Gravitational waves in a nutshell—5

Standard sirens. Searches for continuous waves. Searches for stochastic waves.

Michele Vallisneri, Jet Propulsion Laboratory

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Recap from last week

1. Accurate waveform modeling from binary inspirals proceeds from a separation of scales (compact body, binary, wave), and can be formulated as a tower of effective field theories (worldline + external field, binary + potential field, composite object + radiation field).

2. High-energy degrees of freedom are integrated out from action, or absorbed in low-energy operators with matched coefficients. Feynman rules, power counting, and dimensional regularization allow efficient computation of perturbative expansions.

3. Missing complete theories of gravity with inspiral models, tests of GR have focused on consistency and phenomenological parametric constraints, probing generation, propagation, and polarization.

4. The interpretation of current negative results is weak, and the validation of any positive results would be daunting.

5. Large SNRs will be required to probe expected small modifications to waveforms. Tests that avoid that (e.g., dark-energy constraints based on speed of gravity) can be very powerful.

Standard candles

1. FLRW metric $$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)]$$

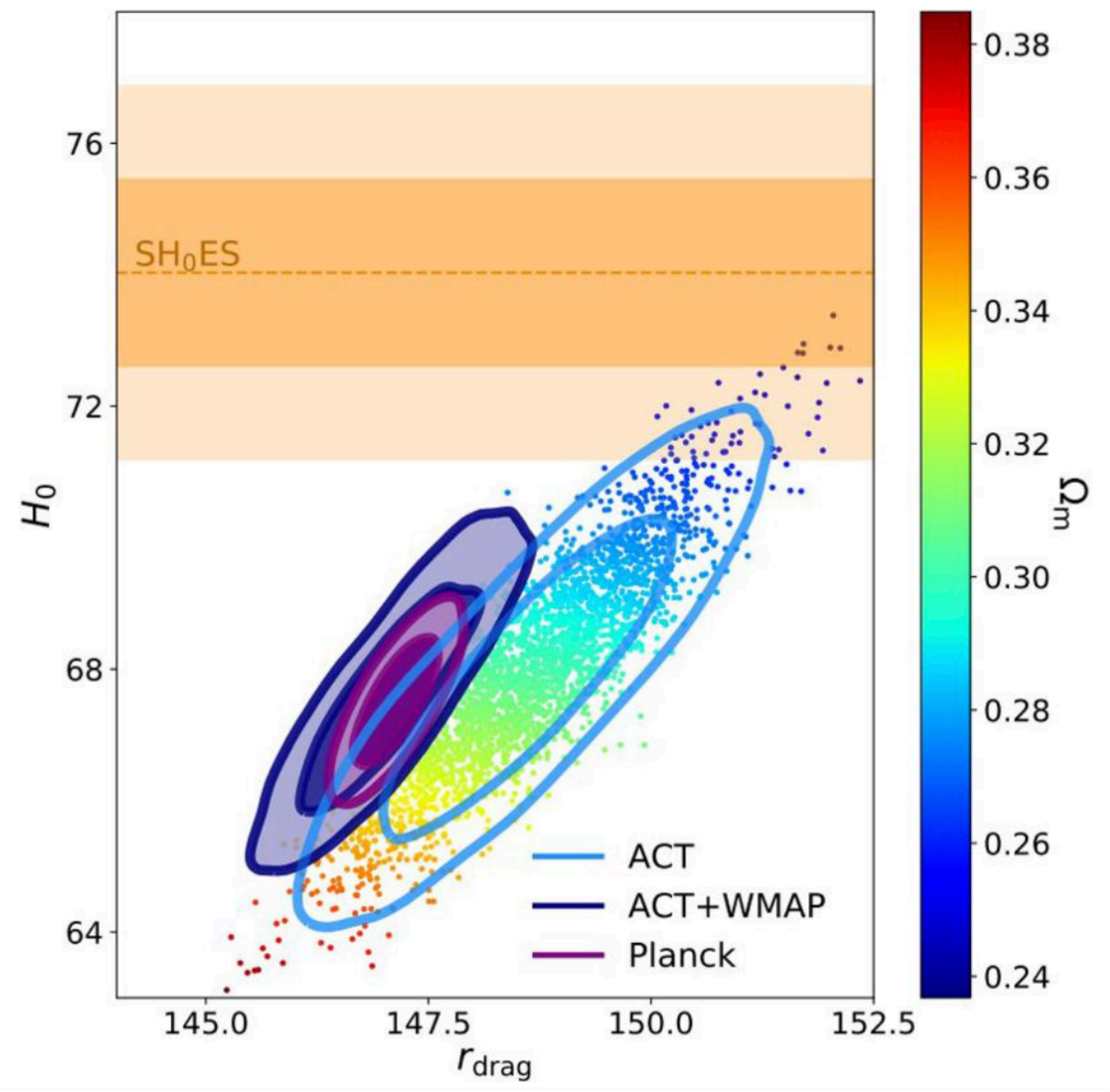
2. Cosmological redshift $$1 + z = \frac{a(t_{\rm obs})}{a(t_{\rm src})} \qquad f_{\rm obs} = \frac{f_{\rm src}}{1+z}$$

3. Luminosity distance $$F_{\rm obs} = \frac{L_{\rm src}}{4\pi\, d_L^2} = \frac{L_{\rm src}}{4\pi\, a^2\,(t_{\rm obs})\, r^2\,(1+z)^2}$$

4. Expansion history $$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')} \simeq \frac{z}{H_0} + O(z^2)$$
$$H(z) = \frac{\dot{a}(z)}{a(z)} = \sqrt{\Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}$$

5. Standard candles: given $F_{\rm obs}$ and $L_{\rm src}$, determine $H_0 = 74\,{\rm km\,s^{-1}\,Mpc^{-1}}$

6. Cosmic tension: CMB yields $H_0 = 68\,{\rm km\,s^{-1}\,Mpc^{-1}}$



[ACT]

Standard sirens

1. Linearized Einstein equations on the FLRW background:

$$ds^2 = a^2(\eta)\left[-d\eta^2 + dr^2 + r^2\, d\Omega^2\right] + h_{ij}\, dx^i dx^j$$

$$\nabla^2 h_{ij} - \partial_\eta^2 h_{ij} + O(f_{\rm gw}/H_0) = 16\,\pi\,\tau_{ij}^{\rm TT}$$

2. Wave ansatz:

$$h \propto \frac{g(\eta - r)}{a(\eta)\,r} = \frac{g(t - r/c)}{a(t_0)\,r}$$

normalizing $\eta = t$ now

3. GW amplitude as interpreted by observer

$$h \propto \frac{4}{a(t_0)\,r} M_c^{5/3}\left(\pi f_{\rm gw}(t_{\rm ret})\right)^{2/3} = \frac{4(1+z)}{d_{\rm L}} M_c^{5/3}\left(\pi\,(1+z) f_{\rm gw}^{\rm obs}(t_{\rm ret})\right)^{2/3}$$

$$= \frac{4}{d_{\rm L}}(M_c^{\rm obs})^{5/3}\left(\pi f_{\rm gw}^{\rm obs}(t_{\rm ret})\right)^{2/3}$$

$M_c^{\rm obs} = (1+z)\,M_c$

4. GW phasing as interpreted by observer

$$f_{\rm gw} \sim M_c^{-5/8}\tau^{-3/8} \;\Rightarrow\; f_{\rm gw}^{\rm obs}\,(1+z) \sim \left(\frac{M_c^{\rm obs}}{1+z}\right)^{-5/8}\left(\frac{\tau^{\rm obs}}{1+z}\right)^{-3/8}$$

$$\Rightarrow\; f_{\rm gw}^{\rm obs} \sim (M_c^{\rm obs})^{-5/8}(\tau^{\rm obs})^{-3/8}$$

5. Therefore $h_{\rm inspiral}$ is the same as for local sources, replacing d with $d_{\rm L}$, M_c with $M_c^{\rm obs} = M_c(1+z)$

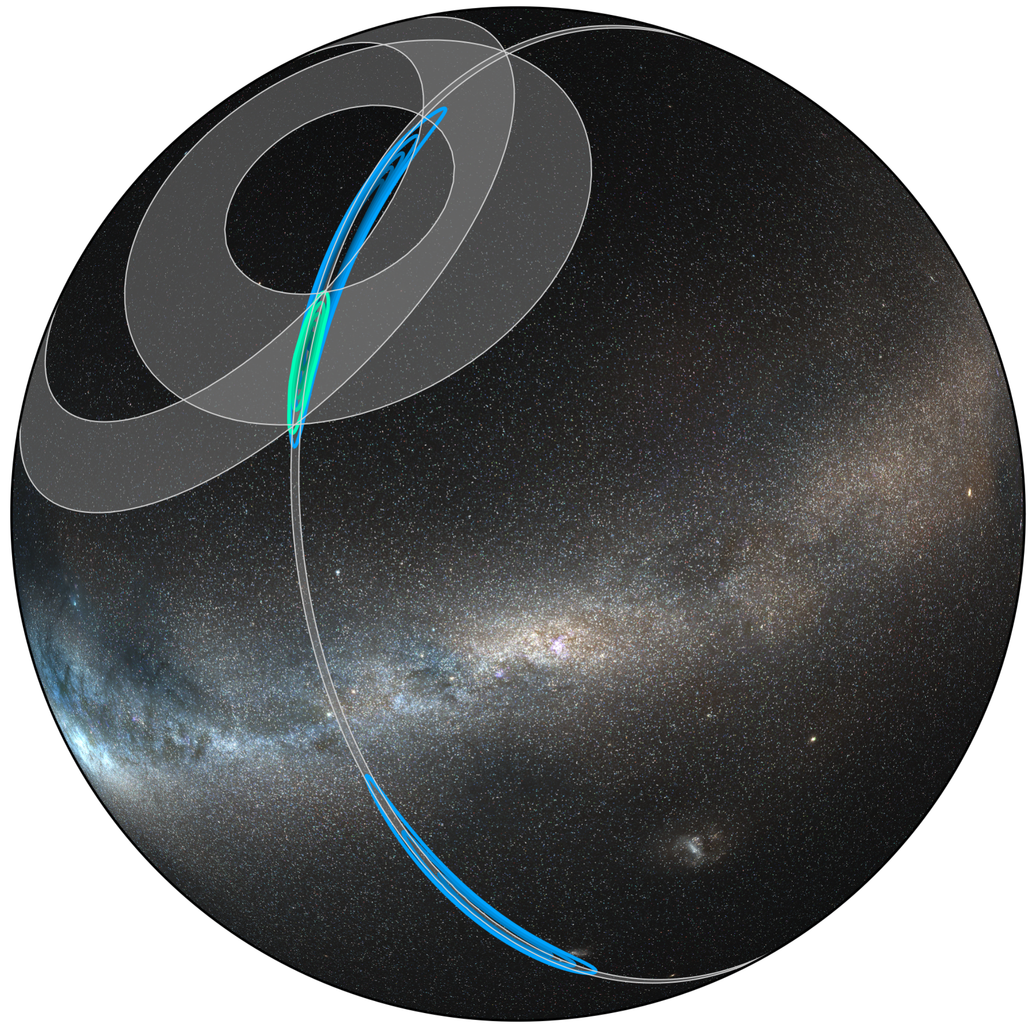
6. Standard sirens: given $h_{\rm inspiral}$, measure $d_{\rm L}$ and z, constrain cosmic history

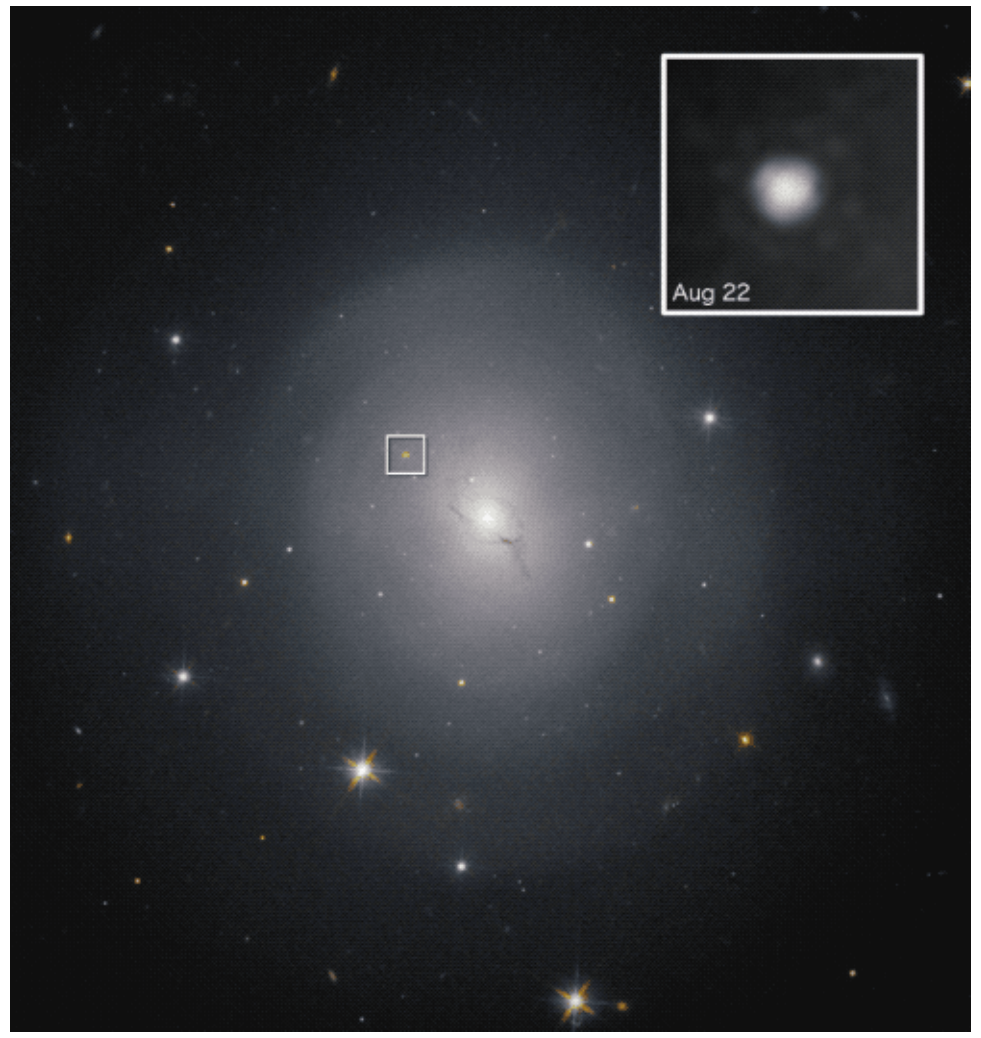
Techniques

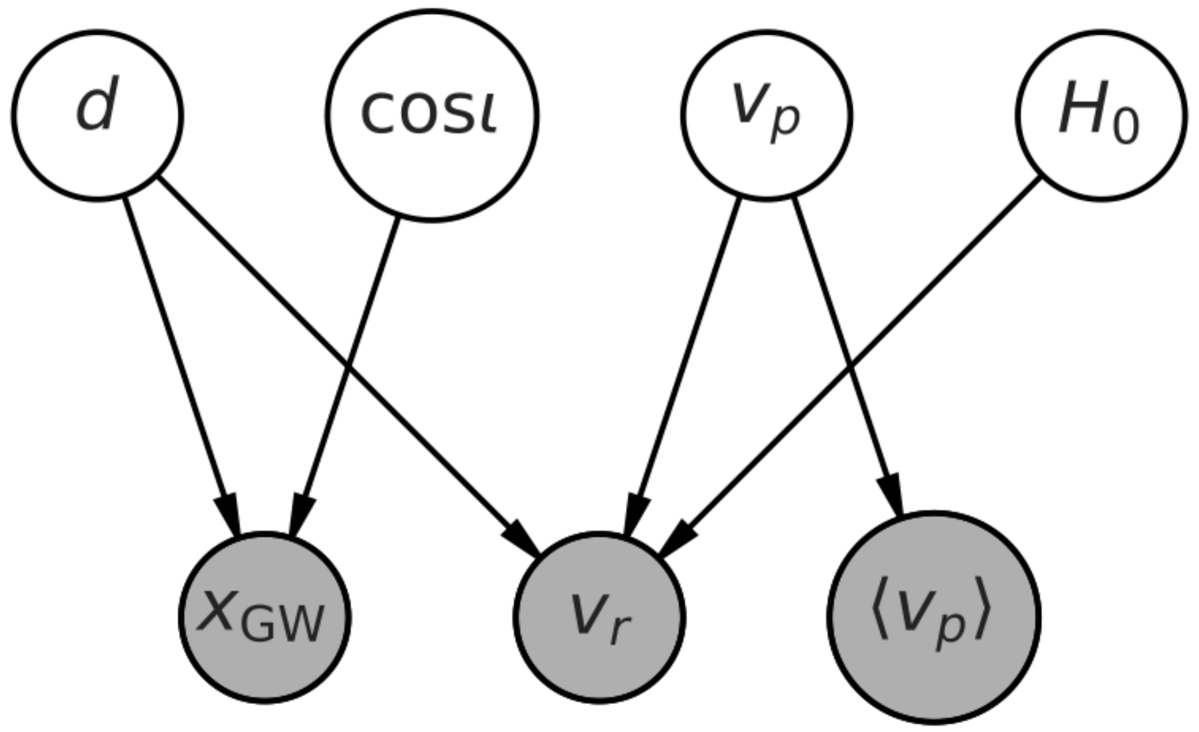
1. For sources with EM counterparts, identify host galaxy and obtain redshift [Holz & Hughes 2005]	We got one!
2. Obtain redshift from localization + galaxy catalog [Schutz 1986]	Better be a good catalog!
3. Fit cosmological parameters and source distribution together [Mastrogiovanni et al. 2021]	Maybe that bump?
4. Compare redshifted mass distribution with astrophysical distribution [Chernoff & Finn 1993]	Better be a good distribution!
5. Cross-correlation of GW-source distribution with galaxies with redshift [Oguri 2016]	A CMB-inspired version of catalog method?
6. Use NS tidal distortions [Messenger & Read 2012]	Introduces scale R^5, formally 5PN but enhanced by compactness $(R/m)^5$

GW170817

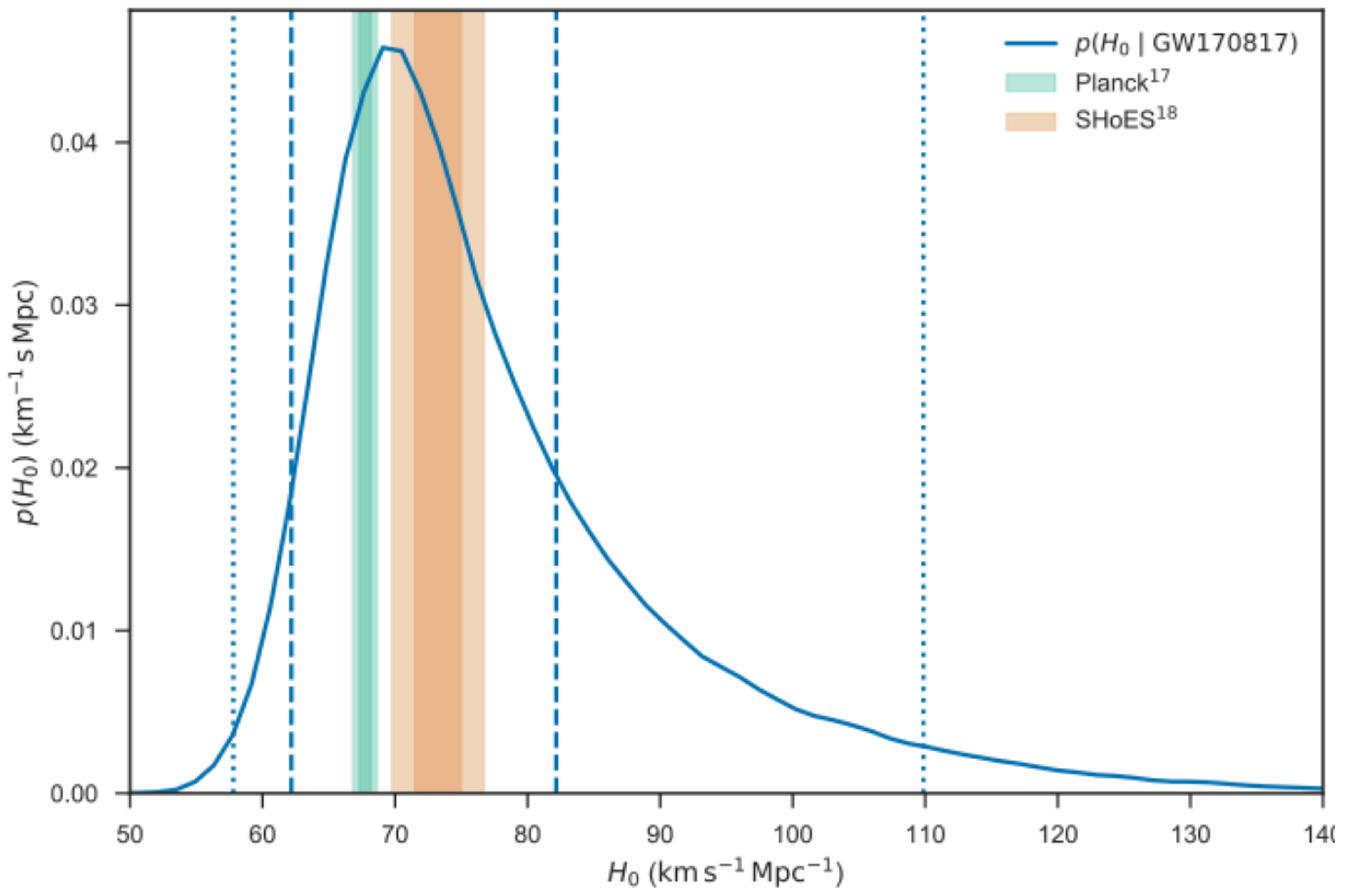
- Optically identified as residing in NGC 4993.
- Obtain $d_L = 43.8^{+2.9}_{-6.9}\ \mathrm{Mpc}$ from GW, fixing sky position. Posterior for ι is almost edge-on.
- (Viva Virgo!)
- Estimate $v_H = 3017 \pm 166\,\mathrm{km\,s^{-1}}$, including $310\,\mathrm{km\,s^{-1}}$ peculiar velocity.

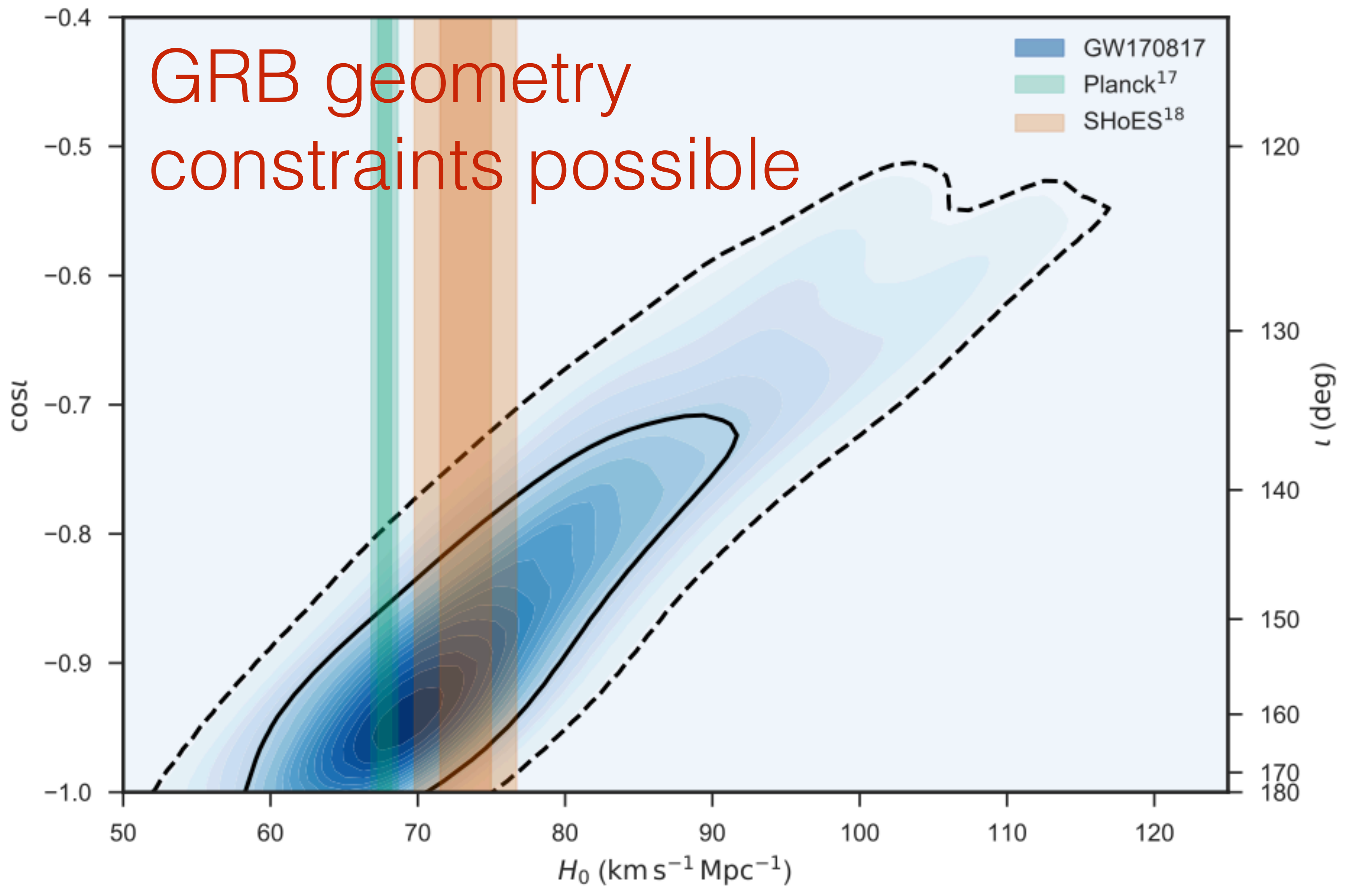


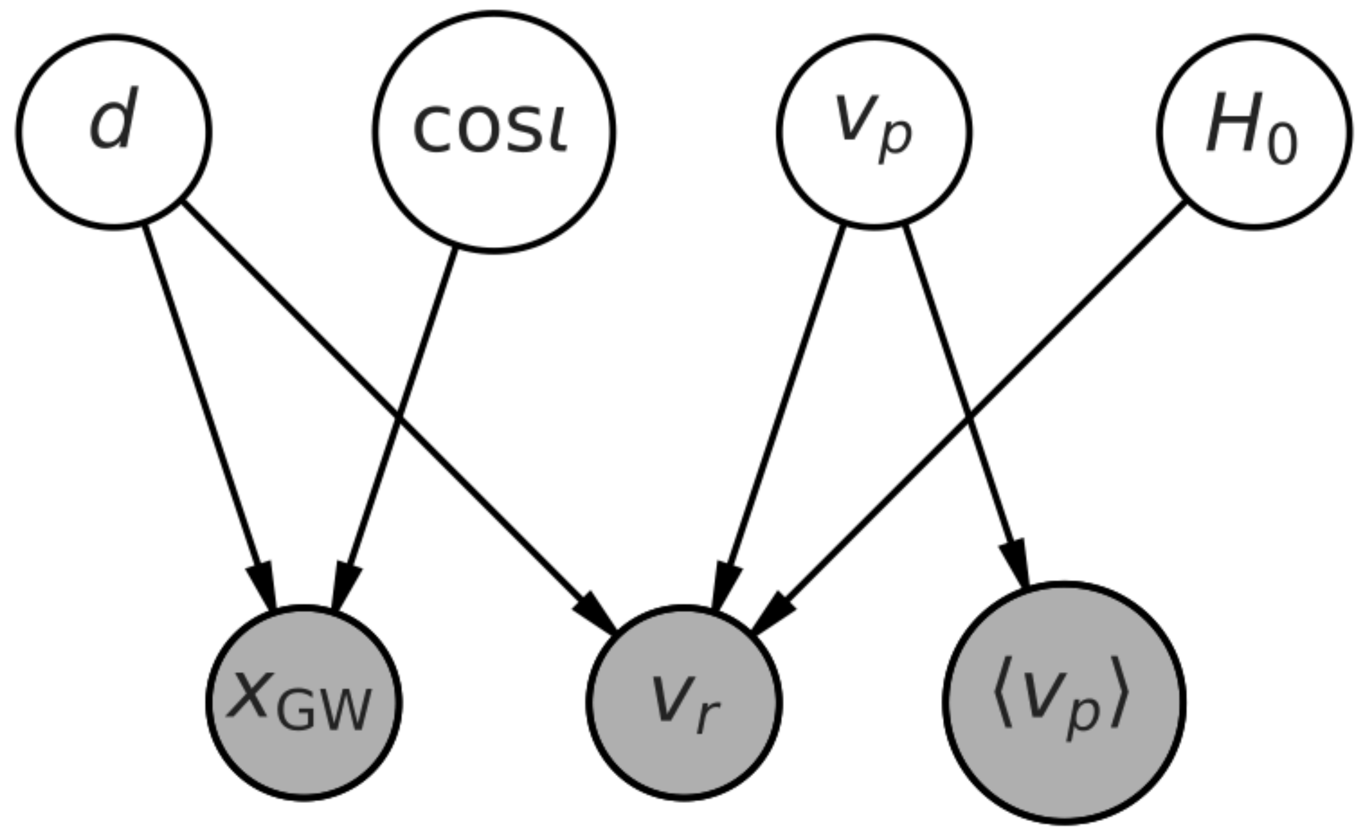




consider also selection effects...







$$p(x_{\rm gw}, v_r, \langle v_p \rangle, d, \cos\iota, v_p, H_0) = p(x_{\rm gw} \,|\, d, \cos\iota)\, p(v_r \,|\, d, v_p, H_0)\, p(\langle v_p \rangle \,|\, v_p) \times p(d)\, p(\cos\iota)\, p(v_p)\, p(H_0)$$

$$= p(x_{\rm gw}, v_r, \langle v_p \rangle \,|\, d, \cos\iota, v_p, H_0) \times p(d, \cos\iota, v_p, H_0)$$

data likelihood prior

$$p(H_0 \,|\, x_{\rm gw}, v_r, \langle v_p \rangle) = \frac{p(H_0, x_{\rm gw}, v_r, \langle v_p \rangle)}{p(x_{\rm gw}, v_r, \langle v_p \rangle)} = \frac{\int p(\text{all})\, \mathrm{d}D\, \mathrm{d}\cos\iota\, \mathrm{d}v_p}{\int p(\text{all})\, \mathrm{d}D\, \mathrm{d}\cos\iota\, \mathrm{d}v_p\, \mathrm{d}H_0}$$

$$= \frac{\int p_{\rm like}(\text{data} \,|\, \text{pars}) \times p_{\rm prior}(\text{pars})\, \mathrm{d}(\text{pars}')}{\text{marginal likelihood}}$$

GWTC-3 — Hierarchical inference

$$p(\Phi|\{x\}, N_{\rm obs}) = p(\Phi) \prod_{i=1}^{N_{\rm obs}} \frac{\int p(x_i|\Phi,\theta) p_{\rm pop}(\theta|\Phi) d\theta}{\int p_{\rm det}(\theta,\Phi) p_{\rm pop}(\theta|\Phi) d\theta}$$

$\Phi_m, H_0, \Omega_m, w_0$

probability of detecting $h(\theta)$, accounts for selection effects

$$m_i = \frac{m_i^{\rm det}}{1 + z(D_{\rm L}; H_0, \Omega_{\rm m}, w_0)}$$

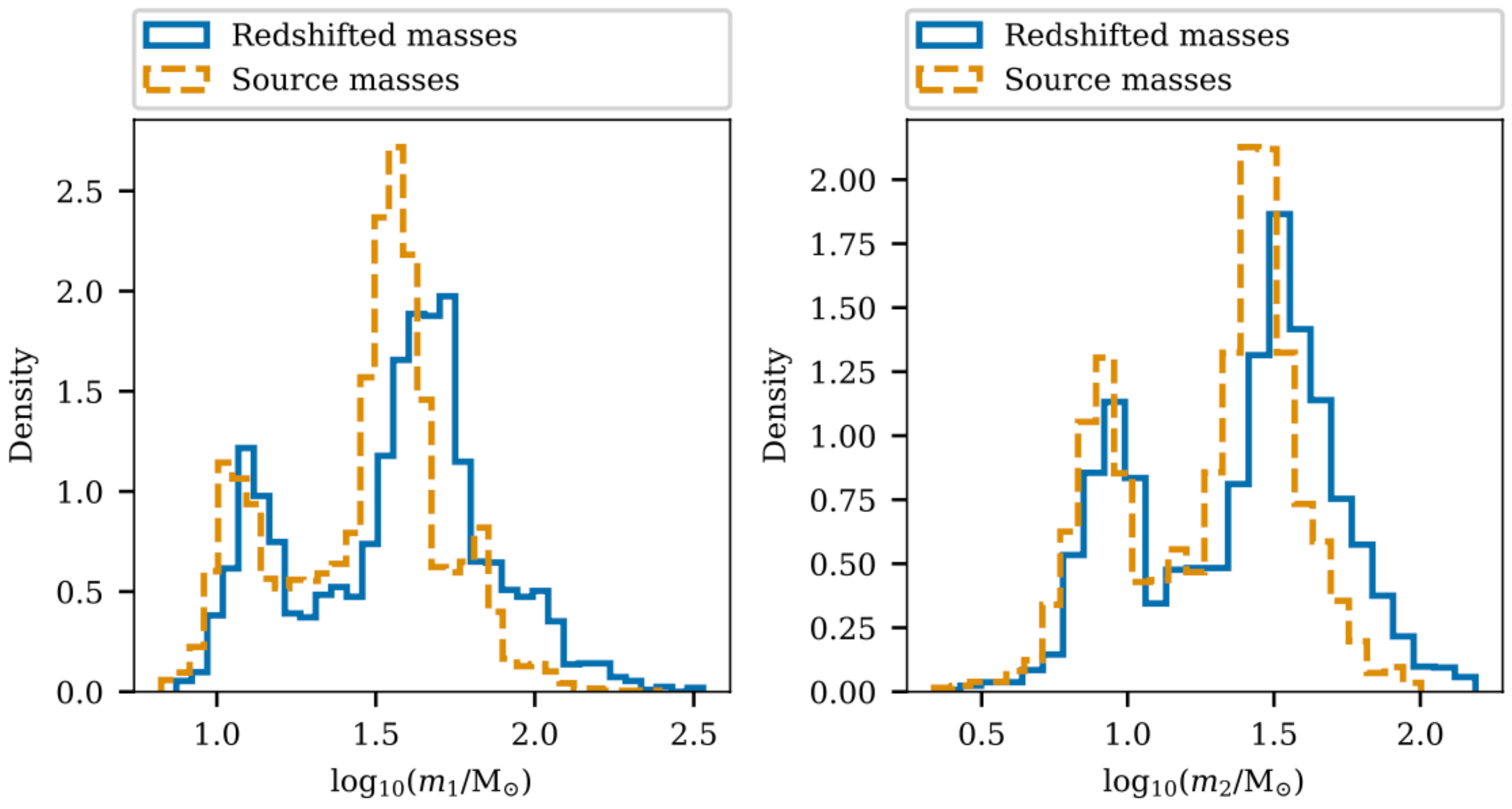
redshift evolution of merger rate

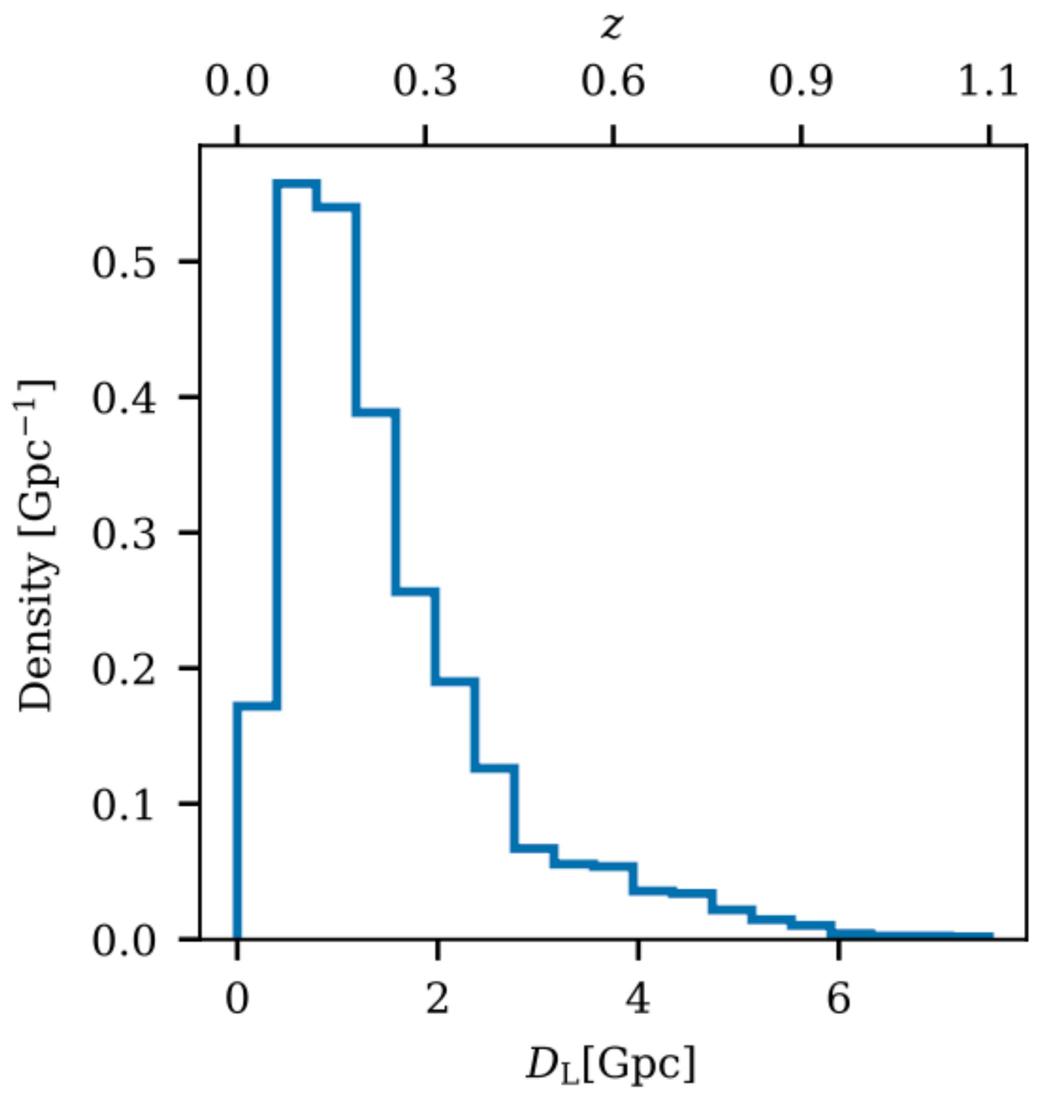
$$p_{\rm pop}(\theta|\Phi_m, H_0, \Omega_{\rm m}, w_0) = C\, p(m_1, m_2|\Phi_m) \psi(z|\gamma, k, z_{\rm p}) \times \frac{p(z|H_0, w_0, \Omega_{\rm m})}{1+z}$$

three phenomenological models:

constant in comoving volume

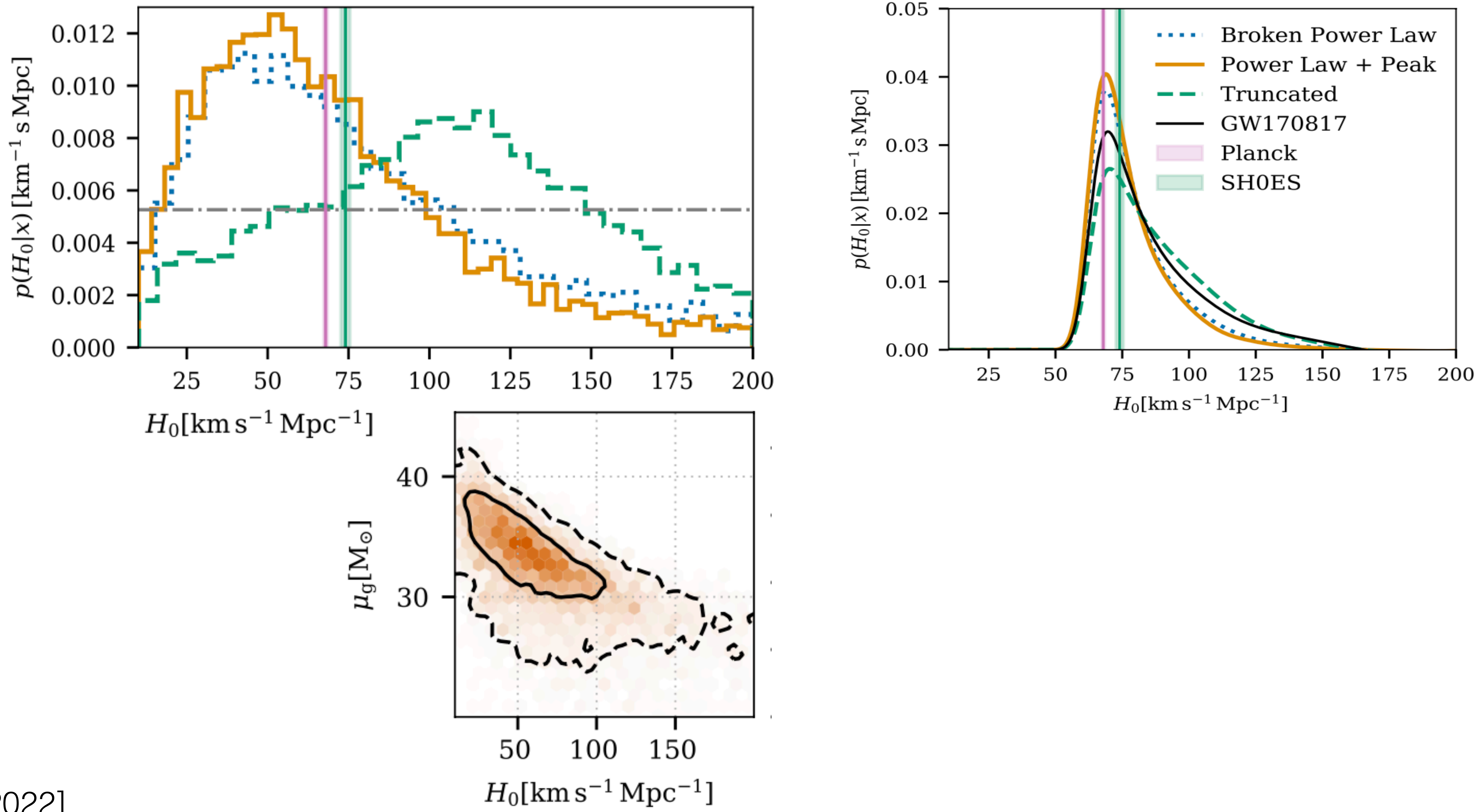
Mass model	$\log_{10} \mathcal{B}$
Truncated	−1.9
Power Law + Peak	0.0
Broken Power Law	−0.5





[LVC 2022]

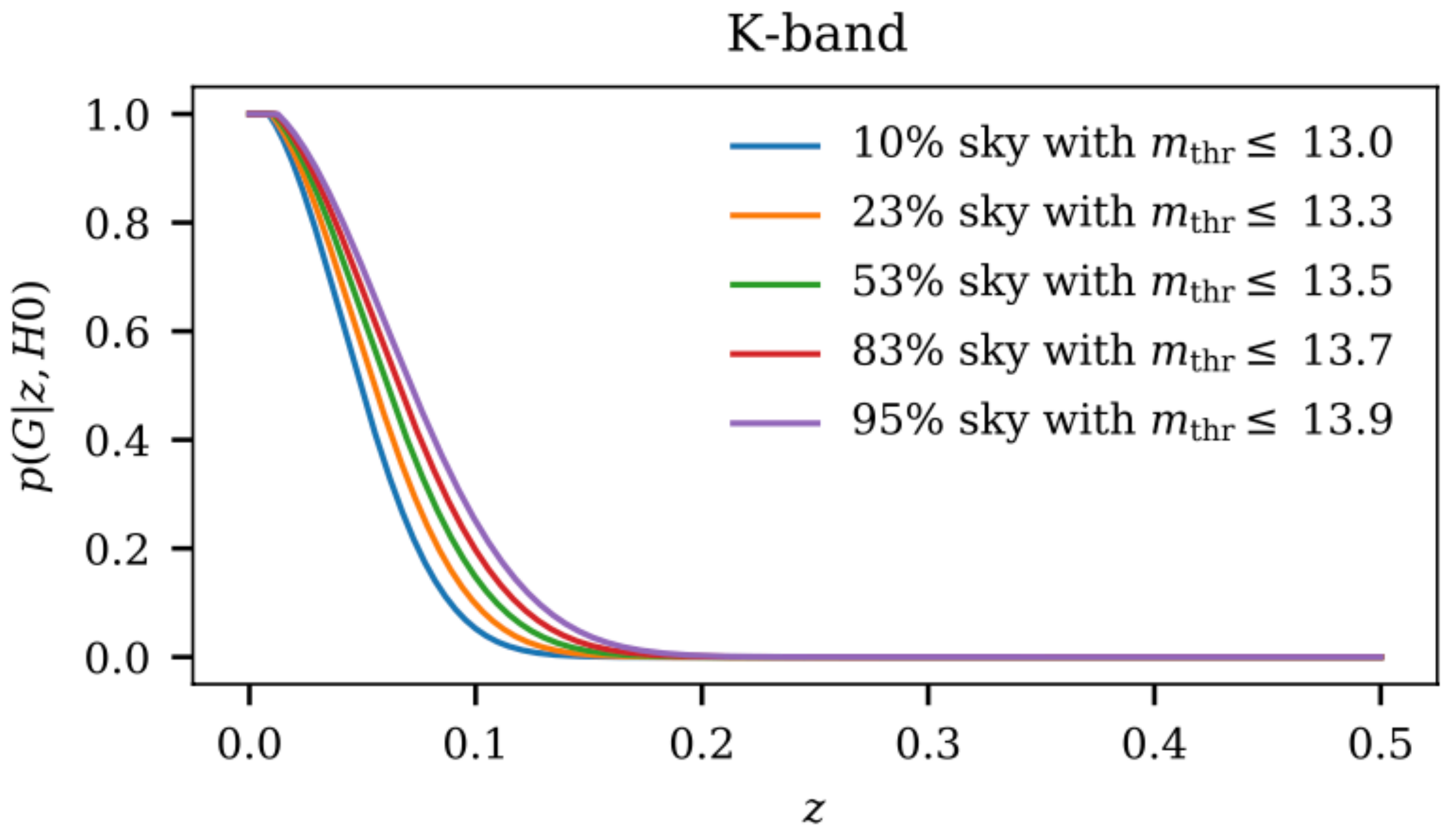
GWTC-3 — Hierarchical inference



[LVC 2022]

GWTC-3 — Galaxy catalog

- GLADE+ = GWGC + 2MASS XSC + 2MPZ + WISExSCOZ + SDSS-DR16Q
- full sky, est. 20% complete to 800 Mpc
- photometric redshifts with $\sigma_z \sim 0.033$
- probability of hosting event proportional to K-band luminosity



$$P(G|z, H_0) = \frac{\int_{L_{\rm thr}(m_{\rm thr}, z, H_0)}^{L_{\rm max}} \phi(L) L \mathrm{d}L}{\int_{L_{\rm min}}^{L_{\rm max}} \phi(L) L \mathrm{d}L}$$

probability that event is in/out catalog

$$p(H_0|x, N_{\rm obs}, \Phi_m) = p(H_0) p(N_{\rm obs}|H_0, \Phi_m) \times \prod_{i=1}^{N_{\rm obs}} \sum_{g \in [G, \bar{G}]} p(x_i|\hat{d}, H_0, \Phi_m, g) p(g|H_0, \Phi_m, \hat{d})$$

probability that galaxy is in catalog

- Φ_m prior is fixed to the median values from hierarchical inference (using power law + peak model)

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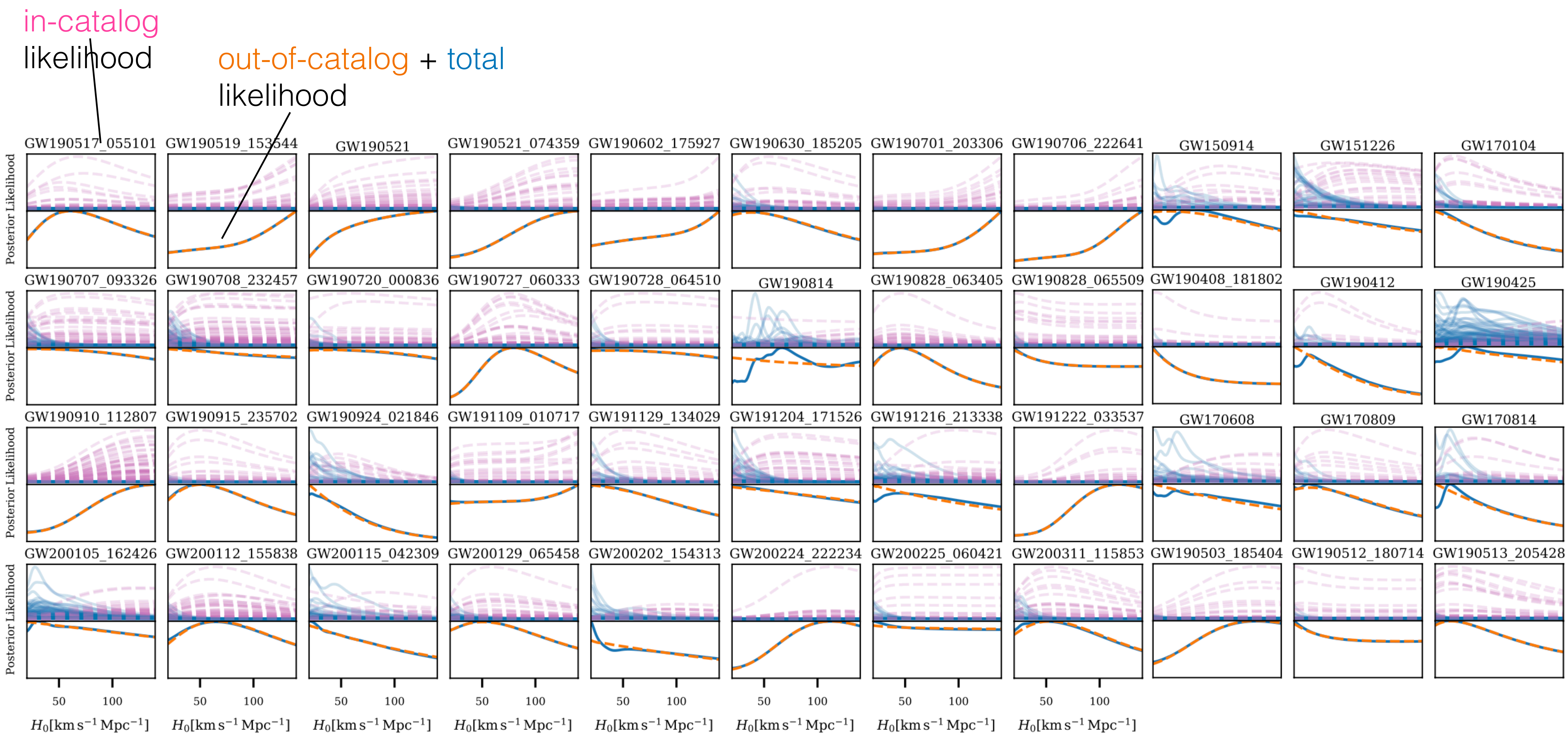
GWTC-3 — Galaxy catalog



[LVC 2022]

Searches for continuous GWs (non-axisymmetric NSs)

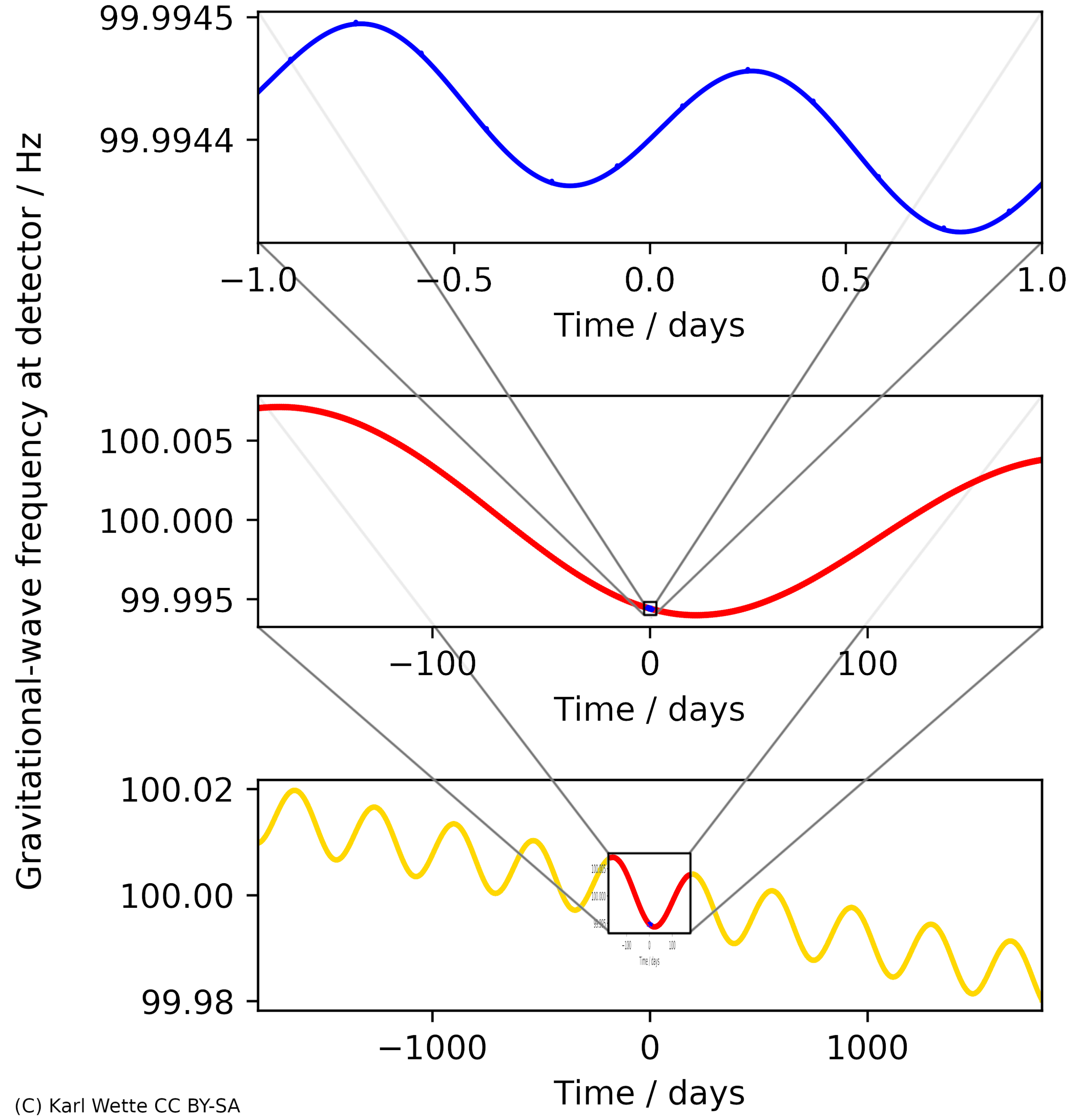
- 10^8 radio-quiet NSs in Galaxy
- Very compact objects (R ~ 10 km), may be spinning rapidly
- Non-axisymmetric "mountain" (~ cm, ppm) $\epsilon = (I_{xx} - I_{yy})/I_{zz}$
- Long-lived quasi-monochromatic signals (except Roemer/Doppler, spin-up/spin-down)
- Too many templates for straight matched filtering. Hierarchical pipelines match shorter segments, then combine them.
- Same searches address very light ($10^{-5} M_\odot$) PBHs

$$h(t) = h_0[F_+(t,\alpha,\delta,\psi)\frac{1+\cos^2\iota}{2}\cos\phi(t) + F_\times(t,\alpha,\delta,\psi)\cos\iota\sin\phi(t)]$$

$$h_0 = \frac{4\pi^2 G}{c^4}\frac{\epsilon I_{zz} f^2}{d} \approx 1.06\times10^{-26}\left(\frac{\epsilon}{10^{-6}}\right)\times\left(\frac{I_{zz}}{10^{38}\,\mathrm{kg\,m^2}}\right)\left(\frac{f}{100\,\mathrm{Hz}}\right)^2\left(\frac{1\,\mathrm{kpc}}{d}\right)$$

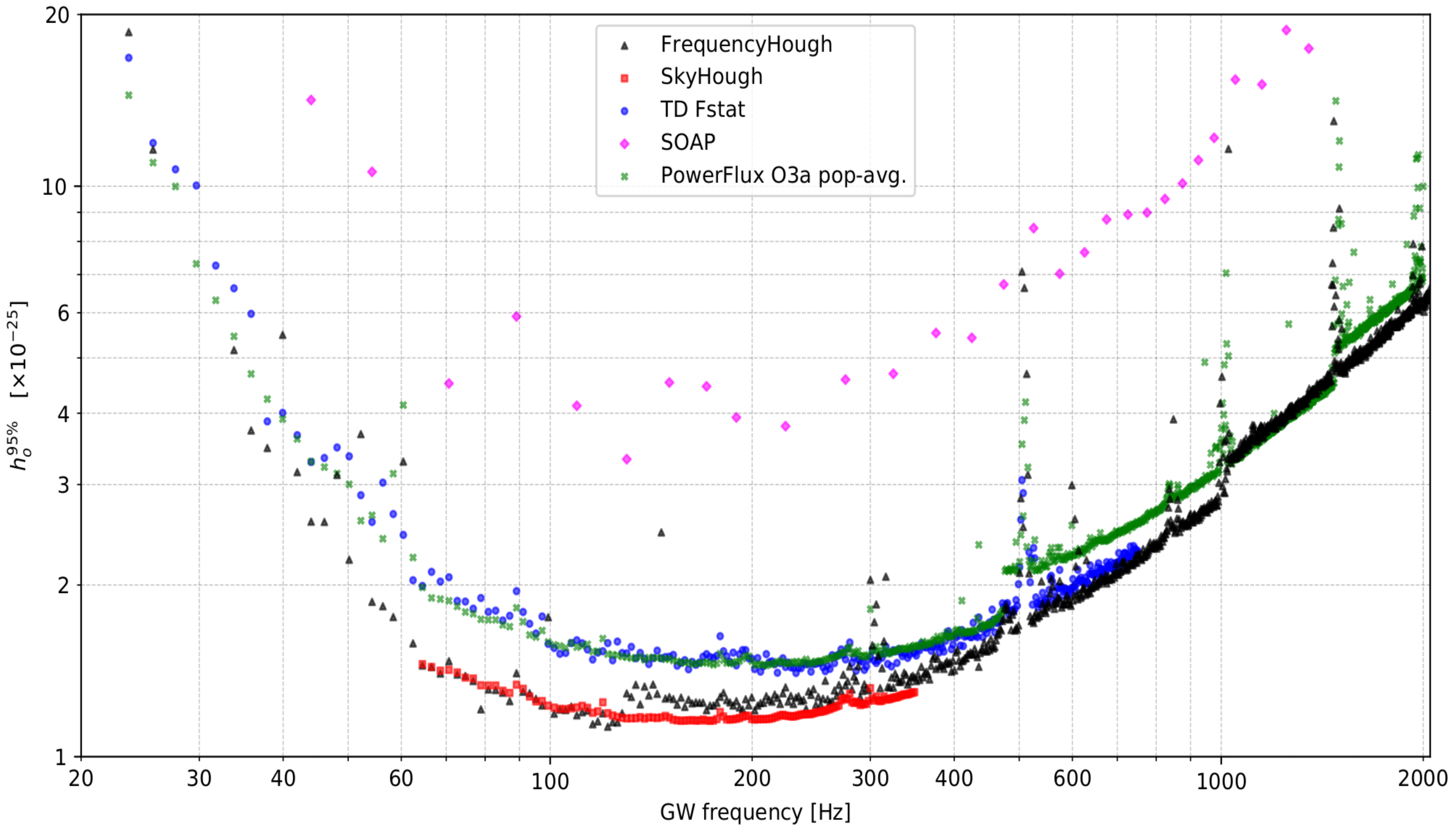
$$\phi(\tau) = \phi_o + 2\pi[f(\tau-\tau_r) + \frac{\dot f}{2!}(\tau-\tau_r)^2] \qquad \tau(t) = t + \frac{\vec r(t)\cdot\vec n}{c} + \Delta_{E\odot} - \Delta_{S\odot}$$

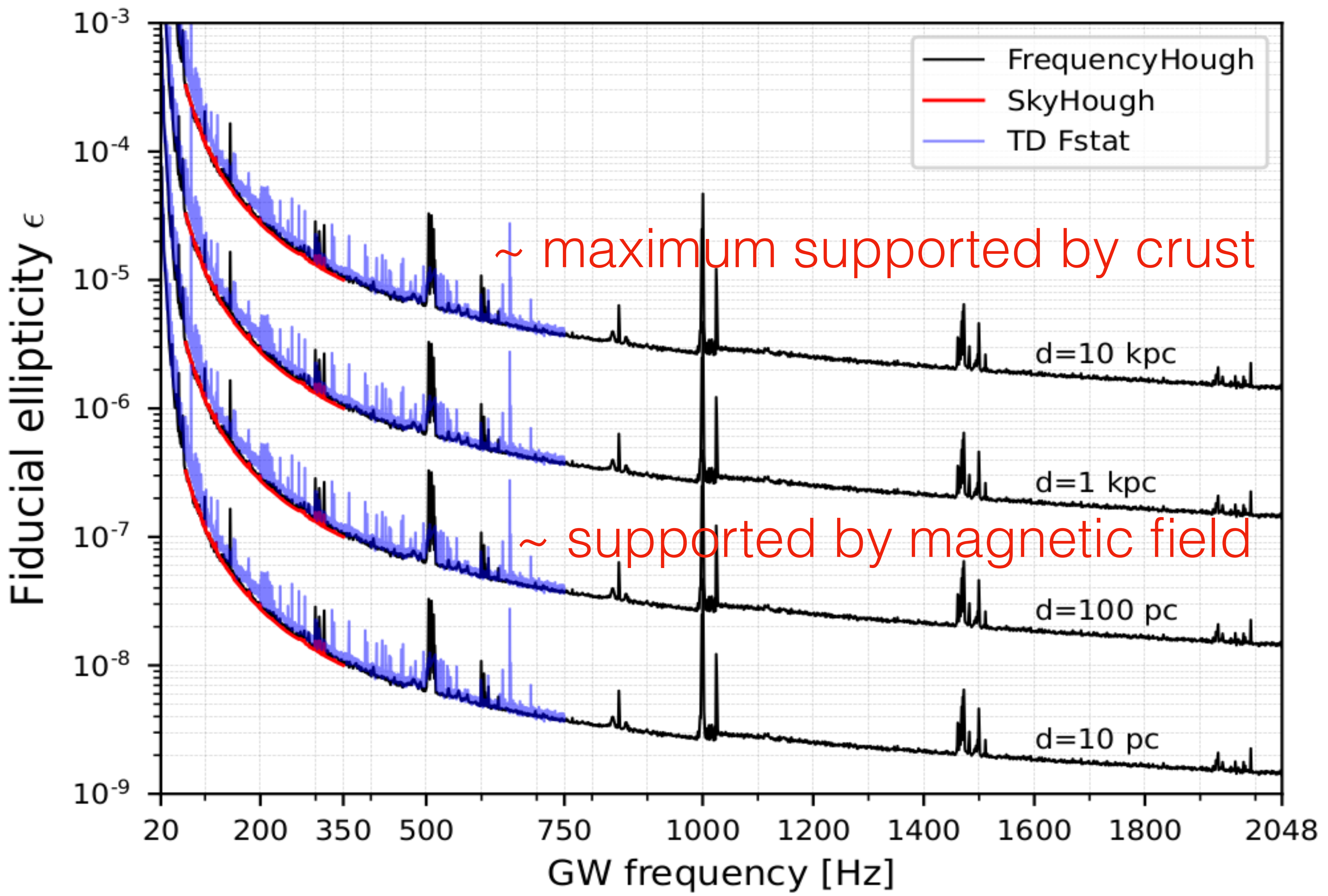
up to 10^{-8}



O3 — All-sky, broad-frequency search for non-axisymmetric NSs

no significant detection — showing 95% upper limits





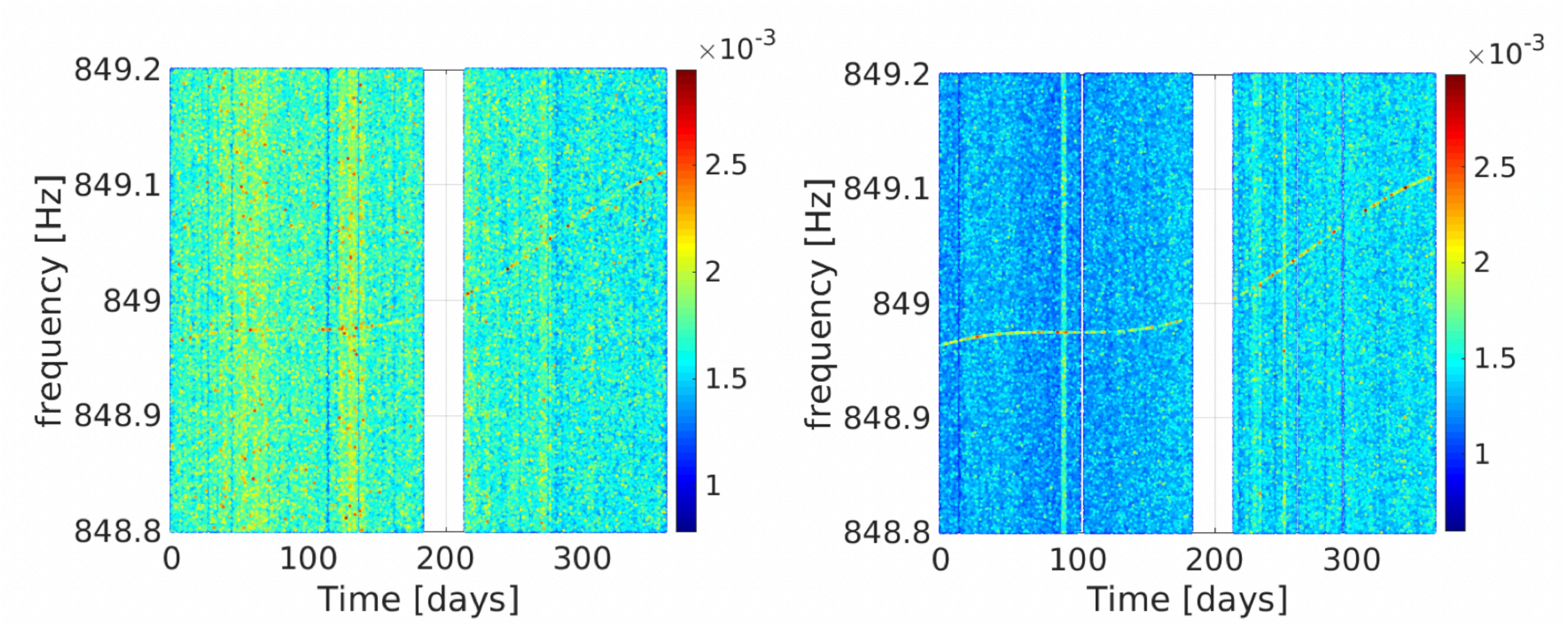
FrequencyHough, SkyHough: short Fourier transform + Hough transform + follow-up

F-statistic: linear-component matched filter on short segments + coherent stacking + follow-up

SOAP: short Fourier transform + Viterbi algorithm + convolutional neural network

[LVC 2022]

O3 — All-sky, broad-frequency search for non-axisymmetric NSs



a hardware injection outlier (a glimpse of the future?)

O1-O3 — All-sky, all-frequency search for persistent GWs

- Past stochastic searches focused on broadband emission from pointlike/diffused sources across the sky, and on narrowband signals from known source locations.
- All-sky/narrowband search is most sensitive.
- Combine 192-s SFTs to obtain cross-spectral density. Fold data into a single sidereal day.
- Identify contaminated bins (~ 30%) with data quality and coherence studies

$$\Omega_{\mathrm{GW}}(f,\hat{\mathbf{n}}) \equiv \frac{f}{\rho_c}\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}f} = \frac{2\pi^2}{3H_0^2} f^3 \mathcal{P}(f,\hat{\mathbf{n}}) \qquad \mathcal{P}(f,\hat{\mathbf{n}}) = \sum_p \mathcal{P}_p(f)\, e_p(\hat{\mathbf{n}})$$

ML anisotropy estimator

$$\hat{\boldsymbol{\mathcal{P}}}(f) = \boldsymbol{\Gamma}(f)^{-1}\,\mathbf{X}(f)$$

$$\Gamma_{pp'}(f) \equiv \frac{\tau\Delta f}{2}\,\mathfrak{Re}\sum_{\mathcal{I},t} \frac{\gamma^{\mathcal{I}*}_{ft,p}\,\gamma^{\mathcal{I}}_{ft,p'}}{P_{\mathcal{I}_1}(t;f)\,P_{\mathcal{I}_2}(t;f)} \qquad X_p(f) = \tau\Delta f\,\mathfrak{Re}\sum_{\mathcal{I},t} \frac{\gamma^{\mathcal{I}*}_{ft,p}\,C^{\mathcal{I}}(t;f)}{P_{\mathcal{I}_1}(t;f)\,P_{\mathcal{I}_2}(t;f)}$$

Fisher matrix

dirty map

$$\gamma^{\mathcal{I}}_{ft,p} :\equiv \sum_A F^A_{\mathcal{I}_1}(\hat{\mathbf{n}}_p,t)\,F^A_{\mathcal{I}_2}(\hat{\mathbf{n}}_p,t)\,e^{2\pi i f\,\hat{\mathbf{n}}_p\cdot\boldsymbol{\Delta}\mathbf{x}_{\mathcal{I}}(t)/c}$$

overlap reduction function

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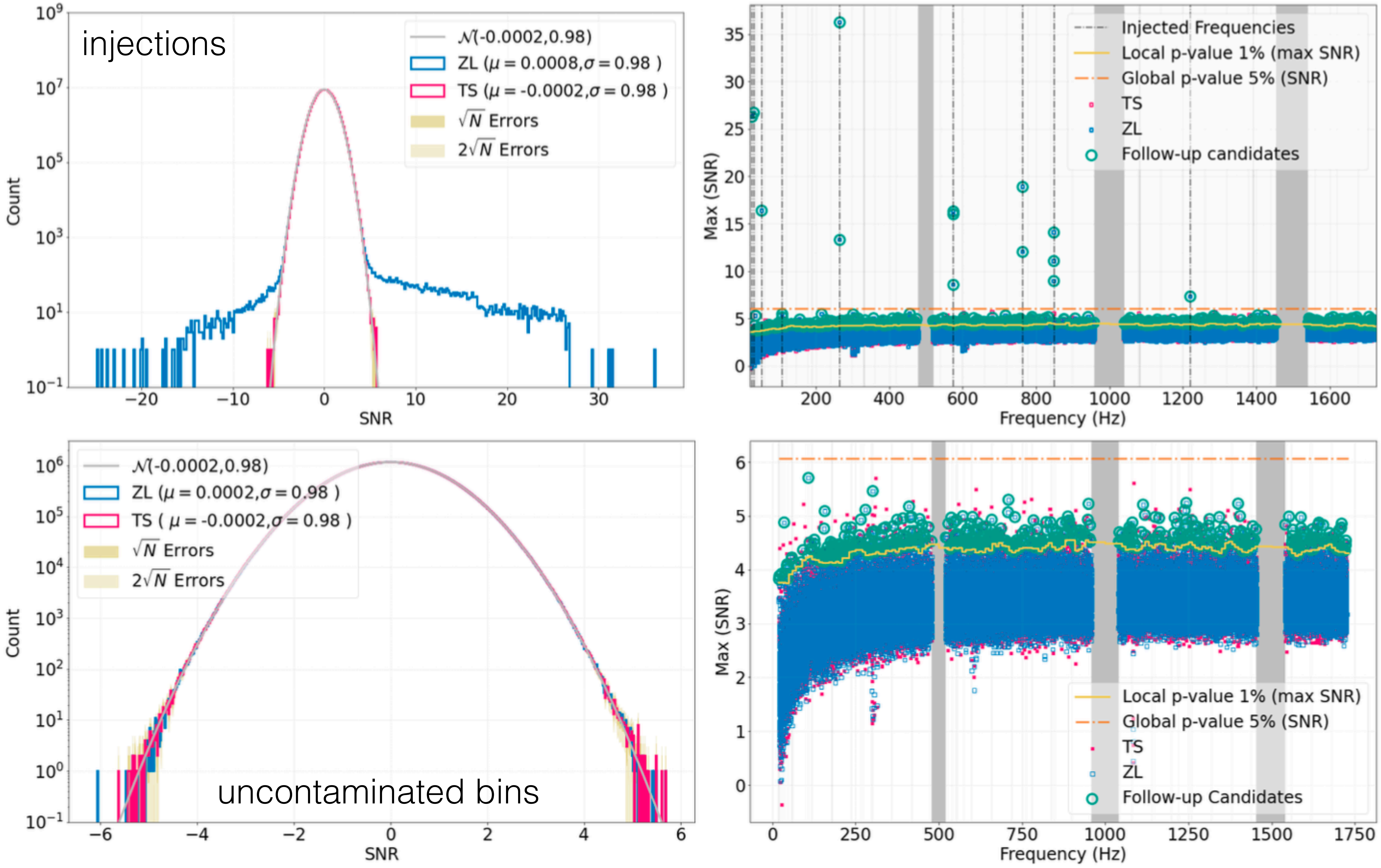
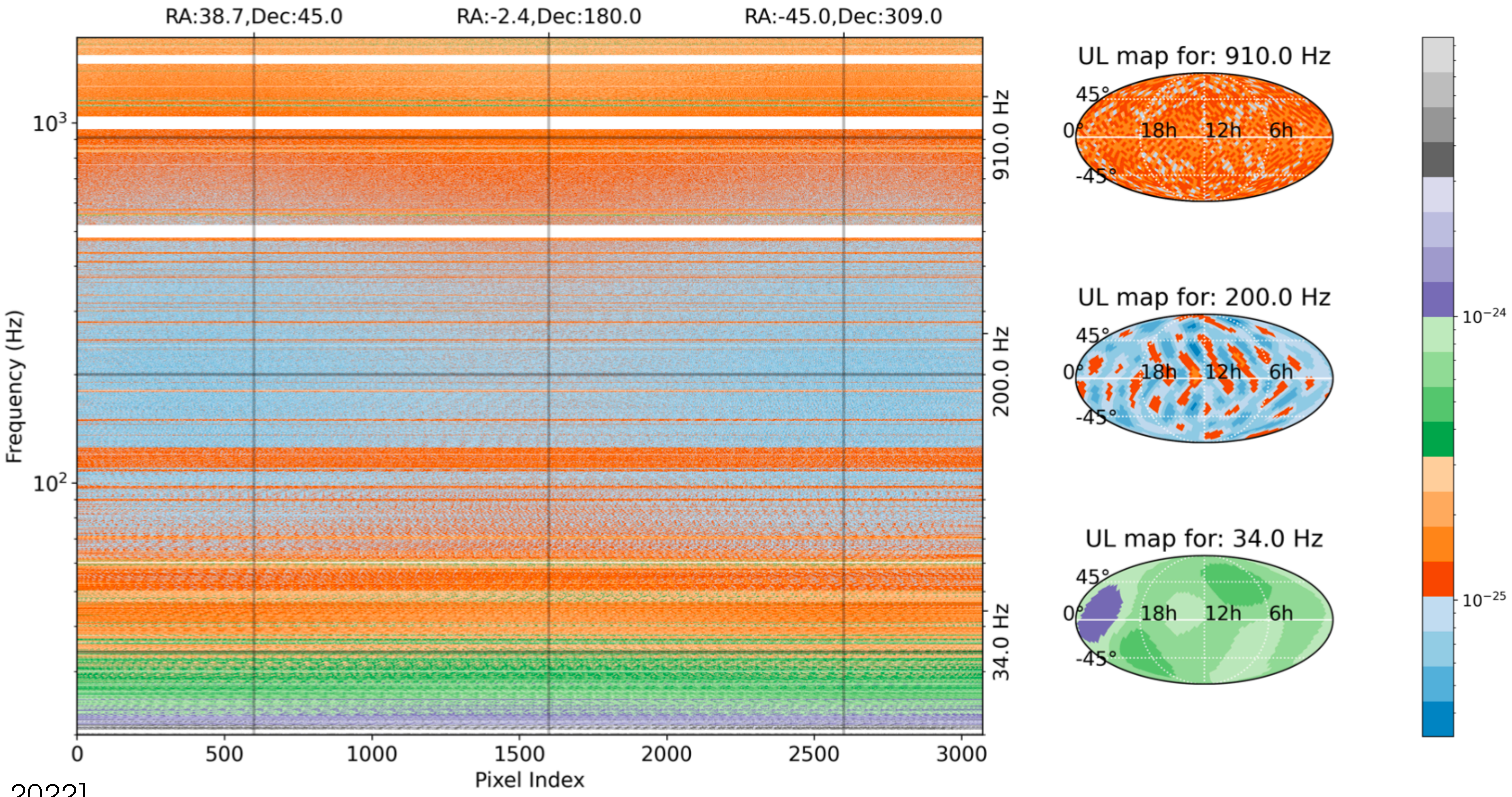
background study of anisotropy estimator



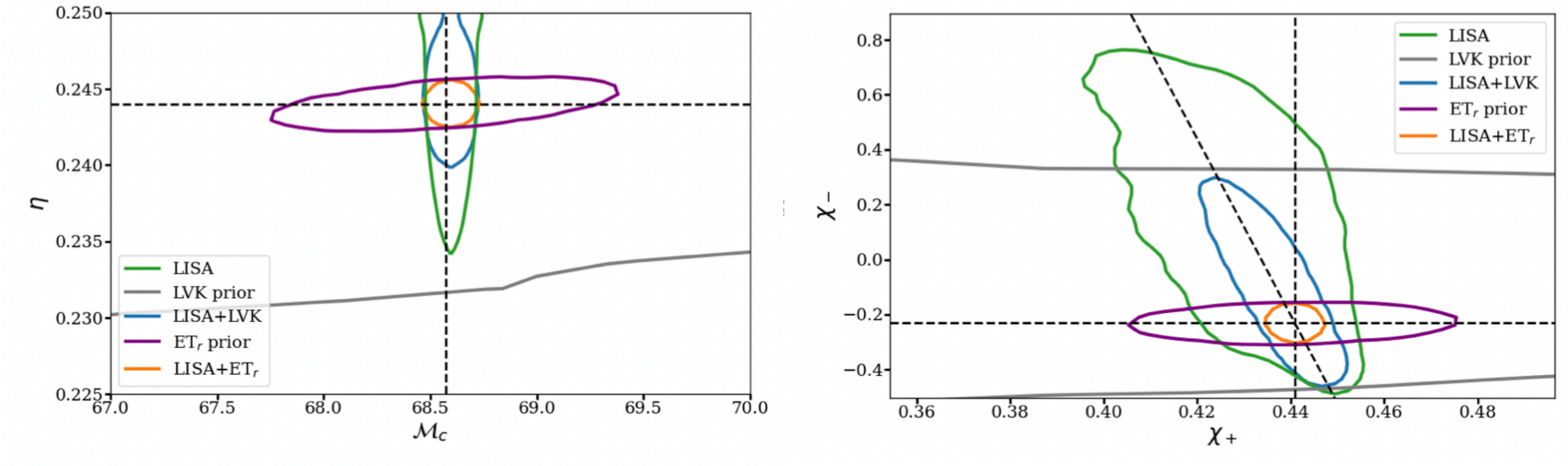
upper limits



[LVC 2022]

LISA detectability of GWTC-3 binaries

parameter estimation for GW191109_010717



Toubiana, Babak, Marsat, Ossokine 2022


That's all Folks!

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