IPhT Lectures on Naturalness

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January 28, 2022

Abstract

This is a summary of what we think we know, thought we knew and despair of ever knowing about the Higgs boson mass m_h. We also discuss in parallel the Cosmological Constant $\Lambda_{\rm CC}$, but giving fewer details. Understanding the values of these two parameters are two of the most fascinating open problems in particle physics. They have kept many of us awake at night for the past 30 years.

Contents

1 Introduction

Accidental cancellations between unrelated parameters often signal that our description of Nature is incomplete. A well-known example is the rest energy of the electron in classical electrodynamics. In natural units ($\hbar = c = 1$) we have

$$m_e = m_{e,0} + \frac{e^2}{4\pi r_e}\,. \tag{1.1}$$

The first term on the right-hand side is the bare electron mass in the Lagrangian. The second accounts for the energy stored in the electric field generated by the electron.

Experimentally we know that $m_e \approx 0.511$ MeV. Cheating a little for illustrative purposes we can use our modern knowledge of the electron radius $r_e \lesssim (100\ \text{GeV})^{-1}$ to cut-off the divergence of the Coulomb self-energy. This corresponds to not having observed deviations from a point-like behavior at LEP [1]. Putting together these two measurements we conclude that only an accidental cancellation between the two terms on the right-hand side of (1.1) can explain the observed value of the electron mass.

This apparent fine-tuning is hiding something deep. At the length scales in our calculation classical electrodynamics breaks down and we need to include quantum effects to obtain the correct result. Restoring units, we can not ignore quantum mechanics below

$$c\Delta t \lesssim \frac{\hbar c}{\Delta E} \approx \frac{\hbar}{m_e c}\,, \tag{1.2}$$

or in natural units for $r_e \lesssim 1/m_e$. So the result of our classical calculation is not correct. If we include the contribution of photons and positrons from vacuum fluctuations [2], the term that diverges as $1/r_e$ is cancelled by virtue of a new symmetry. The chiral symmetry that emerges in quantum electrodynamics as m_e goes to zero. Only a term logarithmic in $1/r_e$ and proportional to $m_{e,0}$ survives, as dictated by the selection rules of this new symmetry,

$$m_e = m_{e,0}\left[1 + \frac{3\alpha}{4\pi}\log\frac{1}{m_e r_e}\right]\,. \tag{1.3}$$

Now we have a correction of less than 10% even for an electron that stays point-like up to the Planck scale. Incidentally, pushing classical electrodynamics beyond its limits of validity has other surprising consequences, including the emergence of an acausal behavior for the electron on time scales of $\mathcal{O}(e^2/m_e)$ [3].

Setting violations of causality aside, we have just seen that what appeared as an accidental cancellation was pointing to a more fundamental description of our physical system in terms of quantum mechanics.

This is not the only case in which apparent coincidences are signaling the emergence of a new paradigm. A second classic example that has a completely different resolution is that of planetary orbits in the solar system. In 1596 Johannes Kepler published the *Mysterium Cosmographicum*, where he showed that each of the five Platonic solids can be uniquely inscribed into and circumscribed by a sphere. If ordered in a specific pattern (octahedron, icosahedron, dodecahedron, tetrahedron, cube) the spheres reproduced, within the experimental accuracy of the time, the orbits of the six known planets, from Mercury to Saturn. This seems a striking coincidence that requires finely tuned values of unrelated parameters. Alternatively, as Kepler did, one could see it as an example of God's refined aesthetic sense.

Today we know that the explanation is different, but still paradigm-shifting. Not only we are not unique in any way, but we are just a tiny speck of dust in an unimaginably vast universe. This kind of approximate accidents become likely if we think about the staggering number of other stars, planets and solar systems over which we have to integrate small probabilities.

I hope that these two examples convinced you that fine-tuning problems in physics are worthy of attention, as they often lead to the emergence of a new understanding of the Universe. Today we are facing two problems of this kind and they might have answers that are just as deep as the historical examples given above.

The first and most dramatic of the two puzzles concerns the size of the cosmological constant:

$$\Lambda_{\rm CC} \approx (10^{-3}\ {\rm eV})^4\,. \tag{1.4}$$

This number is much smaller than all the particle physics scales that we know (except neutrino masses) and should naively contribute to it. In these lectures we will often talk about this problem and its most concrete solutions, but we will not discuss it in depth. I refer the interested reader to the reviews [4, 5, 6, 7] and their references for more details.

In the following I describe in great detail another fine-tuning problem in modern theoretical physics, the one related to the Higgs boson mass, also known as the *hierarchy problem* [8, **?**, 9, 10, 11]. To state it precisely we first have to make sense of the illusory divergences of field theory. We have already encountered one example in the Coulomb self-energy of the electron as $r_e \to 0$.

To this end, in the next section I very briefly introduce Effective Field Theoris (EFTs). In Section 3 we try to state the problem precisely. We first see in 3.1 why, within the SM, the value of the Higgs mass is puzzling, but not at all problematic. We then move on to theories where the Higgs mass is calculable and there is an actual problem 3.2. This leads to the so-called "little Hierarchy Problem" discussed in Section 3.3.

2 Effective Field Theories and Symmetry

In this Section we give an elementary introduction to spurion fields and effective field theories. The expert reader can skip ahead to Section 3. The only unusual element in our exposition is the infinite higher spin symmetry of the scalar Lagrangian discussed in Section 2.2.

2.1 Broken Symmetries and Spurion Fields

In QFT it is very useful to promote mass parameters and couplings to spurion fields (spurions for short) to enforce the selection rules of broken symmetries. You are probably familiar with this procedure from your first courses as an undergraduate, but let's recall one example that will be useful in the following.

Take the QED Lagrangian with gauge coupling g. We have an interacting Dirac fermion, described by the fields e, e^c, and a gague boson, the photon A_μ. We can write their Lagrangian as

$$\mathcal{L}_{\rm QED} = ie^\dagger\overline{\sigma}^\mu\partial_\mu e + ie^{c\dagger}\overline{\sigma}^\mu\partial_\mu e^c + m_{e,\rm tree}(ee^c + \text{h.c.}) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - gA_\mu\left(e^\dagger\overline{\sigma}^\mu e + e^{c\dagger}\overline{\sigma}^\mu e^c\right) \tag{2.1}$$

The chiral symmetry of the fermions

$$e \to e^{i\alpha_e}e\,, \quad e^c \to e^{i\alpha_e}e^c\,, \tag{2.2}$$

is broken by the electron mass term. It is useful to promote the mass to a spurion by giving it fictitious transformation properties that restore the symmetry

$$m_{e,\rm tree} \to e^{-2i\alpha_e}m_{e,\rm tree}\,. \tag{2.3}$$

This shows immediately how to enforce the selection rules of the symmetry: observable quantities must be invariant if we transform $m_{e,\rm tree}$ together with the fields.

It also immediately gives the following prediction for the renormalized electron mass

$$m_{e,R} = m_{e,\rm tree}\left[...\right]\,. \tag{2.4}$$

We don't know what is in the parenthesis without doing a calculation, but we know from the selection rules of the chiral symmetry that whatever is on the right-hand side of the equation must transform as the left-hand side. The only spurion in the theory that transforms as $m_{e,\rm tree} \to e^{-2i\alpha_e}m_{e,\rm tree}$ is the tree-level mass itself.

Eq. (2.4) is a very powerful result. If you have another mass scale appearing at much higher energies (let's call it M), for instance the proton mass, you know from this simple exercise that it can affect the electron mass at most logarithmically

$$m_{e,R} = m_{e,\rm tree}\left[a\log M + ...\right]\,, \tag{2.5}$$

so it's not surprising to find a very light fermion in a theory of Nature with much larger mass scales. Note that the symmetry is broken, but it still allows us to draw very useful conclusions from its selection rules.

2.2 Effective Field Theory and Dilations

Imagine to know that your theory is valid up to some energy scale M_*. If you only need to make a prediction for measured quantities at $E \ll M_*$, it is not necessary to include in your calculation all the details of the dynamics at the high scale. For example, you can describe the energy levels of the Hydrogen atom with excellent accuracy, without knowing anything about the mass of the top quark. The error that you are making is of order $\alpha m_e/m_t$ and if your experimental precision is inferior, this is perfectly acceptable. Some of the low-energy parameters that you need for the calculation are more sensitive to m_t, for example the proton mass m_p and the fine structure constant α. However these are all quantities that you can measure at low energy, forgetting about their ultraviolet (UV) origin.

If we could not describe the low-energy dynamics only in terms of low-energy degrees of freedom, at least to some finite precision, we would not have been able to make predictions for any physical system. So the fact that UV sensitive quantities can all be fixed through low-energy measurements must be independent of our specific example.

However this does not mean that every trace of the UV dynamics disappears in the low-energy theory. There are very non-trivial consequences of UV physics that survive at low energy. One classic example is the spin-statistics theorem. In non-relativistic quantum mechanics it is just a (measured) fact of life, but in quantum field theory it emerges from causality. Other than symmetry constraints, the UV dynamics also leaves behind small corrections to low-energy observables (the $\alpha m_e/m_t$ error in the case of the Hydrogen atom). Therefore if we had a systematic way of building a low-energy theory from a more complete theory we would have accomplished two remarkable tasks. We would have considerably simplified our low-energy calculations and at the same time we would have a way to reconstruct, at least to some extent, the UV dynamics from low-energy measurements. Effective Field Theory is precisely the systematic construction that we are looking for. In the rest of this section I often follow [12].

To see EFTs at work, take a scale M and split the degrees of freedom in the path integral into two parts, the high-energy and the low-energy modes,

$$\begin{aligned} \int \mathcal{D}\phi e^{iS(\phi)} &= \int \mathcal{D}\phi_L \mathcal{D}\phi_H e^{iS(\phi_L,\phi_H)}\,, & (2.6)\\ E_{\phi_H} &> M\,, \\ E_{\phi_L} &< M\,. & (2.7)\end{aligned}$$

If we know how to do the path integral over the high-energy modes we obtain a description of the system in terms of the low-energy degrees of freedom

$$\int \mathcal{D}\phi_L \mathcal{D}\phi_H e^{iS(\phi_L,\phi_H)} = \int \mathcal{D}\phi_L e^{iS_M(\phi_L)}\,. \tag{2.8}$$

This is all we need and we have not even restricted the validity of the theory. In principle we can use $S_M(\phi_L)$ to make predictions up to M. In practice this suggest that we have not really gained anything and in fact most of the time the path integral can not be solved exactly. However we can at least consider the previous equation as a definition of the low-energy action

$$e^{iS_M(\phi_L)} \equiv \int \mathcal{D}\phi_H e^{iS(\phi_L,\phi_H)}\,. \tag{2.9}$$

In some cases this gives us a way to compute $S_M(\phi_L)$ in a perturbative expansion. Even when this is not possible, we can always write $S_M(\phi_L)$ as an infinite sum of operators built from the low-frequency fields and consistent with all the low-energy symmetries of the problem,

$$S_M(\phi_L) = \int d^dx \sum_i c_i \mathcal{O}_i(\phi_L)\,. \tag{2.10}$$

Note that some of these operators are non-local by a $1/M$ amount, since we have integrated out fields with $E_{\phi_H} > M$. So this is also an expansion in derivatives. It might seem that this infinite sum requires the full knowledge of $S(\phi_L, \phi_H)$ to be useful. However here the power of *broken* symmetries comes to our rescue.

This is familiar from quantum and classical mechanics. Even in systems that are not rotationally invariant, for example, selection rules of the rotational symmetry are extremely useful to predict relations between matrix elements. If you prefer a quantum field theory equivalent you can think about Isospin in QCD and its breaking by the quark masses or flavor symmetries in the Standard Model (SM) and their breaking by the Yukawa matrices. In our case we need an even simpler symmetry. We can just use dimensional analysis, which should more appropriately be called the selection rules of the dilatation operator [13].

If we set $\hbar = c = 1$, our operators have some dimension δ_i in units of energy $[\mathcal{O}_i] = E^{\delta_i}$. Since the action is dimensionless ($\hbar = 1$) we must have $[c_i] = E^{d-\delta_i}$.

The largest scale in our low-energy theory is M and we can always write

$$\begin{aligned} c_i &= \gamma_0 M^{d-\delta_i} + \gamma_1 M_1^{d-\delta_i} + \gamma_2 M_2^{d-\delta_i} + ... = g_i M^{d-\delta_i}\,, \\ g_i &\equiv \gamma_0 + \gamma_1 \left(\frac{M_1}{M}\right)^{d-\delta_i} + ... \end{aligned} \tag{2.11}$$

where $M_1, M_2, ... < M$. This just means that even if the c_i receive contributions from multiple scales we can always parametrize them in terms of the largest scale in the theory times some dimensionless coefficient. From simple dimensional analysis we expect $g_i = \mathcal{O}(1)$ unless some extra symmetry is at work. The selection rules of the dilatation operator are what determined the form of c_i, i.e. all contributions must have dimensions $E^{d-\delta_i}$ and the largest one can be at most $\sim M^{d-\delta_i}$.

Now we are in a position to estimate the contribution of each term in the sum (2.10) to low-energy observables. Using again dimensional analysis we have

$$\int d^d x \mathcal{O}_i \approx E^{\delta_i - d}\,, \tag{2.12}$$

so each term in the sum contributes to a low-energy measurement an amount

$$c_i \int d^d x \mathcal{O}_i \approx g_i \left(\frac{E}{M}\right)^{\delta_i - d}\,. \tag{2.13}$$

We see immediately that operators with $\delta_i > d$ are suppressed when $E \ll M$, so if we are interested in a finite level of precision we need only a finite number of operators for our calculation. Not surprisingly operators with $\delta_i > d$ are called *irrelevant*, those with $\delta_i < d$ *relevant* and the ones with $\delta_i = d$ *marginal*.

Here resides the power of Effective Field Theory. We have just seen that ignoring completely the high energy dynamics, we can write a finite set of operators based on the fields and symmetries that we observe at low energy and make predictions to an arbitrary level of accuracy. If our experimental precision is sufficient we can even probe operators suppressed by powers of $1/M$ and obtain information on the scale at which new phenomena should appear.

This is not all. The very simple construction that we have just seen can do something else for us. Given low-energy observations it can tell us if they arise from a "reasonable" high-energy theory. In other words it tells us if we should be surprised or not. For example we can imagine that at low energy we measure the theory of a free massless scalar[1]

$$\mathcal{L} = \frac{(\partial\phi)^2}{2} \tag{2.14}$$

and we know from our experimental observations that it is valid at least up to $E \approx M$. Is this surprising from an EFT perspective? The answer is no. We can easily imagine that the UV theory possesses a shift symmetry, $\phi \to \phi + c$, that prevents interactions from being generated when we integrate out high-frequency modes. Of course we expect higher order terms consistent with the symmetry, as for example $(\partial^2\phi)^2/M^2$, but measuring them might be beyond our experimental capabilities.

What about a free massive scalar with $m \ll M$?

$$\mathcal{L} = \frac{(\partial\phi)^2}{2} - \frac{m^2\phi^2}{2}\,. \tag{2.15}$$

The answer is still no. There is nothing surprising in this Lagrangian and this can be seen in at least two ways. I will discuss the most unusual one that I have learned from [13]. In momentum space the Lagrangian

$$\phi(-k)\left(k^2 - m^2\right)\phi(k) \tag{2.16}$$

has an infinite number of symmetries $\phi(k) \to e^{i\alpha(k)}\phi(k)$ with $\alpha(-k) = -\alpha(k)$. To better understand it, we can expand α in odd powers of k,

$$\alpha(k) = a_\mu k^\mu + a_{\mu\nu\rho}k^\mu k^\nu k^\rho + \ldots \tag{2.17}$$

and notice that the linear term corresponds to translations. Its generator in position space is just $i\partial_\mu$ and the corresponding conserved current is the stress-energy tensor $T^{\mu\nu}$. The higher order terms are generated by higher powers of derivatives and are associated with higher-spin currents. The algebra is trivial (for example $[\partial, \partial^3] = 0$) and obviously does not contain dilations or special conformal transformations. This symmetry is broken by higher-point interactions and preserves the form of the free Lagrangian.

Finally it is time to consider a surprising example:

$$\mathcal{L} = \frac{(\partial\phi)^2}{2} - \frac{\lambda\phi^4}{4}\,. \tag{2.18}$$

In this theory both the shift symmetry and the momentum-space symmetry of the free action are broken by λ. So we expect a mass term of order $m^2 \sim \frac{\lambda M^2}{16\pi^2}$. The linear dependence on λ can be deduced using the selection rules of the broken symmetries of the free action[2], while that on M^2 is as usual dictated by the dilation symmetry. We can also explicitly obtain m^2 by integrating out high-momentum modes of ϕ at one-loop from the diagram obtained by contracting two of the ϕ legs in the ϕ^4 vertex.

[1] Here and in the following when Lorentz contractions are obvious I suppress the corresponding indices.
[2] As an exercise you can check how the higher-spin symmetry of the free action enforces $m^2 \propto \lambda$.

Not observing this mass term is indeed surprising and as we will see in the following it is a simple example of a fine-tuning problem. You might object that (2.18) is scale invariant and a mass term should not be generated when integrating out high-momentum modes of ϕ. However the symmetry is broken by the scale M that limits the validity of (2.18). For example it could be (or be proportional to) the physical mass of a new particle that interacts with ϕ or be the scale at which λ hits a Landau pole. Even if the theory at M was transitioning smoothly to a UV fixed point we would still expect contributions to m^2 of $\mathcal{O}(\lambda M^2)$ [14].

Before concluding, it is worth to mention that the way I presented this simple EFT construction rests on completely solid ground. Integrating out one small momentum shell at a time ($M-dM < E < M$, then $M-2dM < E < M-dM$, ...) we generate a flow in the space of possible actions

$$\frac{\partial S_M}{\partial M} = \mathcal{F}(S_M)\,. \tag{2.19}$$

In this picture $\mathcal{F}$ is a smooth function of the couplings and there are no divergences anywhere (we are always integrating between an IR and a UV cutoff). If we expand this differential equation around a solution, irrelevant operators correspond to negative eigenvalues, meaning that the flow is erasing information while going to low energy and converging towards zero. I refer to [12] for more details.

Finally, you might wonder how to assign operator dimensions. For small deviations from a free action (i.e. small couplings) we can assign operator dimensions starting from kinetic terms. For example

$$S = \int d^d x \frac{(\partial\phi)^2}{2}\,, \tag{2.20}$$

implies that $[\phi] = E^{(d-2)/2}$. Then for operators built out of ϕ and its derivatives we can deduce the eigenvalues of the flow $(d-\delta_i)$ by our simple dimensional analysis arguments. At strong coupling we have to take into account also the running of operator dimensions, but this does not invalidate our categorization of operators, it only moves some of them from one category to another.

This suggests the modern interpretation of quantum field theory that is still absent from many textbooks. We can think of any quantum field theory as an EFT valid up to some scale M. Renormalization is just the flow of the action from M to the energy at which we make our measurements. The flow is generated by integrating out high-momentum modes. There are no divergences that need to be cancelled by counterterms. There are only matching calculations between different effective theories to be performed at physical scales. We can always consider these scales one by one, first we have M than maybe new physics appears again at $10M$ and so on. From a pragmatic point of view this is just the most efficient way of describing our finite experimental knowledge. However this also hints to the more radical possibility that there isn't any quantum field theory valid to arbitrarily high energies.

This concludes our brief introduction to EFTs. Through some of the examples in this Section we have already seen the essence of the hierarchy problem. It is the absence of a term in the action predicted by symmetry. However it is worth to be more precise and make some explicit calculation in theories where there is no problem (the Standard Model) and where there is an actual problem (the supersymmetric Standard Model).

3 The Hierarchy Problem

3.1 The Higgs Mass in the Standard Model

There is no real problem associated to m_h in the Standard Model (SM). However, the mere fact of discovering what looks like a fundamental scalar at energies much smaller than $M_{\rm Pl}$ should give us pause. In this Section we make these statements more precise, partially following the very nice exposition in [13].

If we follow 't Hooft naturalness criterion [11] m_h is puzzling, because as $m_h \to 0$ no new symmetry appears in the SM Lagrangian, but we observe $m_h \ll M_{\rm Pl}$, at odds with the selection rules of dilations. We can be more precise on this point by using the higher-spin symmetry introduced in the previous Section. This symmetry is broken by interactions with more than two legs

$$\mathcal{L} = \sum_n \delta^{(n)}(k_1 + ... + k_n)\Gamma^{(n)}(k_1, ..., k_n)\tilde{\phi}(k_1)...\tilde{\phi}(k_n)\,. \tag{3.1}$$

Under the phase shift below Eq. (2.16) we have

$$\Gamma^{(n)}(k_1, ..., k_n) \to \Gamma^{(n)}(k_1, ..., k_n)e^{i\sum_j \alpha(k_j)} \tag{3.2}$$

if $n = 2$ momentum conservation gives $k_1 = -k_2$ and the vertex is invariant. All higher point interactions break the symmetry. The Higgs boson in the SM has plenty of interactions that break this symmetry. Its selection rules together with those of spacetime dilations

$$\begin{aligned} x^\mu &\to s x^\mu\,, \\ m_h &\to s^{-1} m_h\,, \end{aligned} \tag{3.3}$$

allow us to estimate the expected value of the Higgs mass in the SM. First we briefly recall the SM Lagrangian.

3.1.1 The Standard Model Lagrangian

The SM is a gauge theory that describes strong and electroweak interactions via the symmetry groups $SU(3) \times SU(2)_L \times U(1)_Y$. The fermions representations can be summarized as follows

$$\Psi = \left(Q(3,2)_{1/6}, L(1,2)_{-1/2}, u^c(\bar{3},1)_{-2/3}, d^c(\bar{3},1)_{1/3}, e^c(1,1)_1\right)\,, \tag{3.4}$$

where the first number in parenthesis indicates the $SU(3)$ representation, the second one the $SU(2)_L$ one and the subscript the hypercharge. We have suppressed the flavor indexes that label different fermion families for simplicity, but we will restore them below. In most of this work we use the notation in Eq. (3.4) where the SM fermions are described by left-handed Weyl spinors. We occasionally use also a Dirac notation related to the previous one by $Q = (u_L\ d_L)$ and $u^c = \bar{u}_R$.

The three forces described by the Standard Model are included in the Lagrangian as gauge interactions

$$\mathcal{L}_{\rm SMg} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^{\mu\nu i} - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a} + i\overline{\Psi}\gamma^\mu D_\mu \Psi\,. \tag{3.5}$$

The first three operators are the kinetic terms of the gauge bosons that mediate these interactions

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \,, \\ W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g_W \epsilon^{ijk} W^j_\mu W^k_\nu \,, \\ G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^a_\mu G^b_\nu \,, \end{aligned} \tag{3.6}$$

where B_μ, W^i_μ, G^a_μ are the quantum fields describing the gauge bosons, g_W and g_s are the weak and strong gauge couplings and ϵ and f are the structure constants of $SU(2)_L$ and $SU(3)$, respectively. The last operator in the Lagrangian contains the kinetic term of the fermions and their gauge interactions

$$i\overline{\Psi}\gamma^\mu D_\mu \Psi = i\overline{\Psi}\gamma^\mu \left(\partial_\mu + i g_s \sum_{a=1}^{8} \lambda^a G^a_\mu + i g_W \sum_{i=1}^{3} T^i W^i_\mu + i g_Y Y B_\mu \right) \Psi \,. \tag{3.7}$$

In the previous expression λ^a are the generators of $SU(3)$, T^i those of $SU(2)_L$ and Y is the hypercharge. They act in block-diagonal form on Ψ, which is a vector of irreducible representations.

To complete this picture we need to introduce mass terms for fermions and weak gauge bosons. The difficulty resides in the fact that a mass term for the fermions clashes with $SU(2)_L$ invariance, since it couples left-handed fields with right-handed ones

$$m\overline{\psi}\psi = m(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L) \,. \tag{3.8}$$

The problem with a mass term for W^i_μ is that it is not invariant under the gauge shift $W \to W + \partial\alpha$ that preserves the right counting of degrees of freedom, i.e. a massive gauge boson has one extra degree of freedom compared to the massless gauge boson described by $\mathcal{L}_{\rm SMg}$. The introduction of a single $SU(2)_L$ spin zero doublet

$$H(1,2)_{1/2} \tag{3.9}$$

solves both problems. In the following we will call this field the Higgs boson. H couples to gauge bosons

$$\mathcal{L}_H \supset |D_\mu H|^2 \tag{3.10}$$

and can have the following Yukawa couplings with SM fermions

$$\mathcal{L}_Y = -Y_u Q H u^c - Y_d Q H^\dagger d^c - Y_e L H^\dagger e^c + \text{h.c.} \tag{3.11}$$

$Y_{u,d,e}$ are 3×3 matrices in flavor space, Lorentz and gauge indexes are left implied. If we imagine a non-trivial form for the Higgs potential

$$\mathcal{L}_H = |D_\mu H|^2 + m_h^2 |H|^2 - \frac{\lambda}{2} |H|^2 \,, \tag{3.12}$$

where $m_h^2 > 0$, the ground state of the theory is at a non-zero value of the field

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \,, \quad v = \frac{m_h}{\sqrt{\lambda}} \,. \tag{3.13}$$

In the following we will often use $\langle h \rangle$ to denote the vacuum expectation value of the Higgs boson. h is the spin-0 degree of freedom that remains in the H doublet in unitary gauge. Choosing new field variables we can write H as

$$H = e^{i\frac{\sigma^i \pi^i}{2}} \begin{pmatrix} 0 \\ h \end{pmatrix} . \tag{3.14}$$

A $SU(2)_L$ gauge transformation allows us to get rid of the π^i's, giving

$$H_{\text{unitary}} = \begin{pmatrix} 0 \\ h \end{pmatrix} . \tag{3.15}$$

This is the scalar that was produced at CMS and ATLAS and was observed to have $m_h \simeq 125$ GeV. The degrees of freedom described by π^i of course do not disappear from the theory, they make up the extra degrees of freedom that allows three of the $SU(2)_L \times U(1)_Y$ gauge bosons to become massive. If we expand the SM Lagrangian, including H,

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{SMg}} + \mathcal{L}_Y + \mathcal{L}_H \, , \tag{3.16}$$

around its true ground state, we find a mass term for the gauge bosons

$$|D_\mu H|^2 = m_W^2 W_\mu^+ W^{\mu -} + m_Z^2 Z_\mu Z^\mu + ... \, , \tag{3.17}$$

where

$$W_\mu^\pm = \frac{W_\mu^1 \pm i W_\mu^2}{\sqrt{2}} \, , \quad m_W = \frac{g_W v}{\sqrt{2}} \, , \tag{3.18}$$

and

$$\begin{aligned} Z_\mu &= \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu \, , \quad \theta_W = \arctan\frac{g_Y}{g_W} \, , \\ m_Z &= \frac{\sqrt{g_W^2 + g_Y^2} v}{\sqrt{2}} \, . \end{aligned} \tag{3.19}$$

The fermions get a mass through their Yukawa couplings to H. We can first write them using singular value decomposition

$$Y_a = U_L^a Y_a^D U_R^{a\dagger} \, , \tag{3.20}$$

then we can use the large flavor symmetry of the gauge Lagrangian $SU(3)^5 \times U(1)^4$ to rotate away $U_R^{a\dagger}$. We can do the same for U_L^a and be left with flavor-diagonal Yuakwas that give mass to the fermions

$$\begin{aligned} \mathcal{L}_Y &= -Y_u^D Q H u^c - Y_d^D Q H^\dagger d^c - Y_e^D L H^\dagger e^c + \text{h.c.} \\ &= -Y_u^D v Q u^c - Y_d^D v Q d^c - Y_e^D v L e^c + ... \end{aligned} \tag{3.21}$$

The largest of these masses, given by the largest coupling of the Higgs boson to a SM particle (including gauge couplings) is that of the top quark. After diagonalization of Y_u we have

$$\mathcal{L}_Y \supset y_t Q H t^c + \text{h.c.} = y_t t(h+v)t^c + \text{h.c.} = m_t \left(\frac{h}{m_t} + 1\right) t t^c + \text{h.c.} \tag{3.22}$$

We will use this part of $\mathcal{L}_Y$ quite often in what follows. Note that a single unitary matrix, known as the Cabbibo-Kobayashi-Maskawa (CKM) matrix [15, 16]

$$V \equiv U_L^u U_L^{d\dagger}\,, \tag{3.23}$$

remains in the Lagrangian and mixes different mass eigenstates through weak interactions, after we properly diagonalize the Yukawas,

$$\mathcal{L}_{\text{SMg}} \supset g_W W_\mu^+ \left(\overline{u}_L V \gamma^\mu d_L\right) + \text{h.c.} \tag{3.24}$$

The reason being that weak interactions couple the quark doublet Q with itself, so they respect a single $SU(3)$ flavor symmetry in the left-handed quark sector. To fully rotate away the $U^a_{L,R}$ we need a separate $SU(3)$ for each quark (i.e. u_L, d_L, u_R and d_R), but the Lagrangian is symmetric only under $SU(3)^5 = SU(3)_Q \times SU(3)_{u_R} \times SU(3)_{d_R} \times SU(3)_L \times SU(3)_{e_R}$.

The quark and lepton masses are strongly hierarchical. The heaviest particle is the top quark. Its Yukawa coupling is $y_t = \mathcal{O}(1)$. The lightest SM fermions (electron, u and d quarks) have couplings to the Higgs of $\mathcal{O}(10^{-5})$. At scales comparable to m_h the particles that are most strongly coupled to the Higgs after the top quark are W and Z bosons, since their gauge couplings are $\mathcal{O}(0.5)$.

To conclude this Section we note that there is an operator allowed by all the symmetries introduced above that we have not yet discussed:

$$\frac{\alpha_s \theta}{8\pi} G^a_{\mu\nu} \widetilde{G}^{\mu\nu a}\,, \tag{3.25}$$

where $\widetilde{G}^{\mu\nu a} = (1/2)\epsilon^{\mu\nu\rho\sigma} G^a_{\rho\sigma}$ and $\epsilon^{\mu\nu\rho\sigma}$ is the completely antisymmetric symbol. This operator is a total derivative and perturbatively it does not have a measurable effect. However, at scales where QCD confines, the interference between classical solutions for the gluon field that fall off sufficiently slowly at infinity and quark masses induces observable effects that depend on θ. From measurements of the neutron electric dipole moment we can conclude that $\theta \lesssim 10^{-10}$ [17]. In what follows we will return to this operator for two reasons: 1) The smallness of θ is puzzling from the point of view of dimensional analysis (this is the so-called *strong CP problem*) 2) This is the only operator in the SM whose vacuum expectation value is sensitive to $\langle h \rangle$ and this will play an important role in the explanations that we propose for the value of m_h.

A last aspect of the SM Lagrangian that is worth mentioning, are neutrino masses. The Lagrangian that we have written so far preserves a $U(1)^4$ symmetry even after Yukawa couplings are included. We have a $U(1)_B$, i.e. a phase rotation of all the quarks that are therefore said to carry baryon number and a $U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$, i.e. the phase of each lepton family can be chosen arbitrarily. This latter symmetry is not observed in Nature, only the total lepton number (i.e. the diagonal subgroup of $U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$) is conserved in current experiments. Observing the violation of individual lepton numbers has lead to experimentally establish the existence of neutrino masses [18].

We have two possibilities to include them in the SM. We can add a neutral singlet $N(1,1)_0$. N, together with the neutral component of L, forms a Dirac fermion with mass given by the Yukawa interaction

$$\mathcal{L}_{Y\nu} = -y_N \overline{L} H N + \text{h.c.} \tag{3.26}$$

Alternatively we can include a small Majorana mass for the neutral component of L that would break also the total lepton number. The simplest $SU(2)_L$ invariant operator that can generate this mass term is

$$\mathcal{L}_{M\nu} = \frac{(HL)^2}{\Lambda_N} + \text{h.c.} \tag{3.27}$$

where Λ_N is an unknown scale such that $m_\nu \sim v^2/\Lambda_N$. These two options can be distinguished experimentally, given that they preserve different symmetry groups, but we do not yet have the required sensitivity [18].

3.1.2 All We Can Say About m_h in the Standard Model

We now have all the ingrendients to estimate m_h in the SM EFT based on symmetry. Spacetime dilations $x \to sx$ tell us that

$$\begin{aligned} S &= \int d^4x \left(m_h^2 |H|^2 + m_t t t^c\right) \to \int d^4x \left(s^2 m_h^2 |H|^2 + s m_t t t^c\right) \\ m_h^2 &\to s^{-2} m_h^2\,, \\ m_t &\to s^{-1} m_t\,, \end{aligned} \tag{3.28}$$

where we have kept only the largest mass scale in the SM besides m_h. So from dilations we can conclude that

$$m_h^2 \sim m_t^2 + ... \tag{3.29}$$

where the ellipses represent smaller mass scales in the theory. The selection rules of the higher-spin symmetry that we discussed in Sections 2.2 and 3.1 can be enforced by the spurion transformation

$$y_t \to e^{-i\alpha_h(p)} y_t\,, \tag{3.30}$$

since

$$\begin{aligned} \tilde{h}(p) &\to e^{i\alpha_h(p)} \tilde{h}(p) \\ y_t h t t^c &\to y_t e^{i\alpha_h(p)} h t t^c\,. \end{aligned} \tag{3.31}$$

As before, we are considering only the largest SM coupling of the Higgs boson, y_t. We can then conclude that

$$m_h^2 \sim \frac{|y_t|^2}{16\pi^2} m_t^2 + ... \tag{3.32}$$

The $16\pi^2$ derives from restoring units to $\hbar$. Since $[y_t] = \hbar^{-1/2}$ and $[m_h] = [m_t]$ we need a loop factor to get the right dimensions.

Purely within the SM there is no tension at all, given that $m_t \simeq 174$ GeV. There is, however, a much larger mass scale associated with gravity[3], so naively we expect[4]

$$m_h^2 \sim M_{\rm Pl}^2 + ... \tag{3.33}$$

Why is this expectation not realized in Nature? We give possible answers to this question in the rest of these lectures. However, at this stage it is perhaps more pertinent to ask: what does this estimate really mean? We have stated multiple times that there is neither a problem in the SM, nor a way to compute m_h. This estimate is actually telling us what happens in a higher energy theory where m_h can be computed. This type of theory is particularly relevant because string theory, our current best shot at describing quantum gravity, falls in this category if it requires supersymmetry. In the next two Sections we see explicitly what happens if we try to compute m_h in the SM and then in a supersymmetric theory, but first it's useful to look at a second formulation of the problem.

The symmetry estimate of m_h that we have just performed can be rephrased following [19]. This formulation is completely equivalent to our symmetry arguments. Consider two widely separated scales, $\Lambda_{\rm UV} \gg \Lambda_{\rm IR}$. For definiteness $\Lambda_{\rm UV} \simeq 10^{16}$ GeV could be the scale where a non-supersymmetric Grand Unified Theory (GUT) is realized, while $\Lambda_{\rm IR}$ could be the Fermi scale. If there are no other intermediate scales, the energy dependence of physical quantities at scales $\Lambda_{\rm IR} \ll E \ll \Lambda_{\rm UV}$ is weak and we can approximate this intermediate regime with a CFT. This approximate CFT is nothing but the free SM. From the CFT viewpoint, the stability of the hierarchy between $\Lambda_{\rm IR}$ and $\Lambda_{\rm UV}$ depends on the dimensionality of the scalar operators describing the perturbations of the CFT Lagrangian around the fixed point.

If the theory contains an operator $\mathcal{O}_\Delta$ with dimension $\Delta < 4$, we expect, from the same symmetry considerations in Section 3.1, that UV physics generates

$$\mathcal{L}_p = c\Lambda_{\rm UV}^{4-\Delta}\mathcal{O}_\Delta \,, \tag{3.34}$$

with $c = \mathcal{O}(1)$. This gives the IR scale

$$\Lambda_{\rm IR} = c^{\frac{1}{4-\Delta}}\Lambda_{\rm UV} \,. \tag{3.35}$$

If $4 - \Delta = \epsilon \simeq 0$, we can have an exponential hierarchy

$$\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}} = c^{\frac{1}{\epsilon}} \,, \tag{3.36}$$

also for $c = \mathcal{O}(1)$. This is the case, for instance, for the QCD scale. The corresponding deformation, the glueball field $G^a_{\mu\nu}G^{\mu\nu a}$ is marginally relevant. Its scaling dimension deviates from 4 only from small loop corrections $\Delta_g \simeq 4 - a g_s^2$ and becomes 4 at the gaussian fixed point.

[3] In the next lectures we comment on the use of $M_{\rm Pl}$ as a mass scale in our estimate.

[4] Note that if we restore units to $\hbar$, $M_{\rm Pl}$, defined as $S = \int d^4x\sqrt{-g}(M_{\rm Pl}^2/)R$ has the dimensions of a vev $[M_{\rm Pl}^2] = [x^{-2}] = [m^2\hbar]$ so in our estimate we are imagining a $\mathcal{O}(1)$ coupling in front. This definition also implies $M_{\rm Pl} \equiv 1/(8\pi G_N)$ and $v^2/M_{\rm Pl}^2 \simeq 10^{32}$.

However, if the perturbation is relevant, as is the case for the Higgs mass, $4 - \Delta \simeq 2$, then

$$\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}} \simeq \sqrt{c}\,, \tag{3.37}$$

and $\Lambda_{\rm IR}/\Lambda_{\rm UV} \ll 1$, requires a tiny c, at odds with our expectations from dimensional analysis stated in the previous Section. Let's see this intuition at work in the SM. What happens if we try to turn these estimates into an actual calculation?

3.1.3 In the Standard Model m_h is not Calculable

In this Section I depart from the nice physical picture of EFTs that we followed in Section 2.2. I introduce a much more opaque, but more practical way of computing quantities in a EFT. Instead of starting in the UV, at the largest scale M where your theory is valid, and then integrate out momentum shells one by one, I start in the IR and pretend that I can extend my theory to arbitrarily high energies. Computationally this is much more convenient, but physically it is much less clear and gives the impression that we have to get rid of infinities by adding to the theory arbitrary counterterms.

The physical picture is the one discussed by Wilson: you measure your parameters at some fixed scale M and then by integrating out momentum shells you generate a flow (i.e. the parameters in the low energy theory differ from those measured at M by a calculable amount). There are no infinities and everything is smooth in the couplings.

However most quantum field theory books and courses insist in giving precedence to a different way of computing in a EFT, over Wilson's picture. To dispel the most common misconceptions on the hierarchy problems I am forced to stick to this less physical (but more efficient) way of doing computations.

The Calculation that Everybody Likes Let's see what happens if we try to compute m_h in a simplified version of the SM, where we set to zero all but the leading coupling of the Higgs

$$\mathcal{L}_{\rm SM,toy} = \frac{(\partial_\mu h)^2}{2} - \frac{m_h^2 h^2}{2} + i t^\dagger \overline{\sigma}^\mu t + i t^{c\dagger} \overline{\sigma}^\mu t^c + y_t (h+v)(t t^c + \text{h.c.})\,. \tag{3.38}$$

We first follow the standard point of view on renormalization that consists in computing loops in the low energy theory and regulating the theory with counterterms. Since we are interested in the h two-point function we add to the theory two counterterms δ_ϕ and δ_m, needed to make sense of the calculation

$$\mathcal{L}_{\rm SM,toy} \supset \frac{(\partial_\mu h)^2}{2} - \frac{m_h^2 h^2}{2} = (1+\delta_\phi)\frac{(\partial_\mu h_R)^2}{2} - (1+\delta_m+\delta_\phi)\frac{m_R^2 h_R^2}{2}\,. \tag{3.39}$$

If you are uneasy with adding these vertexes by hand, you're not the only one, I am too, and I think that the Wilsonian calculation in the previous Section provides a far superior physical picture. Here we're forced to add counterterms because we're trying to do something unphysical: extrapolate a theory valid to some finite energy all the way up to infinite energy.

In the counterterm equation we introduced m_R, the so-called renormalized mass. It is a finite quantity, but it is not always a physical quantity. m_R depends on the chosen renormalization scheme and it might or might not coincide with the pole mass (i.e. what we can actually measure).

At one loop the Higgs two-point function is

$$i\Sigma_2(p^2) = (iy_t)^2 \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}\left[(\not{p} + \not{k} + m_t)(\not{k} + m_t)\right]}{[(p+k)^2 - m_t^2 + i\epsilon][k^2 - m_t^2 + i\epsilon]} \,. \tag{3.40}$$

Using standard techniques to evaluate the Feynman integral in dimensional regularization ($d = 4 - 2\epsilon$) we get

$$\begin{aligned} i\Sigma_2(p^2) &= -\frac{y_t^2}{4\pi^2} \left\{ \frac{6m_t^2}{\epsilon} - \frac{p^2}{\epsilon} + m_t^2 - \frac{p^2}{6} \right. \\ &+ \left. \int_0^1 dx \left[3p^2 x(1-x) - 3m_t^2\right] \log \frac{m_t^2 - p^2 x(1-x)}{4\pi\mu^2 e^{-\gamma_E}} \right\} . \end{aligned} \tag{3.41}$$

The scale μ was added to restore the right dimension of Σ_2 when we do the integral in $d = 4 - 2\epsilon$. If we resum all 1PI diagrams the Higgs propagator at one-loop order becomes

$$iG_h(p^2) = \frac{i}{p^2 - m_R^2 + \Sigma_2(p^2) + p^2\delta_\phi - (\delta_m + \delta_\phi)m_R^2} \,. \tag{3.42}$$

The most physical way to assign a value to the counterterms is to require the pole of the propagator to be at the measured value of the mass (that we call $\hat{m}_h^2$) and the residue of the pole to be equal to i. This is the so-called on-shell subtraction scheme and can be easily translated into the following equations

$$\begin{aligned} \hat{m}_h^2 &= m_R^2 - \Sigma_2(\hat{m}_h^2) - \hat{m}_h^2\delta_\phi + (\delta_m + \delta_\phi)m_R^2 \quad \text{Pole} \\ 1 &= \frac{1}{1 + \left.\frac{d\Sigma_2(p^2)}{dp^2}\right|_{p^2=\hat{m}_h^2} + \delta_\phi} \quad \text{Residue} \end{aligned} \tag{3.43}$$

This is not yet enough to specify all free parameters. We have introduced two counterterms $\delta_{\phi,m}$ and we have the two above conditions from our measurements, but we are left with the freedom of choosing m_R. In the on-shell subtraction scheme one takes $m_R^2 = \hat{m}_h^2$. This is true at leading order (i.e. at tree-level), but it becomes an assumption at the order of our calculation. This arbitrary choice does not affect the physics and, as we see below, different choices give the same result. In this renormalization scheme we have, from Eq. (3.43),

$$\delta_m = \frac{\Sigma_2(\hat{m}_h^2)}{\hat{m}_h^2} \,, \qquad \delta_\phi = -\left.\frac{d\Sigma_2(p^2)}{dp^2}\right|_{p^2=\hat{m}_h^2} \,. \tag{3.44}$$

Substituting into the propagator we get something finite

$$iG_h(p^2) = \frac{i}{p^2 - \hat{m}_h^2 + \Sigma(p^2)} \,, \quad \Sigma(p^2) = \frac{y_t^2}{4\pi^2}\left[\frac{(p^2 - \hat{m}_h^2)^2}{20m_t^2} + \mathcal{O}\left(\frac{\hat{m}_h^6}{m_t^4}\right)\right] \tag{3.45}$$

but also kind of useless. We have one free parameter (m_h^2 or equivalently m_R^2) and we can make one measurement ($\hat{m}_h^2$) to fix it. We can't predict m_h^2 in terms of parameters that we can measure independently of the pole in the Higgs propagator. Even if you add back all other couplings in the SM, the conclusion remains exactly the same and, of course, it doesn't change if we take $m_t \gg m_h$. In this case we would expect a problem from our dimensional analysis estimate ($m_h^2 \sim y_t^2 m_t^2/(16\pi^2)$), but since m_h^2 or equivalently m_R^2 are not calculable in the SM, neither is $\hat{m}_h^2$. Measuring $\hat{m}_h^2$ fixes m_R^2. Once we make this measurement the limit $m_t \to \infty$ is smooth, nothing happens to the propagator in Eq. (3.45). If we do not make this measurement we have a bunch of free parameters left in the theory (m_R^2, δ_m and δ_ϕ) and we cannot make predictions.

Unfortunately there is no way of writing

$$\hat{m}_h^2 = f(\hat{e}, \hat{m}_W, \hat{m}_t, \hat{y}_t, ...)\,, \tag{3.46}$$

with only measured quantities on the right-hand side. When we say that the Higgs mass is not calculable in the SM this is what we mean. Since it is not calculable there is no way to formulate a sharp physical problem in terms of observables when a mass scale M coupled to H becomes large. Dimensional analysis tells us that there is something wrong $m_h^2 \sim M^2$, but an actual calculation does not show any patology.

One could find some manifestations of this tension, in the fact that the EFT looks peculiar, but there is no way to find an actual physical problem. One peculiarity is that the $\overline{\text{MS}}$ mass is very different from the pole mass.

In the $\overline{\text{MS}}$ scheme one takes the counterterms equal to the infinite part of the loop integrals plus an $\mathcal{O}(1)$ number that cancels annoying constants (i.e. $\log 4\pi$ and γ_E). This choice is purely conventional, it deserves a name because it enormously simplifies QCD calculations. It obviously gives the same result for observables as the previously discussed on-shell scheme. In $\overline{\text{MS}}$ we have

$$\delta_\phi = -\frac{y_t^2}{4\pi^2}\left(\frac{1}{\epsilon} + \log 4\pi e^{-\gamma_E}\right)\,, \quad \delta_m = -\frac{y_t^2}{4\pi^2}\frac{1}{\epsilon}\left(\frac{6m_t^2}{\hat{m}_h^2} - 1\right)\left(1 + \frac{\epsilon}{2}\log 4\pi e^{-\gamma_E}\right) \tag{3.47}$$

In this scheme the renormalized mass m_R^2 is unphysical and depends on the arbitrary subtraction scale μ. It is conventionally called the $\overline{\text{MS}}$ mass and, given the choice of counterterms above, it reads

$$\begin{aligned} m^2_{h,\overline{\text{MS}}}(\mu) \;\equiv\;& m_R^2 = \hat{m}_h^2 + \Sigma_2(\hat{m}_h^2) - \delta_m \hat{m}_h^2 = \\ & \hat{m}_h^2 - \frac{y_t^2}{4\pi^2}\left[m_t^2 - \frac{\hat{m}_h^2}{6} + 3\int_0^1 dx\,\left[\hat{m}_h^2 x(1-x) - m_t^2\right]\log\frac{m_t^2 - \hat{m}_h^2 x(1-x)}{\mu^2}\right] \end{aligned} \tag{3.48}$$

where we have used $m_R^2 = \hat{m}_h^2$ at tree-level. If $\hat{m}_t \gg \hat{m}_h$ there is a big difference between $m^2_{h,\overline{\text{MS}}}(\mu)$ and the pole mass $\hat{m}_h^2$. What does this mean physically? Purely within this theory it means nothing, since $m^2_{h,\overline{\text{MS}}}(\mu)$ is not physical. It shows that the IR Lagrangian parameters are strongly sensitive to UV physics, i.e. if you change m_t by a small fractional amount, $m^2_{h,\overline{\text{MS}}}(\mu)$, changes a lot. This is telling us something about our theory, but not something physical, since $m^2_{h,\overline{\text{MS}}}(\mu)$ cannot be measured.

Note that we have not technically completed our calculation, as we should have also computed one-loop corrections to m_t and traded m_t and y_t for their measured values $\hat{m}_t$ and $\hat{y}_t$. This does not qualitatively change our conclusions, since in this toy model $m_t = \hat{m}_t + \mathcal{O}(y_t^2/16\pi^2)$ and the same is true for y_t, so the $y_t^2 m_t^2$ contribution in our result becomes a $\hat{y}_t^2 \hat{m}_t^2$ plus loop corrections.

The Calculation that Everybody Likes Even More In the previous discussion we have drawn some conclusions by taking the limit $\hat{m}_t \gg \hat{m}_h$. However, in this limit the calculation breaks down due to the large logarithms $\log \hat{m}_t^2/\hat{m}_h^2$ that appear in $\Sigma_2(p^2 = m_h^2)$ when we try to make measurements at low energies compared to $\hat{m}_t$. The same happens in the $\overline{\text{MS}}$ scheme.

Let us imagine that we live in a fictitious universe where the top quark is vector-like. It has a large mass independent of $y_t v$. Our starting point is still Eq. (3.38), valid at high energies, plus a vector-like mass for the top $M_t t t^c$. To avoid the large logs we can compute low energy observables from an effective theory with the top quark integrated out

$$\mathcal{L}_{\text{EFT,toy}} = \frac{(\partial_\mu h)^2}{2} - \frac{m_E^2 h^2}{2} - \frac{\lambda_E h^4}{4} + \ldots \tag{3.49}$$

We have omitted higher dimensional operators suppressed by the large top mass. λ_E is generated at one-loop when one integrates out the top.

To get the right predictions at low energy we have to match this low energy theory to the full theory in Eq. (3.38), i.e. we have to choose the parameters of the low energy theory to give the same result on observables at some arbitrary matching scale μ_M. This insures that the low energy theory gives the right predictions also at lower energies. At tree-level this is trivial and independent of μ_M,

$$\hat{m}_h^2 = m_E^2 \ \text{EFT}\,, \quad \hat{m}_h^2 = m_h^2 \ \text{Full Theory} \quad \rightarrow \quad m_E^2 = m_h^2\,. \tag{3.50}$$

For λ_E the matching condition is simply $\lambda_E = 0$, but we won't need it in what follows. At one-loop we have to choose our renormalization scheme. Let's take $\overline{\text{MS}}$ where we have already done the calculation. In the full theory

$$\hat{m}_h^2 = m_{h,\overline{\text{MS}}}^2(\mu) + \frac{y_t^2}{4\pi^2}\left[M_t^2 - \frac{\hat{m}_h^2}{6} + 3\int_0^1 dx\,\left[\hat{m}_h^2 x(1-x) - M_t^2\right]\log\frac{M_t^2 - \hat{m}_h^2 x(1-x)}{\mu^2}\right] \quad \text{Full Theory}\,. \tag{3.51}$$

In the effective theory at one loop we still have

$$\hat{m}_h^2 = m_E^2 \quad \text{EFT} \tag{3.52}$$

because λ is a one-loop quantity that gives a two-loops correction to the mass. Therefore we have

$$m_E^2(\mu_M) = m_E^2 = m_{h,\overline{\text{MS}}}^2(\mu_M) + \frac{y_t^2}{4\pi^2}\left[M_t^2 - \frac{\hat{m}_h^2}{6} + 3\int_0^1 dx\,\left[\hat{m}_h^2 x(1-x) - M_t^2\right]\log\frac{M_t^2 - \hat{m}_h^2 x(1-x)}{\mu_M^2}\right] \tag{3.53}$$

Again we don't have any physical problem, but we observe the same peculiar feature as before. The value of the $\overline{\text{MS}}$ mass in the low energy theory, that in this case is also the measurable pole mass, is strongly sensitive to the values of high energy mass scales (i.e. m_t). However we still haven't found a physical problem, because the numerical value of m_E^2 is fixed by a low energy measurement and its prediction from the full theory depends on the unknown (and uncomputable) parameter $m^2_{h,\overline{\text{MS}}}(\mu_M)$. In principle we can calculate the value of $m^2_{h,\overline{\text{MS}}}(\mu_M)$, but only a posteriori, from the measurement of $m_E^2 = \hat{m}_h^2$. We would have a problem only if we could determine the value of $m^2_{h,\overline{\text{MS}}}(\mu_M)$ from some independent measurement and then find that $m^2_{h,\overline{\text{MS}}}(\mu_M)$ and $\hat{M}_t^2$ finely cancel to give $\hat{m}_h^2 \ll \hat{M}_t^2$. However in this theory no such measurement exists and as a consequence there is also no hierarchy problem, even if $\hat{M}_t^2 \gg \hat{m}_h^2$.

3.1.4 m_W is Calculable

After what we just said, you might think that no mass is calculable, but this is far from true. Here I give you one example from the SM that is useful to understand theories where the Higgs mass is calculable. In this Section I follow [20], where you will find more details on the calculation.

Following Schwartz's notation we call the measured parameters

$$\hat{e}, \hat{m}_Z, \hat{s}_W, \ldots \tag{3.54}$$

as we did in the previous Section, while we call

$$e, m_Z, s_W, \ldots \tag{3.55}$$

the Lagrangian $\overline{\text{MS}}$ parameters. We want to check if the measured W boson mass is consistent with the SM prediction. In the case of $\hat{m}_h$ we couldn't do it, because there is no prediction. For our calculation we need to make only three measurements: the fine structure constant, the Z-mass and the muon lifetime. From these we determine the Lagrangian parameters e, m_Z, s_W that are enough to compute $\hat{m}_W$.

W **mass** The W-boson propagator is given schematically by the diagrams in Fig. 2

$$-\frac{i g^{\mu\nu}}{p^2 - m_W^2 - \Pi_{WW}(p^2)} + \mathcal{O}(p^\mu p^\nu)\,. \tag{3.56}$$

We ignore the gauge-dependent correction to the $p^\mu p^\nu$ term because measurements at LEP and subsequent colliders were made with SM fermions in the external legs.

The measured W mass can be read from the pole, as we did for the Higgs mass

$$\hat{m}_W^2 = m_W^2 + \text{Re}[\Pi_{WW}(\hat{m}_W^2)]\,. \tag{3.57}$$

Π_{WW} is given by

$$i\Pi_{WW}(p^2) = i\mathcal{M}(W(p) \to W(p))\,, \tag{3.58}$$

and $\mathcal{M}$ is the invariant matrix element of the bubble diagram. We use the same notation for other electroweak bosons in what follows.

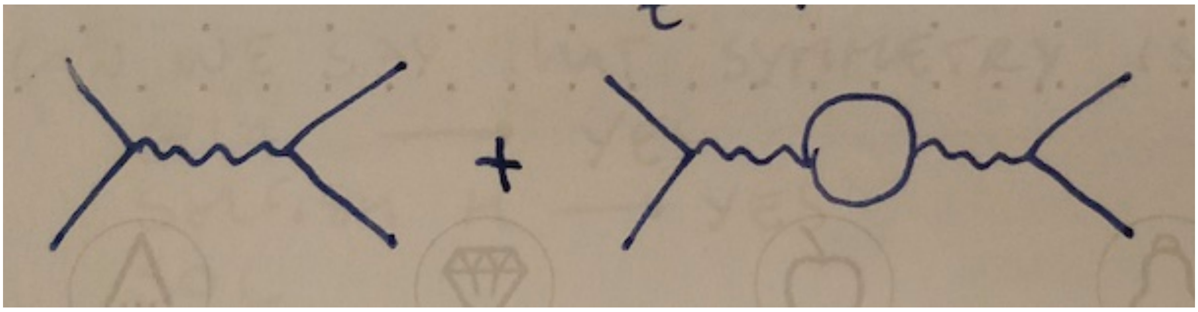

Figure 1: $e^+e^- \to \mu^+\mu^-$ cross section from photon exchange

We can then use the relation between Lagrangian parameters $m_W^2 = c_W^2 m_Z^2$ to express $\hat{m}_W^2$ in terms of other quantities in the SM Lagrangian

$$\hat{m}_W^2 = c_W^2 m_Z^2 + \Pi_{WW}(\hat{m}_W^2)\,. \tag{3.59}$$

If we can measure c_W and m_Z independently of m_W we can compute m_W in the SM and make a prediction that we can compare with experiment. Let's see how.

Gauge coupling To get c_W we need first to measure the EW gauge coupling. The value of $\hat{e}$ can be extracted from the cross section for $e^+e^- \to \mu^+\mu^-$ from photon exchange at center of mass energy s. In terms of observable quantities (we leave the hat implied for the center of mass energy that doesn't have a corresponding non-physical parameter in the Lagrangian)

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{\hat{e}^4(s)}{12\pi s}\,. \tag{3.60}$$

At one-loop order this cross section is determined by the class of diagrams in Fig. 1, giving

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{e^4(s)}{12\pi s}\left(\frac{s}{s-\Pi_{\gamma\gamma}(s)}\right)^2\,. \tag{3.61}$$

We do not include vertex renormalization and electron wavefunction renormalization because at zero momentum transfer they cancel (see for instance [**?**]). The measurement is most precise at small external momentum. Then we will run the coupling with the SM renormalization group equations to get it at $\hat{m}_W$ (or $\hat{m}_Z$ where most precision SM measurements took place at LEP).

Equating the two expressions we get (at one electromagnetic loop order)

$$e^2(s) = \hat{e}^2(s)\left[1 - \frac{\Pi_{\gamma\gamma}(s)}{s}\right] + \mathcal{O}(\alpha^2)\,. \tag{3.62}$$

$e^2(\hat{m}_Z)$ can be evaluated from the $g_e - 2$ measurement

$$\hat{\alpha}(0) = (137.035999074 \pm 0.000000044)^{-1} \tag{3.63}$$

after running up to the electroweak scale. We will use the result of this running

$$\hat{e}^2(\hat{m}_Z) = 4\pi\hat{\alpha}(\hat{m}_Z) = 4\pi(127.944 \pm 0.014)^{-1}\,. \tag{3.64}$$

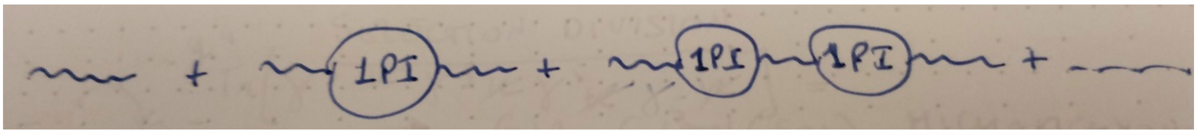

Figure 2: Two-point function

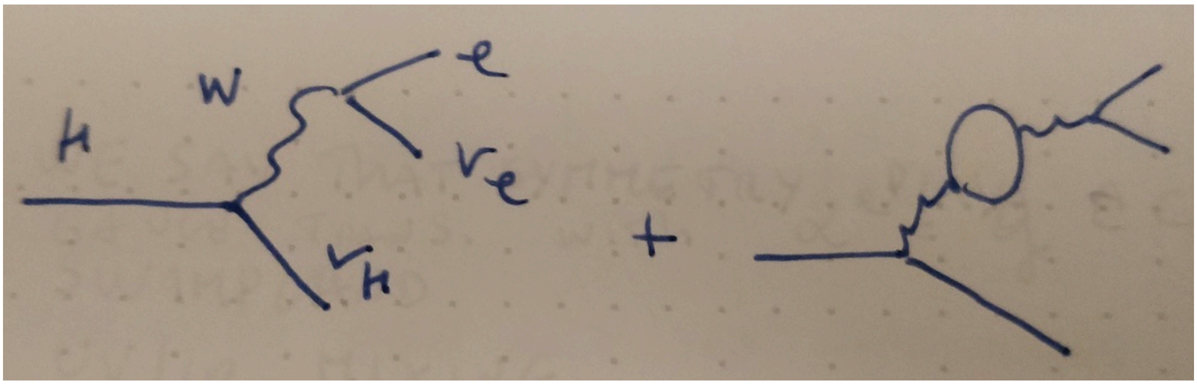

Figure 3: Muon Decay

Z-boson mass By fitting the dilepton invariant mass lineshape, LEP measures the location of the pole in the Z-boson two point function. The Z-boson propagator is given by the diagrams in Fig. 2

$$-\frac{ig^{\mu\nu}}{p^2 - m_Z^2 - \Pi_{ZZ}(p^2)} + \mathcal{O}(p^\mu p^\nu)\,. \tag{3.65}$$

The pole of the above propagator is at

$$\hat{m}_Z = m_Z^2 + \mathrm{Re}[\Pi_{ZZ}(\hat{m}_Z^2)]\,, \tag{3.66}$$

where we have eliminated the contribution of $\Pi_{ZZ}(p^2)$ to the decay width by taking its real part. At this order we can easily invert the above relation

$$m_Z^2 = \hat{m}_Z^2\left(1 - \frac{\mathrm{Re}[\Pi_{ZZ}(\hat{m}_Z^2)]}{\hat{m}_Z^2}\right)\,, \tag{3.67}$$

and use the LEP measurement

$$\hat{m}_Z = (91.1876 \pm 0.0021)\ \mathrm{GeV}\,. \tag{3.68}$$

Weak Mixing Angle We extract the weak mixing angle from the muon lifetime. Its precise measurement allows us to determine the Fermi constant.

The muon decay width at one electroweak loop can be computed from the diagrams in Fig. 3 giving

$$\frac{\hat{G}_F}{\sqrt{2}} = -\frac{e^2}{8s_W^2}\frac{1}{p^2 - m_W^2 - \Pi_{WW}(p^2)}\bigg|_{p^2\simeq 0} = \frac{e^2}{8s_W^2 c_W^2 m_Z^2}\left(1 - \frac{\mathrm{Re}[\Pi_{WW}(0)]}{m_W^2}\right) \tag{3.69}$$

Inverting this equation requires the two previous results for e and m_Z, and the relation

$$\hat{s}_W^2(1-\hat{s}_W^2) = \frac{\pi\hat{\alpha}(m_Z)}{\sqrt{2}\hat{G}_F \hat{m}_Z^2} \qquad \to \hat{s}_W^2 = 0.234289\,. \tag{3.70}$$

Putting all together, after some trigonometry we get

$$s_W^2 = \hat{s}_W^2\left[1 + \frac{\hat{c}_W^2}{\hat{c}_W^2 - \hat{s}_W^2}\left(\frac{\mathrm{Re}[\Pi_{ZZ}(\hat{m}_Z^2)]}{\hat{m}_Z^2} - \frac{\Pi_{\gamma\gamma}(\hat{m}_h^2)}{\hat{m}_h^2} - \frac{\mathrm{Re}[\Pi_{WW}(0)]}{\hat{m}_W^2}\right)\right] + \mathcal{O}(\alpha^2) \tag{3.71}$$

Prediction of the W mass We can now use the relations derived in the previous paragraphs between the Lagrangian parameters in Eq. (3.59) and the observed $\hat{e}, \hat{m}_Z$ and $\hat{c}_W$ to express $\hat{m}_W^2$ in terms of measured quantities

$$\begin{aligned}\hat{m}_W^2 &= c_W^2 m_Z^2 + \Pi_{WW}(\hat{m}_W^2) = \hat{c}_W^2\hat{m}_Z^2\left(1 - \frac{\hat{s}_W^2}{\hat{c}_W^2 - \hat{s}_W^2}\Pi_R + \frac{\mathrm{Re}[\Pi_{ZZ}(\hat{m}_Z^2)]}{\hat{m}_Z^2} + \frac{\mathrm{Re}[\Pi_{WW}(\hat{c}_W^2\hat{m}_Z^2)]}{\hat{c}_W^2\hat{m}_Z^2}\right)\\ \Pi_R &\equiv \left(\frac{\mathrm{Re}[\Pi_{ZZ}(\hat{m}_Z^2)]}{\hat{m}_Z^2} - \frac{\Pi_{\gamma\gamma}(\hat{m}_h^2)}{\hat{m}_h^2} - \frac{\mathrm{Re}[\Pi_{WW}(0)]}{\hat{m}_W^2}\right)\,.\end{aligned} \tag{3.72}$$

Here lies the crucial difference with respect to $\hat{m}_h$. We can write the measurable quantity $\hat{m}_W^2$ in terms of three other quantities that we have measured independently $\hat{s}_W^2, \hat{e}$ and $\hat{m}_Z$. Finally we can make a prediction

$$\hat{m}_W^{\mathrm{SM}} \simeq 80.368\ \mathrm{GeV}\,, \tag{3.73}$$

and compare it with experiment

$$\hat{m}_W^{\mathrm{exp}} = (80.377 \pm 0.012)\ \mathrm{GeV}\,. \tag{3.74}$$

3.1.5 What Have we Learned in the Standard Model?

We did much work to conclude that the selection rules of known symmetries predict

$$m_h^2 \sim \frac{|y_t|^2 m_t^2}{16\pi^2}\,, \tag{3.75}$$

in the SM without gravity and instead predict

$$m_h^2 \sim M_{\mathrm{Pl}}^2\,, \tag{3.76}$$

in the SM with gravity. This second estimate is in great tension with the measured value of m_h.

We have done an even larger amount of work to show that in the SM these estimates cannot be turned into an actual calculation and do not correspond to any physical problem. We can never express m_h^2 as a function of other parameters that we can measure. Even if we include gravity we reach the same conclusion of the toy model with only the top quark. Schematically, if we add gravity, the Higgs measured mass becomes

$$\hat{m}_h^2 = m_R^2 + \Sigma_t(\hat{y}_t, \hat{m}_t, \hat{m}_h^2) + \Sigma_G(\hat{M}_{\rm Pl}^2, \hat{m}_h^2)\,, \tag{3.77}$$

both Σ_t and Σ_G can be computed in terms of observables (and we did it for Σ_t), but m_R^2 remains a free parameter that can be fixed only by measuring the Higgs mass.

What have we learned from all this? The dimensional analysis estimates are telling us something important, but not something important about the SM. They are telling us that we have to be careful when we extend the SM to high energies. The UV theory that extends the SM has to somehow change the symmetries that led to Eq. (3.76) before hitting $M_{\rm Pl}$.

This is a truly useful kind of observation: a low energy measurement ($\hat{m}_h^2 \simeq 125$ GeV) of a relevant operator is giving us important clues on how to extend the theory at high energies.

In the Introduction we have seen two issues similar to the one that we just encountered for the Higgs mass. What we discussed for the electron mass is almost exactly the same problem. We have a quantity that cannot be computed in the low energy theory (because $m_{e,0}$ is free and can only be measured), but is sensitive to high energies ($\hat{m}_e \sim 1/r_e$). This is saying nothing about the consistency of classical electrodynamics. Within that low energy theory you can measure m_e and be done with it. The theory is perfectly consistent and allows you to make predictions that you can compare with experiment.

This tension turned out to teach us something important about the high energy theory that extends classical electrodynamics. At $r_e \sim 1/\hat{m}_e$ a new symmetry emerges and in this case even a new theory of Nature (quantum field theory).

As for the electron, we can associate also to the Higgs mass a scale where we expect something new to happen. If we do not want accidental cancellations between different parts of the low energy calculation (between m_R^2 and $\Sigma_2(\hat{m}_h^2)$ for example) we need the calculation to be modified at a scale

$$m_h^2 \sim \frac{y_t^2 M_{\rm new}^2}{16\pi^2} \to M_{\rm new} \simeq {\rm TeV}\,. \tag{3.78}$$

This kind of rough intuition has paid off several times in the past. Essentially, every time we had a similar potential "fine-tuning" problem between parameters, it was resolved before it became a problem, i.e. at $M < M_{\rm new}$. We have seen this briefly for the electron self-energy, but you might wonder if the same is true also in quantum field theory. The answer is a resounding yes and it even led to the prediction of a new particle and a Nobel prize. Let's see two examples. Charged and neutral pions have a mass difference induced by the coupling of the charged pion to a photon. We can estimate the size of this coupling in the chiral Lagrangian using Naive Dimensional Analysis (NDA) [21]

$$\Lambda_{\rm QCD}^2 f_\pi^2 \frac{\pi^\dagger \overleftrightarrow{\partial}_\mu \pi}{f_\pi^2 \Lambda_{\rm QCD}} \frac{e A^\mu}{\Lambda_{\rm QCD}} = e A^\mu \pi^\dagger \overleftrightarrow{\partial}_\mu \pi\,. \tag{3.79}$$

From this vertex we can estimate the mass difference by cutting off the loop diagram that corrects the $\pi^\pm$ two-point function with a one photon exchange

$$m_{\pi^+}^2 - m_{\pi^0}^2 \sim e^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - m_\pi^2)^2} \sim \frac{\alpha}{4\pi} M_{\rm new}^2 \,. \tag{3.80}$$

We used a hard momentum cutoff at $M_{\rm new}$, the scale where we expect the calculation to be modified. From the measured mass difference we have

$$M_{\rm new}^{\Delta {\rm m}_\pi} \simeq 850 \text{ MeV} \,, \tag{3.81}$$

and indeed we observe the ρ meson at $m_\rho \simeq 770$ MeV. This new particle modifies the calculation. Slightly above this scale pions cease to be a good description of the dynamics which becomes that of perturbative QCD. What protects the pion mass compared to larger scales in the theory is the approximate scale invariance of QCD, as we discuss in the next Sections.

A second, more striking, example is that of the Kaons mass difference that allowed to predict the existence of the charm quark. Kaons are spin zero bound states of a strange and a down quark $K^0 \sim d\bar{s}$. For pedagogical purposes imagine to know the interactions of the three lightest quarks with $SU(2)_L$. Among other vertices you will find

$$\mathcal{L}_{SM} \supset -\frac{g}{\sqrt{2}} W^\mu \left[\bar{u}\gamma_\mu \frac{(1-\gamma_5)}{2} (d \cos\theta_c + s \sin\theta_c) \right] \,, \tag{3.82}$$

From this Lagrangian you can easily write a box diagram (one loop with internal W's and u quarks) that mixes $d\bar{s}$ with $\bar{d}s$, i.e. mixes the two mesons K^0 with $\bar{K}^0$. If you integrate out the W and you match this calculation to the effective theory of mesons (the chiral Lagrangian of QCD that becomes a good description of the SM at energies where the QCD coupling is large, i.e. below $\sim$ GeV), you will find a mass mixing between the two Kaons

$$\begin{pmatrix} \overline{K}^0 & K^0 \end{pmatrix} \begin{pmatrix} m_K^2 & \delta m_K^2 \\ \delta m_K^2 & m_K^2 \end{pmatrix} \begin{pmatrix} \overline{K}^0 \\ K^0 \end{pmatrix} \,. \tag{3.83}$$

The loop diagram that you just computed to get δm_K^2 is sensitive to high energies. Let's cut it off again at $M_{\rm new}$. In this case we get

$$\frac{m_{K_L^0} - m_{K_S^0}}{m_{K_L^0}} = \frac{G_F^2 f_K^2}{6\pi^2} \cos^2\theta_c \sin^2\theta_c M_{\rm new}^2 , \quad f_K \approx 114 \text{ MeV} \,. \tag{3.84}$$

Numerically

$$M_{\rm new} \simeq 2 \text{ GeV} \,. \tag{3.85}$$

At $m_c \simeq$ GeV a new particle appears, the charm quark, with interactions

$$\mathcal{L}_{SM} \supset -\frac{g}{\sqrt{2}} W^\mu \left[\bar{c}\gamma_\mu \frac{(1-\gamma_5)}{2} (-d \sin\theta_c + s \cos\theta_c) \right] \,. \tag{3.86}$$

This new interaction enters the calculation of $m_{K_L^0} - m_{K_S^0}$ cancelling the sensitivity to high energies. This is how the charm quark was predicted [] (modulo some abuse of our modern knowledge to make the exposition more streamlined).

To summarize, we have seen two examples in QFT of quantities that are not calculable in a low energy theory (the pions' and Kaons' mass differences), because they are sensitive to high energies. If we had done a proper calculation, as we did for m_h we would have found

$$\begin{aligned} \hat{m}_{\pi^+}^2 - \hat{m}_{\pi^0}^2 &= \Delta m_\pi^2 + \Sigma_\pi(\hat{m}_\pi, \hat{e}, \hat{f}_\pi) \,. \\ m_{K_L^0} - m_{K_S^0} &= \Delta m_K^2 + \Sigma_K(\hat{m}_K, \hat{G}_f, \hat{\theta}_c, \hat{f}_K) \,, \end{aligned} \tag{3.87}$$

with Δm_π^2 and Δm_K^2 unknown Lagrangian parameters and $\Sigma_{\pi,K}$ calculable (at least on the lattice). Instead we tried to deduce a scale M_{new} by comparing the calculable part in the EFT with the measured value of the observable. We asked that M_{new} be small enough that the calculable part be at most comparable to the measured quantity. We always found a modification of the EFT calculation below M_{new}. This is good news for the hierarchy problem. In the SM we expect $M_{\text{new}} \simeq$ TeV as shown in Eq. (3.78).

3.2 Theories Where m_h is Calculable and the Actual Hierarchy Problem

3.2.1 Supersymmetry

We can consider the Minimal Supersymmetric Standard Model (MSSM) which extends the SM with the minimal field content needed to realize supersymmetry. The algebra of $N = 1$ supersymmetry (i.e. the simplest version with two spinorial generators) is

$$\begin{aligned} \{Q_\alpha, Q_{\dot{\alpha}}^\dagger\} &= 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu \,, \\ \{Q_\alpha, Q_\beta\} &= \{Q_{\dot{\alpha}}^\dagger, Q_{\dot{\beta}}^\dagger\} = 0 \,, \\ [Q_\alpha, P^\mu] &= \left[Q_{\dot{\alpha}}^\dagger, P^\mu\right] = 0 \\ [M^{\mu\nu}, Q_\alpha] &= \frac{(\sigma^{\mu\nu})_\alpha^{\ \beta}}{2} Q_\beta \,, \\ \left[M^{\mu\nu}, Q_{\dot{\alpha}}^\dagger\right] &= \frac{(\overline{\sigma}^{\mu\nu})_{\dot{\alpha}}^{\ \dot{\beta}}}{2} Q_{\dot{\beta}}^\dagger \end{aligned} \tag{3.88}$$

with the addition of the usual Poincaré algebra that we left implicit. Supersymmetry (SUSY) is not the main focus of this work and we refer the reader to [22, 23, 24] for the derivation and significance of this result. For our purposes it is sufficient to notice two powerful consequences of this symmetry algebra:

The Cosmological Constant is Calculable (and protected by SUSY) See Section 5.

The Higgs Mass is Calculable (and protected by SUSY) The existence of two spinorial generators implies that particles of different spin belong to the same supersymmetric multiplet. Let me call $|F\rangle$ fermions and $|B\rangle$ bosons, then

$$Q|F\rangle = |B\rangle\,, \quad Q^\dagger|B\rangle = |F\rangle\,, \tag{3.89}$$

given the algebra, applying $QQ^\dagger$ is a translation

$$QQ^\dagger|B(x)\rangle = |B(x+a)\rangle\,, \tag{3.90}$$

additionally $[Q, P^2] = 0$ and $[Q^\dagger, P^2] = 0$, as again implied by the algebra. Putting all together we have that there must be fermions and bosons with the same mass (that form a multiplet of the algebra)

$$\begin{aligned} \langle B|P^2|B\rangle &= m_B^2\langle B|B\rangle \\ \langle B|P^2|B\rangle &= \langle F|Q^\dagger P^2 Q|F\rangle = \langle F|P^2|F\rangle = m_F^2\langle F|F\rangle\,. \end{aligned} \tag{3.91}$$

In practice to make the SM supersymmetric we need a new $s=0$ scalar for each fermion, and a new $s = 1/2$ fermion for each complex scalar (note that the algebra implies also that Q and $Q^\dagger$ have spin 1/2). Stated more compactly, $P_\mu P^\mu$ is a Casimir of the algebra and the fermionic nature of the generators forces particles with integer and half-integer spin in the same multiplet. This relates m_h to the mass of a fermion $m_{\tilde{h}}$. As we have seen in the previous sections, fermion masses break chiral symmetries. If the mass itself is the only source of breaking, the selection rules of these symmetries tell us that the result of any calculation of the mass must be in the form

$$\hat{m}_h = \hat{m}_{\tilde{h}} = m_{\tilde{h},\,\mathrm{tree}}\,[...]\ . \tag{3.92}$$

There cannot be power law sensitivity to high scales as in the SM. This is of course not the whole story, because the sensitivity to high scales could still be logarithmic

$$\hat{m}_h = \hat{m}_{\tilde{h}} = m_{\tilde{h},\,\mathrm{tree}}\,[\log\Lambda + ...]\ . \tag{3.93}$$

In addition to that, supersymmetry is broken. However the Higgs mass might still be calculable, even taking these two extra subtleties into account. If supersymmetry is only softly broken, by a dimensionful parameter M_S, we must include also contributions of the type

$$\hat{m}_h = \hat{m}_{\tilde{h}} + M_S = M_S + m_{\tilde{h},\mathrm{tree}}\,[\log\Lambda + ...]\ . \tag{3.94}$$

The supersymmetry breaking parameters that we paramterized with M_S can in principle be measured independently of $\hat{m}_{\tilde{h}}$ and so can Λ and everything else in the ellipses, as we show below. Stated in terms of our language of choice: M_S is the largest scale allowed in the estimate of m_h by the selection rules of supersymmetry. Higher scales do not break SUSY and cannot enter the calculation of the Higgs mass as a power-law correction. Note that this is true only if supersymmetry is broken softly (i.e. by dimensionful parameters like M_S). Imagine that we do not supersymmetrize the top sector, then y_t breaks supersymmetry and the estimate of the Higgs mass is

$$m_{\tilde{h}}^2 \sim |y_t|^2\frac{M^2}{16\pi^2}\,, \tag{3.95}$$

where M is any large scale coupled to the top sector and it doesn't have to break SUSY. In this case we are back in the SM case: m_h^2 is not calculable and sensitive to unknown large scales. Summarizing, in a theory with *softly* broken supersymmetry the Higgs mass receives contributions only up to a finite scale where SUSY is broken and we can compute it as a function of supersymmetric and supersymmetry breaking parameters that we can measure independently of m_h. More explicitly the estimates based on selection rules in Section 3.1.2 become

$$m_h^2 \sim \max[M_S^2, m_{\tilde{h}}^2]\,, \tag{3.96}$$

where M_S is a supersymmetry breaking mass. It could also become

$$m_h^2 \sim \max\left[\frac{|y_t|^2}{16\pi^2}M_S^2, m_{\tilde{h}}^2\right]\,, \tag{3.97}$$

if SUSY breaking is communicated to the Higgs only through top loops. It is technically natural for M_S to be much smaller than the Planck scale, since M_S is breaking supersymmetry (i.e. it transforms differently as a SUSY spurion compared to $M_{\rm Pl}$).

In the SM there is always an unknown contribution from high energies. In the previous Section we have parametrized our ignorance of high energies in terms of the $1/\epsilon$ pole, but we could have done it with a dimensionful cutoff Λ. There is no way to measure this unknown contribution, we don't even know at what energy it is saturated. We can only absorb this unknown quantity into a low energy measurement, as we did in the previous Sections. The measurement of $\hat{m}_h$ was fixing the bare Lagrangian parameter m_R plus the unknown $1/\epsilon$ terms. In supersymmetry the calculation is saturated at the SUSY breaking scale[5] Λ_S, where we can measure m_h via its relation to other Lagrangian parameters (as we did for the W mass). After this explanation we can turn to the actual computation. To do it we need to introduce a few more ingredients.

Supersymmetry does not allow us to write both up-type and down-type Yukawa couplings with a single Higgs boson. We have two introduce two new doublets $H_{u,d}$. The extra doublet is also needed to cancel gauge anomalies induced by the supersymmetric partner of the Higgs, the Higgsino.

Furthermore, we need to take into account supersymmetry breaking. We have not observed this plethora of new particles realizing the symmetry, so in Nature the symmetry must be broken at some scale. For this reason we will include also soft (i.e. dimensionful) supersymmetry breaking in what follows.

In this theory the tree-level potential of the two Higgs doublets reads

$$\begin{aligned}\mathcal{L}_{SH} &= -(m_{H_u}^2+|\mu|^2)|H_u|^2-(m_{H_d}^2+|\mu|^2)|H_d|^2-(B\mu H_u H_d+\text{h.c.})\\ &- \frac{g_W^2+g_Y^2}{8}(|H_u|^2-|H_d|^2)^2-\frac{g_W^2}{2}|H_d^\dagger H_u|^2\,.\end{aligned} \tag{3.98}$$

In addition to the three degrees of freedom that make W and Z massive, these theory contains four mass eigenstates: a charged scalar H^+, a CP-odd scalar A and two CP-even scalars H, h.

[5]The scale of SUSY breaking Λ_S does not need to coincide with the scale of the masses M_S. In fact there is a theorem that one can get a spectrum compatible with experiment only if SUSY breaking is mediated to our sector via loop corrections [22].

In this theory the weak scale can be computed in terms of measurable parameters only. At tree-level we have

$$\hat{v}_{\rm tree}^2 = \frac{2}{\hat{g}_W^2 + \hat{g}_Y^2}\left(\frac{|\hat{m}_{H_u}^2 - \hat{m}_{H_d}^2|}{\sqrt{1-4(\hat{v}_u\hat{v}_d/\hat{v}^2)^2}} - \hat{m}_{H_u}^2 - \hat{m}_{H_d}^2 - 2|\mu|^2\right) . \tag{3.99}$$

where $v_{u,d}$ are the vacuum expectation values of the two neutral components of $H_{u,d}$ that satisfy $v_u^2 + v_d^2 = v^2$. As you can see, we have only measured quantities on both sides. We can measure $\hat{m}_{H_{u,d}}^2$ and $\hat{\mu}$ from the masses of other Higgs bosons and from the Higgsinos, the fermionic partners of the Higgs bosons. If we found $m_{H_{u,d}}^2, \mu^2 \gg v^2$, we would have to explain the cancellation needed to get $v \simeq 174$ GeV.

To complete the calculation we can also include the most important loop contribution that is generated by diagrams containing the supersymmetric partner of the top quark, the stop,

$$\begin{aligned} \mathcal{L}_{\tilde{t}H}^{\rm SUSY} &= -|y_t|^2|H_u|^2(|\widetilde{Q}_t|^2 + |\widetilde{t}^c|^2) - \left[\mu^* y_t H_d^{0*}\widetilde{Q}_t^\dagger \widetilde{t}^c + \text{h.c.}\right] , \\ \mathcal{L}_{\tilde{t}H} &= -m_{\widetilde{Q}_t}^2|\widetilde{Q}_t| - m_{\widetilde{t}^c}^2|\widetilde{t}^c|^2 + \left[y_t A_t \widetilde{Q}_t^\dagger H_u^0 \widetilde{t}^c + \text{h.c.}\right] . \end{aligned} \tag{3.100}$$

In the second line we have also included SUSY breaking terms. The two EW eigenstates $\widetilde{Q}_t, \widetilde{t}^c$ combine into two mass eigenstates $\tilde{t}_{1,2}$ that are mixtures of the partners of the left-handed and right-handed top quarks. For simplicity we have ignored the possible mass mixing with other generation of squarks.

The dominant loop correction to v^2 from the previous Lagrangian is

$$\delta m_{H_u}^2 = -\frac{3\hat{y}_t^2}{8\pi^2}\left(|\hat{m}_{\widetilde{Q}_t}|^2 + |\hat{m}_{\widetilde{t}^c}|^2 + |\hat{A}_t|^2\right)\log\frac{\hat{\Lambda}_S}{\rm TeV} , \tag{3.101}$$

As $|m_{\widetilde{Q}_t}|^2, |m_{\widetilde{t}^c}|^2$ or $|A_t|^2$ grow much beyond v^2, we have to explain where the cancellation between them is coming from. An intuitive measure of this fine-tuning is often taken to be [25, 26]

$$\Delta \equiv 2\frac{\delta m_h^2}{m_{h,\rm exp}^2} , \tag{3.102}$$

where $m_{h,\rm exp}^2 \simeq (125~\text{GeV})^2$ and δm_h^2 is any individual contribution to the calculation.

A tuning exists in every theory where the Higgs mass can be calculated. If the new symmetry that makes it calculable is realized only at scales much higher than m_h we need a fine-tuning to explain its value.

If we apply our EFT intuition, we expect the parameters entering the m_h calculation to have a roughly uniform (or power-law [27]) distribution in an $\mathcal{O}(1)$ interval around the typical supersymmetry breaking mass. Observing a tuning is thus a real problem, in the sense that it is signalling that we are making wrong assumptions in our description of Nature. The EFT intuition that has been tremendously successful so far can be wrong in two ways: 1) either Nature accepts some amount of tuning 2) or the "natural" values for supersymmetry breaking parameters have a very different distribution compared to what we naively expect.

Both options leave us with more open questions: in the first case, why do only m_h and the CC appear to be tuned while all other parameters of Nature follow our EFT intuition? In the second one, how does a UV theory that tunes supersymmetry breaking parameters to the right value look like? The answer to the first question is unknown, but we will answer the second one in the next Sections.

Before going there, I find useful two more expressions for m_h^2 that you will encounter in supersymmetry. From now on (and for the rest of these notes) we leave the hats over measured quantities implied.

Given the results from the LHC that has observed an approximately SM-like Higgs boson, we can consider a model where h (one of the two CP-even eigenstates) is approximately decoupled from the rest of the Higgs sector and compute its mass. At tree-level we have

$$m_{h,\mathrm{tree}}^2 = \frac{1}{2}\left(m_A^2 + m_Z^2 - \sqrt{(m_A^2+m_Z^2)^2 - 4m_A^2 m_Z^2 \frac{(v_u^2-v_d^2)^2}{v^4}}\right), \tag{3.103}$$

where m_A is the mass of the CP-odd Higgs. At one loop we can add the leading top and stop contribution

$$m_h^2 = m_{h,\mathrm{tree}}^2 + \frac{3G_F}{\sqrt{2}\pi^2}\left[m_t^4(Q_1)\log\frac{M_s^2}{m_t^2} + m_t^4(Q_2)\frac{X_t^2}{M_s^2}\left(1 - \frac{X_t^2}{12M_s^2}\right)\right] + \ldots \tag{3.104}$$

where the ellipses represent subleading contributions. Here, $M_s^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$, $Q_1 = \sqrt{m_t M_s}$, $Q_2 = M_s$, $X_t = A_t - \mu(v_d/v_u)$ and $m_t(Q)$ is the running top mass. A_t is a supersymmetry-breaking parameter that enters the stop mass matrix [22].

As before, all the parameters in the previous expression can be measured independently of m_h and we left the hats implied. If supersymmetry breaking occurs at scales much higher than m_h the parameters entering its calculation (m_A, X_t and M_s) can give the observed result only if they are precisely tuned to give an approximate cancellation in Eq. (3.104). The larger they are, the larger is the cancellation. However, this is not immediately manifest from Eq.s (3.103) and (3.104), if we take $m_A \to \infty$ or $m_{\tilde{t}_{1,2}} \to \infty$, nothing bad happens.

It is because in those equations we have already tuned! We have expressed m_h as a function of m_t and m_Z which in turn are proportional to the weak scale $v = \sqrt{v_u^2 + v_d^2}$. However, in the MSSM we can compute the weak scale as a function of parameters that can be measured independently of v, as we have seen in Eq.s (3.99) and (3.101). This is exactly what we did in the SM for m_W. We wrote it as a function of c_W, e and m_Z and we do not see any tuning in its expression. The reason is that we have already implicitly tuned v by fixing m_Z (or rather in the SM per se we have just fixed v by measuring m_Z, as we discussed before there is no tuning).

Another reason to consider Eq.s (3.103) and (3.104) is because they show another tension typical of weakly coupled extensions of the SM. The tree level value for m_h, given by Eq. (3.103) is $m_h \leq m_Z$ (or $m_h \leq g_{\mathrm{weak}} v$ in a different theory with a different weak coupling). If we want $m_h \simeq 125$ GeV we need to make $m_{\tilde{t}_{1,2}}$ large, to enhance the logarithmic contribution in Eq. (3.104). This in turns requires a tuning in Eq. (3.101) that grows as $m_{\tilde{t}_{1,2}}^2$.

3.2.2 Scale Invariance and Composite Higgs

It is instructive to ask if there are any other symmetries besides supersymmetry that make the CC and m_h calculable and potentially make their small value natural. We already know the answer to this question from our exercises with dimensional analysis: scale invariance can do the trick. Both the CC and m_h transform non-trivially under the symmetry if we promote them to spurions

$$\begin{aligned} x &\to sx \\ \Lambda_{\rm CC} &\to s^{-4}\Lambda_{\rm CC}\,, \quad m_h^2 \to s^{-2}m_h^2\,. \end{aligned} \tag{3.105}$$

Incidentally this is the reason why we do not worry about the stability of the value of the QCD scale $\Lambda_{\rm QCD} \sim 100$ MeV with respect to some larger UV scale $\Lambda_{\rm UV}$. The reason is that the QCD Lagrangian without quark masses is approximately scale invariant

$$S_{\rm QCD} = \int d^x \left(-\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a} + i\bar{q}\gamma^\mu D_\mu q - \frac{\alpha_s\theta}{8\pi}G^a_{\mu\nu}\widetilde{G}^{\mu\nu a} \right) . \tag{3.106}$$

It does not contain operators with scaling dimension much smaller than 4. Under a scale transformation

$$x^\mu \to sx^\mu\,, \tag{3.107}$$

at the classical level the operators in Eq. (A.32) all get a factor of s^{-4} which compensates the s^4 factor in the integration measure d^4x. Therefore a scale transformation leaves S invariant

$$S \to S\,. \tag{3.108}$$

If we include quantum corrections, scale invariance is broken by effects of $\mathcal{O}(\alpha_s)$. If we imagine that at $\Lambda_{\rm UV}$ we are close to a conformal fixed point, i.e. the theory is almost scale invariant also at the quantum level: $\alpha_s(\Lambda_{\rm UV}) \ll 1$, then all physical quantities depend on the energy scale at most logarithmically at high energy and it takes many decades of running before QCD confines $\alpha_s(\Lambda_{\rm QCD}) \simeq 1$,

$$\log\frac{\Lambda_{\rm UV}}{\Lambda_{\rm QCD}} = \frac{1}{18}\frac{4\pi}{\alpha_s(\Lambda_{\rm UV})}\,. \tag{3.109}$$

The running is slow because there are no relevant deformations in the theory, i.e. no operators with dimension much smaller than $\Delta \simeq 4$. As a consequence there are no dimensionful coefficients of dimension $\Delta - 4$ much bigger than zero that can affect the running of physical quantities. The scale $\Lambda_{\rm QCD}$ is generated through running, without any dimensionful couplings in the theory. This phenomenon is known as "dimensional transmutation" in the QCD literature.

Quark masses do not change this picture. The selection rules of the chiral symmetries that they break, enforce that their running is also logarithmic

$$\frac{dm_q}{d\log E^2} \sim m_q\,, \tag{3.110}$$

so even if the quark mass operators

$$\mathcal{L}_q = m_q \bar{q} q \,, \tag{3.111}$$

have dimension $\Delta_{\bar{q}q} = 3$ they run as marginal operators of dimension $\Delta = 4$.

This general idea can be applied also to explain the hierarchy $m_h \ll \Lambda_{\rm UV}$. Imagine that at some scale m_* a new strongly interacting sector exists and the Higgs boson is a composite state of this sector. Above m_* there is no Higgs boson and no $\Delta_{|H|^2} = 2$ operator associated to its mass, so we expect $m_h^2 \sim m_*^2$. Above this scale m_h^2 does not receive any quantum correction. The scale m_* can be generated from $\Lambda_{\rm UV}$ from dimensional transmutation.

To make this picture compatible with current data we need a second "elementary" sector that contains all other SM particles. The elementary sector is a weakly-coupled gauge theory, essentially the SM minus the Higgs. In principle the right-handed top quark could also be composite. All other fields can at most weakly mix with operators of the new strongly interacting sector (a possibility that is referred to as partial compositness [28]).

To make this picture consistent, the composite sector must respect a symmetry group G that contains the SM gauge group, or at least the subset of the SM gauge group under which the Higgs is charged, i.e. $G \supset SU(2)_L \times U(1)_Y$. In analogy with QCD, the global group G is generically broken to a subgroup H at the confinement scale m_*.

G is also explicitly broken by the gauging of $SU(2)_L \times U(1)_Y$, since the elementary SM particles do not respect G and interact with the composite ones through electroweak gauge bosons. This is analogous to QCD, where $G = SU(3)_L \times SU(3)_R$ which is explicitly broken by gauging its electromagnetic subgroup $U(1)_Q$.

In principle extra explicit breaking terms, analogous to quark masses in QCD, are possible also for our new composite sector. However, we note that the interaction between composite and elementary sectors must not contain any strongly relevant deformation, otherwise the mechanism that stabilizes the $m_* \ll \Lambda_{\rm UV}$ hierarchy would be invalidated.

At some scale. below m_* we have massless Nambu–Goldstone Bosons (NGB) in the G/H coset. Some of them get a mass from the explicit breaking coming from the SM. At this point we have to make a choice: the Higgs boson can either be a generic state of the composite sector or one of the Goldstone bosons.

The first option was first presented in [10, 8, 29, 30] and is known under the name of *technicolor*. The latest results from particle colliders show a strong tension with experiment. If $m_* \simeq m_h$ we would have already observed some particles from the composite sector, the analogue of QCD hadrons, but we have not observed any of them.

The second option [31, 32, 33, 34] is still alive if we accept some amount of tuning. The first question that we should ask if we follow this route, is why the Higgs boson observed at the LHC is consistent with the elementary H in the SM Lagrangian. If H is really part of a composite sector we would expect significant deviations in its couplings compared to the SM expectations [35].

However in these models there is a free parameter that controls how "elementary" the Higgs looks like. To see this we can split the generators of G, T^A into unbroken generators $T^{\hat{a}}$ that form the algebra of H and broken generators X^a. The adjectives "broken" and "unbroken" are not very physical, in reality the whole symmetry is realized and that is what keeps the Goldstone bosons massless. However it is useful to separate the generators in these two categories because the $T^{\hat{a}}$'s are associated to a part of the symmetry that is realized in the usual linear way, with conservation laws that are manifest. The vacuum $\vec{F}$ satisfies

$$T^{\hat{a}}\vec{F} = 0\,, \quad X^a\vec{F} \neq 0\,. \tag{3.112}$$

A priori we can choose any embedding of H in G. If we act with the elements of G on the generators $\{T^{\hat{a}}, X^a\}$ reshuffling them between broken and unbroken, the theory that we obtain is equivalent to the one that we started with, unless H can be embedded in multiple inequivalent ways in G, namely when different choices of the H algebra generators are not all related by inner automorphisms. In this case dynamics selects the right embedding. Barring this complication, we can choose the $T^{\hat{a}}$ to contain $SU(2)_L \times U(1)_Y$.

If we introduce the Goldstone bosons of G/H in the usual way

$$\vec{\Phi}(x) = e^{i\theta^a(x)X^a}\vec{F}\,, \tag{3.113}$$

it is the vev of θ, $\langle\theta\rangle$, which controls the amount of breaking of the EW gauge group

$$v = |\vec{F}|\sin\langle\theta\rangle \equiv f\sin\langle\theta\rangle\,. \tag{3.114}$$

Geometrically this can be understood as follows: $\vec{F}$ is orthogonal to H ($T^a\vec{F} = 0$). The Goldstone bosons are given by tilting $\vec{F}$ by an angle $\theta^a X^a$ whose sine gives the projection to the orthogonal plane where H lives.

Therefore we have a tunable parameter

$$\xi \equiv \frac{v^2}{f^2}\,, \tag{3.115}$$

that allows us to decouple an approximately SM-like Higgs with vev v from the rest of the Goldstone bosons that live at f. This mechanism is known as vacuum misalignment [31, 32, 33, 34]. ξ can be made small by tuning or through a clever use of symmetry as in little Higgs constructions [36, 37, 38]. The latter, however, require a large Higgs quartic at odds with Higgs mass measurements and complex model building.

Much more could be said on these models and for a more comprehensive overview we refer the reader to [39]. Here we just give one of the simplest examples of how to construct the Higgs sector of the model. Consider the global symmetry group $SO(5)$. It contains as a subgroup $SO(4)$ that is locally isomorphic to $SU(2)_L \times SU(2)_R$. We identify weak interactions with $SU(2)_L$ and hypercharge with T_{3R}, the diagonal generator of $SU(2)_R$. We then consider a scalar Φ in the **5** representation of $SO(5)$ with Lagrangian

$$\mathcal{L} = \frac{1}{2}(D_\mu\Phi)^T D^\mu\Phi - \frac{g_*^2}{8}\left(\Phi^T\Phi - f^2\right)^2\,, \tag{3.116}$$

where we have gauge the subgroup of $SO(5)$ corresponding to SM gauge interactions. One can see immediately that the potential has a flat direction $\Phi^T\Phi = f^2$, this means that there are massless fields. There is a general way to find the massless states which is discussed in Section A.1. It amounts to rewrite the field as

$$\Phi = e^{i\sqrt{2}\frac{\pi^a(x)}{f}X^a}\begin{pmatrix}0\\0\\0\\0\\f+\sigma(x)\end{pmatrix} = (f+\sigma(x))\sin\frac{\Pi}{f}\begin{pmatrix}\frac{\pi^1}{f}\\\frac{\pi^2}{f}\\\frac{\pi^3}{f}\\\frac{\pi^4}{f}\\\tan\frac{\Pi}{f}\end{pmatrix}, \tag{3.117}$$

$$\Pi \equiv \sqrt{\sum_a \pi^a\pi^a}\,. \tag{3.118}$$

When $SO(5)$ acts on Φ it rotates the four fields in $\vec{\Pi} = \{\pi^1, ..., \pi^4\}$ as a 4-plet of the $SO(4)$ where the SM gauge group is embedded. One can see this from the form of the generators in Eq. (**??**). It is therefore natural to identify the SM Higgs boson with

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}\pi^1 + i\pi^2\\ \pi^3 - i\pi^4\end{pmatrix}, \tag{3.119}$$

and $SO(4)$ with the custodial symmetry of the SM. We can then write the Lagrangian as

$$\mathcal{L}_\Pi = \frac{f^2}{2|H|^2}\sin^2\frac{\sqrt{2}|H|}{f}(D_\mu H)^\dagger D^\mu H + \frac{f^2}{8|H|^4}\left(2\frac{|H|^2}{f^2} - \sin^2\frac{\sqrt{2}|H|}{f}\right)(\partial_\mu |H|^2)^2 \tag{3.120}$$

where we have integrated out σ that gets a mass at $g_* f$, while at this level the Higgs boson is still massless. In this theory it is natural for the Higgs mass to be small, it is a Goldstone boson with a shift symmetry in the form $H \to H + c$. The shift symmetry is just the non-linear realization of the original $SO(5)$ symmetry. When we apply a transformation in the "broken" sector to Φ, we are just shifting the Π's, as one can see from Eq. (3.118)

$$\Phi' = e^{i\vec{\alpha}\cdot\vec{X}}\Phi \to \vec{\Pi}' = \vec{\Pi} + \vec{\alpha}\,, \tag{3.121}$$

so the so-called "broken" part of the original symmetry ($SO(5)/SO(4)$) is keeping the Higgs mass equal to zero at low energies ($E \lesssim 4\pi f$). Once we get to energies $E \simeq m_\sigma$ and we reconstruct Eq. (3.116) we have to ask what is stabilizing f from high-energy corrections. This can be realized via an approximately scale-invariant sector as discussed previously. This sector could be strongly coupled and, among other things, generate Eq. (3.116) below its confinement scale.

In practice the "broken" $SO(5)/SO(4)$ symmetry is solving the little hierarchy problem, separating H from other resonances of the composite sector by making it a Goldstone boson, while scale invariance is doing the heavy lifting (separating f from $M_{\rm Pl}$). Gauging $SU(2)_L$ and giving H Yukawa couplings, in general breaks this symmetry, generating a potential for H. We refer the reader to [] for some explicit constructions.

This, of course, is not the end of the story. First of all, it is clear from the previous discussion that these models are in tension with current experimental observations. We have already explored scales about a factor of ten above the Higgs mass, without finding the non-linear interactions and possibly new particles expected at f. In this model the natural value for the Higgs vev is precisely f. It is the only scale in the theory and you'll get this result if you try to compute the Higgs potential from the top Yukawa and gauge interactions. Where are all the non-linear effects in Eq. (3.120)? If we expand it and write the covariant derivative in terms of SM gauge bosons, it's easy to see that we predict deviations in Higgs couplings compared to the SM at the level $m_W^2/g^2 f^2$. To make this work experimentally we need an accidental cancellation in the Higgs potential[6] which translates in about a $\xi \simeq 1\%$ tuning.

Secondly, it is not hard to embed this construction in UV complete models [31, 32, 34] that deliver a suitable Nambu-Goldstone Higgs, with the SM gauge groups contained in H. However, not many attempts have been made to extend these constructions to the fermionic sector [40, 41, 42, 43, 44]. The best examples that we have, which successfully account for searches of flavor violation beyond the SM, are are five-dimensional gauge theories on truncated AdS space [45, 46, 47]. Models with extra space dimensions have had a considerable impact on the field. By the AdS-CFT correspondence they can be shown to be equivalent to the 4D constructions that we just discussed. However it is instructive to spend some time discussing explicit 5D models.

3.2.3 Warped Extra Dimensions

Some of the most interesting explicit realizations of scale invariance protecting the Higgs mass have been presented in the form of 5D theories, with the additional dimension described by truncated AdS space. The first examples were presented in [48, 49].

Consider adding one extra dimension with metric,

$$ds^2 = e^{-2ky} dx_\mu dx^\mu + dy^2, \tag{3.122}$$

confined to the interval $y \in [0, y_{\rm IR}]$. This is a slice of AdS space. The fluctuations around this classical solution are

$$y_{\rm IR} \to y_{\rm IR} + T(x) \quad \eta_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}(x) \equiv \bar{g}_{\mu\nu}(x)\,. \tag{3.123}$$

Off-diagonal fluctuations of the metric are massive and excluded from the low-energy effective theory. Gravity can propagate in the bulk, but the SM is on the brane at $y_{\rm IR}$

$$\int d^4x dy \delta(y - y_{\rm IR}) \mathcal{L}_{SM}\,. \tag{3.124}$$

In the 4D effective theory the Planck mass is

$$M^3 \int d^4x \int_0^{y_{\rm IR}} dy e^{-2ky} \sqrt{-\bar{g}} R_4 \to M_{\rm Pl}^2 = \frac{M^3}{2k}\left(1 - e^{-2ky_{\rm IR}}\right) \approx \frac{M^3}{2k}\,, \tag{3.125}$$

where we have assumed a big hierarchy between $y_{\rm IR}$ and $1/k$. What stabilizes this hierarchy also solves the hierarchy problem because on the SM brane we have

$$\int d^4x \sqrt{-\bar{g}} e^{-4ky_{\rm IR}} \left[e^{2ky_{\rm IR}} g^{\mu\nu} \left(D_\mu H\right)^\dagger D^\nu H + m_{h,0}^2 |H|^2 + ...\right] \tag{3.126}$$

[6] When we compute it, we will typically find $V(h) \supset (a_1 + a_2 + ...) y_t^2 f^2 |H|^2$ and we need $\sum_i a_i \sim 0.01 a_{\rm max}$

After rescaling the kinetic term

$$m_h^2 = e^{-2ky_{\rm IR}} m_{h,0}^2 \,. \tag{3.127}$$

We can describe the mechanism as having a large fundamental scale for gravity, which is redshifted to $\simeq$ TeV on the SM brane (the so-called IR brane). However a description equivalent to large extra dimensions (ED), discussed in Section 4, is also possible. A covariant action satisfies

$$S(\Phi, m) = S\left(\Phi', \frac{m}{w}\right) , \tag{3.128}$$

where Φ' are all the fields after a Weyl rescaling, $g \to w^{-2} g$, $H \to wH$, $\psi \to w^{3/2}\psi$, ...

We can in fact see that this is the same as large ED by assuming that the fundamental mass scale of gravity is at a TeV and by rescaling everything by $e^{-ky_{\rm IR}}$ and getting a blue-shifted Planck mass. Here the volume of the ED is made large by the exponential factor. Note that this shows that also this idea predicts new states close to the TeV scale, as supersymmetry and all other ideas based on symmetry. We return to this point at the end of this Section.

First note that the idea works only if $e^{-2ky_{\rm IR}}$ is very small, and what is really stabilizing the hierarchy ($y_{\rm IR} k > 1$) is also solving the hierarchy problem. If you try to expand the action to find the potential for $T(x)$ you will find that it is zero. The reason is that AdS space is scale invariant. One can see it immediately by noticing that the exponential is just a convenient artifact, but we might have chosen different coordinates

$$z = \frac{e^{-2ky}}{k} \, , \quad ds^2 = \frac{1}{k^2 z^2} \left(dx_\mu dx^\mu + dz^2 \right) \, , \tag{3.129}$$

in this frame it is clear that we need a large ED in some sense and also that scale invariance is solving the hierarchy problem.

If we want to stabilize a large hierarchy of scale we have to break this symmetry as little as possible. Let's see how. If we consider the metric in the previous equation, the UV brane is at $z_{\rm UV} = 1/k = R$ and the IR brane, whose position is parametrized by the dilaton χ of the associated CFT, is at $\chi = 1/z_{\rm IR} \ll k$. For a more comprehensive discussion of the mapping between CFT and AdS description we refer to [50].

To stabilize the dilaton (i.e. fix the position of the IR brane) we add a bulk scalar, as first proposed in [51],

$$S = \int d^4x dz \sqrt{-g} \left(g^{MN} \partial_M \phi \partial_N \phi + \Lambda^5_{\rm bulk} - m_b^2 \phi^2 \right) \, . \tag{3.130}$$

ϕ is often called a Goldberger-Wise (GW) scalar. This addition to the action is equivalent to explicitly breaking conformal invariance in 4D, with an operator with dimension related to m_b^2. We imagine that some unspecified dynamics fixes the vev of ϕ in the IR and in the UV and we define the dimensionless ratios $v_{1,0}$ by dividing the vevs by their natural value

$$v_1 \equiv \frac{\langle \phi(z_{\rm IR}) \rangle}{z_{\rm IR}^{3/2}} \, , \quad v_0 \equiv \frac{\langle \phi(z_{\rm UV}) \rangle}{z_{\rm UV}^{3/2}} \, . \tag{3.131}$$

From Eq. (3.130) we can obtain the equations of motion for ϕ in the bulk

$$\frac{3}{z}\partial_z\phi - \partial_z^2\phi = -m_b^2\frac{\phi}{k^2z^2}\,, \tag{3.132}$$

whose solution is

$$\phi(z) = C_1 z^{2+\sqrt{4+\frac{m_b^2}{k^2}}} + C_2 z^{2-\sqrt{4+\frac{m_b^2}{k^2}}}\,. \tag{3.133}$$

Note that even if the scalar vev grows from the UV to the IR $z_{\rm IR} \gg z_{\rm UV}$, this warped ED can still solve the hierarchy problem. If we have a Higgs on the UV brane, its measured vev in the IR is suppressed by $\sqrt{g_{\rm IR}} = 1/(kz_{\rm IR})^4$ which overcomes the z^2 growth.

$C_{1,2}$ can be fixed using our boundary conditions Eq. (3.131). If we plug Eq. (3.133) back into the action and integrate over z, we can obtain a 4D potential for the dilaton. In the region where $\chi \ll k$, where χ parametrizes the position of the IR brane, while k that of the UV brane, we have

$$V = -\epsilon v_0^2 k^4 + \left[(4+2\epsilon)\chi^4(v_1 - v_0(\chi/k)^\epsilon)^2 - \epsilon v_1^2\chi^4\right] + \mathcal{O}(\chi^8/k^4)\,, \tag{3.134}$$

where for simplicity we took $m_b^2 = 4\epsilon/z_{\rm UV}^2$. This shows explicitly that m_b breaks scale invariance, had we only included the kinetic term for ϕ, we would have generated only scale-invariant χ^4 terms. The trick that allows to stabilize the hierarchy is to assume that scale invariance is broken by a small amount ϵ. The minimum of this potential is at $\chi = k(v_1/v_0)^{1/\epsilon}$. So even a mild hierarchy between fundamental parameters: $\epsilon \simeq 1/20$ and $v_1/v_0 \simeq 1/10$ can give $\chi/k \simeq m_W/M_{\rm Pl}$.

A small hierarchy of vevs can thus generate a big hierarchy of scales. This is equivalent to the discussion of dimensional transmutation in QCD, where the logs from quantum corrections play the role of $\chi^{4+\epsilon}$.

Now that we have discussed the core symmetry that stabilizes the hierarchy and explains naturally the observed value of m_h, we find useful to give some extra details about the zeroth-order predictions of the model. Let us focus on the Kaluza-Klein excitations of the graviton (i.e. the spin-2 massive modes). As we will see in more detail in Section 4, a 4D observer who does not yet have enough energy to see the new extra dimension will first see a tower of modes that arise from integrating the action over the fifth coordinate.

If we insert Eq. (3.129) into the action, treating $\eta_{\mu\nu} \to g_{\mu\nu}$ as a dynamical field, we get

$$S = \frac{4M^3}{k^3}\int_{1/k}^{z_{\rm IR}} \frac{dz}{z^3}\sqrt{g}\left\{R_4(g) + \frac{1}{4}\left[(\partial_z g_{\mu\nu})^2 + (g_{\mu\nu}\partial_z g^{\mu\nu})^2\right]\right\} \tag{3.135}$$

If we further split the metric into the flat space one plus fluctuations $h_{\mu\nu}$ the quadratic part of the action for h reads

$$S = \frac{4M^3}{k^3}\int_{1/k}^{z_{\rm IR}} \frac{dz}{z^3}\left\{h_{\mu\nu}K^{\mu\nu\rho\sigma}h_{\rho\sigma} - h^{\mu\nu}\partial_z^2 h_{\mu\nu} + h^\mu_\nu\partial_z^2 h^\nu_\nu\right\}\,, \tag{3.136}$$

where $K^{\mu\nu\rho\sigma}$ is the operator in Eq. (**??**). If we work in the gauge $\partial_\mu h^{\mu\nu} = h^\mu_\mu = 0$, the 4D equations of motion are

$$\left(\Box + m_n^2\right) h^{(n)}_{\mu\nu} = 0 \tag{3.137}$$

where m_n^2 are the eigenvalues of the operator

$$-z^3\partial_z\left(\frac{1}{z^3}\partial_z\psi_n(z)\right) = m_n^2\psi_n(z)\,, \tag{3.138}$$

obtained from the second term in the action. If we impose the boundary conditions

$$\partial_z\psi_n(z = 1/k) = \partial_z\psi_n(z = z_{\rm IR}) = 0\,, \tag{3.139}$$

then the solution is

$$\psi_n(z) = k^2 z^2\left[J_2(m_n z) + b_n Y_2(z m_n)\right]\,, \tag{3.140}$$

and the eigenvalue equation becomes

$$J_1(m_n/k)Y_1(m_n z_{\rm IR}) - Y_1(m_n/k)J_1(m_n z_{\rm IR}) = 0\,. \tag{3.141}$$

If we are interested in the light modes, those that are visible even at energies $\ll k$, then we can expand the above equation and simply obtain

$$J(m_n z_{\rm IR}) \simeq 0\,, \tag{3.142}$$

so the typical mass of the new states is at $\sim 1/z_{\rm IR}$ that numerically we can estimate from Eq. (3.127)

$$\frac{1}{k z_{\rm IR}} \simeq \frac{m_h}{M_{\rm Pl}}\,, \tag{3.143}$$

to be at best comparable to the weak scale.

3.3 The Little Hierarchy Problem

The discussion in this Chapter shows that the most natural expectation is for something (presumably a new symmetry) to appear well below $M_{\rm Pl}$ to explain the value of m_h that we observe. What is the scale where this symmetry should appear?

Let us call this scale M_S. If the UV completion of the SM is perturbative, we have seen that new particles give the leading contribution to the Higgs mass at one loop

$$\delta m_h^2 \simeq \frac{g_S^2}{16\pi^2}M_S^2\,, \tag{3.144}$$

where g_S stands for a coupling in this theory. For an effective symmetry solution to the problem, SM particles must be part of the symmetric multiplets of the new theory, otherwise we would still expect contributions of $\mathcal{O}(M_{\rm Pl}^2)$ from high scales. This means that the largest g_S in our new theory is at least of the size of $y_t = \mathcal{O}(1)$. In principle we can further imagine that to preserve color there are three partners of the top quark in the new theory, giving $\delta m_h^2 \simeq 3\frac{g_S^2}{16\pi^2}M_S^2$. In the end we arrive at the same estimate as Eq. (3.33) with $M_S = \Lambda_{\rm NP}$.

Similar estimates hold also for non-perturbative completions of the SM. In this case, however, we lose the loop suppression in δm_h^2 and we are led to predict $M_S \simeq m_h$. An explicit discussion of the corrections to m_h for the SM flowing into a CFT can be found in [14].

At this point it is natural to ask a second question: What is the scale where this symmetry can appear? We have probed particle physics well above scales of order $\Lambda_{\rm NP} \simeq 400$ GeV.

The largest scales that we have access to are related to symmetries (or approximate symmetries) of the SM Lagrangian, since these signatures make for zero background searches. If we violate baryon number via the operators

$$\mathcal{L} \supset \frac{u^c u^c d^c e^c}{M^2} + \frac{QQQL}{M^2} + \dots \tag{3.145}$$

we can induce proton decay

$$\Gamma \sim \frac{m_p^5}{M^4}\,. \tag{3.146}$$

Current searches at SuperKamiokande [52] and SNO [53] give

$$\frac{\tau_p}{\mathrm{Br}(p \to e^+\pi^0)} \gtrsim 2.4 \times 10^{34}\ \text{years}\,, \quad \frac{\tau_p}{\mathrm{Br}(p \to \text{invisible})} \gtrsim 2 \times 10^{29}\ \text{years}\,, \tag{3.147}$$

corresponding to

$$M \gtrsim 3 \times 10^{16}\ \text{GeV}\,, \quad M \gtrsim 1.5 \times 10^{15}\ \text{GeV}\,. \tag{3.148}$$

A different form of baryon number violation can induce neutron oscillations

$$\mathcal{L} \supset \frac{(u^c d^c d^c)^2}{M^5}\,, \quad \tau_{n\to\bar{n}} = \delta m \sim \frac{m_n^6}{M^5} \tag{3.149}$$

Also in this case we can probe scales well above 400 GeV [54],

$$\tau_{n\to\bar{n}} > 0.86 \times 10^8\ \text{s} \quad M \gtrsim 3 \times 10^6\ \text{GeV}\,. \tag{3.150}$$

Similar considerations hold for tests of the approximate flavor symmetries of the SM. In the lepton sector the largest scale that we can probe is in the decay $\mu \to e\gamma$ induced for instance by

$$\mathcal{L} \supset \frac{m_\mu}{M^2}\bar{\mu}_L \sigma_{\mu\nu} e_R F^{\mu\nu}\,, \quad \Gamma \sim \frac{m_\mu^5}{M^4}\,. \tag{3.151}$$

Current bounds from MEG [55], give

$$\mathrm{Br}(\mu \to e\gamma) < 4 \times 10^{-13} \quad M \gtrsim 3 \times 10^6\ \text{GeV}\,. \tag{3.152}$$

In the quark sector the largest scales can be probed via tests of CP violation in $K^0 - \overline{K}^0$ mixing, where we can get to scales of about $M \gtrsim 10^8$ GeV [56].

Other tests along these lines include searches for CP violation in EDM searches and a host of other flavor measurements in the lepton and quark sectors.

These results are telling us that if we want to extend the SM at $\Lambda_{\rm NP} \simeq 400$ GeV the new theory better respect all the symmetries and approximate symmetries of the SM. This requires quite a bit of model building, since in general these new theories have many more free parameters with respect to the SM, which do not necessarily have to respect these symmetries. We will see an explicit example in the Section devoted to supersymmetry in the next Chapter.

A sharper tension arises from direct searches for new particles at LEP, the Tevatron and the LHC. By now we have explored a vast number of signatures that cover most options for new particles with gauge couplings to the SM. The null results at these particle colliders point to $\Lambda_{\rm NP} \gtrsim$ few TeV. The application of these results to the hierarchy problem is model dependent and different theories might be affected by slightly different bounds. However the general point that we have not found new physics below a few TeV remains valid. Furthermore, the LHC has explored many of the Higgs couplings to SM particles finding a good consistency with an elementary Higgs as described by the Lagrangian in the previous Chapter, leaving room for deviations of order [57, 58, 59]

$$\frac{\delta g_{h\rm SM}}{g_{h\rm SM}} \lesssim 5\% \div 20\% \tag{3.153}$$

This complicates embedding the Higgs in a larger symmetry structure. In light of all these null experimental searches, even if we completely forget about $M_{\rm Pl}$, there is still a tension between direct and indirect searches for new physics and the simplest explanations for the value of the Higgs mass. This has been known since the times of LEP' [60] and today we call it the "little hierarchy problem'.

4 Lowering the Scale of Gravity

In the previous Sections we have taken $M_{\rm Pl}$ to be the dimensionful scale associated to gravity, i.e. the energy scale where we expect it to become important for particle interactions. However $M_{\rm Pl}$ is a vev and it could be the combination of a small mass and a tiny coupling that combine to give an apparently very large energy scale. If the true fundamental scale of gravity is much lower than $M_{\rm Pl}$ the Higgs hierarchy problem might not exist. This possibility is qualitatively different than having a new symmetry around the weak scale, but it also predicts new phenomena close to m_h, so sadly the experimental tensions discussed in the previous Section apply also to this set of ideas, unless we accept some amount of tuning. Nonetheless it is interesting to see how this option differs from having a new symmetry and what are the similarities.

We can start by looking at the Einstein-Hilbert action

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{\rm Pl}^2}{2} R - \Lambda_{\rm CC} + \mathcal{L}_{\rm matter} \right) . \tag{4.1}$$

We can expand the metric in the regime of validity of this EFT (i.e. low energy and low curvature)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \, , \quad |h| \ll 1 \, . \tag{4.2}$$

Then the action becomes schematically

$$S \sim M_{\rm Pl}^2 \int d^4x \left[\partial h \partial h + h\partial h \partial h + h^2 \partial h \partial h + a T_{\mu\nu} h^{\mu\nu} + ...\right] \tag{4.3}$$

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta \mathcal{L}_{\rm matter}\sqrt{-g}}{\delta g^{\mu\nu}}\,, \tag{4.4}$$

where we have suppressed the indexes of $h_{\mu\nu}$ for convenience. The terms odd in h come from higher curvature corrections $R \sim \text{const} + \partial^2 h$. Canonically normalizing the kinetic term we have

$$S \sim \int d^4x \left[\partial h \partial h + \frac{1}{M_{\rm Pl}} h\partial h \partial h + \frac{1}{M_{\rm Pl}^2} h^2 \partial h \partial h + \frac{a}{M_{\rm Pl}} T_{\mu\nu} h^{\mu\nu} + ...\right]. \tag{4.5}$$

It's hard to define a running coupling in the EFT of gravity for reasons that I'm not going to discuss here. Heuristically you can just notice from the previous action that you are starting with a dimension 6 operator. From loop diagrams you are going to get multiple dimension 8 operators with different numerical coefficients (and possibly signs) $R^2, R_{\mu\nu}R^{\mu\nu}$. Which one are you going to pick? Ref. [61] contains an interesting discussion on this point.

For our purposes it is sufficient to notice that at tree-level the 2-to-2 scattering amplitude of gravitons grows with energy

$$\mathcal{M}^{\rm tree}_{2\to 2} \sim \frac{E^2}{M_{\rm Pl}^2}\,. \tag{4.6}$$

At each loop order one gets and extra power of $G_N E^2$, but we are more interested in another fact. The same scattering process at one loop scales as

$$\delta\mathcal{M}^{\rm 1-loop}_{2\to 2} \sim \frac{N E^4}{16\pi^2 M_{\rm Pl}^4}\,, \tag{4.7}$$

from the action in Eq. (4.5) (after gauge fixing) where N is the number of particles in the loop (coming from $T_{\mu\nu}$ in (4.5)). So it is natural to expect something to happen at

$$E \sim \frac{4\pi M_{\rm Pl}}{\sqrt{N}}\,, \tag{4.8}$$

where the one loop corrections becomes comparable to the tree level result. In some sense we are lowering the fundamental scale of gravity, i.e. gravity becomes important for particle interactions at $M_{\rm Pl}/\sqrt{N}$. Therefore, the easiest way to solve the hierarchy problem is to imagine:

$$N \sim \frac{M_{\rm Pl}^2}{v^2} \approx 10^{32}\,. \tag{4.9}$$

The idea of lowering the fundamental scale of gravity by adding new degrees of freedom was first discussed in relation to the hierarchy problem in [62, 63, 64, 65]. An explicit way of implementing this idea is to introduce extra dimensions compactified at a scale near m_h.

Let us take R to be the typical size of the extra dimensions. If we consider $D = 4 + n$ then Newton's law is modified to

$$F(r) \sim \begin{cases} \frac{m_1 m_2}{M^{n+2} r^{n+2}}\,, & r \ll R \\ \frac{m_1 m_2}{M^{n+2} R^n r^2}\,, & r \gg R \end{cases} \tag{4.10}$$

where M is the fundamental scale of gravity in the theory with $D > 4$. This result is just an application of Gauss' theorem and it shows that

$$M_{\rm Pl}^2 \simeq M^{n+2} R^n\,, \quad R = 10^{\frac{30}{n}-17}\ \text{cm} \left(\frac{\text{TeV}}{M}\right)^{1+\frac{2}{n}}\,. \tag{4.11}$$

This means that gravity might appear weak in 4D, where it has a coupling $G_N \sim 1/M_{\rm Pl}^2$, because it is diluted by multiple extra dimensions where it can propagate. In reality the fundamental scale of gravity might be M and much lower than $M_{\rm Pl}$.

Particle interactions are known up to energy scales $E \sim$ TeV, corresponding to $R \sim 10^{-17}$ cm, so if we want $M \simeq$ TeV, the SM fields must be stuck on a 4D brane. On the contrary we don't know gravity that well below a millimiter and there is no problem if gravity propagates in the extra dimension, realizing the "dilution" of $M_{\rm Pl}$ that we would like to invoke to explain the value of m_h.

If $M \sim$ TeV we have solved the hierarchy problem, but to do so we need R to be large compared to $M_{\rm Pl}^{-1}$. Before seeing this in more detail let's see where the connection with large N comes from. It is already manifest that N in the previous theories is playing the role of the volume in this case. Consider one extra dimension compactified on a circle (here I follow [66]). The metric can be split to

$$g_{MN} = \begin{pmatrix} \eta_{\mu\nu} + h_{\mu\nu} & h_{\mu 5} \\ h_{\mu 5} & h_{55} \end{pmatrix}\,. \tag{4.12}$$

The action of diffeomorphisms is

$$h_{MN} \to h_{MN} + \partial_M \epsilon_N + \partial_N \epsilon_M\,. \tag{4.13}$$

Since the extra dimension is compact $p_5 \sim n/R$, so $\delta h_{55} = 2\partial_5 \epsilon_5 \propto \sum_n n \epsilon_5^{(n)}$. We can eliminate all $n \neq 0$ components of h_{55} and $h_{\mu 5}$ using diff. invariance. We are left with a scalar $\phi \equiv h_{55}^{(0)}$, a four-vector $A_\mu \equiv h_{5\mu}^{(0)}$ and a tower of Kaluza-Klein (KK) gravitons $h_{\mu\nu}^{(n)}$.

To see this explicitly we use the periodicity of the spatial coordinate in the extra dimension to write

$$h_{\mu\nu}(x, x_5) = \sum_{n=-\infty}^{n=+\infty} h_{\mu\nu}^{(n)}(x) e^{\frac{inx_5}{R}}\,, \tag{4.14}$$

then we integrate the Einstein-Hilbert action over x_5 and we are left with

$$S = 2\pi R M^3 \int d^4x \left(h^{\mu\nu}\Box h_{\mu\nu} - h_\mu^\mu \Box h_\nu^\nu + 2h_{\mu\nu}\partial^\mu\partial^\nu h_\rho^\rho - 2h_{\mu\nu}\partial^\mu\partial^\rho h_\rho^\nu + \frac{n^2}{4R^2}\left[h_\mu^\mu h_\nu^\nu - h^{\mu\nu}h_{\mu\nu}\right]\right) + \ldots \tag{4.15}$$

From the above action we can conclude that

$$M_{\rm Pl}^2 = 2\pi R M^3\,, \qquad \left(\Box + \frac{n^2}{R^2}\right) h_{\mu\nu}^{(n)} = 0\,. \tag{4.16}$$

So how many gravitons do we have? When we hit the scale M we have to UV complete gravity also in the extra dimension therefore we can have at most

$$\frac{N^2}{R^2} \sim M^2 \quad N \sim \left(\frac{M_{\rm Pl}}{M}\right)^{\frac{2}{n}} \tag{4.17}$$

gravitons in our EFT. For $n = 1$ and $M \sim$ TeV[7], we recover our large N estimate from Eq. (4.9).

How about the new hierarchy problem $R \gg M_{\rm Pl}^{-1}$? A potential for R arises from the $(4+n)$D cosmological constant Λ_n in the Einstein-Hilbert Lagrangian

$$\int d^{4+n}x\sqrt{-g}\Lambda_n \sim \int d^4x\sqrt{-\bar{g}}\Lambda_n R^n\,. \tag{4.18}$$

In the presence of curvature κ in the extra dimensions we have also

$$M^{2+n}\int d^{4+n}x\sqrt{-g}R \sim -\int d^4x\sqrt{-\bar{g}}\kappa M^{2+n}R^{n-2}\,. \tag{4.19}$$

Summing these two terms we can find a stable potential with a minimum $R_* \sim \sqrt{M^{2+n}/\Lambda_n}$. This means that the radius of curvature is roughly

$$L \sim \sqrt{\frac{M^{n+2}}{\Lambda_n}}\,. \tag{4.20}$$

If we don't want our space to split in separate inflating patches of size L or collapse into black holes we need

$$L \gtrsim R \to \Lambda_n \lesssim M^{4+n}\left(\frac{M}{M_{\rm Pl}}\right)^{4/n} \tag{4.21}$$

Smaller than its natural value M^{4+n}. So we need to tune Λ_n and possibly keep it stable with supersymmetry. Furthermore, to reproduce our observed 4D universe, we need the effective (long distance) 4D CC to approximately vanish

$$\sum_i f_i^4 + R^n\Lambda_n \approx 0\,, \tag{4.22}$$

where f are brane tensions. They are nothing mysterious, just the equivalent of a CC on the 4D brane. Their natural value is $f^4 \approx M^4$. If there are N_w branes

$$\Lambda_n \lesssim N_w M^{4+n}\left(\frac{M}{M_{\rm Pl}}\right)^{4/n}\,, \tag{4.23}$$

[7] Phenomenologically excluded because of modifications of gravity on solar system scales.

so the extra dimension can be large for the same reason that people are large (they carry large baryon number).

We are still tuning, once to get R large (Eq. (4.21)) and a second time to get the observed 4D CC (Eq. (4.22)). However, topologically conserved quantum numbers associated with higher-form bulk gauge fields or a large number of 3-branes in the bulk of the extra dimension can naturally stabilize $R \gg M^{-1}$ [67]. This is another analogy between these large flat extra dimensions and lowering the scale of gravity with a large number of particles N. The number N is radiatively stable as a large $U(1)$ charge is. See also [68, 69] for alternative ways to stabilize a large extra dimension by incorporating a discrete number in its geometry.

The metric in the extra dimensions that we are considering here is flat. We have discussed a dynamical way of stabilizing the radius at the desired value in the context of warped extra-dimensions, where the metric is AdS-like. Not surprisingly this corresponds to introducing a symmetry in our theory that stabilizes the hierarchy between R and $M_{\rm Pl}^{-1}$. The symmetry is scale invariance.

Before concluding this Section, it is useful to point out what is currently the biggest problem with these constructions. If we want to lower the scale of gravity down to a TeV, we are predicting a plethora of new particles at that scale and we have not observed any. We can of course take $N \lesssim 10^{32}$ or $M \gtrsim$ TeV and accept some amount of accidental cancellation between different contributions to m_h.

5 The Cosmological Constant

What we have learned about the Higgs mass applies also to the cosmological constant. In the SM Lagrangian the cosmological constant is the coefficient in front of the identity

$$\mathcal{L} = \Lambda_{\rm CC} \mathbb{1} \,. \tag{5.1}$$

Physically it is the energy density of our vacuum. Given that it is the coefficient of the identity operator, there is no symmetry protecting it. Therefore, any mass scale in the theory can enter its estimate based on the selection rules of dilations. Its spurion transformation is given by

$$\begin{aligned} x &\to sx \\ S &= \int d^4x \Lambda_{\rm CC} \to \int d^4x s^4 \Lambda_{\rm CC} \\ \Lambda_{\rm CC} &\to s^{-4}\Lambda_{\rm CC} \,. \end{aligned} \tag{5.2}$$

So, purely within the SM, we have

$$\Lambda_{\rm CC} \simeq a \frac{m_t^4}{16\pi^2} \,, \tag{5.3}$$

with a a number expected to be $\mathcal{O}(1)$. Note that the dimensions of the cosmological constant are those of a $({\rm vev})^2$ times a $({\rm mass})^2$, i.e. $({\rm mass})^4/({\rm coupling})^2$.

If we try to compute it in the SM, we encounter the exact same features that we discussed for the Higgs mass. Consider for example matching a low energy theory where we integrated out the electron to QED where the electron is propagating. We have a huge matching correction to the CC compared to its observed value

$$\Lambda_{\rm CC}^{\rm obs} = \Lambda_{\rm CC}^{0}(\mu_M) + a' \frac{m_e^4}{16\pi^2} + ... \tag{5.4}$$

since $m_e^4/\Lambda_{\rm CC}^{\rm obs} \simeq 10^{39}$, but physically there is no problem, as $\Lambda_{\rm CC}^{0}(\mu_M)$ contains a parameter that can't be calculated and must be fixed by measuring $\Lambda_{\rm CC}^{\rm obs}$. As you can see, in the case of the CC, the "little hierarchy problem" is not so little, given that $m_t^4/\Lambda_{\rm CC}^{\rm obs} \simeq 10^{60}$. If we extend the SM to a theory such as supersymmetry where the CC is calculable, this matching correction turns into an actual tuning of parameters and a real problem.

The Cosmological Constant is Calculable (and protected by SUSY) We can write the Hamiltonian as

$$H = P^0 = \frac{1}{4}\left(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2\right) . \tag{5.5}$$

Therefore its vacuum expectation value is given by

$$\langle 0|H|0\rangle = \langle 0|V|0\rangle \propto |Q_1|0\rangle|^2 + |Q_2|0\rangle|^2 . \tag{5.6}$$

A non-zero value of the CC means that the SUSY generators act non-trivially on the vacuum. This means that $\Lambda_{\rm CC} = 0$ is a special point in supersymmetry and the supersymmetry selection rules enforce that

$$\Lambda_{\rm CC} \propto \Lambda_S^4 , \tag{5.7}$$

where Λ_S^4 is not just any scale in the theory, but one that breaks supersymmetry.

To conclude this Section, note that the value of the CC becomes dramatically important when we turn on gravity. The cosmological constant interacts with gravitons

$$S = \int d^4x \sqrt{-g}\Lambda_{\rm CC} , \tag{5.8}$$

and determines the maximal size of the observable universe. Experimentally we have a Universe that starts very homogeneous and isotropic, but has the possibility to evolve in time to become more anisotropic and accommodate Hubble expansion. This singles out the FRW metric:

$$ds^2 = -dt^2 + a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega_{(2)}^2\right] , \quad d\Omega_{(2)}^2 = d\theta^2 + \sin^2\theta d\phi^2 , \tag{5.9}$$

with this metric Einstein's equations become ($H \equiv \dot{a}/a$)

$$\begin{aligned} H^2 &= \frac{8\pi G_N \rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \\ \dot{H} + H^2 &= \frac{\ddot{a}}{a} = -\frac{8\pi G_N}{6}(\rho + 3p) + \frac{\Lambda}{3} , \end{aligned} \tag{5.10}$$

where we have taken the energy density of the Universe to be dominated by a perfect fluid with energy density ρ and pressure p (see [70] to know why). In the above equations $\Lambda \equiv 8\pi G_N \Lambda_{\rm CC}$ is what cosmologists like to call the cosmological constant. From these equations it is not hard to see that any traditional form of energy (massive or massless particles) will make the Universe expand $\dot{a} > 0$, that in turn makes the energy density decrease $\dot{\rho}, \dot{p} < 0$ (see [70] again for a derivation). Therefore if we wait long enough the CC dominates

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 &\simeq \frac{\Lambda}{3}\,, \\ \frac{\ddot{a}}{a} &\simeq \frac{\Lambda}{3}\,. \end{aligned} \tag{5.11}$$

If we define a constant Hubble parameter as $H_\Lambda \equiv \Lambda/3$. It is easy to solve these equations

$$a(t) = C_1 e^{H_\Lambda t} + C_2 e^{-H_\Lambda t}\,. \tag{5.12}$$

A positive CC picks the exponentially expanding solution proportional to C_1. Very rapidly, an observer remains in causal contact with a region of size $r \sim 1/H_\Lambda$ and this becomes the size of the observable Universe. This happens because spacetime expands faster than the speed of light. This is a de Sitter (dS) universe. A negative CC picks the other solution, leading to a universe that crunches to a size $r \sim 1/H_\Lambda$. This is an Anti-de Sitter (AdS) universe.

In other words, asking why the CC is $\sim 10^{-60} m_t^4$, amounts to asking why isn't the Universe as small as a subatomic particle.

6 What We Learned about High Energies

At this point we have discussed in great detail what is puzzling about the Higgs boson mass and the CC. In the SM we cannot point to a real problem, both because we cannot compute m_h and $\Lambda_{\rm CC}$ and because it is not clear how to treat $M_{\rm Pl}$, the only other scale of Nature that we know about. If we extend the SM with new symmetries that make m_h and $\Lambda_{\rm CC}$ calculable we encounter a fine-tuning whenever these symmetries are realized at scales much higher than m_h and $\Lambda_{\rm CC}$. This is what I call an actual hierarchy problem.

This short summary still leaves us to wonder what we learned from all the work done so far. To answer this question we are forced to think about the UV and speculate about new regimes that we do not have access to experimentally. This is the beauty and the curse of the hierarchy problem, whether we want it or not, we have to set foot in uncharted territory. However, dven without experimental guidance, we can still use logic alone to write down a comprehensive set of possible explanations for m_h and $\Lambda_{\rm CC}$:

0. A large fine-tuning, spanning tens of orders of magnitude, is a fundamental aspect of Nature. This option is obviously viable also for the cosmological constant.

1. A symmetry that makes m_h calculable exists below $M_{\rm Pl}$ and some amount of fine-tuning is a fundamental aspect of Nature.

2. The fundamental scale of quantum gravity is much smaller than $M_{\rm Pl}$ and close to m_h. Also in this case some amount of fine-tuning is a fundamental aspect of Nature.

3. A landscape of values of m_h is realized in Nature. The value that we observe is selected by an early Universe event that we can not yet observe directly. This option is viable also for the cosmological constant.

4. The Higgs mass and the CC are never calculable. At every scale we have a theory similar to the SM where m_h and $\Lambda_{\rm CC}$ are just an input parameters. Although seemingly harmless, this possibility puts strong constraints on the UV theory realizing it and we don't know a theory of quantum gravity that implements it.

5. There is no mass scale beyond the Standard Model sufficiently strongly coupled to the Higgs to generate a fine-tuning problem. Quantum gravity either does not have a scale [71, 72, 73, 74, 75] or incorporates $M_{\rm Pl}$ non-trivially in the S matrix, leaving no power-law corrections to dimensionful parameters [76].

6. The consistency of quantum gravity leaves non-trivial imprints at low energy either in the form of UV/IR mixing or inconsistent low-energy Lagrangians that look acceptable to the low-energy observer (i.e. large m_h and/or $\Lambda_{\rm CC}$ are in the swampland [77]).

We can anticipate that we do not know any consistent theory of quantum gravity that realizes options 4 or 5. At the time of writing the last option is mostly conjectural, while possibly compatible with string theory, it is far from being implemented in a concrete model.

The only possibilities for which we can write a complete theory and propose experimental tests are 1, 2 and 3 (and in some sense 0). These are also the simplest possibilities conceptually, in the sense that they build upon our well-tested knowledge of quantum field theory. The last three options require a radical modification of particle physics at the scale of quantum gravity.

Regardless of what is your favorite option, thinking about the Higgs mass inevitably leads to learning something new (and in my opinion deep) about Nature. All the options listed above require a decisive extension of our current description of physical phenomena. The two most conservative options require adding either a new symmetry, realized by a host of new particles, or accepting the existence of a vast landscape for m_h. This landscape can be realized either by changing the history of the Universe or accepting the existence of a Multiverse of which we occupy a tiny spec.

The three most speculative possibilities require revising completely quantum field theory and our EFT intuition when it comes to quantum gravity, in case 4 and 5 well beyond what is suggested by string theory. In the next Sections we discuss all these options in greater detail, except for option 1 that was already discussed before.

In the case of the CC there is no point in messing arount with the scale of quantum gravity, even if $M_{\rm Pl} \simeq m_h$ or gravity is scaleless (options 2 and 5) the CC is still enormously tuned. Even option 1 doesn't seem to sensible, as the tuning involved would be at least a part in 10^{60} ($\simeq \Lambda_{\rm CC}/m_t^4$). The only options that are open are: a) a landscape (option 3), b) UV/IR magic (option 6), c) $\Lambda_{\rm CC}$ is never calculable (option 4) or d) there is a huge fine-tuning (option 0). As for the Higgs, b) and c) are fun ideas to consider, but we're still very far from attaching concrete models to them, the only concrete option still open for the CC is having a landscape, most likely in the form of a Multiverse and this is what we discuss next.

7 Landscapes and Multiverses

In Section 1 we have seen that historical fine-tunings were resolved in two ways: 1) by the presence of a new symmetry 2) by multiple realizations of the same observable, some of which could be accidentally tuned. We have not yet encountered anything resembling the second option for the Higgs boson mass. However proposals along these lines exist and we discuss them in this Section. The main reasons to consider them are: 1) The most concrete (and essentially) only explanation for $\Lambda_{\rm CC}$ falls in this category 2) It is rather likely that a Multiverse exists independently of the two problems (if the fundamental theory of quantum gravity lives in more than 4 spacetime dimensions, like string theory).

The basic idea is that the observable Universe is just one patch of a vast Multiverse. Each patch has different values of fundamental parameters, in particular of $\Lambda_{\rm CC}$ and m_h^2. In this context, we have to explain why we live in a patch with a value of $\Lambda_{\rm CC}$ and m_h^2 that appears unnaturally small. The traditional explanation is that only these tuned patches can support observers. These are known as anthropic arguments. We review them for m_h^2 in Section 7.2. First, it is instructive to see how a Multiverse can be populated, generating a landscape of values for $\Lambda_{\rm CC}$.

We start with a special kind of Multiverse, first proposed by Brown and Teitelboim [78, 79], that allows us to naturally build up to what is today considered the most concrete explanation at our disposal for the value of $\Lambda_{\rm CC}$.

It is important to stress that we are still far from formulating a complete theory of the Multiverse. Such a theory would allow us to compute exactly what is the underlying distribution of metastable vacua in Nature and how they are populated. We would then be able to predict how frequent a patch is in the Multiverse, given the observed values of fundamental parameters. Nobody is currently able to do this. The most convincing examples of Multiverses come from string theory. Compactifying its extra dimensions leaves us with a multitude of moduli with 10^{500} possible vacua (or more), most of them have lifetimes longer than that of the observable Universe. If the Universe is eternally inflating all these vacua can be populated by tunneling and live a long and healthy life before decaying to the true ground state.

Starting from this broad picture, concrete toy models of the landscape were proposed, showing that a Multiverse explanation of $\Lambda_{\rm CC}$ and m_h^2 is possible. However we are not able to calculate the distribution of $\Lambda_{\rm CC}$ and m_h^2 in the Multiverse and predict what is likely or unlikely for their observed value. Anthropic arguments allow us to bypass this difficulty, since they identify a small viable range for these parameters. If only a few values, compatible with current measurements of $\Lambda_{\rm CC}$ and m_h^2, allow to have observers, we do not really need to compute how likely different patches are. We will not have a precise prediction for $\Lambda_{\rm CC}$ and m_h^2, but at least we have a reason to expect them to be much smaller than their natural value.

7.1 The Cosmological Constant in the Multiverse

Brown-Teitelboim We can now turn to constructing and populating a Multiverse. Imagine having a 3-form field $A_{\mu\nu\rho}$, totally antisymmetric in its indexes. We can construct its kinetic term starting from the 4-form

$$F_{\mu\nu\rho\sigma} = \partial_\mu A_{\nu\rho\sigma} - \partial_\sigma A_{\mu\nu\rho} + \partial_\rho A_{\sigma\mu\nu} - \partial_\nu A_{\rho\sigma\mu}\,. \tag{7.1}$$

Its most general action, including also gravity, reads

$$S = -\frac{1}{48}\int d^4x\sqrt{-g}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} + S_{\text{boundary}} + S_G\,, \tag{7.2}$$

where the boundary action

$$S_{\text{boundary}} = \frac{1}{3!}\int d^4x\partial_\mu\left(\sqrt{-g}F^{\mu\nu\rho\sigma}A_{\nu\rho\sigma}\right) + M_{\text{Pl}}^2\int_\Sigma d^3x\sqrt{-h}K \tag{7.3}$$

does not have any effect on-shell, but is needed to make the theory consistent. The second term must be included in spacetimes where the manifold is not closed (i.e. it has a boundary). Σ is the boundary of the manifold, h is the induced metric and K the extrinsic curvature. This is the Gibbons-Hawking-York boundary term [80].

The last term in S is the usual Einstein-Hilbert action with a cosmological constant

$$S_G = \int d^4x\sqrt{-g}\left(\frac{M_{\text{Pl}}^2}{2}R - \Lambda_0\right)\,. \tag{7.4}$$

The equations of motion for A are

$$\partial_\mu\left(\sqrt{-g}F^{\mu\nu\rho\sigma}\right) = 0 \tag{7.5}$$

and the only solution

$$F^{\mu\nu\rho\sigma} = c\epsilon^{\mu\nu\rho\sigma}\,, \tag{7.6}$$

where c is a constant of dimension 2. This shows that A is non-dynamical. This is a consequence of the large gauge symmetry of the action, which is invariant under

$$A_{\mu\nu\rho} \to A_{\mu\nu\rho} + \partial_{[\mu}B_{\nu\rho]} \tag{7.7}$$

with $B_{\mu\nu}$ any antisymmetric ($B_{\mu\nu} = -B_{\nu\mu}$) tensor.

Therefore in this theory the cosmological constant is not only Λ_0, but also has a contribution from F^2 in the action

$$\Lambda_{\text{CC}} = \Lambda_0 - \frac{c^2}{2}\,. \tag{7.8}$$

We do not yet have a landscape, but we are close. You might have noticed the analogy between our 3-form and the vector potential in electromagnetism (equivalently between F and the EM field). The only missing ingredient in this analogy is some form of charged matter like the electron. If such an object existed its pair production could discharge the primordial electric field c and change the cosmological constant.

To introduce this object in the theory it is useful to go deeper into the analogy with electromagnetism. Take a particle of unit charge moving along the worldline $x_p^\mu(\tau)$. Its current density is

$$J^\mu = eu^\mu \delta^{(3)}(\vec{x} - \vec{x}_p(\tau))\,, \quad u^\mu = \frac{dx_p(\tau)^\mu}{d\tau}\,, \tag{7.9}$$

and we can write its coupling to electromagnetism as

$$\int d^4x\sqrt{-g}J^\mu A_\mu = e\int d^4x\sqrt{-g}\delta^{(3)}(\vec{x} - \vec{x}_p(\tau))\frac{dx_p^\mu(\tau)}{d\tau}A_\mu = e\int dx_p^\mu A_\mu\,, \tag{7.10}$$

where the last integral is taken along the worldine of the particle. We can now scale this example to one more dimension. A 2-form $A_{\mu\nu} = -A_{\nu\mu}$ will couple to a one dimensional object (rather than a point particle), spanning a worldsheet $x^\mu(\tau, \sigma) \equiv x^\mu(\vec{\xi})$. Instead of a single four-velocity u^μ in this case we have two possible derivatives $\partial x^\mu/\partial\tau$, $\partial x^\nu/\partial\sigma$. Due to the antisymmetry of $A_{\mu\nu}$ there is only one possible Lorentz-invariant coupling

$$A_{\mu\nu}\epsilon^{ab}\frac{\partial x^\mu}{\partial\xi^a}\frac{\partial x^\nu}{\partial\xi^b}\,. \tag{7.11}$$

To complete the analogy with electromagnetism we can integrate over the worldsheet to obtain the action

$$S_{\text{int}} = \frac{e}{2}\int d^2\xi A_{\mu\nu}\epsilon^{ab}\frac{\partial x^\mu}{\partial\xi^a}\frac{\partial x^\nu}{\partial\xi^b}\,. \tag{7.12}$$

It is now straightforward to apply the same reasoning to our 3-form and obtain the action of the brane coupling to it

$$S_{\text{brane}} \supset \frac{e}{3!}\int d^3\xi A_{\mu\nu\rho}\epsilon^{abc}\frac{\partial x^\mu}{\partial\xi^a}\frac{\partial x^\nu}{\partial\xi^b}\frac{\partial x^\rho}{\partial\xi^c}\,. \tag{7.13}$$

To complete the action we need only to generalize the free Lagrangian of a point particle to a brane

$$S_{\text{free}} = -m\int d\tau = -m\int\sqrt{g^{(1)}}dt\,. \tag{7.14}$$

In the last equality we have noted that $\gamma d\tau = dt$ and introduced the one dimensional metric induced on the worldline. It is now easy to generalize the previous expression to

$$S_{\text{brane}} \supset -T\int d^3\xi\sqrt{g^{(3)}}\,, \tag{7.15}$$

the only difference to keep in mind is that T is now a tension of dimension mass/volume. Putting together the two terms in S_{brane} with the action in Eq. (7.2) we can obtain the new equations of motion

$$\partial_\mu c(y)\epsilon^{\mu\nu\rho\sigma} = -e\int d^3\xi\delta^{(4)}(y - x(\vec{\xi}))\epsilon^{abc}\frac{\partial x^\mu}{\partial\xi^a}\frac{\partial x^\nu}{\partial\xi^b}\frac{\partial x^\rho}{\partial\xi^c}\,. \tag{7.16}$$

On both sides of the brane c is constant and it jumps through it by a unit of brane charge e

$$\Delta c = e\,. \tag{7.17}$$

If we have initially a large electric field $c^2 > e^2$, membranes of opposite charge can be spontaneously nucleated. The electric field inside the bubble formed by the brane and the ani-brane is now smaller than that outside. This configuration has lower energy than the outside vacuum, so the bubble walls will expand.

This is the same process as Schwinger pair production in QED. It is a tunneling process akin to a phase transition, governed by the same equations as that of a scalar jumping from a metastable minimum to a deeper minimum.

If we add to the mix eternal inflation we have created a Multiverse where each patch has a different CC. The bubble walls will expand at most at the speed of light, but the volume of the universe grows faster, so configurations with different values of c can coexist.

The smallest splitting between CCs in this Multiverse is

$$\Delta\Lambda = e^2\,. \tag{7.18}$$

One can write down a model where pair production of branes continues until $\Lambda_{\rm CC} > 0$. To nucleate a "brane-bubble" we need its radius to be smaller than the radius of curvature of the AdS space that will be born inside the bubble going from $\Lambda_{\rm CC} > 0$ to $\Lambda_{\rm CC} < 0$. If this is not the case a bubble that can live long and expand never forms and this nucleation process stops dynamically at $\Lambda_{\rm CC} \simeq e^2$. This is the beauty of Brown and Teitelboim's idea.

Eternal inflation is useful for two reasons. Since it gives an exponentially expanding volume, bubble walls, even if they move at the speed of light, never manage to meet, so instead of having a single universe in the ground state, we have multiple bubbles constantly expanding under the effect of inflation. Secondly, but maybe less critically, eternal inflation provides a large volume and a long time for the bubbles to form. The tunneling process is slow

$$t_{\rm nucleation} \simeq e^{S_E} \simeq e^{\frac{M_{\rm Pl}^4}{e^2}}\,, \tag{7.19}$$

even extremely slow if we want $\Delta\Lambda \simeq \Lambda_{\rm CC} \simeq (0.1~{\rm meV})^4$, so we need a long enough period of inflation to populate all the values of the CC. This discussion can be straightforwardly generalized to scanning the Higgs mass if we add a coupling to the 4-form, for instance

$$S \supset \int d^4x\sqrt{-g}\frac{F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}}{48}|H|^2\,. \tag{7.20}$$

This gives at least a proof-of-principle that a Multiverse for $\Lambda_{\rm CC}$ and m_h^2 can exist in Nature. However, if this was really the theory of our universe, we would live in a completely empty one! The CC scans by small jumps of $\mathcal{O}(e^2)$ while the universe is inflating. It takes an exponentially long time to get from $M_{\rm Pl}^4$ to $(0.1~{\rm meV})^4$ and during this time the universe is inflating. Once we arrive at $\Lambda_{\rm CC} \simeq (0.1~{\rm meV})^4$ the universe is empty and we have only this tiny amount of energy density at our disposal to reheat it. This is the problem with Brown and Teitelboim original work [78, 79] that we have just summarized.

Bousso-Polchinski What makes Brown and Teitelboim's construction interesting is that string theory possess the ingredients that we have described in our toy model in some abundance and having more than one 4-form, might lead to an acceptable (non-empty) universe. It is very likely that if string theory is the right theory of quantum gravity a landscape actually exists, but it is still debated if all values of $\Lambda_{\rm CC}$ can exist in this landscape [81, 82].

For instance in M-theory there is a 7-form F_7 in 11D that upon compactification gives rise to several lower-order forms, including two F_4 of the type that we have described [83].

The only extra subtlety is that c in string theory is quantized [84], $c = en$ with $n \in \mathbb{Z}$, because both electric and magnetic sources are present for all gauge fields (see for instance [85] for a pedagogical discussion of Dirac's quantization condition and the quantization of magnetic fluxes).

In this picture, if we have J 4-forms from compactifying higher form fields, the cosmological constant is

$$\Lambda_{\rm CC} = \Lambda_0 - \frac{1}{2}\sum_{i=1}^{J} e_i^2 n_i^2 \,. \tag{7.21}$$

If we now imagine that bubbles with different c_i's are nucleated and expand during eternal inflation we can ask what it takes to get at least one patch where $\Lambda_{\rm CC} = \Lambda_{\rm obs} \simeq (0.1~{\rm meV})^4$. If we had a single 4-form we would need

$$e^2 \simeq \Lambda_{\rm obs} \ll M_{\rm Pl}^4 \,, \tag{7.22}$$

to scan the CC finely enough, as in the previous example. This is technically natural, since if we send $e \to 0$ the 3-form and the brane decouple and we have two free theories with extra symmetries. However it is nice to notice that if we have J fields then we can get away with much smaller couplings [84]

$$\frac{2\pi^{J/2}}{\Gamma(J/2)}\Lambda_0^{J/2}\frac{\Lambda_{\rm obs}}{\Lambda_0} \gtrsim \prod_{i=1}^{J} e_i \,. \tag{7.23}$$

For instance $\Lambda_0 \simeq M_{\rm Pl}^4$ and $J = 100$ gives $e_i \simeq (0.01 M_{\rm Pl})^2$ and indeed string theory predicts a large number of such fields. This is quite remarkable as we can have a tiny CC, but still enough energy density ($e_i \simeq 0.01 M_{\rm Pl}$) to reheat the universe above the experimental bound coming from BBN ($T \gtrsim$ MeV).

One can get Eq. (7.23) by noticing that the possible CCs given by the 4-forms are in a multidimensional grid. To find our universe in this grid, we have to cancel Λ_0 against the 4-forms contributions with a precision $\Lambda_{\rm obs}$, so we are asking if there is any point in this grid contained within the surfaces of two spheres, one of radius $\Lambda_0 - \Lambda_{\rm obs}$ and another of radius $\Lambda_0 + \Lambda_{\rm obs}$. Calculating the volume of this region gives us Eq. (7.23). What we have just summarized is the celebrated Bousso-Polchinski explanation [84] for the value of the CC. The only missing ingredient is the argument that explains why we are in a patch with such a tiny CC. This argument is due to Weinberg [86] and even if this work is mainly about the Higgs boson we find useful to review it here.

Weinberg's Argument If we reduce it to its most basic ingredients the argument runs as follows [87]. If the energy density from the CC, ρ_Λ, dominates, the Universe can have one of two fates: 1) If the CC is negative it takes the Universe a time $\sim \Lambda_{\rm CC}^{1/2}/M_{\rm Pl}^2$ to collapse into an object of size $\sim \Lambda_{\rm CC}^{1/2}/M_{\rm Pl}^2$ and comparable curvature radius 2) If $\Lambda_{\rm CC}$ is positive the Universe expands exponentially with a scale factor $e^{(\Lambda_{\rm CC}^{1/2}/M_{\rm Pl}^2)t}$. All other forms of energy are diluted, leaving an empty Universe.

Therefore if we want to form galaxies we need the matter energy density ρ_m to dominated over ρ_Λ for a long enough time. More precisely, density perturbations grow linearly with the scale factor

$$\frac{\delta\rho}{\rho} \sim a \tag{7.24}$$

if $\rho_m > \rho_\Lambda, \rho_r$, where ρ_r is the energy density in radiation. We can roughly call a galaxy a density perturbation of order one, i.e $\delta\rho/\rho \simeq 1$. Therefore, to form galaxies we need

$$\rho_\Lambda \lesssim \rho_{\rm MR}\left(\frac{\delta\rho_{\rm MR}}{\rho_{\rm MR}}\right)^3 , \tag{7.25}$$

where $\rho_{\rm MR}$ is the matter energy density at matter-radiation equality and $\left(\frac{\delta\rho_{\rm MR}}{\rho_{\rm MR}}\right)^3$ is the amount that this energy density has redshifted before density perturbations growing linearly with a become $\mathcal{O}(1)$. From CMB measurements we know that $\rho_{\rm MR} \simeq {\rm eV}^4$, $\delta\rho_{\rm MR}/\rho_{\rm MR} \simeq 10^{-5}$, so we get

$$\rho_\Lambda \lesssim (0.1{\rm meV})^4 , \tag{7.26}$$

remarkably close to the observed value. If we were more precise, we would find an upper bound about 100 to 1000 times larger than the actual measurement, but it is remarkable how close this simple argument gets to the actual value of the CC.

This idea is quite robust, in the sense that it doesn't rely on a precise definition of observers, we just don't want the universe to be empty or tiny and with a large curvature. However, it must be taken with a grain of salt. As stated above we don't know what the Multiverse really looks like and other parameters, including $\rho_{\rm MR}$ and $\delta\rho_{\rm MR}$ can vary between patches. This is nonetheless a pretty striking proof-of-principle that a Multiverse explanation for the CC might work.

7.2 Anthropic Selection of the Higgs Mass in the Multiverse

We have seen how to populate a vast landscape of values for the Higgs boson mass. However, we still need to explain why we happen to be in a patch with such an improbably small value of m_h.

Nature is full of interesting coincidences. There are a number of parameters that are just at the edge of what is needed to make a certain phenomenon possible. It was argued [88] that the Higgs boson mass might be one of these parameters. If it deviated more than a factor of a few from its observed value, complex chemistry would not be possible. This is traditionally taken as a sign that complex observers like us would not exist in most other patches of the Multiverse. In this sense the selection of the Higgs boson mass might be "anthropic", i.e. we don't see a more likely universe because there we don't exist.

The key observation is that nuclear parameters depend on m_h^2. Let us first consider universes with $m_h^2 < 0$. For the neutron-proton mass difference we have

$$m_n - m_p = (m_d - m_u) + \Delta m_{\text{em}} \approx 3\text{ MeV}\frac{v}{v_{\text{us}}} + \Delta m_{\text{em}} \tag{7.27}$$

For $v \lesssim$ few hundred$\times v_{\text{us}}$, then $m_{d,u} < \Lambda_{\text{QCD}}$ and we can leave $\Delta m_{\text{em}} = -1.7$ MeV fixed at the value that it has in our universe. Also the QCD scale and the mass difference between isospin 1/2 and an isospin 3/2 baryons depend on v,

$$\Lambda_{\text{QCD}} \simeq \Lambda_{\text{QCD,us}}\frac{v^\xi}{v_{\text{us}}^\xi} \tag{7.28}$$

$$m_{3/2} - m_{1/2} \simeq 300\text{ MeV}\frac{v^\xi}{v_{\text{us}}^\xi}\,, \tag{7.29}$$

$$\xi \simeq 0.3 \text{ for } 10^{-2} < \frac{v}{v_{\text{us}}} < 10^4\,. \tag{7.30}$$

There are main more hadronic properties that depend on v. The last one that we need to formulate our anthropic arguments is that the long range nucleon potential is well approximated by single pion exchange. The pion mass is also sensitive to v: $m_\pi^2 \sim f_\pi(m_u + m_d)$. $m_\pi \sim m_{\pi,us}\sqrt{v/v_{\text{us}}}$.

If v decreases, at some point Hydrogen becomes unstable, but other nuclei still exist since $m_p - m_n$ never gets above 1.7 MeV. So this kind of universes might support life. On the contrary if v becomes too big, the nuclear binding energy decreases (from m_π increasing). Besides $m_n - m_p$ increases indefinitely. At some point ($v/v_{\text{us}} \gtrsim 5$) no complex elements, beyond hydrogen, form. The reason is the following: in our universe the nuclear binding energy is negative, i.e. the mass of a nucleus is less than the mass of its constituents by an amount given by the nuclear force minus the EM repulsion, so it is energetically convient for baryons to form nuclei.

When $m_n - m_p$ exceeds the binding energy, the nucleus decays rapidly (if it ever forms). Consider the decay of a nucleus ${}^{\text{A}}_{\text{Z}}\text{X}$ of mass $m({}^{\text{A}}_{\text{Z}}X)$,

$${}^{\text{A}}_{\text{Z}}\text{X} \to {}^{\text{A}}_{\text{Z+1}}\text{X} + e^- + \bar\nu_e\,, \quad m({}^{\text{A}}_{\text{Z}}\text{X}) = m_N({}^{\text{A}}_{\text{Z}}\text{X}) + Zm_e - \sum_{i=1}^{Z} B_{i,e}\,. \tag{7.31}$$

The decay rate is given by

$$\begin{aligned} \Gamma &\sim G_F^2 Q^5 \\ Q &\approx m({}^{\text{A}}_{\text{Z}}\text{X}) - m({}^{\text{A}}_{\text{Z+1}}\text{X}) - m_e \approx m_N({}^{\text{A}}_{\text{Z}}\text{X}) - m_N({}^{\text{A}}_{\text{Z+1}}\text{X}) = (m_n - m_p) - B_N\,. \end{aligned} \tag{7.32}$$

The difference in electron binding energy is very small for high Z atoms and we have neglected it. B_N is the difference of the nuclear binding energies. Note that $-B_N$ is always negative because replacing a neutron with a proton increases the electrostatic repulsion. When $Q > 0$ the decay is allowed and the rate grows rapidly with Q. This sets an upper bound on the magnitude of m_h^2 in universes where $m_h^2 < 0$, exactly what we need to explain the smallness of m_h^2. How about $m_h^2 > 0$ universes?

In $m_h^2 > 0$ universes baryons are washed-out through electroweak sphalerons that convert them to neutrinos unless an asymmetry is produced after the EW phase transition. Molecules do not form until much later times compared to our universe. We need the cosmic microwave background to cool below $\epsilon\alpha^2 m_e \sim \epsilon\alpha^2 y_e \frac{\Lambda_{QCD}^3}{m_H^2}$, $\epsilon \approx 10^{-3}$.

This "biochemical energy" characteristic of molecules, can be estimated from the quantum mechanical model of the hydrogen atom

$$V(r) = \frac{p_e^2}{2m_e} - \frac{\alpha}{r} = \frac{1}{2m_e r^2} - \frac{\alpha}{r}\,. \tag{7.33}$$

The minimum of this potential is at

$$r = \frac{1}{\alpha m_e}\,, \tag{7.34}$$

and the typical kinetic energy of the electron $p_e^2/2m_e \sim \alpha^2 m_e$. We can roughly understand the ϵ suppression factor from the fact that molecules are bigger and more loosely bound than atoms. These arguments more or less rule out also $m_h^2 > 0$ universes as hospitable hamlets for observers relying on complex chemistry.

One aspect of this story that is not always appropriately emphasized, is that the arguments on chemistry outlined above are very detailed. By "detailed" I mean that they rely on a very specific definition of observers. If one starts searching, there are a lot of similar coincidences without which either complex chemistry would not exist or observers similar to us would not exist. The role of the Higgs is not that unique. Personally, I interpret this as a sign that maybe we are not using a good definition of observers, in the sense that it is possible that a much larger class of observers not based on complex chemistry might exist. This would make Higgs anthropic arguments contentless. Of course, until we further progress in the study of life, this discussion will remain at the philosophical level. It is nonetheless interesting to notice that Weinberg's argument for the CC, described at the beginning of this Section, is not at all detailed in this sense. It essentially only requires some amount of entropy in a causally connected patch.

To substantiate my earlier point on Nature being riddled with these coincidences, let me give two examples. I refer the reader to [89] for more fun coincidences.

When four nucleons make ${}^4_2\mathrm{He}$, 0.7% of their mass is converted to energy. If this number was smaller we would have only hydrogen otherwise there would be no hydrogen.

When a star runs out of Hydrogen it collapses until its core temperature reaches 10 keV. Then

$${}^4_2\mathrm{He} + {}^4_2\mathrm{He} \to {}^8_4\mathrm{Be} \tag{7.35}$$

$${}^4_2\mathrm{He} + {}^8_4\mathrm{Be} \to {}^{12}_6\mathrm{C} + 2\gamma \tag{7.36}$$

$${}^4_2\mathrm{He} + {}^{12}_6\mathrm{C} \to {}^{16}_8\mathrm{O} + \gamma \tag{7.37}$$

We need the excited state of Carbon on the right hand side to be between 7.3 and 7.9 MeV to produce sufficient carbon for life to exist, and must be further "fine-tuned" to between 7.596 MeV and 7.716 MeV to produce the amount observed in nature. There is an excited state of oxygen which, if it were slightly higher, would provide a resonance and speed up the reaction. In that case insufficient carbon would exist in nature; it would almost all have converted to oxygen. Hoyle used these facts to predict the existence of the ${}^{12}_6\mathrm{C}$ excited state. The ground state of Carbon is at 7.3367 MeV, below the ${}^4_2\mathrm{He} + {}^8_4\mathrm{Be}$ energy.

7.3 Friendly Landscapes

The anthropic arguments for $\Lambda_{\rm CC}$ and m_h^2 rely on the fact that dimensionless SM parameters, in particular Yukawa couplings (for m_h^2), do not vary appreciably between different patches of the Multiverse. This is not an unlikely occurrence, as can be seen from the explicit construction in [90]. A perhaps less debated, but more important point to keep in mind is that $m_h^2 = 0$ or $\Lambda_{\rm CC} = 0$ are not special points in theories without supersymmetry or scale invariance. Therefore a generic, non-symmetric, landscape will scan m_h^2 and the cosmological constant around their natural value ($m_h^2 \simeq M_*^2$ or $\Lambda_{\rm CC} \simeq M_*^4$ if M_* is the fundamental high scale of our theory) with very few vacua around zero, in general not enough to explain their value.

To illustrate this point, consider the QFT toy model of a landscape in [90]. We imagine a theory with N scalars ϕ_i. Each scalar has a potential V_{ϕ_i} with two minima at $\langle \phi_i \rangle = \phi_{1,2}$ and vacuum energies $V_{1,2}$. We take $V_1 \geq V_2$. The full theory has 2^N vacua described by the potential

$$V = \sum_{i=1}^{N} V_{\phi_i}\,. \tag{7.38}$$

We can label the vacua using a set of integers $\eta_i = \pm 1$. Every choice of $\{\eta\} = \{\eta_1, ..., \eta_N\}$ corresponds to a different CC

$$\begin{aligned} \Lambda_{\{\eta\}} &= N\bar{V} + \sum_{i=1}^{N} \eta_i \Delta V\,, \\ \bar{V} &= \frac{V_1 + V_2}{2}\,, \quad \Delta V = \frac{V_1 - V_2}{2}\,. \end{aligned} \tag{7.39}$$

For simplicity we have taken the same values of $V_{1,2}$ for all the scalars, since it does not affect our conclusions.

The distribution of CCs in the landscape at large N is well approximated by a Gaussian (as expected from the central limit theorem)

$$p(\Lambda) \to \frac{2^N}{\sqrt{2\pi N}\Delta V} e^{-\frac{(\Lambda - N\bar{V})^2}{N\Delta V^2}}\,. \tag{7.40}$$

If we have enough minima to populate only the central region of the Gaussian, the CC is finely scanned in a region $\Lambda = \bar{\Lambda} \pm \delta\Lambda = N\bar{V} \pm \sqrt{N}\Delta V$. If $\bar{V} \simeq \Delta V$, as we expect from dimensional analysis, then

$$\frac{\delta\Lambda}{\bar{\Lambda}} \simeq \frac{1}{\sqrt{N}}\,. \tag{7.41}$$

In particular we are not scanning around zero in the central region of the Gaussian. In this landscape the number of vacua with nearly vanishing vacuum energy is $\simeq 2^N e^{-N\bar{V}^2/\Delta V^2}$. To finely scan the CC around zero we need both $\bar{V}/\Delta V \lesssim \sqrt{\log 2}$ and sufficiently large N. A generic landscape is finely scanning the CC only around $N\bar{V}$.

The situation is different in supersymmetry. Take for instance the odd superpotential

$$W = \lambda\phi^3 - \mu^2\phi\,. \tag{7.42}$$

In this case at the two minima $W_1 = -W_2$ so that $\bar{W} = 0$. Then the landscape generated by N of these superpotentials is scanning the CC in the range

$$-3\frac{|\sqrt{N}\Delta W|^2}{M_{\rm Pl}^2} \lesssim \Lambda \lesssim 0\,. \tag{7.43}$$

In this case supersymmetry is keeping $\Lambda \leq 0$ and a Z_4 R-symmetry that protects the odd structure of the superpotential is ensuring that the distribution of negative CCs has a central value comparable to its standard deviation: $|\sqrt{N}\Delta W|^2/M_{\rm Pl}^2$ [90]. After SUSY breaking, this landscape scans the CC efficiently around zero, because of its symmetries. The situation is analogous for m_h^2.

In summary, even a landscape solution is probably relying on one of the symmetries that we presented in the previous Sections. Maybe they are realized only at very high energies, but this is still an interesting information about Nature. In the next Chapter we will see that the presence of these symmetries (in disguise) is often true also for solutions that explain m_h^2 through some early Universe event. However, this is just a simple toy example and we don't know the actual measure of $\Lambda_{\rm CC}$ and m_h^2 in the landscape, but it is generic enough that it is useful to keep it in mind.

7.4 Crunching

Anthropic arguments are not the only possible explanation for $\Lambda_{\rm CC}$ and m_h^2 in the landscape. Those for m_h^2 are particularly fragile and it would be interesting to see if there are alternatives. The answer is that there are many and here we give the first example. Thinking about a Mulitverse can truly change completely the way we can solve fine-tuning problems.

Here we discuss an idea that solves the hierarchy problem and strong CP problem in one shot [91]. Similar ideas for the CC or the hierarchy problem in isolation can be found in [92, 93, 94].

Consider the following simple Lagrangian

$$\begin{aligned}
V_\pm &= V_{\phi_\pm} + V_{\phi H} \\
V_{\phi_\pm} &= \mp\frac{m_{\phi_\pm}^2}{2}\phi_\pm^2 - \frac{m_{\phi_\pm}^2}{4M_\pm^2}\phi_\pm^4 \\
V_{\phi H} &= -\frac{\alpha_s}{8\pi}\left(\frac{\phi_+}{F_+} + \frac{\phi_-}{F_-} + \theta\right)\widetilde{G}G\,,
\end{aligned} \tag{7.44}$$

where $m_{\phi_\pm}^2 > 0$. In this Section for brevity we use θ to mean what we have called $\bar{\theta}$ in the previous Section. V_ϕ and $V_{\phi H}$ break the shift symmetry $\phi_\pm \to \phi_\pm + c_\pm$ by a small amount $m_{\phi_\pm}, \Lambda_{\rm QCD} \ll F_\pm, M_\pm$. V_ϕ is an EFT description valid at least for $|\phi_\pm| \lesssim M_\pm$. A concrete way to generate Eq. (7.44) and UV complete it is described in [91, 94].

Imagine that this simple model is embedded in a Multiverse that scans the values of θ, m_h^2 and $\Lambda_{\rm CC}$. What happens in the different patches of this Multiverse? Their destiny is determined by the two scalars $\phi_\pm$. If they ever reach the region $|\phi_\pm| \gtrsim M_\pm$, they can roll down their potential and eventually dominate $\Lambda_{\rm CC}$. Sooner or later these universes acquire a large negative CC and crunch, becoming tiny micro-universes with very large curvature. Only universes where $\phi_\pm$ are both stabilized in the region $|\phi_\pm| \lesssim M_\pm$ can live a long lifetime and expand, provided that their CC has been tuned to a small value. If $\phi_\pm$ are not stabilized to this region, the universe crunches even if all contributions to the $\Lambda_{\rm CC}$, excluding $\phi_\pm$, are tuned to be small.

To see what universes survive, let's look at the $\phi_\pm$ potentials. V_{ϕ_-}, taken in isolation, has a minimum near the origin, while V_{ϕ_+} has a local maximum. If we ignore the coupling to QCD, ϕ_+ eventually rolls down its potential and crunches all universes in the Multiverse. Below the QCD phase transition a new contribution to the $\phi_\pm$ potential is generated by $V_{\phi H}$ (as discussed in the previous Section) that might save some universes. To understand the motion of $\phi_\pm$we write their equation of motion in a FRW universe

$$\ddot{\phi}_\pm + 3H\dot{\phi}_\pm + \frac{dV_\pm}{d\phi_\pm} = 0\,. \tag{7.45}$$

If $3H\dot{\phi}_\pm \gg \frac{dV_\pm}{d\phi_\pm}$ the scalars are kept into place by Hubble friction $\dot{\phi}_\pm \simeq 0$. Therefore if we want any universe to survive we need $m_{\phi_+} \lesssim H(\Lambda_{\rm QCD})$, i.e. we need to leave a chance to QCD to stabilize the ϕ_+ potential. Let's assume that this is true and see what happens after the QCD phase transition.

Let us focus on the region $|\phi_\pm| \lesssim M_\pm$, where ϕ_- has a safe minimum and ϕ_+ might have a chance to get one. Furthermore, we make the technically natural choice $M_\pm/F_\pm \ll 1$ (M and F break different symmetries). Then, as discussed in Appendix A, at temperatures below the QCD phase transition we have

$$V_{\phi H} \simeq \frac{\Lambda^4(\langle h\rangle)}{2}\left(\theta + \frac{\phi_-}{F_-} + \frac{\phi_+}{F_+}\right)^2 + \ldots\,. \tag{7.46}$$

For simplicity we have also expanded for $\theta \ll 1$, but our arguments hold also for $\theta = \mathcal{O}(1)$. For $m_{u,d} \lesssim 4\pi f_\pi$ the scale of the potential reads [95] (see also Appendix A)

$$\Lambda^4(\langle h\rangle) = m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u+m_d)^2}\,. \tag{7.47}$$

Note that Λ^4 is a monotonic function of the Higgs vev $\langle h\rangle$ (so also of m_h^2). What does this new contribution to the potential do? To understand it, we make a few simplifying assumptions (not crucial for the mechanism to work, but extremely useful pedagogically): 1) We take $1/m_{\phi_-} \ll 1/m_{\phi_+}$, so that ϕ_- starts rolling first. 2) We take $M_-/F_- \lesssim \theta + M_+/F_+$, so that the tadpole term dominates the ϕ_- potential in Eq. (7.46) in the region $|\phi_-| \lesssim M_-$). Hence, the cross-interactions between $\phi_\pm$ generated by QCD are negligible and the minimization problem factorizes into two separate ones for the two scalars.

With these simplifying assumptions we will see after a minimal amount of work that only universes that have both the Higgs vacuum expectation value (vev) $\langle h\rangle$ and the QCD θ-angle in specific ranges

$$\mu_S \le \langle h\rangle \le \mu_B\,, \quad \theta \le \theta_{\rm max}\,, \tag{7.48}$$

survive the crunch. Therefore today only universes with a small θ and small and negative m_h^2 can exist (and potentially have expanded and grown to accomodate galaxies and observers). We are using Weinberg's argument for the CC to explain the observed values of m_h^2 and θ!

Let's see how this works in practice. V_{ϕ_-} has a safe minimum that can prevent crunching, but the tadpole generated by QCD

$$V_{\phi H} \supset \frac{\Lambda^4(\langle h \rangle)}{2} \theta \frac{\phi_-}{F_-} \tag{7.49}$$

can destroy it, if it is too large. This sets un upper bound on the QCD scale

$$\Lambda^4(\langle h \rangle) \lesssim \frac{m_{\phi_-}^2 M_- F_-}{(\theta + M_+/F_+)} \,. \tag{7.50}$$

Since $\Lambda^4(\langle h \rangle)$ is a monotonic function of $\langle h \rangle$ we have set an upper bound on m_h^2. The case of ϕ_+ is in some sense opposite. V_{ϕ_+} doesn't have any safe minimum. To generate one in the region $|\phi_+| \lesssim M_+$ we need: 1) The positive mass term generated by QCD

$$V_{\phi H} \supset \frac{\Lambda^4(\langle h \rangle)}{2} \frac{\phi_+^2}{F_+^2} \tag{7.51}$$

must be large enough to overcome the negative mass term in V_{ϕ_+}. Secondly, the potential generated by QCD, must be "aligned" with the original ϕ_+ potential, i.e. the QCD mass must be important near the origin of V_{ϕ_+} otherwise it won't ever generate a minimum, if it turns on in the region where V_{ϕ_+} is dominated by the quartic coupling. This can be achieved only if θ is small

$$\frac{M_+}{F_+} \gtrsim \theta + \frac{M_-}{F_-} \,. \tag{7.52}$$

In summary, universes where

$$\mu_S \leq \langle h \rangle \leq \mu_B \,, \quad \theta \leq \theta_{\max} \,, \tag{7.53}$$

have local minima for $\phi_\pm$ in the region $|\phi_\pm| \lesssim M_\pm$. All other universe crunch (at the latest) after the QCD phase transition. So having a small, but non-zero, Higgs vev and a small θ angle is a necessary condition for a universe to survive. We have thus explained why today we observe tiny values for m_h^2 and θ compared to their natural expectations. Note that a potential survivor can still crunch if it starts its life in the region $|\phi_\pm| \gtrsim M_\pm$, so we need a bigger Multiverse than usual to get at least one surviving universe today. If you want more details on this idea I refer you to [91]. As far as I know this is the only model that solves the hierarchy and strong CP problems together. Even if you didn't like the particular implementation, there's an important lesson to learn: m_h^2 and θ are fundamentally linked by an important property of the SM: $\widetilde{G}G$ is the only local operator in the SM whose vev depends on m_h^2. It turns out that it's also the only one whose vev depends on θ. The significance of this fact has been mentioned briefly in Section 7.7.1, but I recommend reading [96] for more details.

7.5 Landscapes that are not Multiverses

What we really need from a Multiverse is to realize an exponentially large number of values of m_h^2 and/or $\Lambda_{\rm CC}$, including exponentially tuned values. In the case of a Multiverse these values exist in causally disconnected spacetime patches that live for cosmologically long times. This is not the only way to scan over different values of the two parameters and in this Section we see how to do it in different ways.

7.5.1 Abbott

Here we show how to dynamically relax the CC to its small observed value, following an idea proposed by Abbott [97]. Spoiler: this idea predicts a completely empty Universe, very different from our own. The idea is still instructive and, other than realizing a landscape different from a Multiverse, it inspired a modern solution to the Higgs hierarchy problem.

Imagine having a scalar ϕ with the following potential

$$\begin{aligned} V(\phi) &= V_0 + V_1(\phi) + V_2(\phi) \\ V_1(\phi) &= -\frac{\alpha_g}{2\pi}\frac{\phi}{f}{\rm Tr}[F\widetilde{F}]\,, && (7.54) \\ V_2(\phi) &= \epsilon\frac{\phi}{f} + {\rm h.c.}\,, && (7.55) \end{aligned}$$

where V_0 is a cosmological constant, F is the field strength of the gauge bosons of a non-Abelian group G with gauge coupling α_g. G confines in the IR at a scale Λ_g. $V_{1,2}$ break different symmetries. V_1 breaks the continuous shift symmetry $\phi \to \phi + c$ down to a discrete subgroup $\phi \to \phi + 2\pi n f$. V_2 breaks the shift symmetry completely and it also breaks the discrete symmetry $\phi \to -\phi$. So it's technically natural to take $\epsilon \ll \Lambda_g^4$, where Λ_g^4 is the typical size of V_1 after G confines (see Section ??).

Imagine that initially ϕ sits high up in its potential, such that $V(\phi_I) \gg (\Lambda_g M_{\rm Pl})^2$. If $V(\phi_I)$ dominates the energy density of the Universe we are in a dS phase with Hubble parameter $H^2 \simeq V(\phi_I)/M_{\rm Pl}^2$ and temperature $T = H/2\pi$. In this phase, the gauge group has not confined yet ($T \gg \Lambda_g$) and the ϕ potential comes entirely from the linear term $V(\phi) = V_0 + V_2(\phi)$. We are in an inflationary universe where ϕ obeys the equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV_2(\phi)}{d\phi} = 0\,, \tag{7.56}$$

If we imagine that $\frac{M_{\rm Pl}}{V(\phi)}\frac{dV_2(\phi)}{d\phi} \ll 1$, ϕ is rolling slowly down its linear potential

$$\dot{\phi} \simeq \frac{1}{3H}\frac{dV_2(\phi)}{d\phi}\,. \tag{7.57}$$

Its kinetic energy remains small compared to the potential energy as long as the slow-roll condition on V_2' is satisfied. At some point ϕ gets to ϕ_c where

$$V(\phi_c) \simeq (\Lambda_g M_{\rm Pl})^2\,, \tag{7.58}$$

when this happens a phase transition occurs in the confining sector and V_1 starts to turn on. For $\phi > \phi_c$ the potential looks like

$$V(\phi) \simeq V_0 + \Lambda_g^4 \cos\frac{\phi}{f} + \left(\epsilon\frac{\phi}{f} + \text{h.c.}\right) . \tag{7.59}$$

Imagine that we are still in the slow roll regime, then as soon as V_1 turns on, ϕ will stop rolling if

$$V_1' \simeq \frac{\Lambda_g^4}{f} \gtrsim V_2' \simeq \frac{\epsilon}{f} . \tag{7.60}$$

Graphically, ϕ gets stuck when it encounters one of the first wiggles of $\cos\frac{\phi}{f}$ because it doesn't have enough kinetic energy to surpass it. So ϕ has stopped when

$$V(\phi_c) \simeq (\Lambda_g M_{\text{Pl}})^2 . \tag{7.61}$$

If this model described Nature, measuring the CC today amounts to measuring Λ_g

$$\Lambda_g^2 \simeq \frac{\Lambda_{\text{CC}}^{\text{obs}}}{M_{\text{Pl}}^2} . \tag{7.62}$$

One can easily show that tunneling between neighboring local minima of ϕ occurs on cosmologically long timescales. This seems a wonderfully simple explanation of $\Lambda_{\text{CC}}^{\text{obs}}$. It is just too bad that it required this Universe is completely empty. This is a consequence of the long periodo of inflation required to get from ϕ_I all the way down to ϕ_c.

Barring dirty tricks that violate the null energy condition, you can't extract enough energy from the final CC ($\Lambda_{\text{CC}}^{\text{obs}} \simeq 0.1$ meV) to explain the observable Universe. This is because we know from measurements of light elements abundances that the Universe existed as a SM bath in thermal equilibrium at least up to temperatures $T \simeq$ few MeV.

7.5.2 The Relaxion

The original relaxion solution [98] can be summarized by this potential valid up to a cut-off M

$$V = \left(-M^2 + g\phi\right)|H|^2 + V_\phi(g\phi) + \frac{\phi}{f}\widetilde{G}^a_{\mu\nu}G^{\mu\nu a} , \tag{7.63}$$

$$V_\phi(g\phi) = g^2\phi^2 + gM^2\phi + \ldots , \tag{7.64}$$

accompanied by an exponentially large number of e-folds of low scale inflation ($H_I \sim \Lambda_{\text{QCD}}$, where H_I is the Hubble parameter during inflation). All terms in the potential proportional to g break the shift symmetry $\phi \to \phi + c$, so it is technically natural to take $g \ll M$, since in the limit $g \to 0$ this symmetry is restored.

If we imagine that the relaxion field ϕ starts from $\phi \gtrsim M^2/g$, during inflation it is going to slowly roll down its potential until it arrives at a field value where the Higgs mass crosses zero. When $\phi > M^2/g$, $m_h^2 > 0$, $\langle h \rangle = 0$ and the potential is just a tadpole $g\phi$.

If we are at $T \sim H_I \lesssim \Lambda_{\rm QCD}$ this point is special from the relaxion point of view. It is where the barriers of size $f_\pi^2 m_\pi^2$ generated by $\frac{\phi}{f}\widetilde{G}^a_{\mu\nu}G^{\mu\nu a}$, start to appear, since they are proportional to the Higgs vev, $m_\pi \propto m_u + m_d \propto v$.

If inflation is still ongoing (i.e. the relaxion kinetic energy is negligible), the rolling of ϕ is going to stop when the slope of

$$\frac{\phi}{f}\widetilde{G}^a_{\mu\nu}G^{\mu\nu a} \sim f_\pi^2 m_\pi^2 \cos\frac{\phi}{f} \tag{7.65}$$

equals the slope of the other part of the potential $gM^2\phi$. This happens at

$$g \approx \frac{f_\pi^2 m_\pi^2}{fM^2} \approx 10^{-21}\ \text{GeV}\left(\frac{10^9\ \text{GeV}}{f}\right)\left(\frac{10\ \text{TeV}}{M}\right)^2 . \tag{7.66}$$

The value of f is chosen to respect current bounds on axion interactions and we have taken a low value of the cut-off M. Following our EFT discussion, it is technically natural to take g so small, since it is breaking the shift symmetry of ϕ. However the value of g implies trans-Planckian field excursions

$$\Delta\phi \gtrsim M^2/g \gg M_{\rm Pl} \tag{7.67}$$

that in our EFT formulation are allowed, but are usually problematic when gravity is taken into account [99, 100, 101, 77, 82, 81]. As mentioned above the solution also requires an exponentially large number of e-folds

$$N = \int dt H_I = \int d\phi \frac{H_I}{\dot\phi} \approx \Delta\phi\frac{H_I}{\dot\phi} \approx \Delta\phi\frac{H_I^2}{V'} \approx \frac{H_I^2}{g^2} . \tag{7.68}$$

The other conditions on the inflationary sector are

$$H_I > M^2/M_{\rm Pl} , \tag{7.69}$$

to inflate throughout the rolling of ϕ, and

$$H_I \lesssim \Lambda_{\rm QCD} , \tag{7.70}$$

to have wiggles when the Higgs mass crosses zero and finally

$$H_I^3 \lesssim V' , \tag{7.71}$$

so that classical rolling $\phi_c \simeq V'/H_I$ dominates over quantum brownian motion $\phi_Q \simeq H_I^2$. Let us try to attach some numbers to these requirements, we can take $f \gtrsim 10^8$ GeV from cooling of SN1987A [102] and $M \gtrsim 10$ TeV from explaining at least the little hierarchy problem, this gives

$$\begin{aligned} g &\lesssim \frac{f_\pi^2 m_\pi^2}{fM^2} \simeq 10^{-21}\ \text{GeV} , \\ \Delta\phi &\gtrsim 10^{10} M_{\rm Pl} , \\ N &\gtrsim \frac{H_I^2}{g^2} \simeq 10^{36} . \end{aligned} \tag{7.72}$$

These numbers are all technically natural, but pretty extreme. Subsequent efforts were able to dispose of super-Planckian field excursions and loosen the requirements of the inflationary sector, introducing additional fields in the model [103, 104, 105, 106, 107, 108, 109]. We should also keep in mind that one has to add also an appropriate reheating sector that does not spoil the mechanism.

Note also that we have not solved the strong CP problem. In this model $\theta \sim \mathcal{O}(1)$. If we want to solve it without new fields, g needs to be smaller by a factor of $\theta \sim 10^{-10}$ [98]. Alternatively we can include in the theory a second strongly interacting gauge group, under which new vector-like leptons getting an $\mathcal{O}(1)$ fraction of their mass from the Higgs are charged.

The mechanism at the core of the relaxion was proposed by Abbott to explain the value of the cosmological constant [97]. The first relaxion paper has had the merit to creatively apply this idea to m_h, although it must be pointed out that many of the ideas that have later taken root in the literature (for instance using $\mathrm{Tr}[G\widetilde{G}]$ as a trigger or looking for dynamics that can explain m_h in the multiverse) were already presents in Dvali's first attempts in the early '00s [110, 111, 112] that we discuss in Section **??**. Having said this, without the relaxion, many of us, including myself would not have started thinking about the problem in these terms and I think that it deserves its success. As is always the case in science, today much progress has been made and we know how to use similar ideas without the extreme requirements (or complex model building) that the relaxion demands.

7.5.3 Nnaturalness

We imagine that multiple copies of the SM exist and that they have different values of the Higgs mass [113]. In this Section we distinguish between the Lagrangian parameter m_H^2 that multiplies the $|H|^2$ operator and the physical Higgs mass m_h^2. The difference is at most a factor of $\sqrt{2}$, but it is useful to keep in mind if you ever try to reproduce the results in [113].

The point $m_H^2 = 0$ is not special in any way, so we have both sectors with $m_H^2 > 0$ and sectors with $m_H^2 < 0$. We take a uniform distribution for m_H^2, so if the theory has N sectors and a cut-off M, the lightest Higgs is at $m_H \approx M/\sqrt{N}$ and each sector has a mass

$$m_{h_i}^2 = m_{h_i^2}^{\min}(2i+1)\,, \quad -\frac{N}{2} \leq i \leq \frac{N}{2}\,. \tag{7.73}$$

We identify the sector with the smallest (in absolute value) $m_H^2 < 0$ with the SM that we observe and imagine that all the other sectors are coupled to us only through gravity. Obviously in this setup it is expected to have sectors with a Higgs mass that appears unnaturally small and arises from a cancellation. We just need to have enough sectors, given a cut-off M. However even a relatively low cut-off $M \approx 10$ TeV, requires a large number of new sectors $N \approx 10^4$ to get at least one with the observed Higgs mass. If we are brave, and consider $N \approx 10^{16}$ Nnaturalness can solve the whole hierarchy problem, bringing the scale of gravity down to $M_{\mathrm{Pl}}/\sqrt{N} \simeq 10^{10}$ GeV and the naturalness cutoff on m_h all the way up to the same scale.

A simple physical picture for this setup is that the new sectors are localized to branes which are displaced from one another in an extra dimension. A scalar that lives in the bulk and is coupled to $|H|^2$ has a non-trivial profile in the extra dimension and is responsible for the variation of m_h^2

from brane to brane. In this scenario, the lack of direct coupling is due to locality in the extra dimension, there is just a tiny coupling induced by gravity and potentially this new scalar.

It seems that we have already explained the size of the Higgs mass with this "brute force" approach, however there is still one experimental fact that we have not taken into account. Why is most of the energy density contained in the sector with the smallest negative m_H^2? The observed value of $\Delta N_{\rm eff}$ (all the energy density gravitationally coupled to us normalized to that contained in one SM neutrino) has an upper bound of approximately 0.5 at the epoch of recombination [114].

$$\Delta N_{\rm eff} = \frac{\rho - \rho_{SM}}{\rho_\nu} \, . \tag{7.74}$$

We can not simply give to our sector special couplings to the inflaton or to whatever reheats the Universe, otherwise we would not have really solved the problem. We would still need to explain why the smallest negative m_H^2 sector is also the one that couples to the inflaton. Nnaturalness explains the smallness of the observed Higgs mass only if all the sectors are treated democratically.

To obtain the observed value of $\Delta N_{\rm eff}$ we have to imagine that at some point the energy density was dominated by a gauge-singlet field, the *reheaton*. For illustrative purposes I take it to be a scalar ϕ. Then we can couple ϕ to all the Higgs bosons with the most relevant coupling that we can write down

$$a \sum_i \phi |H_i|^2 \tag{7.75}$$

and let ϕ decays reheat the SM and all other sectors. If $m_\phi \lesssim m_{H_i}, \forall i$ we can compute the decays in the EFT where we have integrated out all the Higgs bosons. The leading operators that we need to consider are[8]

$$\frac{a}{m_{h_i}} y_\psi \phi \bar\psi \psi \, , \quad \text{if } m_{H_i}^2 < 0 \tag{7.76}$$

$$\frac{a}{m_{H_i}^2} \phi F^2 \, , \quad \text{if } m_{H_i}^2 > 0 \, . \tag{7.77}$$

Here F is the field strength of any $SU(2)_L \times U(1)_Y$ gauge boson and this operator is allowed because only QCD is breaking the electroweak symmetry in sectors with $m_{H_i}^2 > 0$, where the Higgs boson does not have a vev. So $m_W, m_Z \sim \Lambda_{QCD} \ll m_{H_i}$. It is useful to distinguish between m_{h_i} the physical Higgs mass and the coefficient of $|H_i|^2$ in the Lagrangian, m_{H_i}. They coincide only for sectors with $m_{H_i}^2 > 0$.

From the operators above it is clear that even with equal couplings to all sectors the reaheaton decays preferentially to the lightest one with $m_{H_i}^2 < 0$ since

$$\Gamma_{m_{H_i}^2 < 0} \sim \frac{a^2 m_\phi}{m_{h_i}^2} \tag{7.78}$$

$$\Gamma_{m_{H_i}^2 > 0} \sim \frac{a^2 m_\phi^3}{m_{H_i}^4} \, . \tag{7.79}$$

[8] As an exercise check this explicitly. What other operators that can lead to ϕ decays are present in the $m_{H_i}^2 > 0$ sectors up to dimension five?

The energy density in each sector is then given by

$$\rho_i \;=\; \rho \mathrm{BR}_{\phi\to \mathrm{i}} = \rho\frac{\Gamma_i}{\Gamma} \quad \text{naively} \quad \Delta N_{\rm eff} \approx \sum_i \frac{\Gamma_i}{\Gamma} \sim \sum_i \frac{1}{i} \sim \log N\,. \tag{7.80}$$

In reality the mass thresholds in the SM and the very hierarchical nature of Yukawa couplings helps us a lot. Once v_{H_i} is such that $2m_{b_i} > m_\phi$ the decay rate drops significantly

$$\Delta N_{\rm eff} \;\approx\; \sum_i \frac{\Gamma_i}{\Gamma} \sim \sum_{i=1}^{N_b} \frac{1}{2\,i+1} + \frac{y_c^2}{y_b^2}\sum_{i=N_b+1}^{N_c} \frac{1}{2\,i+1} \simeq \frac{1}{2}\left(\log 2N_b + \frac{y_c^2}{y_b^2}\log\frac{N_c}{N_b}\right)\,, \tag{7.81}$$

$$N_{b,c} \;=\; \left(\frac{m_\phi^2}{8\,m_{b,c}^2} - \frac{1}{2}\right) \tag{7.82}$$

This is not quite enough to show that Nnaturalness satisfies all experimental constraints, but it does and the core of the reason is this parametric argument. For more details and potential smoking-gun signals I refer to the original paper [113], and the works that (among other things) explore its cosmological signatures [115, 116, 117, 118, 119].

It is nice to notice that Nnaturalness can work only because scalars can have relevant couplings to other scalars $\phi|H|^2|$ and marginal couplings to fermions $\phi\psi\psi^c$, this is what permits decay widths that scale as inverse powers of m_h, $\Gamma_i \sim 1/m_{h_i}^n$. Interestingly, these same couplings are what causes a hierarchy problem.

7.6 Statistical Selection of the Higgs Mass in the Multiverse

The very first attempts at explaining the Higgs mass cosmologically were made, to the best of my knowledge, in [110, 111, 112]. The idea was to create a Multiverse with an exponential accumulation of vacua near $m_h^2 = 0$. The basic mechanism can be described as follows. We have our usual 4-form

$$F_4 = dA_3\,, \tag{7.83}$$

introduced in Section 7, where we have used a shorthand for Eq. (7.2) employing the definition of the exterior derivative. The theory contains the terms relevant for the mechanism

$$S \supset \int d^4x\sqrt{-g}\left[-\frac{F_4^2}{48} + M_{\rm Pl}^2\left(-1+\frac{F_4^2}{M_{\rm Pl}^2}\right)h^2\right] + q(h)\int d^3\xi A_{\mu\nu\rho}\frac{\partial x^\mu}{\partial\xi^a}\frac{\partial x^\nu}{\partial\xi^b}\frac{\partial x^\rho}{\partial\xi^c}\epsilon^{abc}\,. \tag{7.84}$$

The nucleation of bubbles can proceed as in Section 7, following the Brown-Teitelboim idea. The crucial difference is that

$$q(h) = \frac{h^N}{M_{\rm Pl}^{N-2}}\,, \tag{7.85}$$

this can be enforced via a discrete symmetry [111, 112]. After every nucleation, the brane charge decreases. If $N > 2$,

$$\frac{\Delta\langle h\rangle^2}{\langle h\rangle^2} \propto \langle h\rangle^{N-2}\,, \tag{7.86}$$

Table 1: default

the vast majority of vacua have $\langle h \rangle$ close to zero. We have illustrated the idea for a scalar, but it can be generalized to an $SU(2)_L$ doublet. To populate these vacua, through nucleation of branes (which is an exponentially slow semiclassical process) eternal inflation is needed. This introduces the problem of measuring how likely a certain vacuum is. Even if we have a theory with exponentially more vacua at $\langle h \rangle \simeq 0$ compared to large values of the vev, we still can not compute how likely these vacua are in the Multiverse, due to the well-known measure problem of eternal inflation [120]. This is a common problem of models that aim to explain m_h using "statistical" arguments, i.e. by populating a special landscape where small m_h^2 is more likely. Other interesting examples include [121, 122, 123]. They most rely on the fact that regions at the top of the potential, i.e. with larger positive vacuum energy, inflate more, presumably becoming more likely in the Multiverse. However, they run into the problem of measure just described.

7.7 What All Landscapes Have in Common

A number of innovative ideas that trace the origin of the weak scale to early times in the history of the Universe are present in the literature [111, 112, 98, 113, 124, 121, 122, 125, 126, 93, 96, 123, 94] and in the previous Sections we have briefly reviewed some of them. I want to stress that we have described only the original ideas for each option, but often subsequent works have further developed the models, surpassing some of the drawbacks of the original theories. One prime example is the Relaxion [98] that has given rise to a series of model building works that (sometimes strongly) relax the requirements of the original model, some selected examples include [107, 103, 108, 127, 128, 129]. However obstruction to fully UV-completing the model still exist [130].

Taken at face value the ideas that we have reviewed in the context of a landscape seem widely different, selecting the weak scale by unrelated mechanisms and predicting different phenomenology. However, very schematically, they all possess a symmetric sector, where a large hierarchy of scales is natural, a SM landscape and a coupling between the two. This is shown in Fig. 4.

The coupling between the symmetric sector and the SM does not destabilize the hierarchy of scales in the symmetric sector. At late times, a cosmological event triggered by the Higgs vev and the coupling between SM and symmetric sector selects the observed value of the weak scale (right panel of Fig. 4). To further illustrate this sketch, in Table 1 I show how the models discussed so far, decompose into the three basic ingredients in Fig. 4.

It is useful, mostly as a bookkeeping device, to divide these ideas in three broad categories: 1) *Anthropic Selection* [131, 132, 88, 133]. Observers can arise only if $\langle h \rangle \simeq v$. 2) *Statistical Selection* [111, 112, 121, 122, 123]. Given some measure, the Multiverse is dominated by patches where $\langle h \rangle \simeq v$. 3) *Dynamical Selection* [124, 96, 98, 113, 125, 126, 93, 94]. Only non-empty[9]

[9] The simplest definition of an empty a patch is given by a universe where a positive CC always dominates the energy density. However for our purposes it is sufficient that, as explained below, observers can only exist for a

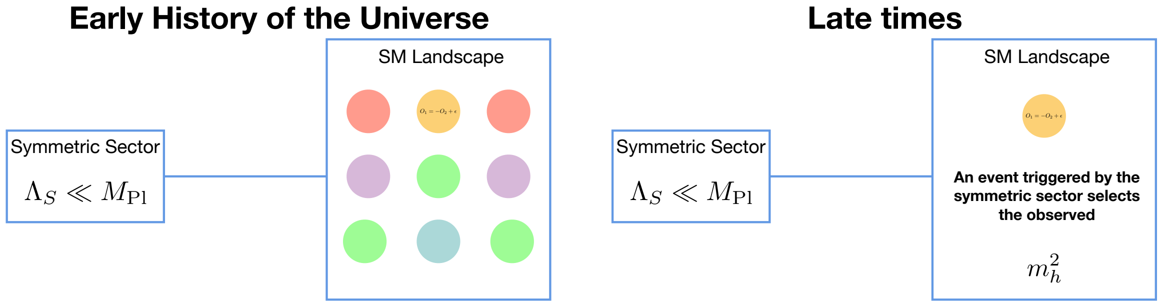

Figure 4: Models of cosmological selection of the weak scale. A symmetric sector, where a large hierarchy of scales is technically natural, is weakly coupled to a landscape of values of m_h^2. The SM landscape contains tuned values of m_h^2 including the observed one and is populated early in the history of the Universe. At a later time a cosmological event selects the observed value of m_h^2 through the coupling to the symmetric sector.

patches where $\langle h\rangle \simeq v$ live for cosmologically long times. This division is arbitrary and debatable, I find it useful mainly because it groups together ideas with similar experimental consequences.

Anthropic and statistical selection do not require new observable physics coupled to the SM. The mechanism that populates the landscape and generates its structure can take place at unobservably high energies or be due to non-dynamical fields with extremely feeble couplings to the SM [111, 112, 125, 123].

Dynamical selection instead predicts observable signals through the coupling between us and the symmetric sector. Conceptually there might be no difference between some of the ideas that I have grouped under the "dynamical selection" label and anthropic ones. For example, selection via crunching is leveraging Weinberg's anthropic argument for $\Lambda_{\rm CC}$. However the difference is very clear and quantitatively sharp when it comes to experimental signatures, since dynamical mechanisms can potentially be detected in the near future. Additionally, if compared to statistical selection, this class of models does not suffer from measure problems. A more elaborate discussion on this categorization can be found in [91]. A one paragraph summary is that dynamical selection mechanisms are 1) More robust that the others because they do not rely on a precise definition of observers or on counting patches during eternal inflation 2) More interesting experimentally, because there are potential avenues towards detection that can be explored within our professional lifetimes. These experimental signatures arise from the two main ingredients of the selection mechanism: 1) one or more new scalars or pseudo-scalars with masses inversely proportional to the cutoff of the theory and 2) an operator whose vev changes at $\mathcal{O}(1)$ when Higgs vev changes at $\mathcal{O}(1)$. These operators are coupled to the new scalar(s) and were first identified explicitly and called *triggers* in [96]. When the Higgs vev (and thus the operator vev) crosses certain upper or lower bounds, a cosmological event is *triggered* via the coupling to the new scalar(s).

sufficiently short time. We can consider empty also patches where the CC is positive and larger than a certain threshold $\Lambda > \Lambda_{\rm min}$. In these patches we can have a period of radiation and/or matter domination that lasts at most $\sim M_{\rm Pl}/\sqrt{\Lambda_{\rm min}}$. For an empty patch this time has to be much shorter compared to the age of our universe. In most models this time is much shorter than typical particle physics scales ($\ll 1/v$).

For a discussion of the first ingredient (light scalars) we mostly refer to [91], but let me give you also a very rough argument. In these models the weak scale is selected because two parts of the potential of these scalars (or their derivatives) become comparable when $\langle h\rangle \simeq v$. Let's consider a single scalar, since generalizations are simple. In most of these models we have

$$V = V_\phi + V_{H\phi}\,, \tag{7.87}$$

where $V_{H\phi}$ is generated by a coupling to the SM Higgs sector. It can also be an indirect coupling, as for example $(\phi/f)G\widetilde{G}$, but it generates a potential that depends monotonically on the Higgs vev $\langle h\rangle$. In general we can parametrize the two terms in V as

$$\begin{aligned} V_\phi &= m_\phi^2 M^2 f\left(\frac{\phi}{M}\right), \\ V_{H\phi} &= m_h^2 \langle h\rangle^2 h\left(\frac{\phi}{M}, \frac{h}{M}\right). \end{aligned} \tag{7.88}$$

To run this rough argument we have chosen a single cutoff M for the Higgs sector and the new scalar ϕ. Of course, in general this does not need to be the case.

When $V_\phi \simeq V_{H\phi}$ (in crunching mechanisms for instance) or $dV_\phi/d\phi \simeq \partial V_{H\phi}/\partial\phi$ (in relaxion models) some cosmological event is triggered, leading to the selection of the weak scale. If we want $\langle h\rangle \simeq v \simeq 174$ GeV to be selected, we need a naturally small mass for the new scalars

$$m_\phi^2 \simeq \frac{v^4}{M^2}\,. \tag{7.89}$$

We get this same condition from $V_\phi \simeq V_{H\phi}$ and $dV_\phi/d\phi \simeq \partial V_{H\phi}/\partial\phi$ (or higher derivatives). We have been sloppy in the distinction between masses and vevs, taking $m_h^2 \simeq v^2$. If we want to solve the hierarchy problem up to high scales $M \simeq M_{\rm Pl}$ the new scalars can become very light. This simple argument explains why we need a (approximately) symmetric sector that stabilizes the hierarchy $m_\phi \ll M$.

The new scalars often have all the right properties to be dark matter and the coupling to the SM predicts some distinct phenomenological features in these cases (see [96, 94] for general considerations, and [129, 134, 135, 136] for relaxion models).

Note also that exceptions to our simple argument exist, either because the weak scale is not selected by comparing two different terms in the potential of a new scalar, but rather directly its mass to that of SM particles [113] or because it occurs via a non-dynamical field [125].

In the next Section we discuss the second ingredient of these models, the so-called *trigger* operators. These objects are very interesting phenomenologically because they generate model-independent and testable experimental consequences of m_h^2 cosmological selection. They are also important conceptually because they are a generic feature of SM or BSM physics. They exist and are related to m_h^2 even in absence of a Multiverse or a landscape.

7.7.1 Trigger Operators

The coupling in Fig. 4 between the symmetric sector and the SM, in dynamical models, is given by a local operator O_T whose vev depends strongly on the Higgs vev

$$\frac{d\log\langle O_T\rangle}{d\log\langle h\rangle} = \mathcal{O}(1)\,. \tag{7.90}$$

Interestingly only one such object exists in the SM and it is related to another fine-tuning problem (the value of the neutron EDM). Let's first see why the vast majority of local operators in the SM do not have the above property. To compute their vev we add to the theory a probe scalar ϕ with a shift symmetry $\phi \to \phi + c$ broken only by a weak coupling ξ to the operator $O_{\rm SM}$ whose vev we are interested in. Our Lagrangian then contains the terms

$$\mathcal{L} \supset \mathcal{L}_{\rm SM} + \xi\phi O_{\rm SM}\,. \tag{7.91}$$

If we integrate out all massive SM particles, we generate a potential for ϕ in the form

$$V(\xi\phi) = \xi\phi\langle O_{\rm SM}\rangle + \ldots \tag{7.92}$$

The coefficient of the linear term in ϕ is the vev that we want to compute. Let us take for example $O_{\rm SM} = |H|^2$. The above procedure gives at one loop

$$V(\xi\phi) = \xi\phi\left(v^2 + c\frac{\Lambda_H^2}{16\pi^2}\right) + \ldots \tag{7.93}$$

Here Λ_H is a scale above which the SM becomes supersymmetric or scale invariant that we have used to cutoff the loop, and c is a $\mathcal{O}(1)$ number. If there is a hierarchy problem at all ($\Lambda_H \gg 4\pi\langle h\rangle$), the vev of $|H|^2$ is almost insensitive to the weak scale

$$\frac{d\log\langle O_T\rangle}{d\log\langle h\rangle} \simeq \frac{16\pi^2\langle h\rangle^2}{\Lambda_H^2} \ll 1\,. \tag{7.94}$$

The same is true for any gauge-invariant local operator in the SM, because we can always close the loop (or the loops) obtained from all the SM legs in $O_{\rm SM}$, if we use an insertion of $O_{\rm SM}$ itself coming from $\mathcal{L}_{\rm SM}$ in Eq. (7.91). Since $O_{\rm SM}$ is a gauge-invariant local operator, it exists in the SM Lagrangian. If you are not convinced, take for example $O_{\rm SM} = QHu^c$. In this case we have at two loops

$$V(\xi\phi) = \xi\phi\left(f_\pi^2\Lambda_{\rm QCD}v + c\frac{\Lambda_H^4}{(16\pi^2)^2}\right) + \ldots \tag{7.95}$$

If you are still not convinced, you can try with any other operator in the SM or read the more precise discussion in [113]. The reason why $G\widetilde{G}$ escapes this logic is that it can be written as a total derivative

$$\phi G\widetilde{G} = \phi\partial_\mu K^\mu\,. \tag{7.96}$$

If we integrate by parts, it's clear that perturbatively $G\widetilde{G}$ can never generate a potential for ϕ. The shift symmetry of ϕ in the theory

$$\mathcal{L} \supset \mathcal{L}_{\rm SM} + \frac{\alpha_s}{8\pi}(\xi\phi + \theta)G\widetilde{G}\,, \tag{7.97}$$

is broken only non-perturbatively at the QCD scale, as detailed in Appendix A. High-energy contributions are correctly captured by 't Hooft instanton calculation [] and are suppressed by factors of $e^{-1/g_s^2(M_{\rm UV})}$. They are typically negligible compared to the Higgs-dependent part of the vev

$$\langle G\widetilde{G}\rangle \simeq \theta m_\pi^2 f_\pi^2\,. \tag{7.98}$$

If we assume SM-like running up to the Planck scale, terms like $M_{\rm Pl}^4 e^{-1/g_s^2(M_{\rm Pl})}$ are completely negligible. The same is true in traditional supersymmetric grand unified models.

Beyond the SM, only two successful examples of trigger operators exist if we want to explain an arbirtrarly large hierarchy $M \gg m_h$. They are

$$H_1 H_2 \tag{7.99}$$

if the theory posses at least the $\mathbb{Z}_2$ symmetry $H_1H_2 \to -H_1H_2$, and

$$F\widetilde{F} \tag{7.100}$$

for a new confining gauge group G, if new fermions that get an $\mathcal{O}(1)$ fraction of their mass from $\langle h\rangle$ exist and are charged under G. The difficulty in finding other operators lies in the fact that we can't add new mass scales to the SM that are bigger than m_h, otherwise $\frac{d\log\langle O\rangle}{d\log\langle h\rangle} \sim (m_h/M)^n \ll 1$. So the H_1H_2 trigger predicts a new Higgs doublet with components that have masses below a few hundred GeV and one of them strictly lighter than m_h [113]. Similarly $F\widetilde{F}$ predicts new vector-like leptons close to the weak scale [98]. Essentially these are the only two operators that are still experimentally viable.

$G\widetilde{G}$ is the most challenging trigger of all, because the operator already exist in the SM and we can only detect the feeble axion-like interactions of the new scalars in the symmetric sector that are coupled to it.

To conclude, note that if we want to explain just a "little" hierarchy $m_h \ll M \ll M_{\rm Pl}$, other candidate triggers exist. A complete list is still missing, but the operators that can do the trick are those whose vev is protected by an approximate SM symmetry, for example $(Qu^c)(Qd^c)/M^2$ that is protected by a subgroub of the SM flavor symmetry. This subgroup is only broken by the small up and down Yukawas and, as a consequence, $\langle (Qu^c)(Qd^c)\rangle \simeq f_\pi^4\Lambda_{\rm QCD}^2 + y_u y_d \Lambda_H^6/(16\pi^2)^3$. The tree-level part can dominate up to Λ_H large enough to solve the little hierarchy problem.

For some more details on trigger operators and their phenomenology you can read [96, 94].

8 Comments on Scaleless Gravity

What if gravity did not have a scale? In that case, our estimate $m_h^2 \sim M_{\rm Pl}^2$ would not hold. Consider the action

$$S = \int d^4x\sqrt{-g}\left[\frac{R^2}{6f_0^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} - \xi_S|S|^2R + \mathcal{L}_{\rm matter}\right]\,, \quad \xi_S\langle S\rangle^2 = \frac{M_{\rm Pl}^2}{16\pi}\,. \tag{8.1}$$

Other terms of dimension greater than 4 are pure derivatives or can be redefined away. The second term is the square of the Weyl (or conformal) tensor obtained by subtracting all traces from the Riemann tensor. We have imagined that $\mathcal{L}_{\text{matter}}$ contains a potential for the scalar S, giving it the vev in the above equation.

Schematically this action gives an EOM for the graviton of the type

$$\Box h + \frac{1}{M^2}\Box^2 h = 0 \to \frac{1}{M^2 p^2 - p^4} = \frac{1}{M^2}\left(\frac{1}{p^2} - \frac{1}{p^2 - M^2}\right) . \tag{8.2}$$

Therefore this theory contains a ghost. It is not yet clear that we can make sense of it [137, 138, 139, 140]. However, there is at least another way to make gravity scale-less from the point of view of the Higgs boson. This second option does not pose problems of consistency of the theory, but the only known example of this behavior is in 2D where gravity is non-dynamical and very different than 4D.

In gravity local diffeomorphisms are a gauge symmetry and correlation functions are not good observables (but note that this is only a non-perturbative problem). We can only measure the S matrix or correlation functions along a worldline $x^\mu(\tau)$. Although the number $x^\mu(\tau)$ is arbitrary, it unambigously identifies a point on the spacetime manifold and we can measure

$$\langle 0|O(x^\mu(\tau_1))...O(x^\mu(\tau_n))|0\rangle . \tag{8.3}$$

The S matrix is defined at infinity where gauge symmetries are not redundancies anymore, they change states in the Hilbert space to different states, so the large gauge symmetry of gravity does not pose problems in the definition of S. It only imposes honest (global) symmetry contraints on its matrix elements.

How can we see the hierarchy problem in terms of these observables? Nobody really knows, so it is possible that our estimate $m_h^2 \sim M_{\text{Pl}}^2$ rooted in QFT intuition was too quick. There is one example in 2D [76], where M_{Pl}^2 enters the S matrix only through a phase, not affecting the pole structure of S. The gravitational S matrix is obtained from the flat space one by multiplication by the phase factor

$$\hat{S}_n(p_i) = e^{i\frac{1}{M_{\text{Pl}}^2}\sum_{i<j}\epsilon_{\alpha\beta}p_i^\alpha p_j^\beta} . \tag{8.4}$$

The most attractive feature of these very special theories is that they implement explicitly the idea that in absence of local off-shell observables the hierarchy problem might not be a problem. Its most unattractive feature is that gravity in 2D does not have a propagating massless spin-2 degree of freedom and this result looks very much like just eikonal scattering, i.e. scattering at high energies and large impact parameter $b = J/\sqrt{s}$. Here J is the angular momentum in a partial wave expansion of the amplitude and s the usual Mandelstam variable. By large we mean

$$b \gg \frac{E}{M_{\text{Pl}}^2}, \quad E > M_{\text{Pl}} . \tag{8.5}$$

In this regime also in 4D the effect of gravity is encoded in terms of a phase

$$e^{-i\frac{s}{4M_{\text{Pl}}^2}\log(b/R_{\text{IR}})} , \tag{8.6}$$

where $R_{\rm IR}$ is a IR cutoff that regulates infrared divergences. This type of scattering is indeed the only remnant of gravity in 2D.

If we ignore the problems with the previous examples (i.e. the ghost and the difficulty of extending the second idea to 4D), and power through, we still have two problems to solve. First of all, these theories still have a large scale (larger than $M_{\rm Pl}$) given by the Landau pole from the running of hypercharge in the SM. To avoid it we need new particles charged under $U(1)_Y$. Secondly, all BSM questions raised in Section 3.1.1 have to be answered without introducing new scales that are too strongly coupled to the Higgs. Rather than a problem, this is a feature of this class of ideas, which in principle can be falsified by discovering new scales coupled to H. Some of the phenomenological implications of this scenario were worked out in [141, 142, 143].

9 UV/IR Mixing

In addition to the attempts at formulating a scaleless theory of gravity, there are two more ways in which gravity might behave differently compared to what discussed in the previous Sections, where it was simply providing a new dimensionful scale to deal with in QFT.

The first one is quite direct and violates our EFT intuition on the Higgs mass. The Higgs boson mass squared is given by an integral over multiple energy scales

$$m_h^2(\Lambda_{\rm IR}) = m_h^2(\Lambda_{\rm UV}) + \int_{\Lambda_{\rm IR}}^{\Lambda_{\rm UV}} d\Lambda\, \delta m_h^2(\Lambda)\,. \tag{9.1}$$

It is possible that high energy effects are not independent of low energy ones and what appears as an accidental cancellation between $m_h^2(\Lambda_{\rm UV})$ and the integral in Eq. (9.1), is explained by the full theory of quantum gravity.

This brief discussion might have appeared vague. This is not an accident. To the best of my knowledge there is no concrete proposal to implement the previous idea. The closest we got are examples in quantum field theory on non-commutative spacetimes [144]. In this proposal IR and UV effects are related in a precise way, but the explicit breaking of Lorentz symmetry, inherent in the theory, obstructs incorporating the mechanism in the SM, given current experimental results. Still we find interesting to review the basics of this idea here, since it could be a toy model for something more subtle actually going on in Nature. Consider

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}\,, \tag{9.2}$$

where $\theta_{\mu\nu} = -\theta_{\nu\mu}$. This is relating long-distance and short-distance effects

$$\Delta\hat{x}_\mu \Delta\hat{x}_\nu \geq \frac{|\theta_{\mu\nu}|}{2}\,, \tag{9.3}$$

but is also breaking Lorentz invariance. The tensor $\theta_{\mu\nu}$ is breaking Lorentz invariance as a uniform magnetic field breaks rotational invariance, by defining a preferred direction in space(time).

It is possible to show that a quantum field theory on these spacetimes can be written in terms of commuting coordinates, if we additionally introduce the product [145, 146]

$$f(x) \star g(x) = \exp\left(\frac{i}{2}\theta_{\mu\nu}\partial_y^\mu \partial_z^\nu\right) f(y)g(z)\Big|_{y=z=x}\,. \tag{9.4}$$

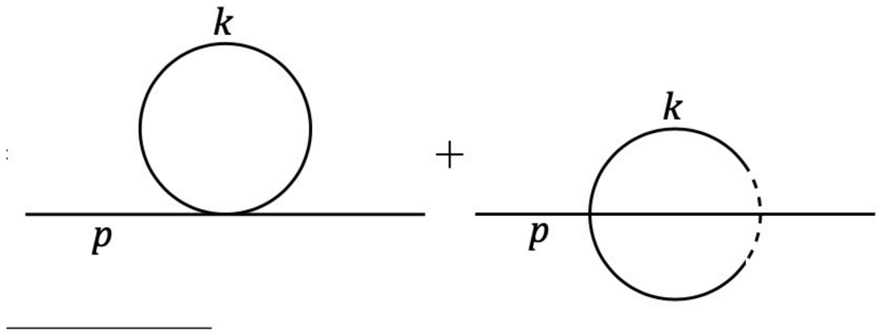

Figure 5: Contributions to the two-point function of a scalar in a non-commutative ϕ^4 theory. Figure taken from [144].

This trick allows to show that noncommutative quantization does not affect the free part of the tree-level action due to momentum conservation and the antisymmetry of $\theta_{\mu\nu}$ that make the new exponential factor $= 1$ for the quadratic terms in the Lagrangian.

Interactions are modified to

$$\mathcal{L}_{\rm int}^{\rm NC} = \frac{\lambda_n}{n!}\phi(x) \star \phi(x) \star ... \star \phi(x)\,. \tag{9.5}$$

The corresponding action in momentum space looks like

$$S_{\rm int}^{\rm NC} = \frac{\lambda_n}{n!}\int \prod_{i=1}^{n} d^4k_i \phi(k_1)...\phi(k_n)\delta^{(4)}(k_1 + ... + k_n)\exp\left(\frac{i}{2}\sum_{j<i}^{n}\theta_{\mu\nu}k_i^\mu k_j^\nu\right)\,. \tag{9.6}$$

If we expand for $\theta \ll 1$ this looks a like a perfectly normal EFT with a set of irrelevant operators. If instead we keep the full exponential, an interesting UV/IR duality emerges.

The antisymmetry of $\theta_{\mu\nu}$ together with the momentum-conserving δ-function in each vertex, allow to considerably simplify calculations in these theories. If the graph is planar, including any tree-level graph, all exponential factors from loops can be eliminated. The only contributions to the new phase factor containing θ come from external lines and their ordering. In non-planar graphs, internal lines that cross can also contribute. A proof can be found in [147]. In practice at tree-level these theories are identical to commutative QFTs. At loop level, it is easy to evaluate integrands, but integrations can give surprising results.

Consider the scalar ϕ^4 theory studied in [148], in Euclidean signature[10]

$$S_4 = \int d^4x\left(\frac{\partial_\mu\phi\partial^\mu\phi}{2} + \frac{m^2}{2}\phi^2 + \frac{g^2}{24}\phi\star\phi\star\phi\star\phi\right)\,. \tag{9.7}$$

At one loop the two-point function of a scalar with external momentum p^μ receives two contri-

[10]There are subtleties related to unitarity in non-commutative Lorentzian theories that do not affect our main point. We refer the reader to [144] for a more complete discussion with relevant references.

butions

$$\begin{aligned}\Gamma_p^{(2)} &= \frac{g^2}{3(2\pi)^4}\int \frac{d^4k}{k^2+m^2}\,,\\ \Gamma_{np}^{(2)} &= \frac{g^2}{6(2\pi)^4}\int \frac{d^4k}{k^2+m^2}e^{ik^\mu\theta_{\mu\nu}p^\nu}\,,\end{aligned}\tag{9.8}$$

given by the diagrams in Fig. 5. The first integral can be evaluated by standard techniques using a momentum cutoff to give

$$\Gamma_p^{(2)} = \frac{g^2}{48\pi^2}\left(\Lambda^2 - m^2\log\frac{\Lambda^2}{m^2} + \mathcal{O}(1)\right).\tag{9.9}$$

For the second integral we introduce the Schwinger parameter α

$$\frac{1}{k^2+m^2} = \int_0^\infty d\alpha e^{-\alpha(k^2+m^2)}\,,\tag{9.10}$$

complete the square at the exponent, perform the d^4k integral and then add the regulator $e^{-1/(\Lambda^2\alpha)}$

$$\Gamma_{np}^{(2)} = \frac{g^2}{96\pi^2}\int\frac{d\alpha}{\alpha^2}e^{-\alpha m^2-\frac{p^\mu\theta_{\mu\rho}\theta^\rho_\nu p^\nu}{4\alpha}-\frac{1}{\Lambda^2\alpha}}\,.\tag{9.11}$$

Evaluating the integral gives

$$\begin{aligned}\Gamma_{np}^{(2)} &= \frac{g^2}{96\pi^2}\left(\Lambda_{\text{eff}}^2 - m^2\log\frac{\Lambda_{\text{eff}}^2}{m^2}+\mathcal{O}(1)\right)\,,\\ \Lambda_{\text{eff}}^2 &= \frac{1}{\frac{1}{\Lambda^2}+\frac{p\circ p}{4}}\,.\end{aligned}\tag{9.12}$$

Intriguingly $\Gamma_{np}^{(2)}$ is finite for $\Lambda\to\infty$. Have we really regulated UV divergences using the fuzziness of spacetime? Not exactly, since the UV pole has not entirely disappeared. It just does not commute with a new IR pole that did not exist in the tree-level theory. Let's see this by introducing a counter-term that regulates the usual UV divergence: $M^2 = m^2 + \frac{g^2}{48\pi^2}\left(\Lambda_{\text{eff}}^2 - m^2\log\frac{\Lambda_{\text{eff}}^2}{m^2}\right)$. Then we can write the 1PI effective action at one-loop as

$$S_{1PI} = \int\frac{d^4p}{(2\pi)^4}\phi(-p)\left[p^2 - M^2 + \frac{g^2}{96\pi^2\left(\frac{1}{\Lambda^2}+\frac{p\circ p}{4}\right)} - \frac{g^2M^2}{96\pi^2}\log\frac{\Lambda_{\text{eff}}^2}{M^2}\right]\phi(p)\tag{9.13}$$

where we have defined $p\circ p \equiv -p^\mu\theta_{\mu\rho}\theta^\rho_\nu p^\nu$. If we take $\Lambda\to\infty$ first, we have a new pole when

$$p\circ p = -\frac{g^2}{24\pi^2m^2}\,,\tag{9.14}$$

we seem to have generated a new particle of mass $\frac{g^2\Lambda_\theta^4}{24\pi^2m^2}$ from UV dynamics. Here we have assumed that $1/\Lambda_\theta^2$ is the only eigenvalue of θ. Similarly, taking $p\circ p\to 0$ leaves us with a UV divergence. This remains true in different regularization schemes [144].

A Wilsonian effective theorist would write a low energy Lagrangian that is finite in the $\Lambda \to \infty$ limit. This Lagrangian must contain a new field that accounts for the IR pole that appears in this limit. This means adding to S_4 the terms

$$\Delta S_4(\Lambda) = \int d^4x \left(\frac{1}{2}\partial\chi \circ \partial\chi + \frac{\Lambda^2}{8}(\partial \circ \partial\chi)^2 + \frac{i}{\sqrt{24\pi^2}} g\chi\phi \right) . \tag{9.15}$$

It is not clear at all to me (or to anyone, as far as I know) that this is the right perspective. Sure, this Lagrangian respects the Wilsonian tenet that the correlations functions computed from the action converge smoothly to their $\Lambda \to \infty$ limits[11]. However, χ does not look at all like a normal low-energy field. For instance, we can't simply write an effective Lagrangian for χ by integrating out ϕ, since its non-standard kinetic term prevents diagonalization of the quadratic terms in the Lagrangian. Furthermore, the new pole breaks unitarity in this theory [149] and finally the only interaction of χ is linear mixing, which means that its action is not renormalized (any divergences are absorbed by ϕ counterterms).

The Wilsonian point of view might indeed by inadequate to understand these theories, since it is based on a "UV first" logic, which is the right point of view is still source of debate [144].

We can content ourselves to note an intriguing fact. Given any finite UV scale Λ we have generated a new stable IR scale in the form of an IR cutoff $\sim \Lambda_\theta^2/\Lambda$. Even more intriguingly, for $\Lambda \to \infty$, the theory is finite but we have a new IR pole in a two-point function at $p^2 \sim g^2\Lambda_\theta^4/m^2$, which can be naturally much smaller than m^2.

This is exactly what we need to solve the hierarchy problem. However, to have a real solution we still have to deal with Lorentz violation (in the Wilsonian picture χ propagates only in non-commuting directions) and better understand unitarity in this theory. For a more complete discussion we refer the reader to [144].

The second way in which gravity might surprise us, is possibly even more speculative, but not completely unfamiliar from an EFT perspective. It is well-known that a UV theory might leave non-trivial constraints at low energy, which the low energy physicists can only accept as facts of life. The prime example is the Spin-Statics theorem in quantum mechanics that in QFT is seen as a consequence of Lorentz invariance and causality.

String theory might offer a more dramatic realization of this idea. It is possible that many perfectly sensible, local, Lorentz-invariant EFTs are in the so-called "swampland", i.e. they are incompatible with quantum gravity. A number of string theory examples make this intuition precise. For example the same modulus usually controls the mass of multiple towers of new states. A classic example are KK and winding modes in string theory, whose masses scale as

$$M_{\text{KK}} \sim e^{\alpha\phi} , \quad M_{\text{winding}} \sim e^{-\alpha\phi} \tag{9.16}$$

where ϕ is a modulus. The two towers are related by T-duality. Given the pervasive nature of dualities in string theory this has led to the so-called Swampland Distance Conjecture [77]

- Consider a theory, coupled to gravity, with a moduli space M which is parametrized by the expectation values of some field ϕ_i which have no potential. Starting from any point $P \in M$ there exists another point $Q \in M$ such that the geodesic distance between P and Q, denoted $d(P, Q)$, is infinite.

[11] This can be verified by integrating out χ at tree-level, since the action is quadratic in χ.

- There exists an infinite tower of states, with an associated mass scale m, such that

$$m(Q) \sim m(P)e^{-\alpha d(P,Q)}\,, \tag{9.17}$$

where α is some positive constant.

This means that considering large field excursion might break our EFT, even if at low energy we would not suspect that.

Another example is the Weak Gravity Conjecture. When we compactify string theory we might obtain gauge fields at low energy that arise either from the high-dimensional components of the gravitational field, or from higher form fields (for instance $B_{\mu\nu}$). In both cases the low dimensional gauge coupling is a function of the moduli that determine the size and geometry of the compactified dimensions. There is therefore a relation between the mass of charged states (KK and winding modes), coming from this compactification and their charge under the gauge group. Explicit examples corroborate the following Weak Gravity Conjecture [150, 151]:

- Consider a theory, coupled to gravity, with a $U(1)$ gauge symmetry with gauge coupling g

$$S = \int d^dx\sqrt{-g}\left[M^{d-2}\frac{R^d}{2} - \frac{1}{4g^2}F^2\right]\,. \tag{9.18}$$

Then

- **Electric**: There exists a particle in the theory with mass m and charge q such that

$$m \leq \sqrt{\frac{d-2}{d-3}}gqM^{\frac{d-2}{2}}\,. \tag{9.19}$$

- **Magnetic**: The cutoff of this EFT is bounded from above by

$$\Lambda \lesssim gM^{\frac{d-2}{2}}\,. \tag{9.20}$$

A third interesting example is the Refined de Sitter Conjecture [81, 82]

- The scalar potential of a theory coupled to gravity must satisfy either

$$|\nabla V| \geq \frac{c}{M_{\text{Pl}}}V\,, \tag{9.21}$$

or

$$\min\left(\nabla_i\nabla_j V\right) \leq -\frac{c'}{M_{\text{Pl}}^2}V \tag{9.22}$$

For $c, c' = \mathcal{O}(1)$. This conjecture comes from the calculation of the de Sitter entropy plus the distance conjecture. If a scalar rolls too far down its potential, the tower of states that becomes light changes the entropy, making it incompatible with what we know about de Sitter space [100, 81, 82].

All these examples have a few features in common: 1) They describe highly non-trivial constraints on the EFT that the low energy physicist could not have imagined 2) They are (conjectures)^2. They arise from string theory (which we do not know for sure to be the right theory of quantum gravity), within string theory they come from a handful of examples that correspond to limits where we have the theory under control. If we want to apply them to phenomenology they become (conjectures)^3 in the sense that we typically have to take an extra step. For instance, by adding to the distance conjecture the statement that all low energy scalars are moduli.

Having said this, the fact that $m_h^2 = 0$ is special from the point of view of quantum gravity is not impossible, and we can keep it in mind as an intriguing possibility for future work and speculation. For an idea in this direction see [152].

10 Generalized Symmetries

In this Section we discuss an example in $2d$ where the hierarchy problem is solved by a non-invertible symmetry. The symmetry is not at all manifest at the Lagrangian level. This example cannot be generalized to our world in $4d$ for reasons discussed in [], but it is still very instructive if we take it as inspiration for what we might have missed while staring at the SM Lagrangian.

The symmetry that I am alluding two is well-known in statistical mechanics as the Kramers Wannier duality of the Ising model. We first review the standard derivation of the duality, then we show that the model admits a low energy limit described by a Higgs-like scalar and finally we rephrase everything in terms of generalized symmetries.

10.1 Kramers Wannier Duality

Consider an array of spins $\sigma_i = \pm 1$ on a square lattice in two space dimensions with Hamiltonian

$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j \,. \tag{10.1}$$

The sum over $\langle ij \rangle$ extends on all pairs of nearest neighbors. The partition function of the system is

$$Z = \sum_{\{\sigma\}} e^{\beta \sum_{\langle ij \rangle} \sigma_i \sigma_j} \,, \tag{10.2}$$

where it's more convenient to define β as

$$\beta \equiv J/k_B T \,, \tag{10.3}$$

rather than the usual inverse temperature, and $\{\sigma\}$ denotes a sum over all spin configurations. Eq. (10.2) can be manipulated to obtain a mapping between the high- and low-temperature regimes of the model. We can think of this as a duality between two different theories that describe the same 2D Ising model or as a symmetry of the model that transforms non-trivially

From the above equation we can easily write the usual covariant derivative in terms of Wilson lines []. We can also write the usual kinetic term for the gauge bosons as a trace in the schematic form $\mathrm{Tr}[U_1 U_2 U_3^\dagger U_4^\dagger]$, if the paths of each Wilson line are chosen so that the 4 paths form a closed loop. We refer to [] for more details.

It is straightforward to adapt the previous discussion to a discrete lattice, now U_{ij} connects two sites i and j following a path l through the lattice links, as we did in Section 10.1. If $s_{i,j}$ are scalar lattice variables, living on the lattice sites, that transform in the fundamental representation of the gauge symmetry, the object

$$s_i^* U_{ij} s_j \tag{B.4}$$

is gauge invariant. This is the simplest heuristic explanation of how to realize a discrete gauge theory on the lattice. You might have noticed from the discussion in Section 10.1 that this symmetry is doing nothing locally and it introduces no new propagating degrees of freedom. However it can still have some physical effects. **to be completed**

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