus collisions[6] have focused on the determination of the flow direction, which is the direction of maximum kinetic energy flow[7]. The angle between the flow direction and the collision axis, or flow angle, has been measured. It decreases with increasing beam energy, reflecting the fact that longitudinal momenta become larger than transverse momenta. At ultrarelativistic energies, it is very small and therefore cannot be measured. The flow direction gives an experimental determination of the reaction plane, which is the plane spanned by the collision axis and the impact parameter. The latter can also be determined independently by measuring the transverse momentum transfer between target and projectile regions[8]. This method gives equivalent results, and is better suited to ultrarelativistic energies. Although it has not met any success at AGS and SPS yet[2], it has been recently argued[9] that it should work better with heavy nuclei.

These studies privileged the investigation of flow *in* the reaction plane. More recently, collective flow *out of* the reaction plane has been identified and studied[10], thereby completing the picture of hydrodynamical behaviour at Bevalac energies. It turns out that if one views the collision in the plane perpendicular to the flow direction, matter escapes preferentially in the direction orthogonal to the reaction plane; this is indeed the only direction which is not obstructed by either the projectile or target nucleus. This has been referred to[10] as the *squeeze–out* effect.

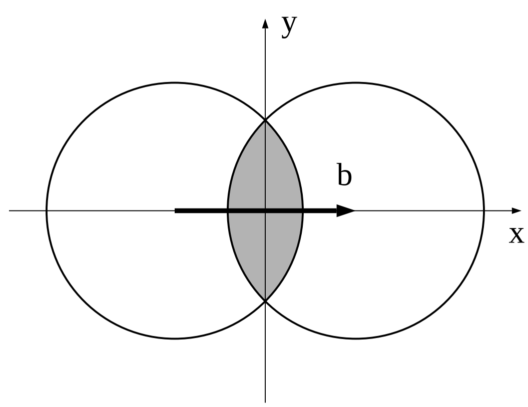

Fig.1 : A peripheral collision viewed in the transverse plane. **b** denotes the impact parameter.

Since the flow direction merges with the collision axis at ultrarelativistic energies, whether or not there is collective behaviour, evidence for collectivity must be sought for in the transverse plane, as in the study of the squeeze–out effect. Consider a peripheral collision viewed in this plane, as in Fig.1, and let us see what happens in the central rapidity region. The shaded area corresponds to the region where nucleon–nucleon collisions take place and mesons are created. Outside this area is the vacuum. The region where the energy is concentrated thus has a larger size in the y–direction (out of the reaction plane) than in the x–direction (impact parameter direction). Now, if a thermal equilibrium is reached, collective flow develops with a velocity proportional to the gradient of pressure, according to Euler's equation of fluid dynamics. Since the pressure gradient is inversely proportional to the size, it is larger along the x–axis than along the y–axis. Matter is thus expected to flow preferentially in the direction of impact parameter, which should result in anisotropy in the transverse momentum. Let us emphasize that only the spatial distribution is initially anisotropic; this anisotropy is carried over to the momentum distribution through pressure, and is therefore characteristic of collective behaviour. Note that this phenomenon can only take place for peripheral collisions since central collisions are azimuthally symmetric. We thus propose anisotropy of the transverse momentum distribution, correlated with impact parameter (more precisely, increasing with impact parameter) as a signature for collective flow in ultrarelativistic collisions.

We predict a larger flow *in* the reaction plane than *out of* the reaction plane; this seems in contradiction with the squeeze–out effect observed at Bevalac. However, the situation is quite different: the origin of squeeze–out is believed to be that both target and projectile nuclei obstruct the way and the matter compressed in the overlap zone can only escape out of the reaction plane. But at ultrarelativistic energies, the time it takes for the nuclei to cross each other (of the order of 1 fm/c) is much smaller than the typical time it takes for transverse flow to develop, which is of the order of the nuclear size (about 10 fm/c). Thus the matter created in the central rapidity region does not "see" the target and projectile nuclei any more. Note further that the main contribution is expected to come from mesons, instead of nucleons at intermediate energies. Let us put it in another way: the squeeze–out effect results from an interaction between the participants which try to escape the fireball and the spectators which bar the way out, while anisotropy results from the interaction of particles in the central rapidity region among themselves. Note that the squeeze–out effect is observed to decrease with increasing incident energy[10].

2.2 A measure of anisotropy

From the measured transverse momenta of M particles $\mathbf{p}(1)$,... $\mathbf{p}(M)$, one constructs the sphericity 2×2 tensor S_{ij} defined by

$$\begin{aligned} S_{ij} &= \sum_{\nu=1}^{M} p_i(\nu) p_j(\nu) \\ &= \begin{pmatrix} \sum_{\nu=1}^{M} p_x^2(\nu) & \sum_{\nu=1}^{M} p_x(\nu) p_y(\nu) \\ \sum_{\nu=1}^{M} p_x(\nu) p_y(\nu) & \sum_{\nu=1}^{M} p_y^2(\nu) \end{pmatrix} \end{aligned} \tag{1}$$

Note that we give here the same weight to all particles. Other possible choices are briefly discussed in section 5. Diagonalization of this matrix yields two eigenvalues f_1 and f_2. Obviously $f_1 = f_2$ for an isotropic emission, while $f_2 = 0$ if all momenta are parallel to the x-axis. A natural measure of the anisotropy is thus

$$\alpha = \frac{|f_1 - f_2|}{f_1 + f_2} \tag{2}$$

Note that this is the only quantity we may construct from S_{ij} if we require it to be dimensionless and azimuthally invariant.

3. Quantitative predictions

In order to give quantitative estimates of the effect, a hydrodynamical model must be used: one assumes that the system behaves as an ideal fluid during some stage of its evolution. The model I use[11] assumes that the central rapidity region is boost invariant. However, one easily shows that release of this assumption does not change the results by more than 15%. Choice must also be made of an equation of state. Here I assume that the system is made of massless non–interacting pions, in which case the equation of state is that of black body radiation, $P \propto T^4$. Results are displayed in Fig.2 for three types of colliding systems. The anisotropy α is computed as a function of the number of participant nucleons N, which is a measure of impact parameter. As expected, the anisotropy vanishes for central collisions and increases up to very peripheral collisions. Its variation with N is almost linear. Clearly, the anisotropy is larger with heavier target and/or projectile, which is a first argument in favor of the use of heavy nuclei.

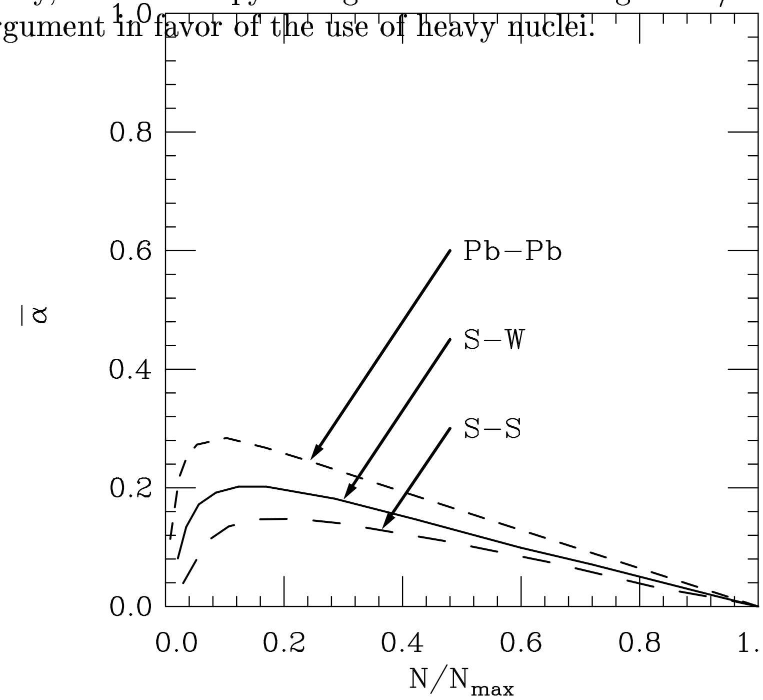

Fig.2 : Anisotropy predicted by fluid dynamical calculations for various colliding systems, as a function of the number of participants N, scaled by its maximum value $N_{\max}$ reached for central collisions.

Note that the equation of state we have used does not contain any temperature scale. Thus the result does not depend on the initial temperature of the system. More realistic equations of state should of course be used. Notably, the value of the speed of sound (which is the parameter that governs the expansion) is believed to be much lower than for black–body radiation. Using a more realistic value, one finds values of α which are slightly smaller, by about 20% or so[5]. Similarly, the occurrence in the system of a strong first order phase transition from a quark–gluon plasma, which slows down the transverse expansion, lowers the value of α, possibly by a factor of two.

4. Finite multiplicity fluctuations

The values of α diplayed in Fig.2 result from a fluid dynamical calculation where the system is supposed to be continuous. However, α is constructed experimentally from a finite number of measured transverse momenta and hence it is subject to statistical fluctuations. Let us discuss how these may alter the results. Experimentally, one selects events with a given multiplicity M (this is an impact parameter selection since M is proportional to the number of participants N) and measures α for each event. One thus obtains a probability distribution for α, which we denote by $\mathrm{d}P/\mathrm{d}\alpha$. We would like this distribution to be nicely peaked around the fluid dynamical value (denoted by $\bar{\alpha}$ in Fig.2) with statistical fluctuations of order $1/\sqrt{M}$. The following questions must therefore be answered: can we interpret a maximum of $\mathrm{d}P/\mathrm{d}\alpha$ at $\alpha \neq 0$ as an effect of flow? Conversely, under what condition does collective flow result in a maximum of $\mathrm{d}P/\mathrm{d}\alpha$ at $\alpha \neq 0$?

4.1 Jacobian correction

Let us assume for sake of simplicity that particles are emitted independently from each other, i.e. that there is no correlation. Then the sphericity tensor (1) is the sum of M independent contributions. In the limit of large M, the central limit theorem thus ensures that the probability law for S_{ij} is gaussian, peaked around its maximum value. However, the anisotropy α is constructed from the eigenvalues of S_{ij} and we know from elementary random matrix theory that eigenvalues "repel each other". In particular, the probability that $f_1 = f_2$ vanishes. This simply means that it is very unlikely to make an isotropic emission with a finite number of particles. The problem is that $\mathrm{d}P/\mathrm{d}\alpha$ vanishes at $\alpha = 0$, so that its maximum always lies at some value of $\alpha \neq 0$: the probability looks peaked at some non–vanishing value of α even is the emission law is isotropic! One encounters the same problem in the determination of the flow angle at intermediate energy. A simple and elegant way to remedy this[12] is to multiply $\mathrm{d}P/\mathrm{d}\alpha$ by the jacobian transforming the sphericity tensor into the relevant quantity (here the anisotropy). After a simple calculation[5], we thus define the following corrected distribution

$$\frac{\mathrm{d}P_{\mathrm{cor}}}{\mathrm{d}\alpha} = \frac{1}{\alpha}\frac{\mathrm{d}P}{\mathrm{d}\alpha} \tag{3}$$

This corrected distribution is always maximum at $\alpha = 0$ for isotropic emission, so that a maximum at $\alpha \neq 0$ can be safely interpreted as a dynamical effect of anisotropy.

4.2 An estimate of fluctuations

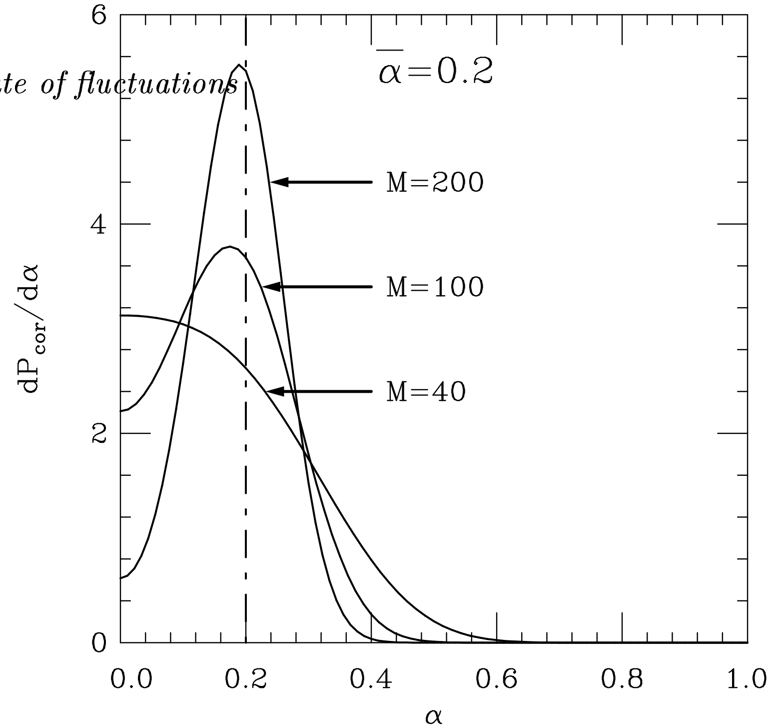

Fig.3 : Corrected probability distributions for α, for various values of the multiplicity. The area is normalised to unity.

One must then study under what condition the converse statement holds, that is whether the anisotropy coming from collective flow results in a maximum of the corrected distribution (3) at $\alpha \neq 0$. Assuming uncorrelated emission and using the central limit theorem to calculate the probability law of S_{ij}, one finds[5] that this condition is

$$\bar{\alpha} > \sqrt{\frac{\delta}{M}} \tag{4}$$

where $\bar{\alpha}$ is the anisotropy in the emission probability (i.e. the anisotropy yielded by collective flow) and $\delta = \langle p_T^4 \rangle / \langle p_T^2 \rangle^2$. This condition expresses the fact that anisotropy is observable only if it is larger that statistical fluctuations of order $1/\sqrt{M}$. An illustration is given in Fig.3, which displays the corrected probability distribution one obtains with various values of the multiplicity. We assumed here that the transverse momentum distribution is gaussian, with 20% anisotropy, as one could reasonably expect if collective flow occurs. One easily checks that $\delta = 2$ for a gaussian distribution, so that Eq.(4) is satisfied only for $M > 50$. Indeed, one sees in Fig.3 that for $M = 40$, anisotropy is hidden by fluctuations so that $dP_{cor}/d\alpha$ is maximal at $\alpha = 0$.

4.3 Why we need heavy nuclei

Since the anisotropy predicted by hydrodynamics is larger for heavy nuclei (see Fig.2) and statistical fluctuations are smaller due to larger multiplicities, heavy nuclei are clearly to be preferred. Here, we show how to estimate the mass numbers and multiplicities required. Note that while statistical fluctuations dominate for peripheral collisions where the multiplicity is low, anisotropy disappears for central collisions. Therefore one must look for the condition under which both effects (statistical and dynamical) balance. Assuming that α decreases linearly with the number of participants (see Fig.2) and thus with the observed multiplicity, condition (4) writes

$$\alpha_{\rm max}\left(1-\frac{M}{M_{\rm max}}\right) > \sqrt{\frac{\delta}{M}} \tag{5}$$

where $M_{\rm max}$ is the value of the multiplicity for a central collision and $\alpha_{\rm max}$ is the maximum value of α for a peripheral collision. A straightforward calculation shows that if this condition is not satisfied for $M = M_{\rm max}/3$, then it cannot be satisfied for any M. The value of the impact parameter such that $M = M_{\rm max}/3$ offers thus the best balance between dynamical effects and statistical fluctuations. Condition (5) is satisfied for this value of M if

$$M_{\rm max} > \frac{27\delta}{4}\frac{1}{\alpha_{\rm max}^2} \tag{6}$$

Taking $\alpha_{\rm max} = 0.25$, which is a rather conservative estimate according to the results of section 3, and $\delta = 2$ as for a gaussian p_T–distribution (the actual value of δ is probably somewhat larger) one obtains $M_{\rm max} > 216$. Such a value of the charged multiplicity requires heavy nuclei but is easily achieved for Pb–Pb collisions where one expects at least 500 charged particles per rapidity unit at CERN.

5. Discussion

Measurement of anisotropy requires of course full azimuthal coverage of the detector. Note that we have constructed the sphericity tensor (1) from the transverse momenta $\mathbf{p}$ and that this knowledge requires particle identification. However, it is clear that the relevant quantity is the azimuthal direction of the momentum rather than its magnitude. Thus p_i could be replaced by $p_i/|\mathbf{p}|$ in the definition of the sphericity tensor. Particle identification is therefore not necessary in this study. Each particle could also be weighted by some power of the transverse energy it deposits in the calorimeter. It has indeed been noticed at Bevalac that collective flow results not only in a larger number of emitted particles in the flow direction, but also in a higher momentum per particle in this direction[10]. The work presented here is very preliminary, and a more careful investigation is necessary in order to determine the most appropriate weight.

Since longitudinal momenta remain much larger than transverse momenta at ultrarelativistic energies, only transverse momentum has been taken into account in our flow analysis. This restriction makes the discussion much simpler than at intermediate energies where more parameters come into play. Although this is nice from a theoretical point of view, the information we get about collective flow is rather scarce since it lies in a single observable, α. We would be more confident in attributing anisotropy to collective flow if we had an independent measure of the reaction plane. Indeed we know that other effects, for instance jets or minijets, also produce anisotropy, and they have not been included in the present study. One will get an independent determination of the reaction plane if the tranverse momentum transfer between projectile and target can be measured with heavy nuclei at CERN[9]. Alternatively, one can measure the reaction plane and α from S_{ij} in two (or more) separated rapidity intervals (if the multiplicity is large enough) and see whether results are correlated.

Although much theoretical work is to be done in order to evaluate other effects contributing to the anisotropy, this new observable could provide the first direct evidence for thermalization in ultrarelativistic collisions. This certainly would be a major step towards the discovery of quark–gluon plasma.

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