Nuclear Physics B394 (1993) 555–664
North-Holland

NUCLEAR
PHYSICS B

Renormalization theory for interacting crumpled manifolds

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Received 19 November 1992
Accepted for publication 10 December 1992

We consider a continuous model of D-dimensional elastic (polymerized) manifold fluctuating in d-dimensional euclidean space, interacting with a single impurity via an attractive or repulsive δ-potential (but without self-avoidance interactions). Except for $D = 1$ (the polymer case), this model cannot be mapped onto a local field theory. We show that the use of intrinsic distance geometry allows for a rigorous construction of the high-temperature perturbative expansion and for analytic continuation in the manifold dimension D. We study the renormalization properties of the model for $0 < D < 2$, and show that for bulk space dimension d smaller that the upper critical dimension $d^\star = 2D/(2-D)$, the perturbative expansion is ultraviolet finite, while ultraviolet divergences occur as poles at $d = d^\star$. The standard proof of perturbative renormalizability for local field theories (the Bogoliubov–Parasiuk–Hepp theorem) does not apply to this model. We prove perturbative renormalizability to all orders by constructing a subtraction operator **R** based on a generalization of the Zimmermann forests formalism, and which makes the theory finite at $d = d^\star$. This subtraction operation corresponds to a renormalization of the coupling constant of the model (strength of the interaction with the impurity). The existence of a Wilson function, of an ϵ-expansion à la Wilson–Fisher around the critical dimension, of scaling laws for $d < d^\star$ in the repulsive case, and of non-trivial critical exponents of the delocalization transition for $d > d^\star$ in the attractive case, is thus established. To our knowledge, this study provides the first proof of renormalizability for a model of extended objects, and should be applicable to the study of self-avoidance interactions for random manifolds.

1. Introduction

One general problem arising in statistical physics is the understanding of the effect of interactions on the thermodynamical properties of extended fluctuating geometrical objects. These objects may be (one-dimensional) lines, like long linear macromolecules or polymers, (two-dimensional) surfaces, like membranes or interfaces, or even (three-dimensional) volumes, like gels. The interactions involve in general two-body attractive or repulsive forces, and one may in general reduce such problems into two different classes: (i) either one deals with self-interactions between distinct points of the same fluctuating object, or mutual

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