Gravitational waves in a nutshell—4

PN/NRGR theory. Tests of GR.

Michele Vallisneri, Jet Propulsion Laboratory

CEA/IPhT, June 24 2022

Recap from last week

1. Binary inspiral waveforms are functions of many astrophysical (masses, spins) and astronomical (position, distance) parameters
2. For GW detection, we select the most likely data model (noise, or noise + signal), based on the assumed spectrum of noise. This is equivalent to matched filtering (requiring maximum correlation of data and theoretical template)
3. Real-world detectors require sophistication (calibration, glitches, non-stationary noise) and the empirical estimation of detection backgrounds with time shifts
4. Source parameter estimation explores the Bayesian posterior with stochastic algorithms such as Markov Chain Monte Carlo. Hierarchical models are used to infer binary population features.
5. The LIGO GWTC-1,2,3 catalogs include ~100 GW signals from BH-BH, NS-BH, and NS-NS coalescences, showing many intriguing trends (no mass gaps; mass bumps; rate evolution with redshift; tilted spins; heavy NS/light BHs; etc.)
6. The pulsar-timing-array data model is built of Gaussian processes; convincing GW detection will follow from identifying Hellings–Downs spatial correlations.

Master BH-binary dynamics and GW emission by separation of scales and high-energy DOF integration

BH to PP: effacement principle

field to binary potential: PN EFT

isolated BHs coupled to gravity	point particles coupled to gravitational field	composite binary coupled to radiation
black-hole excitations; background field	PP worldlines; potential modes, radiation modes	multipole moments; long-wavelength modes

scales: $m \quad \ll \quad r \quad \ll \quad \lambda$

$$\frac{m}{r} \sim v^2 \qquad\qquad \frac{r}{\lambda} \sim \frac{r}{r/v} \sim v$$

Logic of **EFT**: integrate out high-energy physics to include its effects in low-energy observables, **OR** encode high-energy physics in low-energy operators (which are model independent and must respect symmetries) then match or measure their coefficients

worldline EFT for spherical body of size R in curvature ~ L

start from point particle, add all scalar terms that we can build from worldline and metric

$$S_{\rm WL}[z^\mu] = \boxed{-m\int d\tau} + \cancel{C_1\int d\tau\, a^2 + C_2\int d\tau\, R_{\alpha\beta}(z)u^\alpha a^\beta}$$

remove by worldline shift
and field redefinition (in vacuum)

$$+ \cancel{C_R\int d\tau\, R(z) + C_V\int d\tau\, R_{\alpha\beta}(z)u^\alpha u^\beta} + \frac{1}{2}C_E\int d\tau\, \mathcal{E}_{\alpha\beta}(z)\mathcal{E}^{\alpha\beta}(z) + \frac{1}{2}C_B\int d\tau\, \mathcal{B}_{\alpha\beta}(z)\mathcal{B}^{\alpha\beta}(z) + \cdots$$

$$m u^\alpha \nabla_\alpha u^\mu = 0$$

$$\mathcal{E}_{\alpha\beta} = C_{\alpha\mu\beta\nu}u^\mu u^\nu \quad \mathcal{B}_{\alpha\beta} = \epsilon_{\alpha\mu\nu\sigma}C^{\mu\nu}{}_{\beta\rho}u^\sigma u^\rho$$

$\mathscr{E}_{\alpha\beta}$, $\mathscr{B}_{\alpha\beta} \sim 1/L^2$, so C_E, $C_B \sim R^5$, yielding the effacement principle $La^\mu = \frac{R}{m}\left(\frac{R}{L}\right)^4 \sim \left(\frac{R}{m}\right)^5 v^8$

for a BH, the (tidal Love numbers) C_E and C_B are 0, and $La^\mu = \left(\frac{R}{L}\right)^6 \sim v^{12}$

$$S_{\rm WL}[z^\mu] = \int d\tau \Big\{ -m(\tau) + \frac{1}{2}S_{ab}\Omega^{ab} + \frac{1}{2}\sum_{\ell=2}^{\infty}\frac{1}{\ell!}I^{a_1\cdots a_\ell}(\tau)\nabla_{a_1\cdots a_{\ell-2}}\mathcal{E}_{a_{\ell-1}a_\ell}(z)$$

$$-\frac{1}{2}\sum_{\ell=2}^{\infty}\frac{2\ell}{(\ell+1)!}J^{a_1\cdots a_\ell}(\tau)\nabla_{a_1\cdots a_{\ell-2}}\mathcal{B}_{a_{\ell-1}a_\ell}(z) + \cdots \Big\}$$

additional intrinsic moments couple to $\mathscr{E}_{\mu\nu}$, $\mathscr{B}_{\mu\nu}$ and their derivatives

[following Galley 2022]

PN EFT warmup: Poisson to Newton, integrating out the field

$$S[\boldsymbol{z}_1, \boldsymbol{z}_2, \phi] = \int dt \left\{ \sum_{n=1}^{2} \left[-m_n + \frac{1}{2} m_n \dot{\boldsymbol{z}}_n^2(t) - m_n \phi(t, \boldsymbol{z}_n(t)) \right] - \frac{1}{8\pi} \int_{\boldsymbol{x}} \partial_i \phi \partial^i \phi \right\}$$

Euler–Lagrange yield equations of motion

$$-\frac{1}{4\pi} \partial_i \partial^i \phi(t, \boldsymbol{x}) = -\sum_{n=1}^{2} m_n \delta^3(\boldsymbol{x} - \boldsymbol{z}_n(t))$$

$$\ddot{z}_n^i(t) = -\partial^i \phi(t, \boldsymbol{z}_n(t))$$

solved by Green's function

$$G(t, \boldsymbol{x}; t', \boldsymbol{x}') = \frac{\delta(t - t')}{|\boldsymbol{x} - \boldsymbol{x}'|} \qquad -\frac{1}{4\pi} \partial_i \partial^i G(t, \boldsymbol{x}; t', \boldsymbol{x}') = \delta(t - t') \delta^3(\boldsymbol{x} - \boldsymbol{x}')$$

$$\phi(t, \boldsymbol{x}) = \int dt' \int d^3x' \, G(t, \boldsymbol{x}; t', \boldsymbol{x}') \left(-\sum_{n=1}^{2} m_n \delta^3(\boldsymbol{x}' - \boldsymbol{z}_n(t')) \right) = -\sum_{n=1}^{2} \frac{m_n}{|\boldsymbol{x} - \boldsymbol{z}_n(t)|}$$

substituting

$$\ddot{\boldsymbol{r}}(t) = -\frac{m \, \boldsymbol{r}(t)}{|\boldsymbol{r}(t)|^3}$$

[following Galley 2022]

PN EFT warmup: with Feynman diagrams

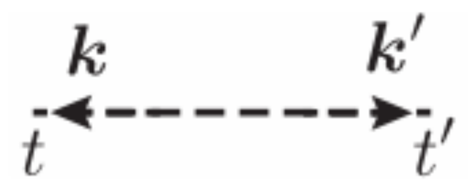
$$S_{\text{eff}}[\boldsymbol{z}_1(t), \boldsymbol{z}_2(t)] = S\big[\boldsymbol{z}_1(t), \boldsymbol{z}_2(t), \phi = \phi_{\text{soln}}[\boldsymbol{z}_1(t), \boldsymbol{z}_2(t)]\big]$$

(momentum space)

$$S[\boldsymbol{z}_1, \boldsymbol{z}_2, \phi] = \int dt \int_{\boldsymbol{k}} \left(-\frac{\boldsymbol{k}^2}{8\pi} \phi_{\boldsymbol{k}}(t) \phi_{-\boldsymbol{k}}(t) - \sum_{n=1}^{2} m_n e^{i\boldsymbol{k}\cdot\boldsymbol{z}_n(t)} \phi_{\boldsymbol{k}}(t) \right) + \int dt \sum_{n=1}^{2} \left(-m_n + \frac{1}{2} m_n \dot{\boldsymbol{z}}_n^2(t) \right)$$

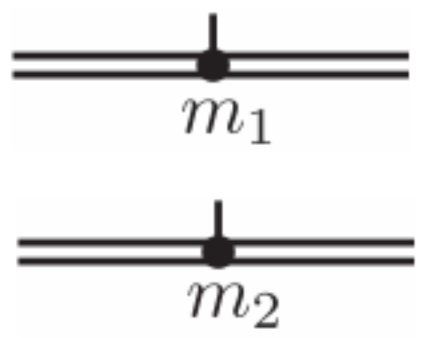
by variation of the action and perturbative expansion

$$\frac{\delta S}{\delta \phi_{\mathbf{k}}} = 0 \quad \Rightarrow \quad -\frac{\delta^2 S}{\delta \phi_{\mathbf{k}} \delta \phi_{\mathbf{k}'}}\bigg|_{\phi=0} \phi_{\mathbf{k}'} = \frac{\delta S}{\delta \phi_{\mathbf{k}}}\bigg|_{\phi=0}$$



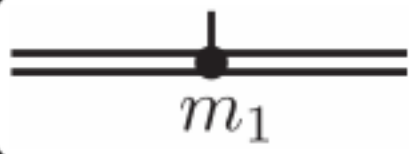
$$-\left(\left[\frac{\delta^2 S}{\delta \phi_{\boldsymbol{k}}(t) \delta \phi_{\boldsymbol{k}'}(t')} \right]_{\varphi=0} \right)^{-1} = G_{\boldsymbol{k},\boldsymbol{k}'}(t, t') = \frac{4\pi}{\boldsymbol{k}^2} \delta(t - t') \delta^3(\boldsymbol{k} + \boldsymbol{k}')$$

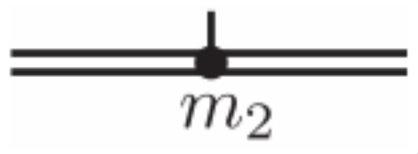
Feynman rules: coefficients of functional expansion of action

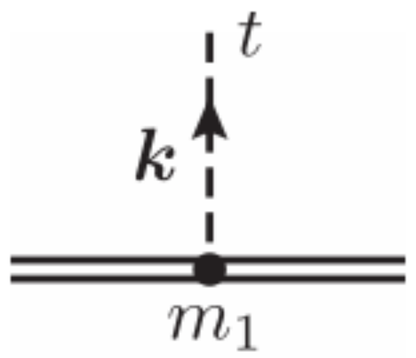


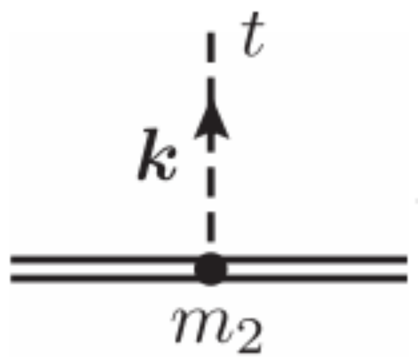
$$+ \quad \left[\frac{\delta S}{\delta \phi_{\boldsymbol{k}}(t)} \right]_{\phi=0} = -m_1 e^{i\boldsymbol{k}\cdot\boldsymbol{z}_1(t)} - m_2 e^{i\boldsymbol{k}\cdot\boldsymbol{z}_2(t)}$$

$$\phi_{\boldsymbol{k}}(t) = \int dt' \int_{\boldsymbol{k}'} \left(\text{[diagram 3]} \right) \times \left(\text{[diagram 4]} + \text{[diagram 5]} \right) = \text{[diagram 6]} + \text{[diagram 7]}$$









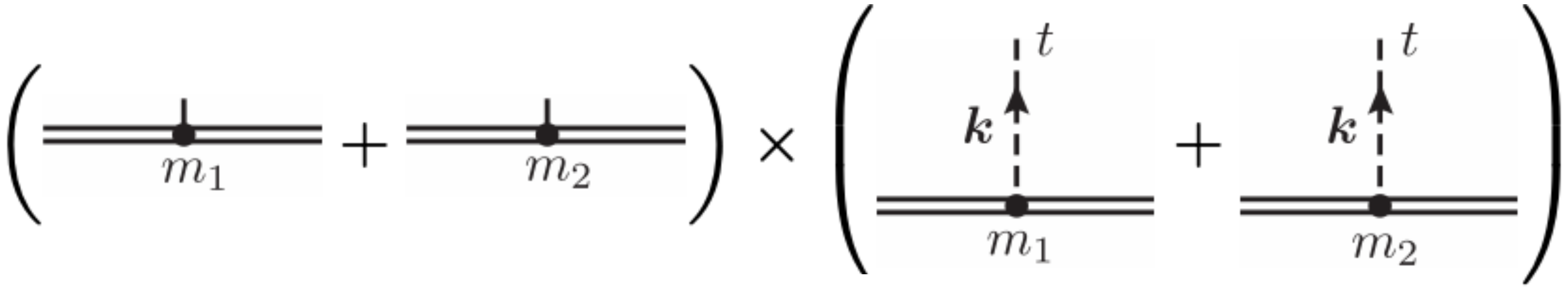
[following Galley 2022]

PN EFT warmup: at the level of the action

$$S_{\text{eff}}[\boldsymbol{z}_1(t),\boldsymbol{z}_2(t)] = S\big[\boldsymbol{z}_1(t),\boldsymbol{z}_2(t),\phi=\phi_{\text{soln}}[\boldsymbol{z}_1(t),\boldsymbol{z}_2(t)]\big]$$

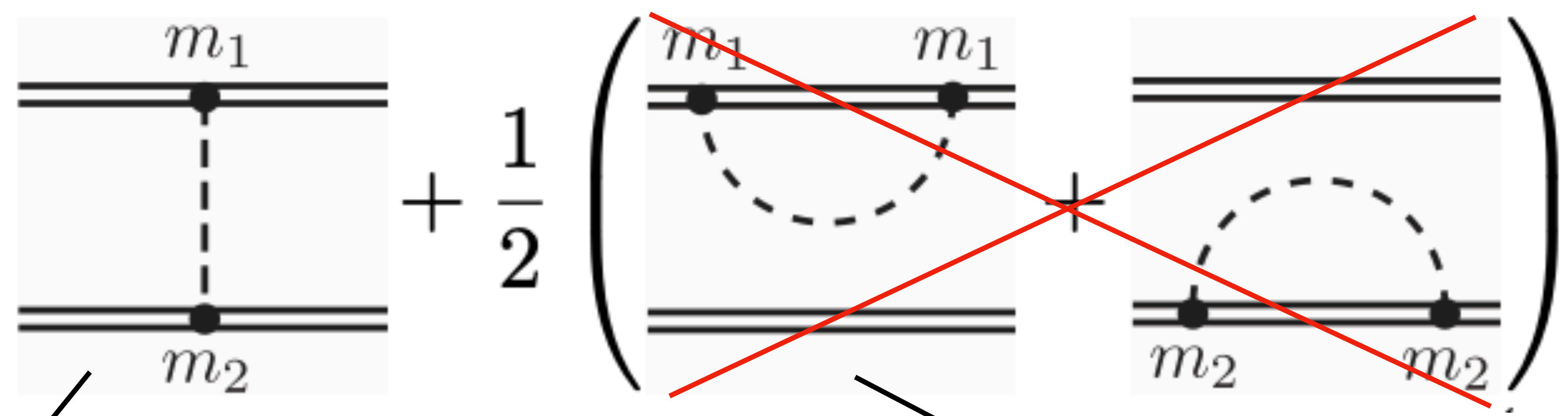
$$S[\boldsymbol{z}_1,\boldsymbol{z}_2,\phi] = \int dt \int_{\boldsymbol{k}} \left(-\frac{\boldsymbol{k}^2}{8\pi}\phi_{\boldsymbol{k}}(t)\phi_{-\boldsymbol{k}}(t) - \sum_{n=1}^{2} m_n e^{i\boldsymbol{k}\cdot\boldsymbol{z}_n(t)}\phi_{\boldsymbol{k}}(t)\right) + \int dt \sum_{n=1}^{2}\left(-m_n + \frac{1}{2}m_n\dot{\boldsymbol{z}}_n^2(t)\right)$$

$$= \frac{1}{2}\int dt \int_{\boldsymbol{k}}$$



$$+ \int dt \sum_{n=1}^{2}\left(-m_n + \frac{1}{2}m_n\dot{\boldsymbol{z}}_n^2(t)\right)$$

$$=$$



$$+ \int dt \sum_{n=1}^{2}\left(-m_n + \frac{1}{2}m_n\dot{\boldsymbol{z}}_n^2(t)\right)$$

draw all connected, tree-level diagrams with no free lines

$$\boxed{\int dt\, dt' \int_{\boldsymbol{k},\boldsymbol{k}'}\left(-m_1 e^{i\boldsymbol{k}\cdot\boldsymbol{z}_1(t)}\right)\left(\frac{4\pi}{\boldsymbol{k}^2}\delta(t-t')\delta^3(\boldsymbol{k}+\boldsymbol{k}')\right)\left(-m_2 e^{i\boldsymbol{k}'\cdot\boldsymbol{z}_2(t')}\right) = \int dt\, \frac{m_1 m_2}{|\boldsymbol{z}_1(t)-\boldsymbol{z}_2(t)|}}$$

$$\boxed{2\pi m_1^2 \int dt \int_{\boldsymbol{k}} \frac{1}{\boldsymbol{k}^2} = 2\pi m_1^2 \int dt \int_{-\infty}^{\infty} \frac{d^3k}{(2\pi)^3}\frac{1}{\boldsymbol{k}^2}}$$

UV divergent: 0 in dimensional regularization or cutoff and introduce counterterm

[following Galley 2022]

PN EFT for real: metric expansion

metric at scale $\gg m$: $g_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu} + h_{\mu\nu} = \bar{g}_{\mu\nu} + H_{\mu\nu}$ $m \ll r \ll \lambda$

perturbation at scale r (for $H_{\mu\nu}$)   perturbation at scale λ (for $h_{\mu\nu}$)

$$S_{\rm orb}[\boldsymbol{z}_1, \boldsymbol{z}_2, H_{\mu\nu}] = -\frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,R(g_{\mu\nu}) - m_1\int_{\boldsymbol{z}_1} d\tau - m_2\int_{\boldsymbol{z}_2} d\tau - \frac{1}{32\pi}\int d^4x\,\sqrt{-\bar{g}}\,\bar{g}_{\alpha\beta}\bar{\Gamma}^\alpha_{\rm orb}\bar{\Gamma}^\beta_{\rm orb}$$

Einstein–Hilbert eff. point particle gauge fixing (harmonic)

$$-\frac{1}{64\pi}\int_x\left(\partial_\mu H_{\alpha\beta}\partial^\mu H^{\alpha\beta} - \frac{1}{2}\partial_\mu H\partial^\mu H + \mathcal{O}(H^2h)\right)$$
$$+\sum_{n=1}^{2}\int dt\left(-m_n + \frac{1}{2}m_n\boldsymbol{v}_n^2 - \frac{1}{8}m_n\boldsymbol{v}_n^4 + \frac{m_n}{2}H_{\alpha\beta}(z_n)v_n^\alpha v_n^\beta - \frac{m_n}{8}\left(H_{\alpha\beta}(z_n)v_n^\alpha v_n^\beta\right)^2 - \frac{m_n}{4}H_{\alpha\beta}(z_n)v_n^\alpha v_n^\beta\boldsymbol{v}_n^2 + \frac{m_n}{2}h_{\alpha\beta}(z_n)v_n^\alpha v_n^\beta + \cdots\right)$$

$$\partial_i H_{\mu\nu} \sim \frac{1}{r}H_{\mu\nu} \quad\text{and}\quad \partial_0 H_{\mu\nu} \sim \Omega H_{\mu\nu} \sim \frac{v}{r}H_{\mu\nu} \ll \partial_i H_{\mu\nu}$$

[following Galley 2022]

PN EFT for real: propagator

$$P_{\alpha\beta\gamma\delta} \equiv \frac{1}{2}\left(\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma} - \eta_{\alpha\beta}\eta_{\gamma\delta}\right)$$

$$P_{\alpha\beta\gamma\delta}H^{\gamma\delta} = H_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta}H$$

$$S_{\rm orb} = -\frac{1}{64\pi}\int_x \left(P^{\alpha\beta\gamma\delta}\partial_i H_{\alpha\beta}\partial^i H_{\gamma\delta} - P^{\alpha\beta\gamma\delta}\partial_0 H_{\alpha\beta}\partial_0 H_{\gamma\delta} + \mathcal{O}(H^2 h, H^3)\right)$$
$$+\sum_{n=1}^{2}\int dt\left(\frac{1}{2}m_n \boldsymbol{v}_n^2 - \frac{1}{8}m_n\boldsymbol{v}_n^4 + \frac{m_n}{2}H_{00}(t,\boldsymbol{z}_n) + m_n H_{0i}(t,\boldsymbol{z}_n)v_n^i + \frac{m_n}{2}H_{ij}(t,\boldsymbol{z}_n)v_n^i v_n^j + \frac{m_n}{4}H_{00}(t,\boldsymbol{z}_n)\boldsymbol{v}_n^2 + \frac{m_n}{8}H_{00}(t,\boldsymbol{z}_n)^2 + \cdots\right)$$

$$-\left[\frac{\delta^2 S_{\rm pot}}{\delta H_{\boldsymbol{k}\,\alpha\beta}(t)\,\delta H_{-\boldsymbol{k}\,\gamma\delta}(t')}\right]^{-1}_{H_{\mu\nu}=0} = \frac{32\pi}{\boldsymbol{k}^2}P_{\alpha\beta\gamma\delta}\sum_{n=0}^{\infty}\frac{1}{n!}\left(-\frac{1}{\boldsymbol{k}^2}\frac{\partial}{\partial t}\frac{\partial}{\partial t'}\right)^n \delta(t-t')$$

propagator with insertions

$=$ 

$+\ \mathcal{O}(v^6)$

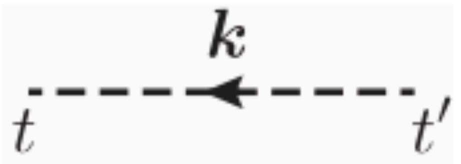
[following Galley 2022]

PN EFT for real: Kol–Smolkin variables and Feynman rules

using Kaluza–Klein variables $ds^2 = -\,\mathrm{e}^{2\phi}(dt - A_i\, dx^i)^2 + \mathrm{e}^{-2\phi}(\delta_{ij} + \sigma_{ij})\, dx^i dx^j$

Newtonian potential — gravito-magnetic field — 3-metric field

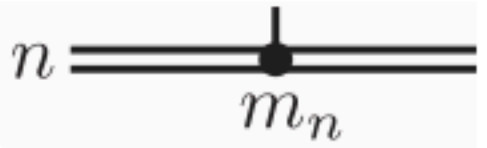
$$S_{\rm pot} = -\frac{1}{16\pi}\int dt \int_{\boldsymbol{x}} \left\{ \frac{d-1}{d-2}\partial_i\phi\partial^i\phi - \frac{1}{2}\delta_{ij}\partial_m A^i \partial^m A^j + \frac{1}{4}P_{ijkl}\partial_m\sigma^{ij}\partial^m\sigma^{kl} + \cdots \right\}$$
$$+ \sum_{n=1}^{2}\int dt \left\{ -m_n + \frac{1}{2}m_n\boldsymbol{v}_n^2 - m_n\phi(t,\boldsymbol{z}_n) - \frac{1}{8}m_n\boldsymbol{v}_n^4 + m_n A_i(t,\boldsymbol{z}_n)v_n^i - \frac{3m_n}{2}\phi(t,\boldsymbol{z}_n)\boldsymbol{v}_n^2 - \frac{1}{2}m_n\phi^2(t,\boldsymbol{z}_n) + \cdots \right\}$$



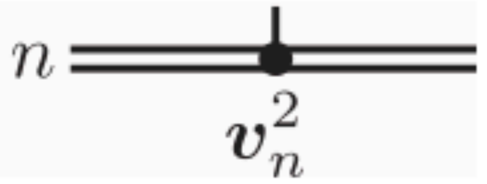
$$= \frac{8\pi}{\boldsymbol{k}^2}\frac{d-2}{d-1}\delta(t-t') \sim rv$$

$$= -\frac{16\pi}{\boldsymbol{k}^2}\delta(t-t')\delta_{ij} \sim rv$$

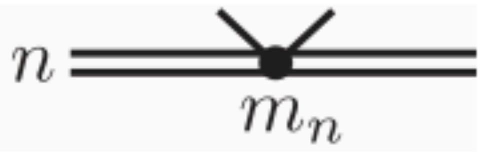
$$= \frac{32\pi}{\boldsymbol{k}^2}\delta(t-t')P_{ijkl} \sim rv$$

$-m_n\phi(t,\boldsymbol{z}_n) \longrightarrow$ 

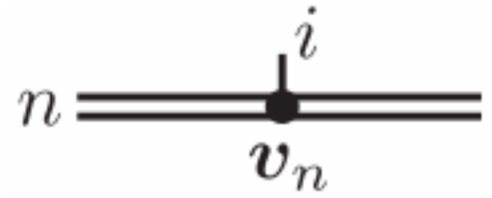
$= -m_n e^{i\boldsymbol{k}\cdot\boldsymbol{z}_n(t)} \sim m$

$-\frac{3}{2}m_n\phi(t,\boldsymbol{z}_n)\boldsymbol{v}_n^2 \longrightarrow$ 

$= -\frac{3}{2}m_n\boldsymbol{v}_n^2 e^{i\boldsymbol{k}\cdot\boldsymbol{z}_n(t)} \sim mv^2$

$-\frac{1}{2}m_n\phi^2(t,\boldsymbol{z}_n) \longrightarrow$ 

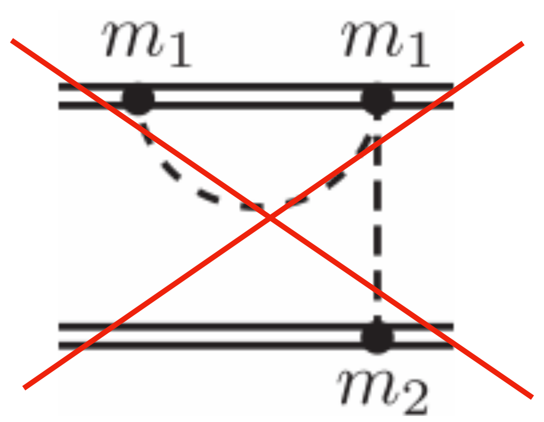
$= -m_n e^{i(\boldsymbol{k}_1+\boldsymbol{k}_2)\cdot\boldsymbol{z}_n(t)} \sim m$

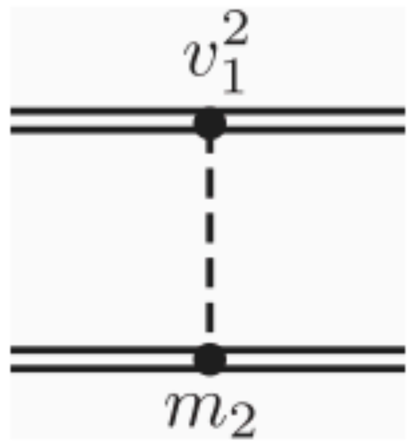
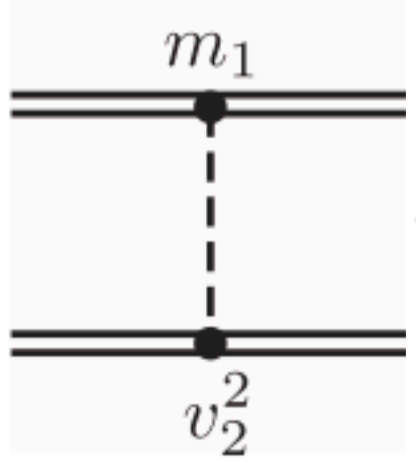
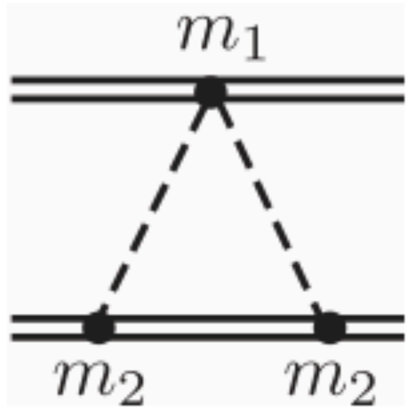
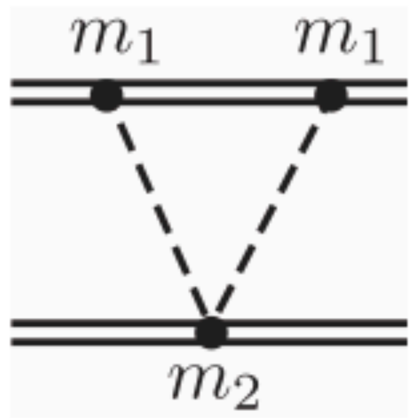
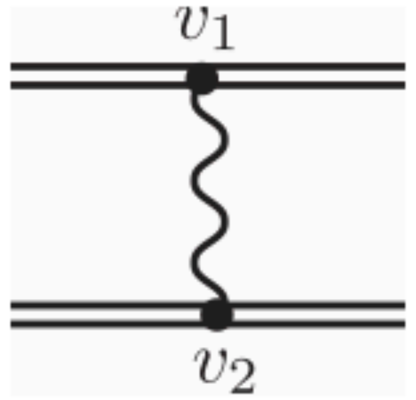
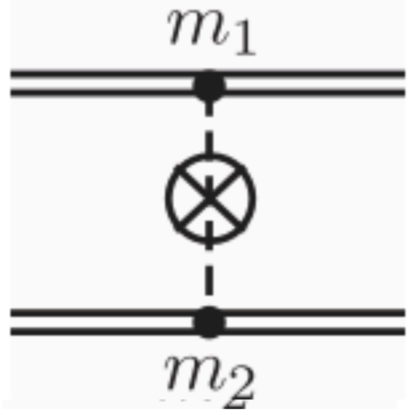
$m_n A_i(t,\boldsymbol{z}_n)v_n^i \longrightarrow$ 

$= m_n v_n^i e^{i\boldsymbol{k}\cdot\boldsymbol{z}_n(t)} \sim mv$

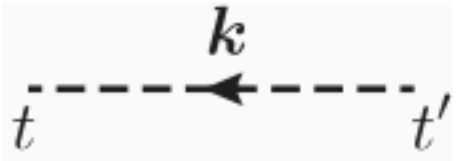
[following Galley 2022]

PN EFT for real: 1PN action



$$S^{\rm eff}_{\rm 1PN}[\boldsymbol{z}_1,\boldsymbol{z}_2] = -\sum_{n=1}^{2}\int dt\,\frac{1}{8}m_n\boldsymbol{v}_n^4 +$$  $+$  $+$  $+$  $+$  $+$ 

$$\int dt dt' \int_{\boldsymbol{k}} \left(-\frac{3}{2}m_1\boldsymbol{v}_1^2 e^{i\boldsymbol{k}\cdot\boldsymbol{z}_1(t)}\right)\left(\frac{4\pi}{\boldsymbol{k}^2}\delta(t-t')\right)\left(-m_2 e^{-i\boldsymbol{k}\cdot\boldsymbol{z}_2(t')}\right) + (1\leftrightarrow 2) = \int dt\,\frac{3m_1m_2}{2r}\left(\boldsymbol{v}_1^2+\boldsymbol{v}_2^2\right)$$

 $= \dfrac{8\pi}{\boldsymbol{k}^2}\dfrac{d-2}{d-1}\delta(t-t') \sim rv$

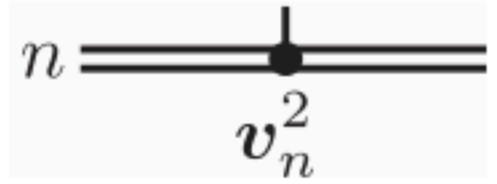
$\dfrac{4\pi}{\boldsymbol{k}^2}\delta(t-t')$

 $= -\dfrac{16\pi}{\boldsymbol{k}^2}\delta(t-t')\delta_{ij} \sim rv$

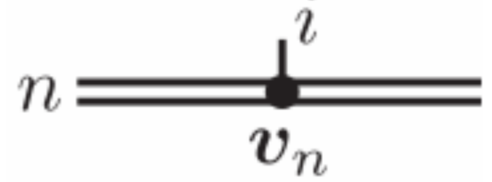
 $= \dfrac{32\pi}{\boldsymbol{k}^2}\delta(t-t')P_{ijkl} \sim rv$

$2\,\boldsymbol{v}_1\cdot\boldsymbol{v}_2$

$-m_n\phi(t,\boldsymbol{z}_n) \longrightarrow$  $= -m_n e^{i\boldsymbol{k}\cdot\boldsymbol{z}_n(t)} \sim m$

$-\dfrac{3}{2}m_n\phi(t,\boldsymbol{z}_n)\boldsymbol{v}_n^2 \longrightarrow$  $= -\dfrac{3}{2}m_n\boldsymbol{v}_n^2 e^{i\boldsymbol{k}\cdot\boldsymbol{z}_n(t)} \sim mv^2$

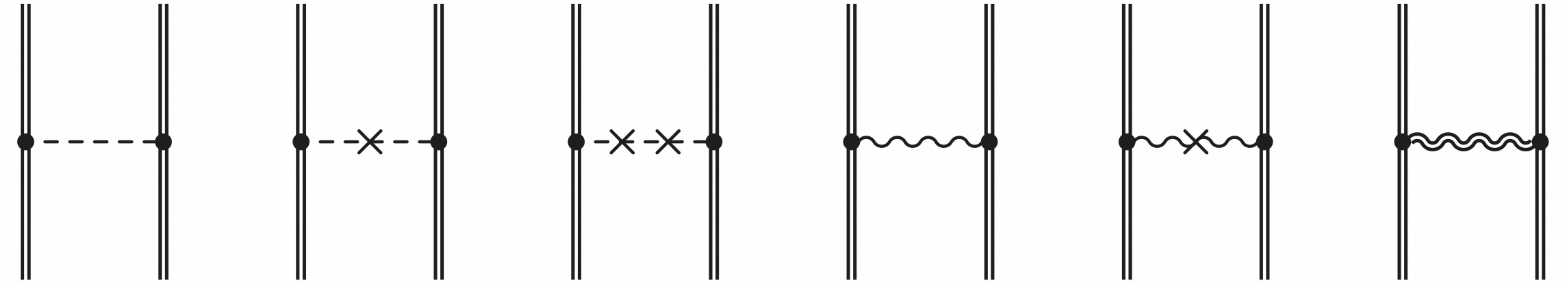
$-\dfrac{1}{2}m_n\phi^2(t,\boldsymbol{z}_n) \longrightarrow$  $= -m_n e^{i(\boldsymbol{k}_1+\boldsymbol{k}_2)\cdot\boldsymbol{z}_n(t)} \sim m$

$m_nA_i(t,\boldsymbol{z}_n)v_n^i \longrightarrow$  $= m_nv_n^i e^{i\boldsymbol{k}\cdot\boldsymbol{z}_n(t)} \sim mv$

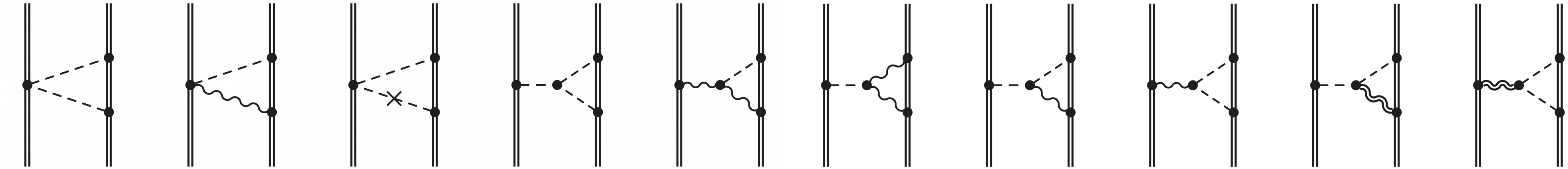
$$\boxed{S^{\rm eff}_{\rm 1PN}[\boldsymbol{z}_1,\boldsymbol{z}_2] = \int dt\left\{\frac{1}{8}\sum_{n=1}^{2}m_n\boldsymbol{v}_n^4 + \frac{m_1m_2}{2r(t)}\left[3\boldsymbol{v}_1^2+3\boldsymbol{v}_2^2-7\boldsymbol{v}_1\cdot\boldsymbol{v}_2-(\boldsymbol{v}_1\cdot\hat{\boldsymbol{n}})(\boldsymbol{v}_2\cdot\hat{\boldsymbol{n}})\right]-\frac{m_1m_2(m_1+m_2)}{2r^2(t)}\right\}}$$

PN EFT for real: 2PN action

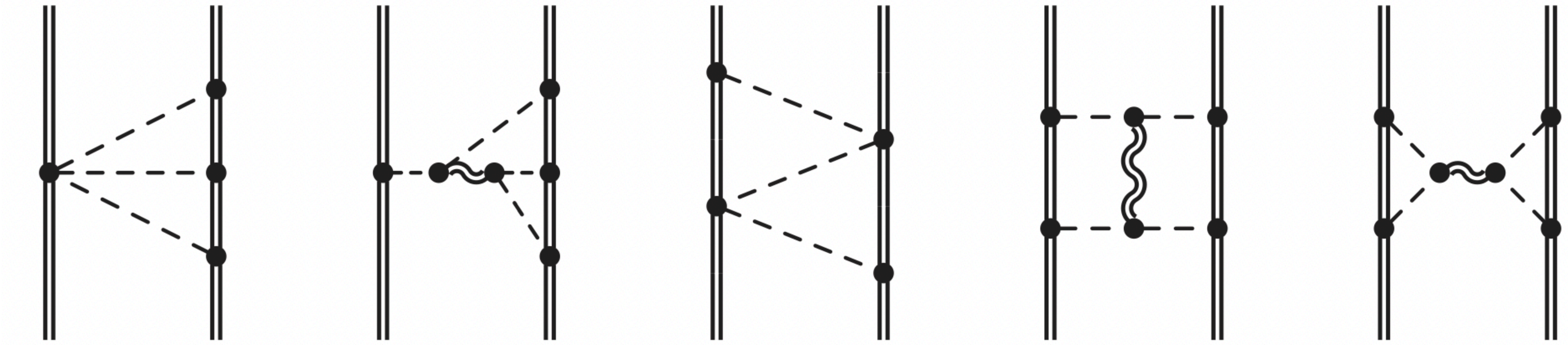
$O(Gv^4)$



$O(G^2v^2)$



$O(G^3v^0)$



[Gilmore and Ross 2008]

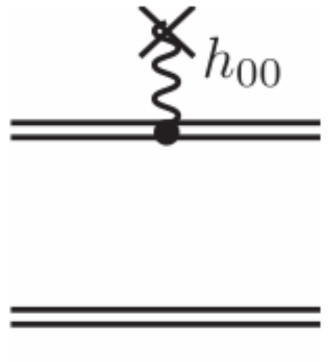
Radiative theory

To compute GWs at $\lambda \gg r$, we couple the radiative field with a composite object (the binary) carrying intrinsic moments determined by orbital dynamics...

$$S_{\rm rad}[z^\mu, h_{\mu\nu}] = -\frac{1}{64\pi}\int_x \{P^{\alpha\beta\gamma\delta}\partial_\mu h_{\alpha\beta}\partial^\mu h_{\gamma\delta} + \mathcal{O}(h^3)\} + \int d\tau \Big\{ -m(\tau) + \frac{1}{2}S_{ab}\Omega^{ab} + \frac{1}{2}I^{ab}(\tau)\mathcal{E}_{ab}(z^\mu) + \cdots \Big\}$$

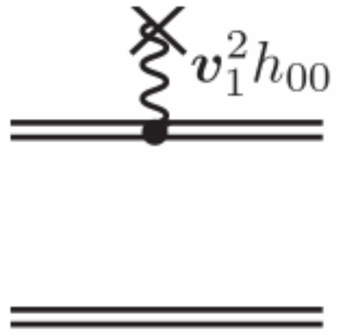
...matching multipoles using the UV stress–energy tensor:

$$S_{\rm UV} = \frac{1}{2}\int d^4x \sqrt{-g}\, h_{\mu\nu}(x) T^{\mu\nu}(x) = \frac{1}{2}\sum_{n=0}^{\infty}\frac{1}{n!}\int_x x^{i_1}\cdots x^{i_n}\partial_{i_1}\cdots\partial_{i_n} h_{\mu\nu}(t,\mathbf{0})\, T^{\mu\nu}(t,\boldsymbol{x})$$

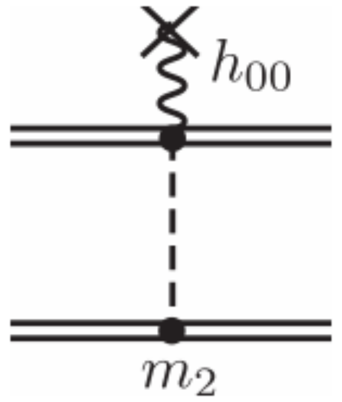


$$+ (1 \leftrightarrow 2) = \frac{1}{2}\sum_{n=1}^{2} m_n \int dt\, h_{00}(t,\mathbf{0}) \qquad T^{00}(t,\boldsymbol{x}) = \delta^3(\boldsymbol{x})\sum_{n=1}^{2} m_n$$

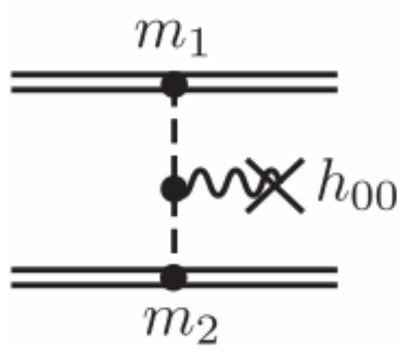
+1PN



$+$



$+ (1 \leftrightarrow 2) +$



$$T^{00}(t,\boldsymbol{x}) = \delta^3(\boldsymbol{x})\left(M + \frac{1}{2}\mu \boldsymbol{v}^2 - \frac{\mu M}{r} + \mathcal{O}(v^4)\right)$$

$$+ (1 \leftrightarrow 2) = \int dt \left(\sum_{n=1}^{2} m_n v_n^i\right) h_{0i}(t,\mathbf{0}) \qquad T^{0i}(t,\boldsymbol{x}) = \delta^3(\boldsymbol{x})\sum_{n=1}^{2} m_n v_n^i \qquad S^{ij}(t) = \sum_{n=1}^{2} m_n \left(z_n^i v_n^j - z_n^j v_n^i\right)$$

$$+ (1 \leftrightarrow 2) = \frac{1}{2}\int dt \left(\sum_{n=1}^{2} m_n v_n^i v_n^j\right) h_{ij}(t,\mathbf{0}) \qquad T^{ij}(t,\boldsymbol{x}) = \delta^3(\boldsymbol{x})\sum_{n=1}^{2} m_n v_n^i v_n^j \qquad I^{ij}(t) = \sum_{n=1}^{2} m_n \left(z_n^i z_n^j - \frac{1}{3}\delta^{ij}\boldsymbol{z}_n^2\right) + \mathcal{O}(v^2)$$

$$
\begin{aligned}
L^{\mathrm{harm}} &= \frac{Gm_1m_2}{2r_{12}} + \frac{m_1v_1^2}{2} \\
&+ \frac{1}{c^2}\left\{ -\frac{G^2m_1^2m_2}{2r_{12}^2} + \frac{m_1v_1^4}{8} + \frac{Gm_1m_2}{r_{12}}\left(-\frac{1}{4}(n_{12}v_1)(n_{12}v_2) + \frac{3}{2}v_1^2 - \frac{7}{4}(v_1v_2) \right) \right\} \\
&+ \frac{1}{c^4}\left\{ \frac{G^3m_1^3m_2}{2r_{12}^3} + \frac{19G^3m_1^2m_2^2}{8r_{12}^3} \right. \\
&\quad + \frac{G^2m_1^2m_2}{r_{12}^2}\left(\frac{7}{2}(n_{12}v_1)^2 - \frac{7}{2}(n_{12}v_1)(n_{12}v_2) + \frac{1}{2}(n_{12}v_2)^2 + \frac{1}{4}v_1^2 - \frac{7}{4}(v_1v_2) + \frac{7}{4}v_2^2 \right) \\
&\quad + \frac{Gm_1m_2}{r_{12}}\left(\frac{3}{16}(n_{12}v_1)^2(n_{12}v_2)^2 - \frac{7}{8}(n_{12}v_2)^2v_1^2 + \frac{7}{8}v_1^4 + \frac{3}{4}(n_{12}v_1)(n_{12}v_2)(v_1v_2) \right. \\
&\qquad \left. - 2v_1^2(v_1v_2) + \frac{1}{8}(v_1v_2)^2 + \frac{15}{16}v_1^2v_2^2 \right) + \frac{m_1v_1^6}{16} \\
&\quad \left. + Gm_1m_2\left(-\frac{7}{4}(a_1v_2)(n_{12}v_2) - \frac{1}{8}(n_{12}a_1)(n_{12}v_2)^2 + \frac{7}{8}(n_{12}a_1)v_2^2 \right) \right\} \\
&+ \frac{1}{c^6}\left\{ \frac{G^2m_1^2m_2}{r_{12}^2}\left(\frac{13}{18}(n_{12}v_1)^4 + \frac{83}{18}(n_{12}v_1)^3(n_{12}v_2) - \frac{35}{6}(n_{12}v_1)^2(n_{12}v_2)^2 - \frac{245}{24}(n_{12}v_1)^2v_1^2 \right. \right. \\
&\qquad + \frac{179}{12}(n_{12}v_1)(n_{12}v_2)v_1^2 - \frac{235}{24}(n_{12}v_2)^2v_1^2 + \frac{373}{48}v_1^4 + \frac{529}{24}(n_{12}v_1)^2(v_1v_2) \\
&\qquad - \frac{97}{6}(n_{12}v_1)(n_{12}v_2)(v_1v_2) - \frac{719}{24}v_1^2(v_1v_2) + \frac{463}{24}(v_1v_2)^2 - \frac{7}{24}(n_{12}v_1)^2v_2^2 \\
&\qquad \left. - \frac{1}{2}(n_{12}v_1)(n_{12}v_2)v_2^2 + \frac{1}{4}(n_{12}v_2)^2v_2^2 + \frac{463}{48}v_1^2v_2^2 - \frac{19}{2}(v_1v_2)v_2^2 + \frac{45}{16}v_2^4 \right) \\
&\quad + \frac{5m_1v_1^8}{128} \\
&\quad + Gm_1m_2\left(\frac{3}{8}(a_1v_2)(n_{12}v_1)(n_{12}v_2)^2 + \frac{5}{12}(a_1v_2)(n_{12}v_2)^3 + \frac{1}{8}(n_{12}a_1)(n_{12}v_1)(n_{12}v_2)^3 \right. \\
&\qquad + \frac{1}{16}(n_{12}a_1)(n_{12}v_2)^4 + \frac{11}{4}(a_1v_1)(n_{12}v_2)v_1^2 - (a_1v_2)(n_{12}v_2)v_1^2 \\
&\qquad - 2(a_1v_1)(n_{12}v_2)(v_1v_2) + \frac{1}{4}(a_1v_2)(n_{12}v_2)(v_1v_2) \\
&\qquad + \frac{3}{8}(n_{12}a_1)(n_{12}v_2)^2(v_1v_2) - \frac{5}{8}(n_{12}a_1)(n_{12}v_1)^2v_2^2 + \frac{15}{8}(a_1v_1)(n_{12}v_2)v_2^2 \\
&\qquad - \frac{15}{8}(a_1v_2)(n_{12}v_2)v_2^2 - \frac{1}{2}(n_{12}a_1)(n_{12}v_1)(n_{12}v_2)v_2^2 \\
&\qquad \left. - \frac{5}{16}(n_{12}a_1)(n_{12}v_2)^2v_2^2 \right) \\
&\quad + \frac{G^2m_1^2m_2}{r_{12}}\left(-\frac{235}{24}(a_2v_1)(n_{12}v_1) - \frac{29}{24}(n_{12}a_2)(n_{12}v_1)^2 - \frac{235}{24}(a_1v_2)(n_{12}v_2) \right. \\
&\qquad - \frac{17}{6}(n_{12}a_1)(n_{12}v_2)^2 + \frac{185}{16}(n_{12}a_1)v_1^2 - \frac{235}{48}(n_{12}a_2)v_1^2 \\
&\qquad \left. - \frac{185}{8}(n_{12}a_1)(v_1v_2) + \frac{20}{3}(n_{12}a_1)v_2^2 \right) \\
&\quad + \frac{Gm_1m_2}{r_{12}}\left(-\frac{5}{32}(n_{12}v_1)^3(n_{12}v_2)^3 + \frac{1}{8}(n_{12}v_1)(n_{12}v_2)^3v_1^2 + \frac{5}{8}(n_{12}v_2)^4v_1^2 \right. \\
&\qquad - \frac{11}{16}(n_{12}v_1)(n_{12}v_2)v_1^4 + \frac{1}{4}(n_{12}v_2)^2v_1^4 + \frac{11}{16}v_1^6 \\
&\qquad - \frac{15}{32}(n_{12}v_1)^2(n_{12}v_2)^2(v_1v_2) + (n_{12}v_1)(n_{12}v_2)v_1^2(v_1v_2) \\
&\qquad + \frac{3}{8}(n_{12}v_2)^2v_1^2(v_1v_2) - \frac{13}{16}v_1^4(v_1v_2) + \frac{5}{16}(n_{12}v_1)(n_{12}v_2)(v_1v_2)^2 \\
&\qquad + \frac{1}{16}(v_1v_2)^3 - \frac{5}{8}(n_{12}v_1)^2v_1^2v_2^2 - \frac{23}{32}(n_{12}v_1)(n_{12}v_2)v_1^2v_2^2 + \frac{1}{16}v_1^4v_2^2 \\
&\qquad \left. - \frac{1}{32}v_1^2(v_1v_2)v_2^2 \right) \\
&\quad - \frac{3G^4m_1^4m_2}{8r_{12}^4} + \frac{G^4m_1^3m_2^2}{r_{12}^4}\left(-\frac{9707}{420} + \frac{22}{3}\ln\left(\frac{r_{12}}{r'_1}\right) \right) \\
&\quad + \frac{G^3m_1^2m_2^2}{r_{12}^3}\left(\frac{383}{24}(n_{12}v_1)^2 - \frac{889}{48}(n_{12}v_1)(n_{12}v_2) - \frac{123}{64}(n_{12}v_1)(n_{12}v_{12})\pi^2 - \frac{305}{72}v_1^2 \right. \\
&\qquad \left. + \frac{41}{64}\pi^2(v_1v_{12}) + \frac{439}{144}(v_1v_2) \right) \\
&\quad + \frac{G^3m_1^3m_2}{r_{12}^3}\left(-\frac{8243}{210}(n_{12}v_1)^2 + \frac{15541}{420}(n_{12}v_1)(n_{12}v_2) + \frac{3}{2}(n_{12}v_2)^2 + \frac{15611}{1260}v_1^2 \right. \\
&\qquad - \frac{17501}{1260}(v_1v_2) + \frac{5}{4}v_2^2 + 22(n_{12}v_1)(n_{12}v_{12})\ln\left(\frac{r_{12}}{r'_1}\right) \\
&\qquad \left. \left. - \frac{22}{3}(v_1v_{12})\ln\left(\frac{r_{12}}{r'_1}\right) \right) \right\} + 1 \leftrightarrow 2 + \mathcal{O}\left(\frac{1}{c^7}\right). \qquad (209)
\end{aligned}
$$

[Blanchet 2014]

PN waveform

$$x = (M\omega)^{2/3} \qquad \nu = m_1 m_2/(m_1+m_2)^2$$

$$\phi = -\frac{x^{-5/2}}{32\nu}\Bigg\{1 + \left(\frac{3715}{1008}+\frac{55}{12}\nu\right)x - 10\pi x^{3/2} + \left(\frac{15293365}{1016064}+\frac{27145}{1008}\nu+\frac{3085}{144}\nu^2\right)x^2 + \left(\frac{38645}{1344}-\frac{65}{16}\nu\right)\pi x^{5/2}\ln\left(\frac{x}{x_0}\right)$$
$$+\left[\frac{12348611926451}{18776862720}-\frac{160}{3}\pi^2-\frac{1712}{21}\gamma_\mathrm{E}-\frac{856}{21}\ln(16\,x)+\left(-\frac{15737765635}{12192768}+\frac{2255}{48}\pi^2\right)\nu+\frac{76055}{6912}\nu^2-\frac{127825}{5184}\nu^3\right]x^3$$
$$+\left(\frac{77096675}{2032128}+\frac{378515}{12096}\nu-\frac{74045}{6048}\nu^2\right)\pi x^{7/2}+\mathcal{O}\left(\frac{1}{c^8}\right)\Bigg\}$$

$$\boxed{h_{+,\times} = \frac{2\,\mu M x}{R}\sum_p x^{p/2}\;{}_{p/2}H_{+,\times}(\psi,\cos\iota,\sin\iota)} \qquad \psi = \phi - 2\,M\,\Omega\log\Omega/\Omega_0$$

$$\underset{0}{H}_+ = -(1+c_i^2)\cos 2\psi - \frac{1}{96}\,s_i^2\,(17+c_i^2)\,,$$

$$\underset{1/2}{H}_+ = -\,s_i\,\Delta\left[\cos\psi\left(\frac{5}{8}+\frac{1}{8}\,c_i^2\right)-\cos 3\psi\left(\frac{9}{8}+\frac{9}{8}\,c_i^2\right)\right]\,,$$

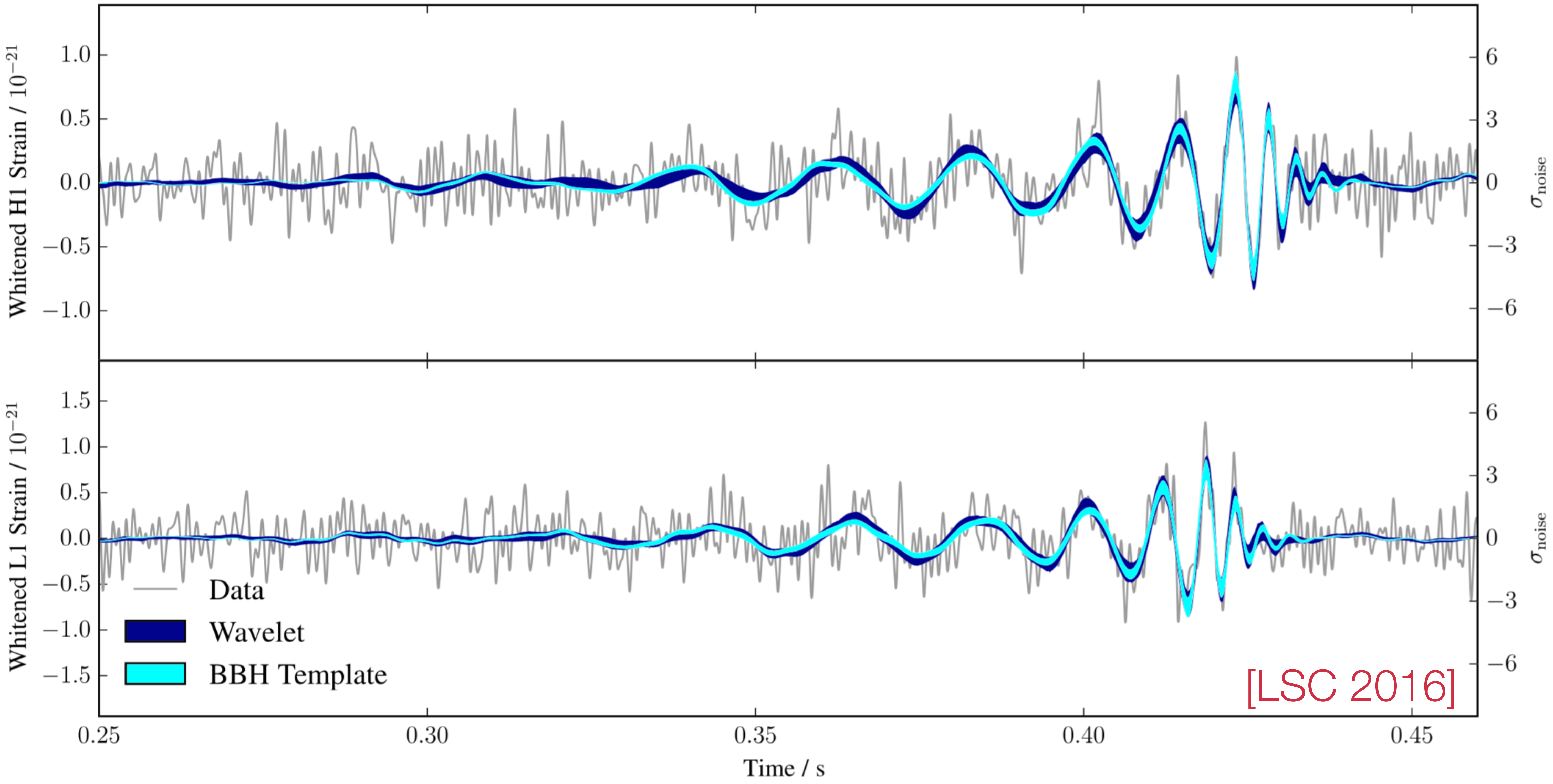
$$\underset{1}{H}_+ = \cos 2\psi\left[\frac{19}{6}+\frac{3}{2}\,c_i^2-\frac{1}{3}\,c_i^4+\nu\left(-\frac{19}{6}+\frac{11}{6}\,c_i^2+c_i^4\right)\right]-\cos 4\psi\left[\frac{4}{3}\,s_i^2(1+c_i^2)(1-3\nu)\right]$$

$$\underset{3/2}{H}_+ = s_i\,\Delta\cos\psi\left[\frac{19}{64}+\frac{5}{16}\,c_i^2-\frac{1}{192}\,c_i^4+\nu\left(-\frac{49}{96}+\frac{1}{8}\,c_i^2+\frac{1}{96}\,c_i^4\right)\right]+\cos 2\psi\left[-2\pi(1+c_i^2)\right]$$
$$+\,s_i\,\Delta\cos 3\psi\left[-\frac{657}{128}-\frac{45}{16}\,c_i^2+\frac{81}{128}\,c_i^4+\nu\left(\frac{225}{64}-\frac{9}{8}\,c_i^2-\frac{81}{64}\,c_i^4\right)\right]+\,s_i\,\Delta\cos 5\psi\left[\frac{625}{384}\,s_i^2(1+c_i^2)(1-2\nu)\right]$$

PN order		$1.4\,M_\odot + 1.4\,M_\odot$	$10\,M_\odot + 1.4\,M_\odot$	$10\,M_\odot + 10\,M_\odot$
N	(inst)	15952.6	3558.9	598.8
1PN	(inst)	439.5	212.4	59.1
1.5PN	(leading tail)	−210.3	−180.9	−51.2
2PN	(inst)	9.9	9.8	4.0
2.5PN	(1PN tail)	−11.7	−20.0	−7.1
3PN	(inst + tail-of-tail)	2.6	2.3	2.2
3.5PN	(2PN tail)	−0.9	−1.8	−0.8

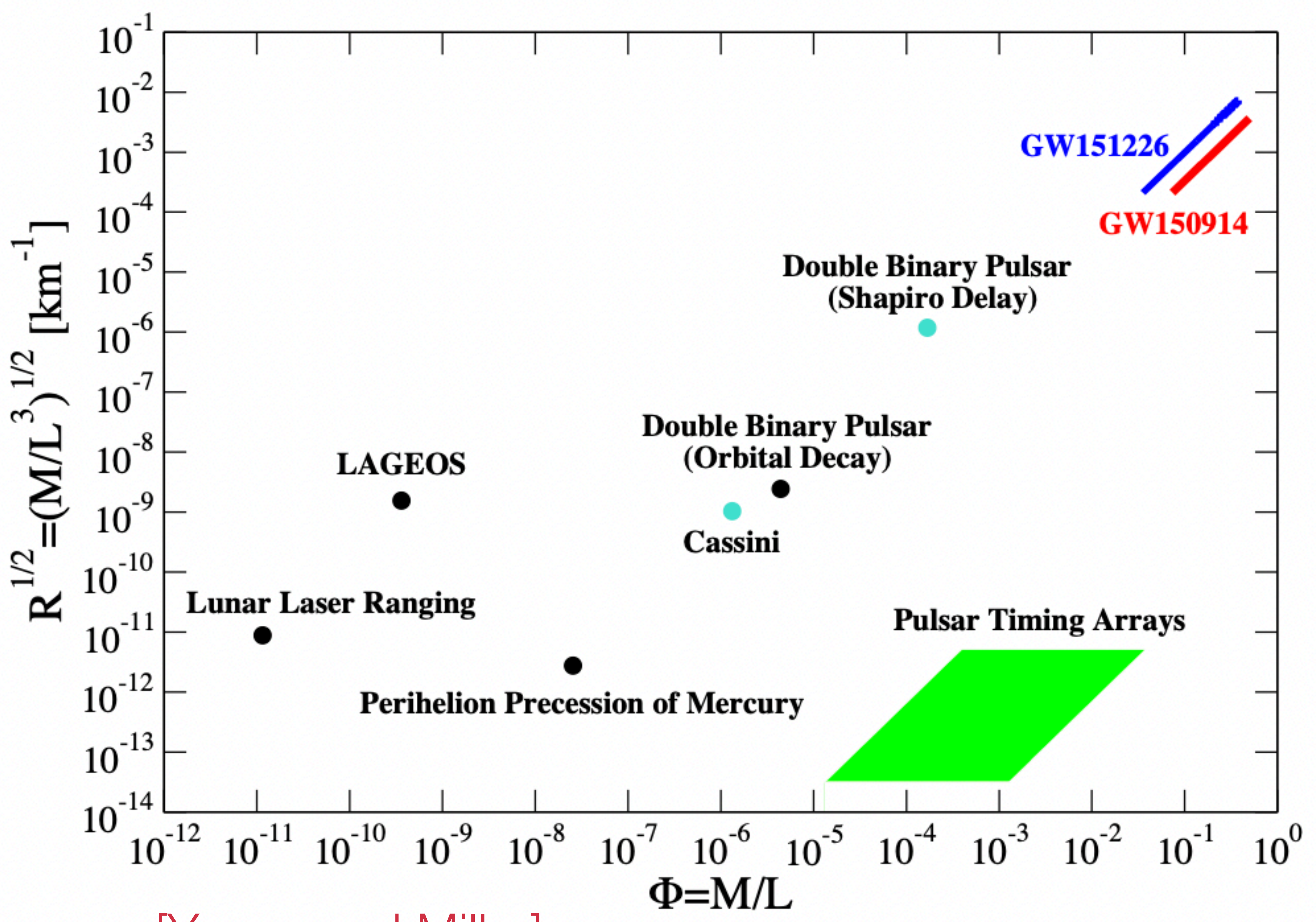
[Blanchet 2014]

Testing GR with GW observations



- Detecting GWs offered a profound confirmation of general relativity
- Using GW observations to test GR with increasing precision, we hope to obtain clues of new physics and establish the scope of GR

Testing GR with GW observations



[Yunes and Miller]

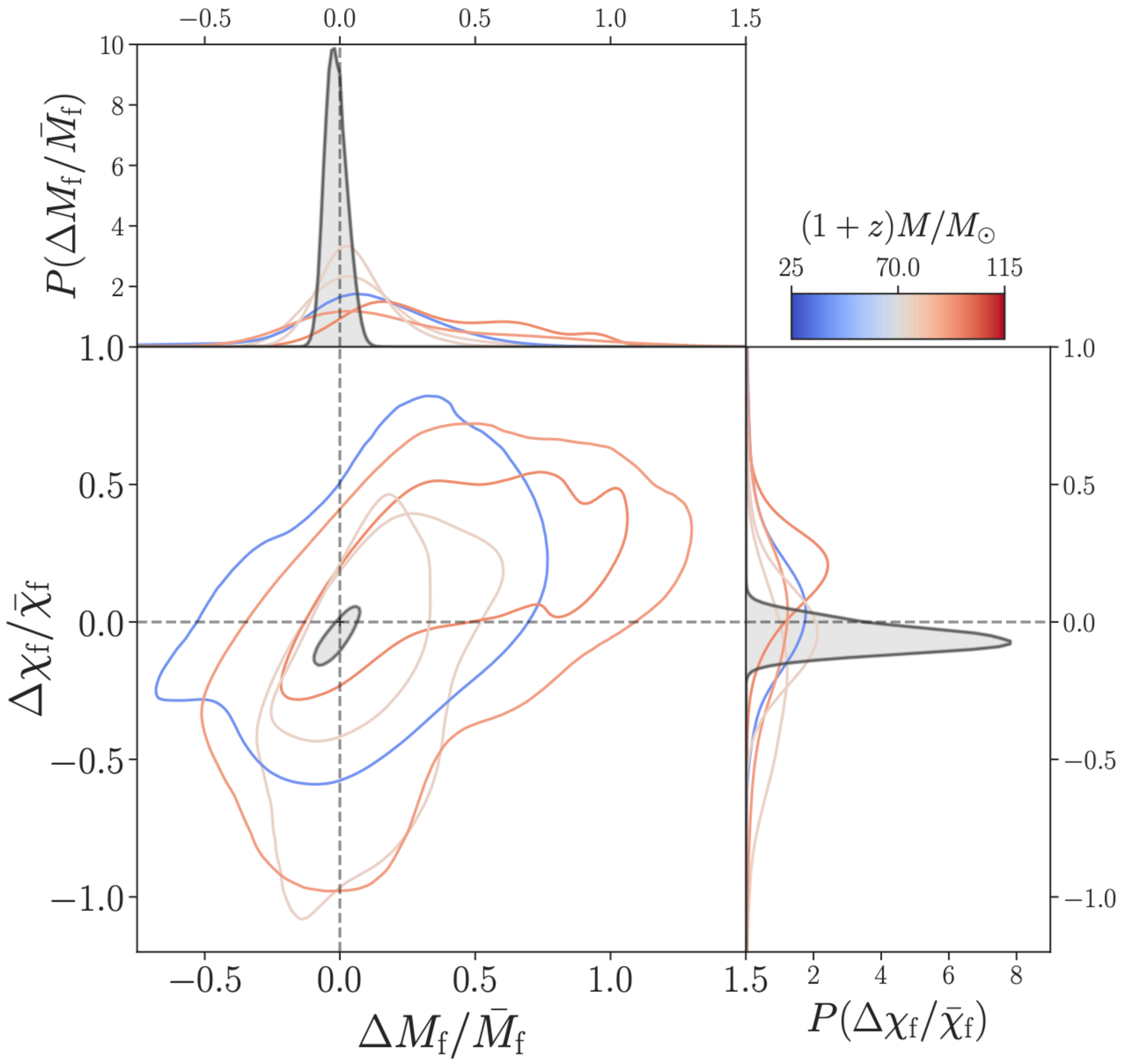
- GW observations of compact binaries probe nonlinear, dynamical GR
- GWs are predicted in any Lorentz-invariance metric theory of gravitation, but they may differ from GR in **generation**, **propagation**, **polarization**
- No simple framework like PPN exists to describe strongly gravitating radiative systems, and we have few models of binaries in modified gravity
- Therefore we default to characterizing the accuracy of GR either parametrically (by extending waveform families in "non-GR" directions) or non-parametrically (with other tests of "consistency").

Testing GR with GW observations

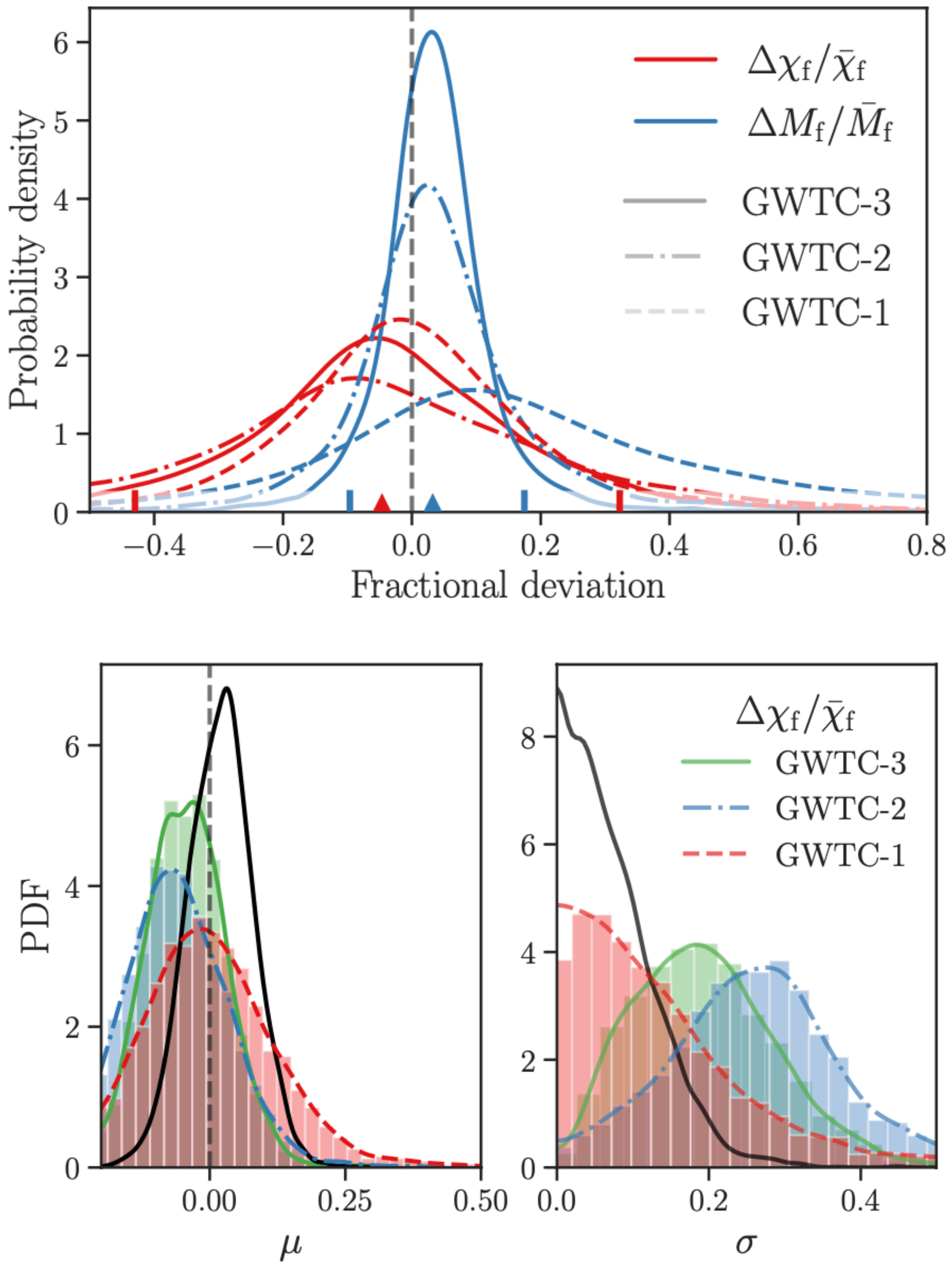
- **Consistency**: a useful sanity check, but hard to interpret statistically. P values are possible with much work. But would we ever believe an inconsistent result?

Consistency: parameters from partial waveforms match

$$\frac{\Delta M_{\mathrm{f}}}{\bar{M}_{\mathrm{f}}} = 2\frac{M_{\mathrm{f}}^{\mathrm{insp}} - M_{\mathrm{f}}^{\mathrm{postinsp}}}{M_{\mathrm{f}}^{\mathrm{insp}} + M_{\mathrm{f}}^{\mathrm{postinsp}}} \qquad \frac{\Delta \chi_{\mathrm{f}}}{\bar{\chi}_{\mathrm{f}}} = 2\frac{\chi_{\mathrm{f}}^{\mathrm{insp}} - \chi_{\mathrm{f}}^{\mathrm{postinsp}}}{\chi_{\mathrm{f}}^{\mathrm{insp}} + \chi_{\mathrm{f}}^{\mathrm{postinsp}}}$$



Identical deviations across events

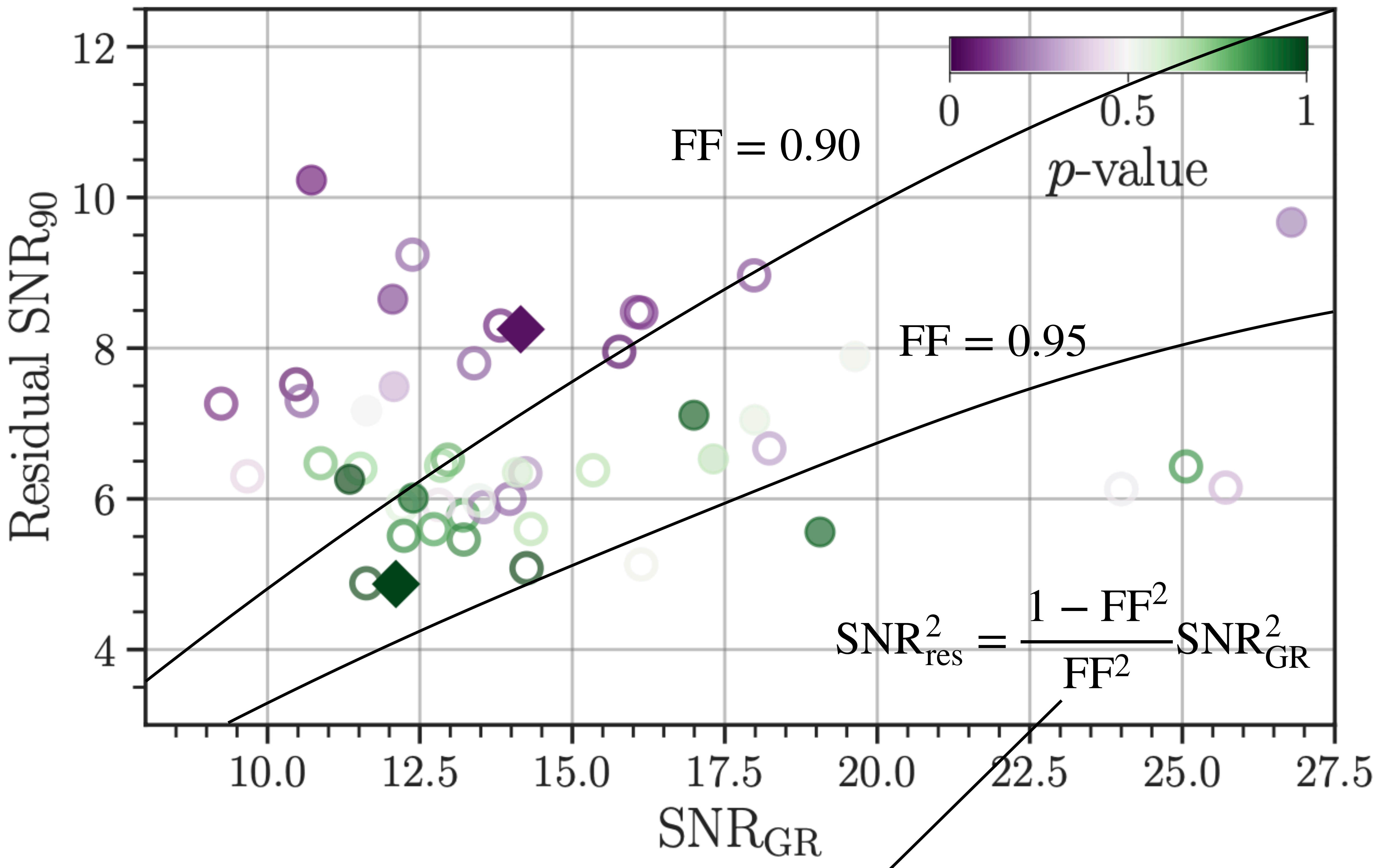


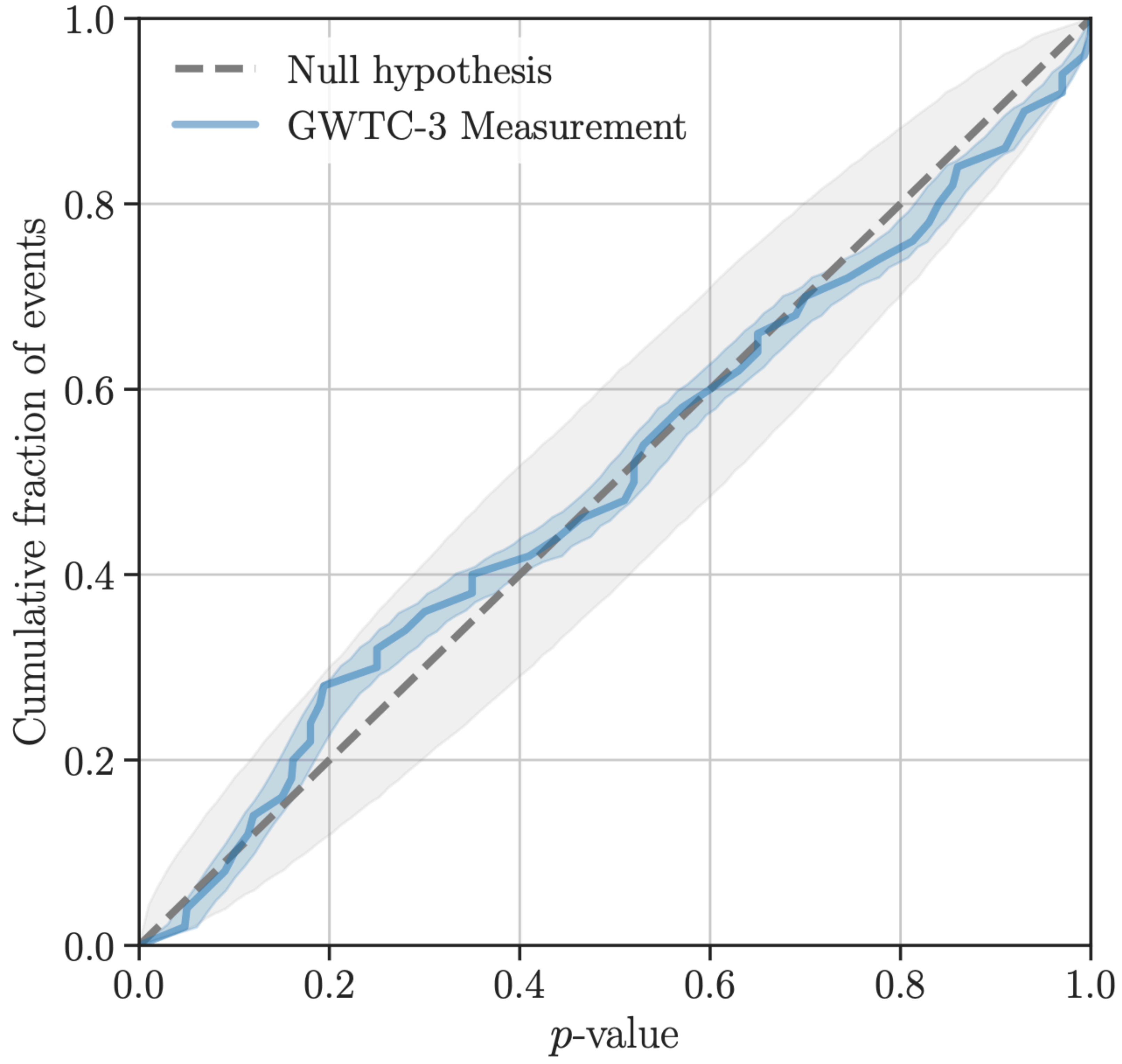
Hierarchical model with mean+variance for fractional deviations

[LVC 2022]

Consistency: residuals look like noise

Subtract best-fit signal model; search for coherent content (wavelet model); compare SNR with background obtained on nearby stretches of data. An actual hypothesis test, which implies that GR is verified to better than a few percent.





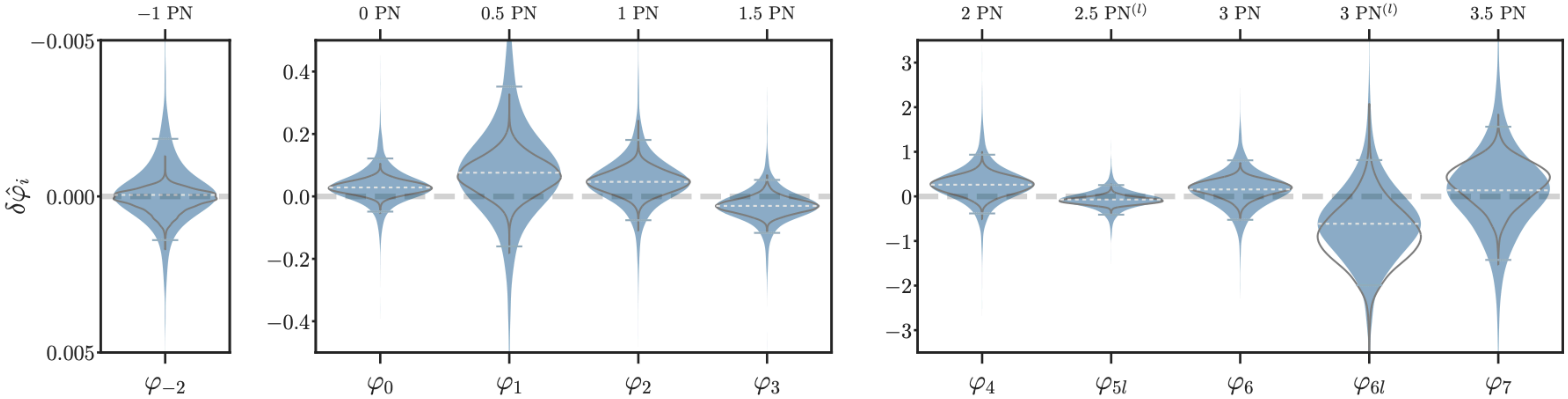
Fitting Factor FF: parameter-maximized waveform overlap

[LVC 2022]

Testing GR with GW observations

- **Consistency**: a useful sanity check, but hard to interpret statistically. P values are possible with much work. But would we ever believe an inconsistent result?
- **Parametric tests**: constraints on GR "constants" (PN coefficients, graviton mass) witness increasing sensitivity, but again hard to interpret. Apparent violations may focus our search for new physics.

Parametric test: PN coefficient perturbations



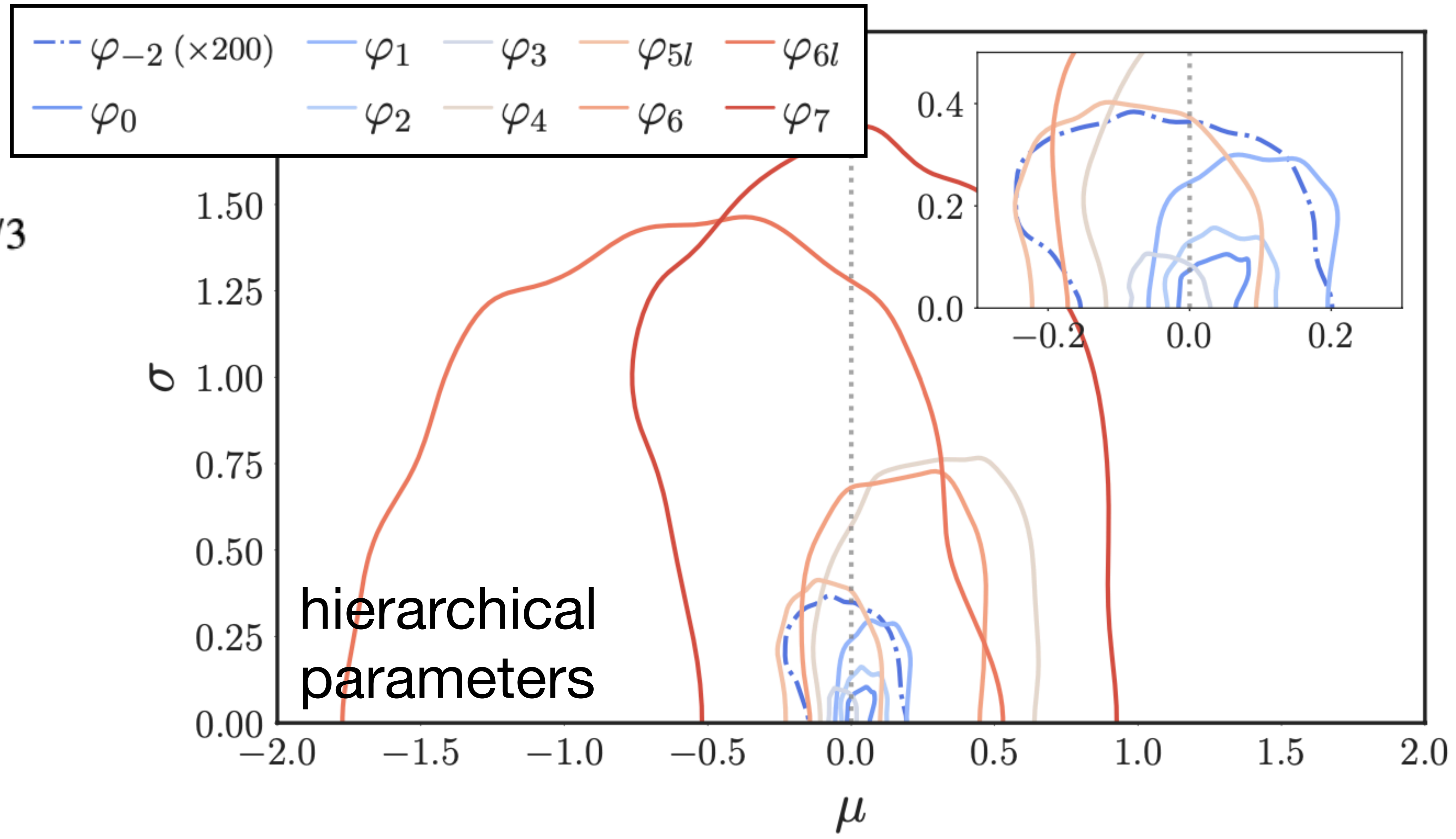
$$\varphi_{\rm PN}(f) = 2\pi f t_{\rm c} - \varphi_{\rm c} - \frac{\pi}{4} + \frac{3}{128\eta}\left(\pi\tilde{f}\right)^{-5/3}\sum_{i=0}^{7}\left[\varphi_i + \varphi_{i\,l}\log(\pi\tilde{f})\right]\left(\pi\tilde{f}\right)^{i/3}$$

$$\{\delta\hat{\varphi}_{-2}, \delta\hat{\varphi}_0, \delta\hat{\varphi}_1, \delta\hat{\varphi}_2, \delta\hat{\varphi}_3, \delta\hat{\varphi}_4, \delta\hat{\varphi}_{5l}, \delta\hat{\varphi}_6, \delta\hat{\varphi}_{6l}, \delta\hat{\varphi}_7\}$$

dipole radiation

massive gravity

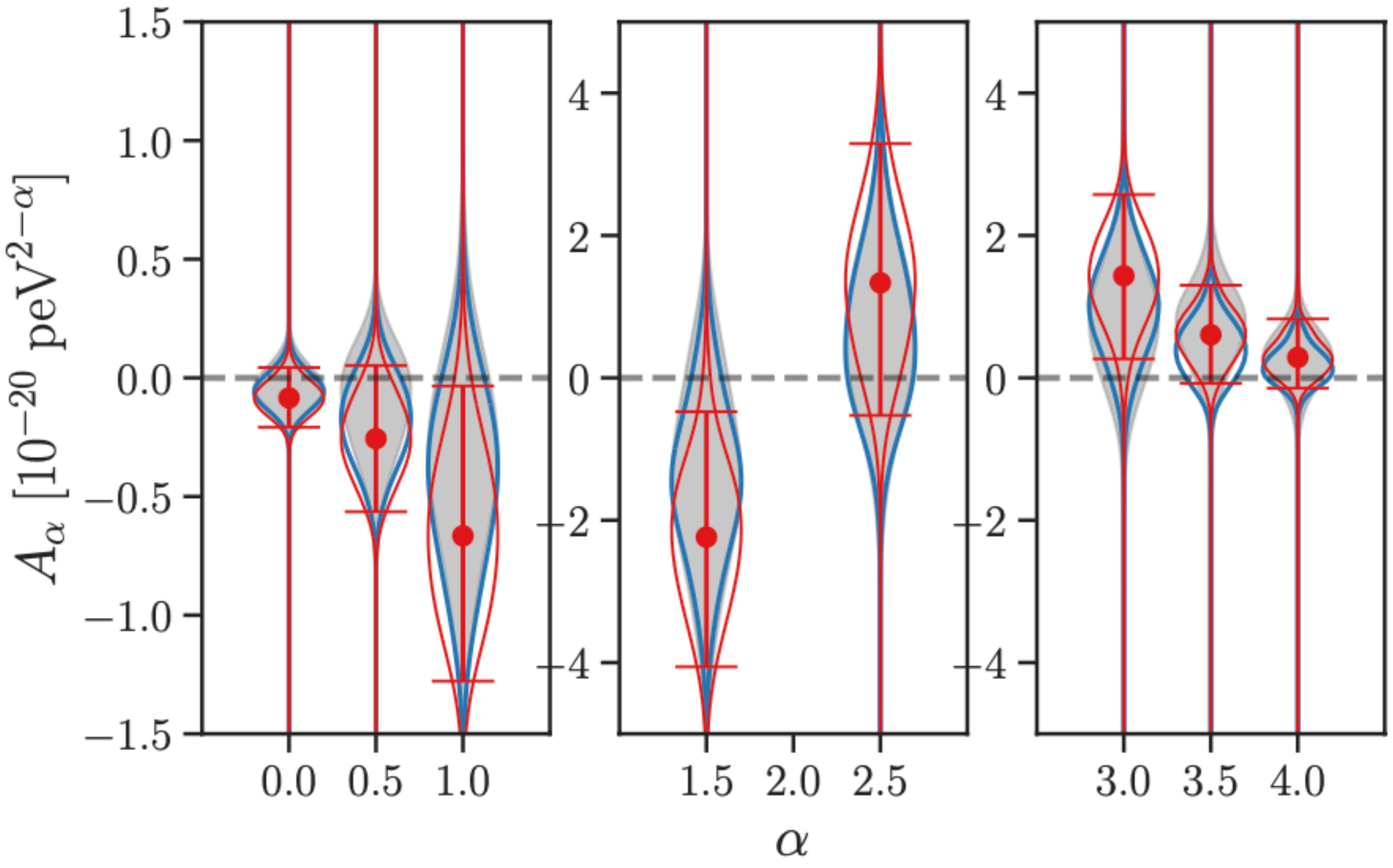
dynamical Chern–Simons



[LVC 2022]

Parametric test: dispersion law

$$E^2 = p^2c^2 + A_\alpha p^\alpha c^\alpha$$



massive graviton: $\alpha = 0$, $m_g c^2 = \sqrt{A_0}$
joint 90% upper limit is $1.27 \times 10^{-23}\,\mathrm{eV}/c^2$

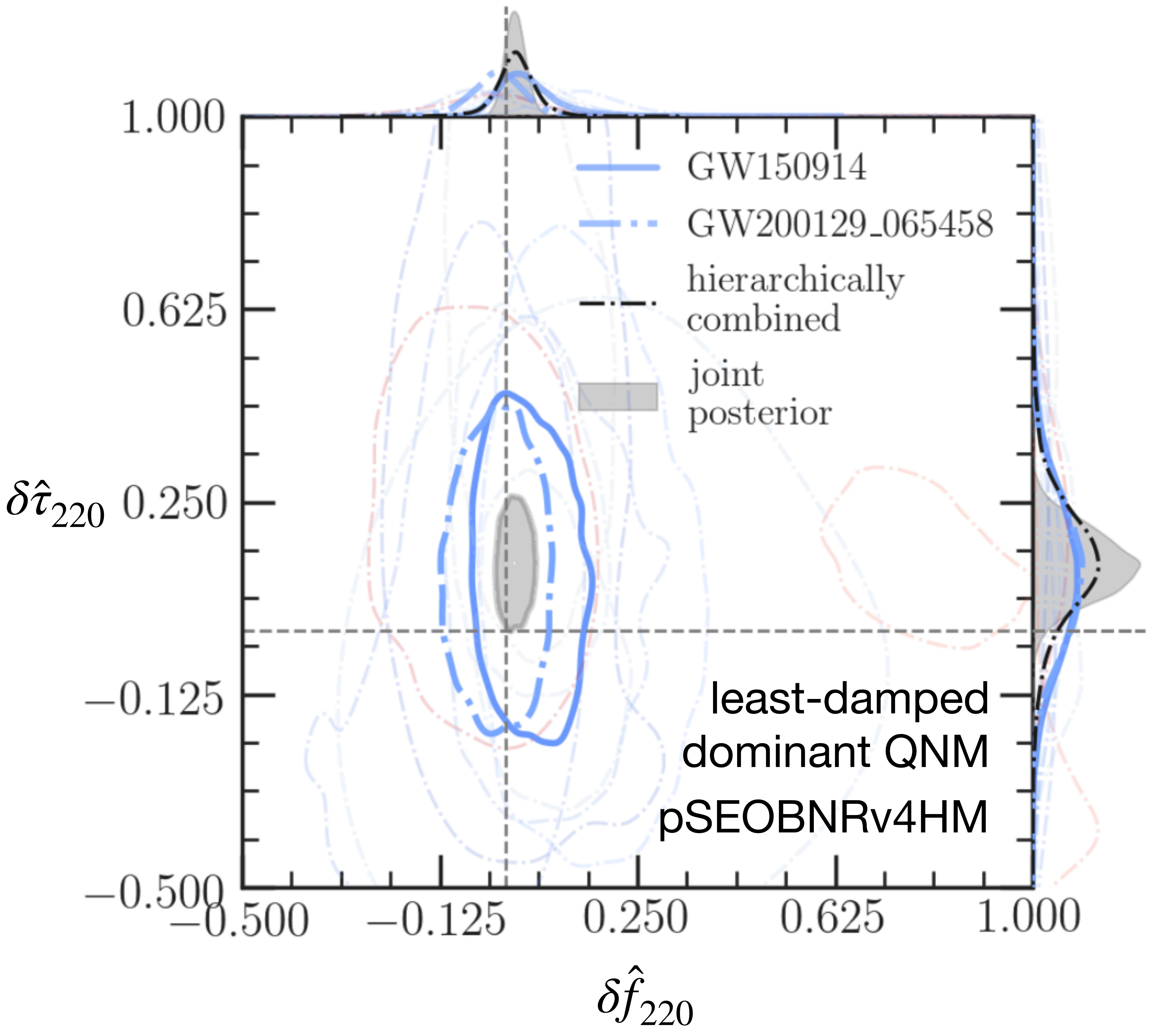
$$\frac{v_{\mathrm{g}}^2}{c^2} = 1 - \frac{m_{\mathrm{g}}^2 c^4}{E^2} \qquad \delta\Psi(f) = \frac{\pi D c}{\lambda_g^2 (1+z) f}$$

[LVC 2022]

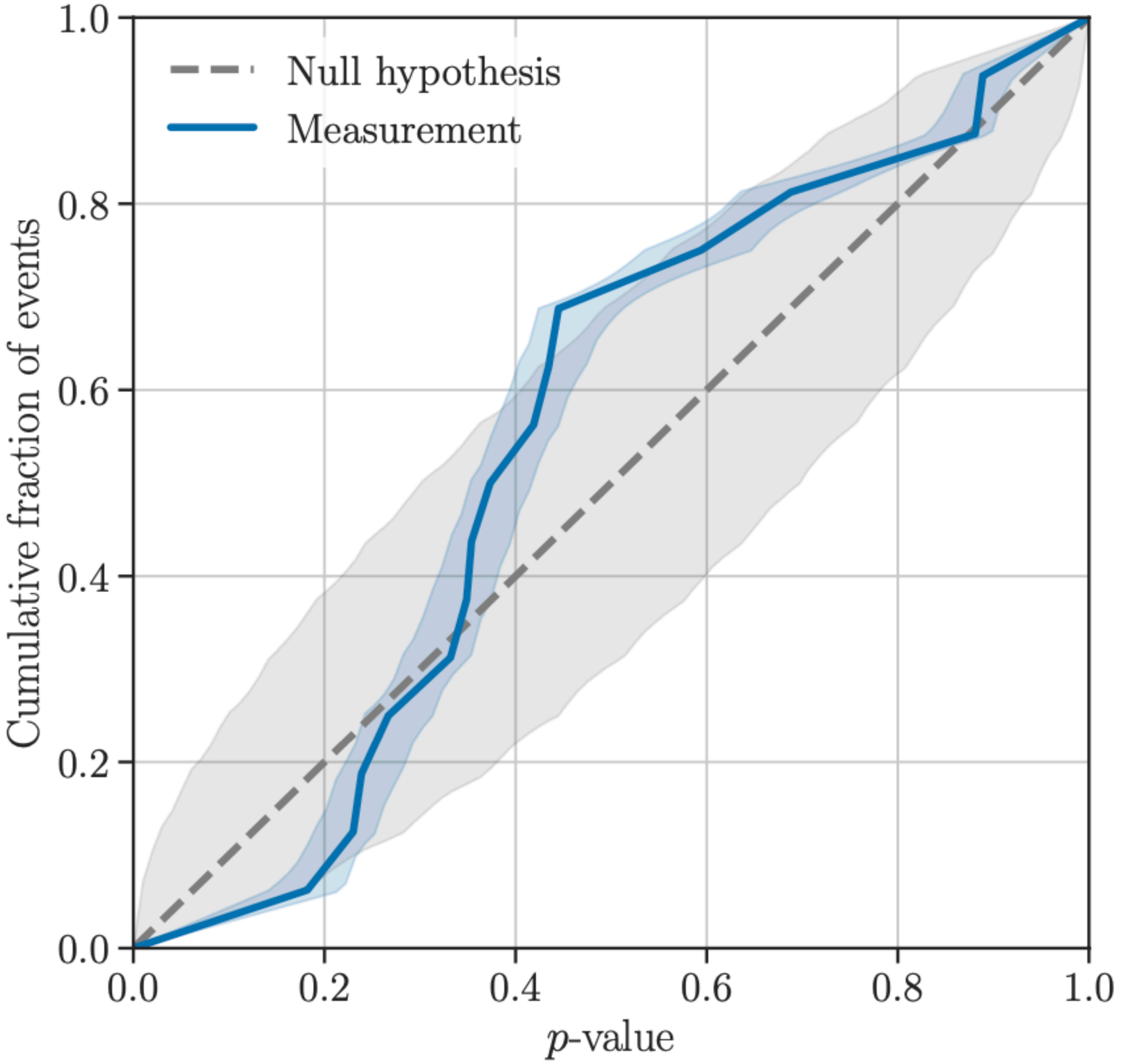
Parametric test: ringdown fundamental mode f/τ

$$h_{\text{ringdown}} = \sum_{lmn} A_{lmn} \mathrm{e}^{2\pi i (f_{lmn} + i/\tau_{lmn})(t - t_0)}$$

$$f_{\ell m0} = f^{\mathrm{GR}}_{\ell m0}\,(1 + \delta\hat{f}_{\ell m0})$$
$$\tau_{\ell m0} = \tau^{\mathrm{GR}}_{\ell m0}\,(1 + \delta\hat{\tau}_{\ell m0})$$



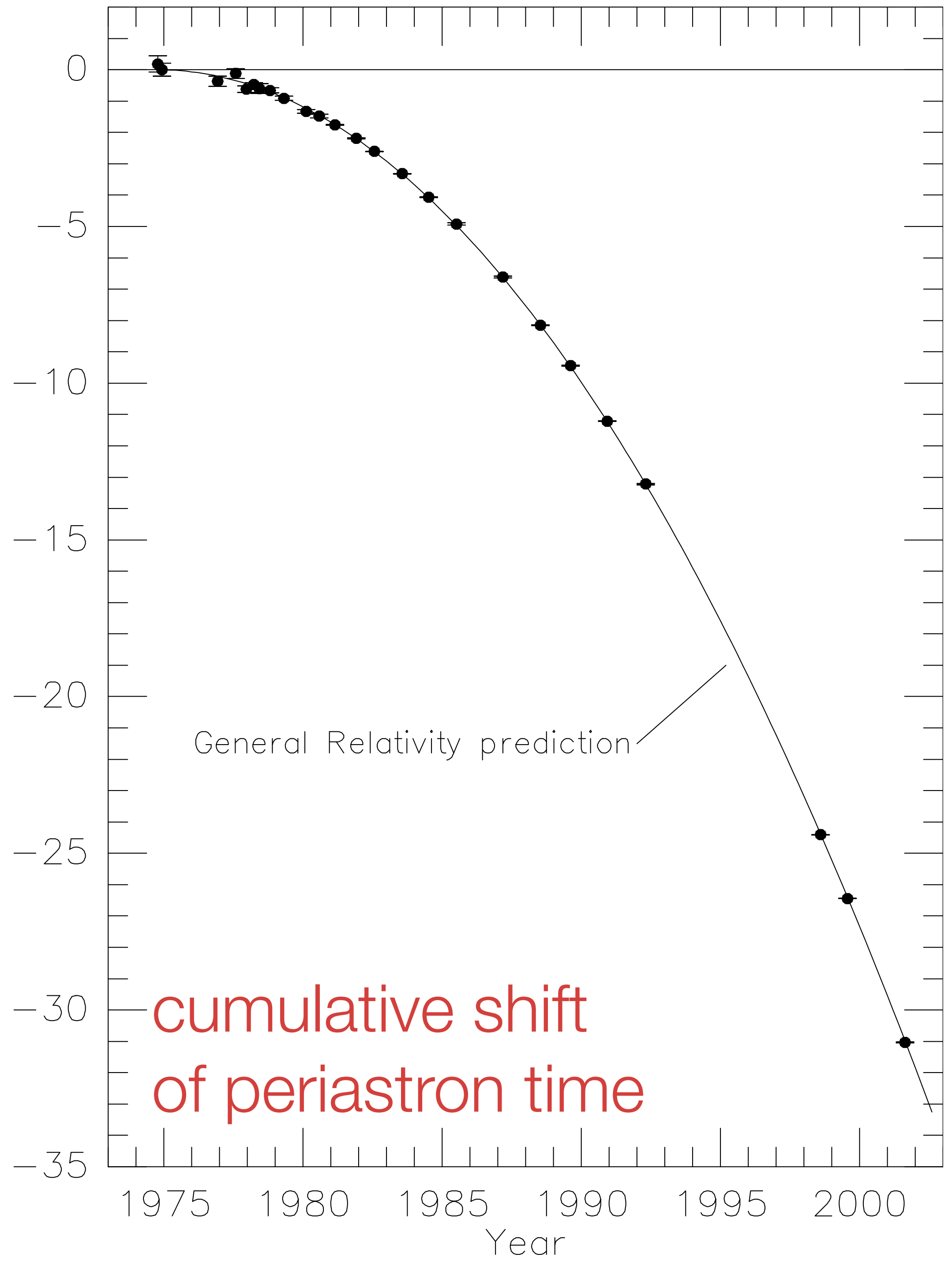
Event	p-value
GW191109_010717	0.35
GW191129_134029	0.35
GW191204_171526	0.37
GW191215_223052	0.23
GW191216_213338	0.88
GW191222_033537	0.89
GW200115_042309	0.44
GW200129_065458	0.33
GW200202_154313	0.43
GW200208_130117	0.24
GW200219_094415	0.18
GW200224_222234	0.59
GW200225_060421	0.69
GW200311_115853	0.42
GW200316_215756	0.27

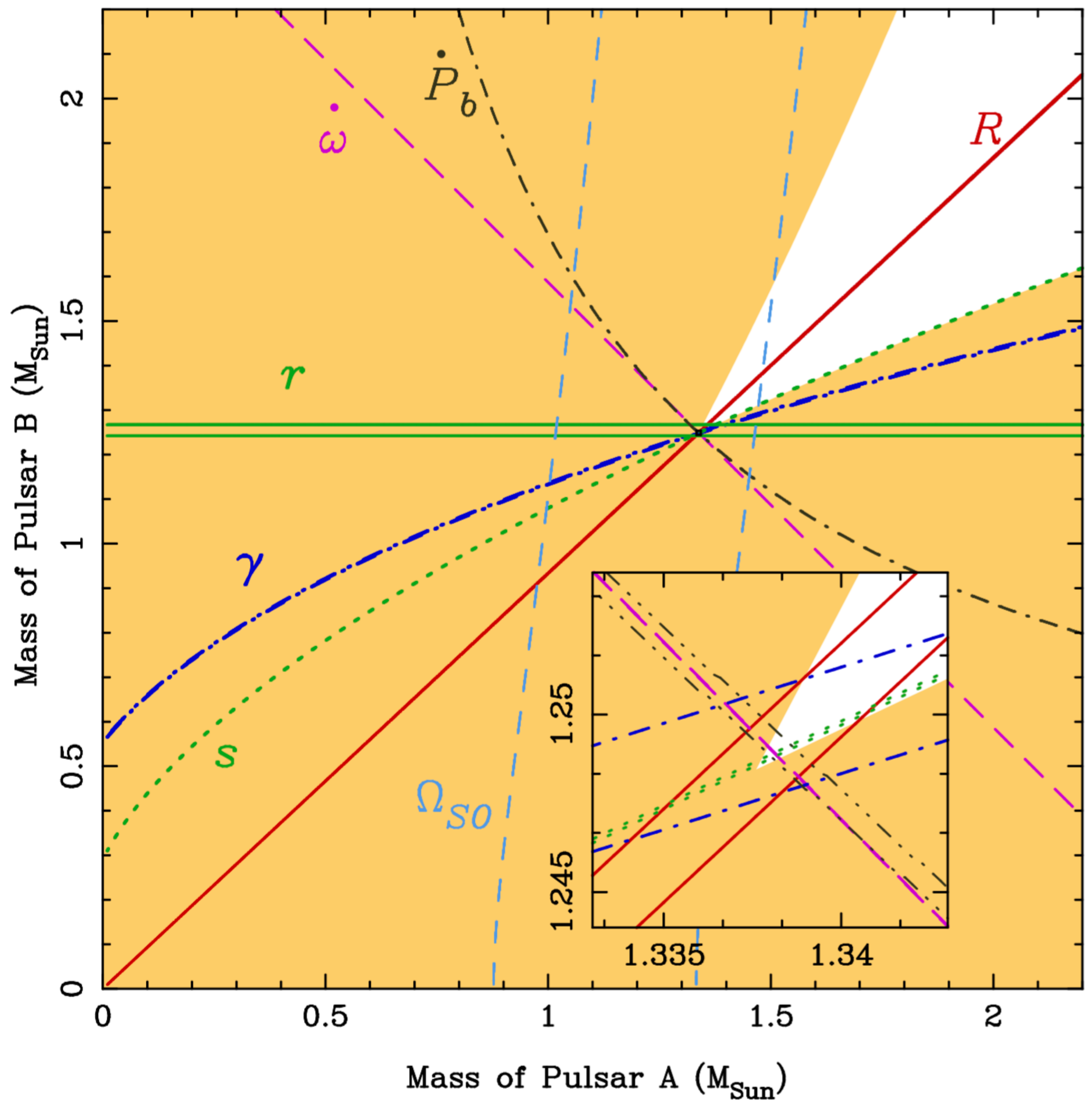


[LVC 2022]

For comparison: pulsar-timing tests of GR concern physical parameters, but also have weak interpretations

PSR B1913+16
[Weisberg 2003]





PSR J0737-3039
[Kramer 2016]

$$\dot{P}_{\rm b} = -\frac{192\pi}{5}\left(\frac{P_{\rm b}}{2\pi}\right)^{-5/3}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)(1-e^2)^{-7/2}\,T_\odot^{5/3}\,m_1 m_2\,M^{-1/3}$$

$$\dot{\omega} = 3\left(\frac{P_{\rm b}}{2\pi}\right)^{-5/3}(T_\odot M)^{2/3}\,(1-e^2)^{-1}, \qquad r = T_\odot\, m_2,$$

$$\gamma = e\left(\frac{P_{\rm b}}{2\pi}\right)^{1/3} T_\odot^{2/3} M^{-4/3}\, m_2(m_1+2m_2), \quad s = x\left(\frac{P_{\rm b}}{2\pi}\right)^{-2/3} T_\odot^{-1/3}\, M^{2/3}\, m_2^{-1}.$$

Testing GR with GW observations

- **Consistency**: a useful sanity check, but hard to interpret statistically. P values are possible with much work. But would we ever believe an inconsistent result?
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GW observations constrain modified gravity very effectively. Starting from the most general Lorentz-invariant scalar-tensor theory with 2nd-order equations of motion (Horndeski) + terms that avoid Ostrogradski instabilities...

$$\mathcal{L} = G_4(\phi, \cancel{X})R + G_2(\phi, X) + \cancel{G_3(\phi, X)\Box\phi} \qquad \Box\phi \equiv \phi^{;\mu}_{;\mu} \quad X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}$$

$$\cancel{-2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - (\phi_{;\mu\nu})^2\right]}$$

$$\cancel{+ G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right]}$$

$$\cancel{- F_4(\phi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}}$$

$$\cancel{- F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu\nu'}\phi_{;\rho\rho'}\phi_{;\sigma\sigma'}}$$

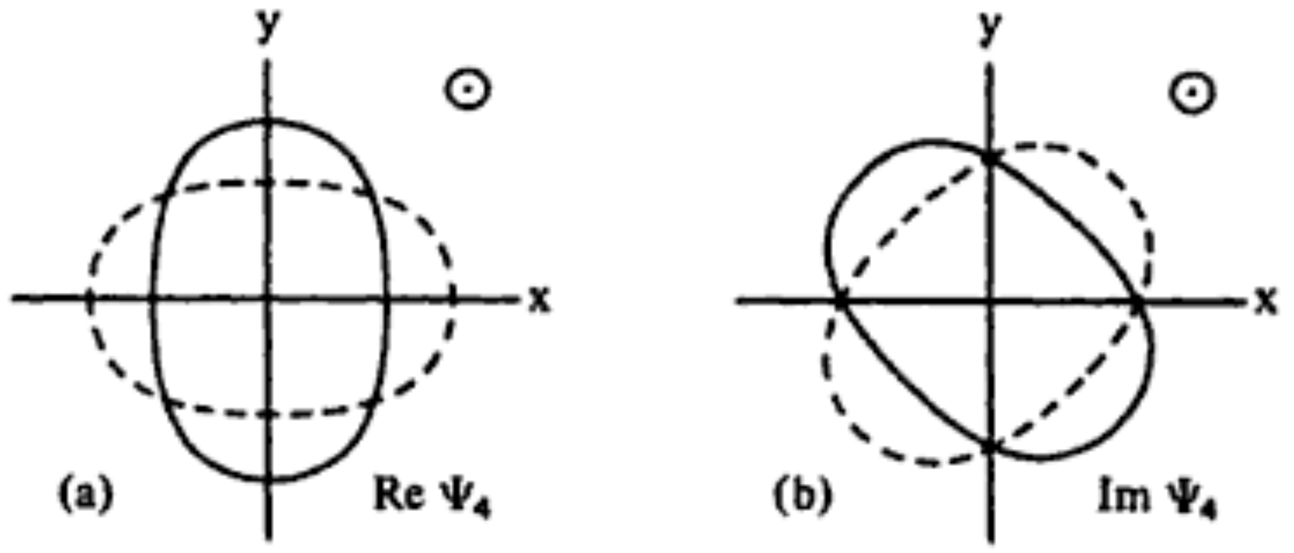
LIGO–Virgo observations

- Speed of GW $|c_T - 1| \lesssim 10^{-15}$
- Perturbative decay and dispersion $|\alpha_H| \lesssim 10^{-10}$
- Resonant graviton decay $10^{-10} \lesssim |\alpha_H| \lesssim 10^{-20}$
- Instabilities due to GW $|\alpha_B| \lesssim 10^{-2}$

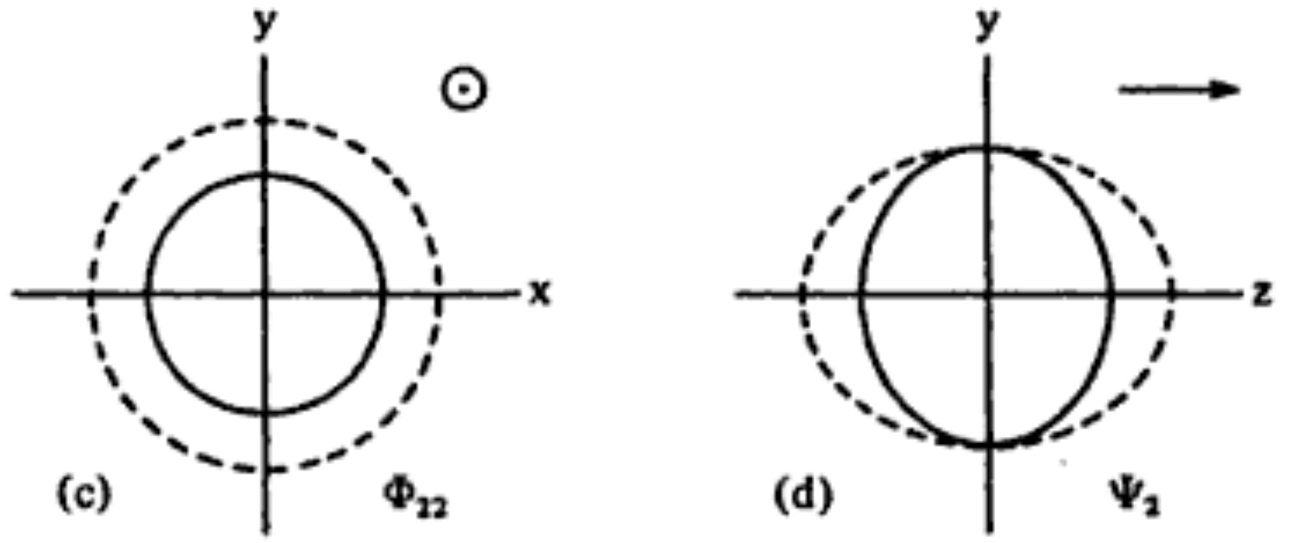
[Vernizzi]

Alternative theories: polarization

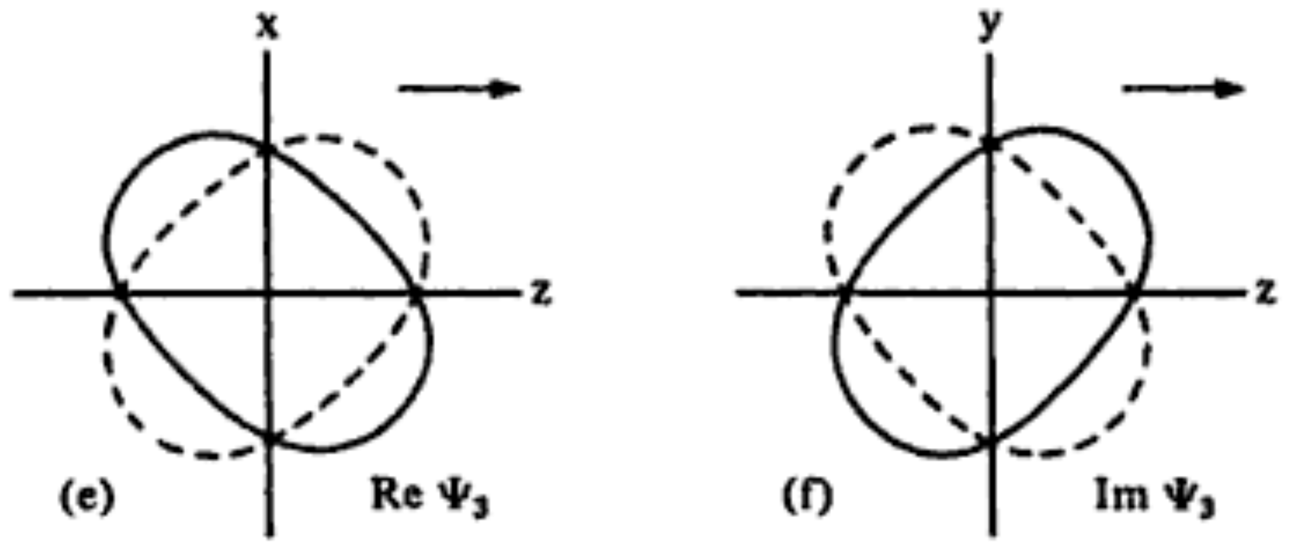
tensor



scalar



vector



Hypothesis	Description	# of basis modes	Mode(s)	Basis mode(s)	Free parameters
$\mathcal{H}_{\mathrm{T},1}$	Pure tensorial	1	$+, \times$	$+$	5
$\mathcal{H}_{\mathrm{V},1}$	Pure vectorial	1	x, y	x	5
$\mathcal{H}_{\mathrm{S},1}$	Pure scalar	1	b	b	2
$\mathcal{H}_{\mathrm{TS},1}$	Tensor–scalar	1	$+, \times, b, l$	$+$	9
$\mathcal{H}_{\mathrm{TV},1}$	Tensor–vector	1	$+, \times, x, y$	$+$	9
$\mathcal{H}_{\mathrm{VS},1}$	Vector–scalar	1	x, y, b, l	x	9
$\mathcal{H}_{\mathrm{TVS},1}$	Tensor–vector–scalar	1	$+, \times, b, l, x, y$	$+$	13
$\mathcal{H}_{\mathrm{T},2}$	Pure tensorial	2	$+, \times$	$+, \times$	2
$\mathcal{H}_{\mathrm{V},2}$	Pure vectorial	2	x, y	x, y	2
$\mathcal{H}_{\mathrm{TS},2}$	Tensor–scalar	2	$+, \times, b, l$	$+, b$	11
$\mathcal{H}_{\mathrm{TV},2}$	Tensor–vector	2	$+, \times, x, y$	$+, x$	11
$\mathcal{H}_{\mathrm{VS},2}$	Vector–scalar	2	x, y, b, l	x, b	11
$\mathcal{H}_{\mathrm{TVS},2}$	Tensor–vector–scalar	2	$+, \times, b, l, x, y$	$+, b$	19

Events	$\log_{10} \mathcal{B}^{\mathrm{S}}_{\mathrm{T}}$	$\log_{10} \mathcal{B}^{\mathrm{V}}_{\mathrm{T}}$	$\log_{10} \mathcal{B}^{\mathrm{TS}}_{\mathrm{T}}$	$\log_{10} \mathcal{B}^{\mathrm{TV}}_{\mathrm{T}}$	$\log_{10} \mathcal{B}^{\mathrm{VS}}_{\mathrm{T}}$	$\log_{10} \mathcal{B}^{\mathrm{TVS}}_{\mathrm{T}}$
O1	-0.04 ± 0.07	0.09 ± 0.07	0.04 ± 0.07	0.09 ± 0.07	0.09 ± 0.07	0.07 ± 0.07
O2	-0.42 ± 0.12	0.04 ± 0.12	0.08 ± 0.12	0.22 ± 0.12	0.09 ± 0.12	0.35 ± 0.12
O3a	-1.85 ± 0.21	-1.04 ± 0.20	0.25 ± 0.20	0.07 ± 0.20	-1.05 ± 0.20	-0.18 ± 0.20
O3b	-1.93 ± 0.17	-0.79 ± 0.17	-0.17 ± 0.17	-0.07 ± 0.17	-0.86 ± 0.17	-0.32 ± 0.17
Combined	-4.24 ± 0.30	-1.70 ± 0.30	0.20 ± 0.30	0.31 ± 0.30	-1.73 ± 0.30	-0.08 ± 0.30

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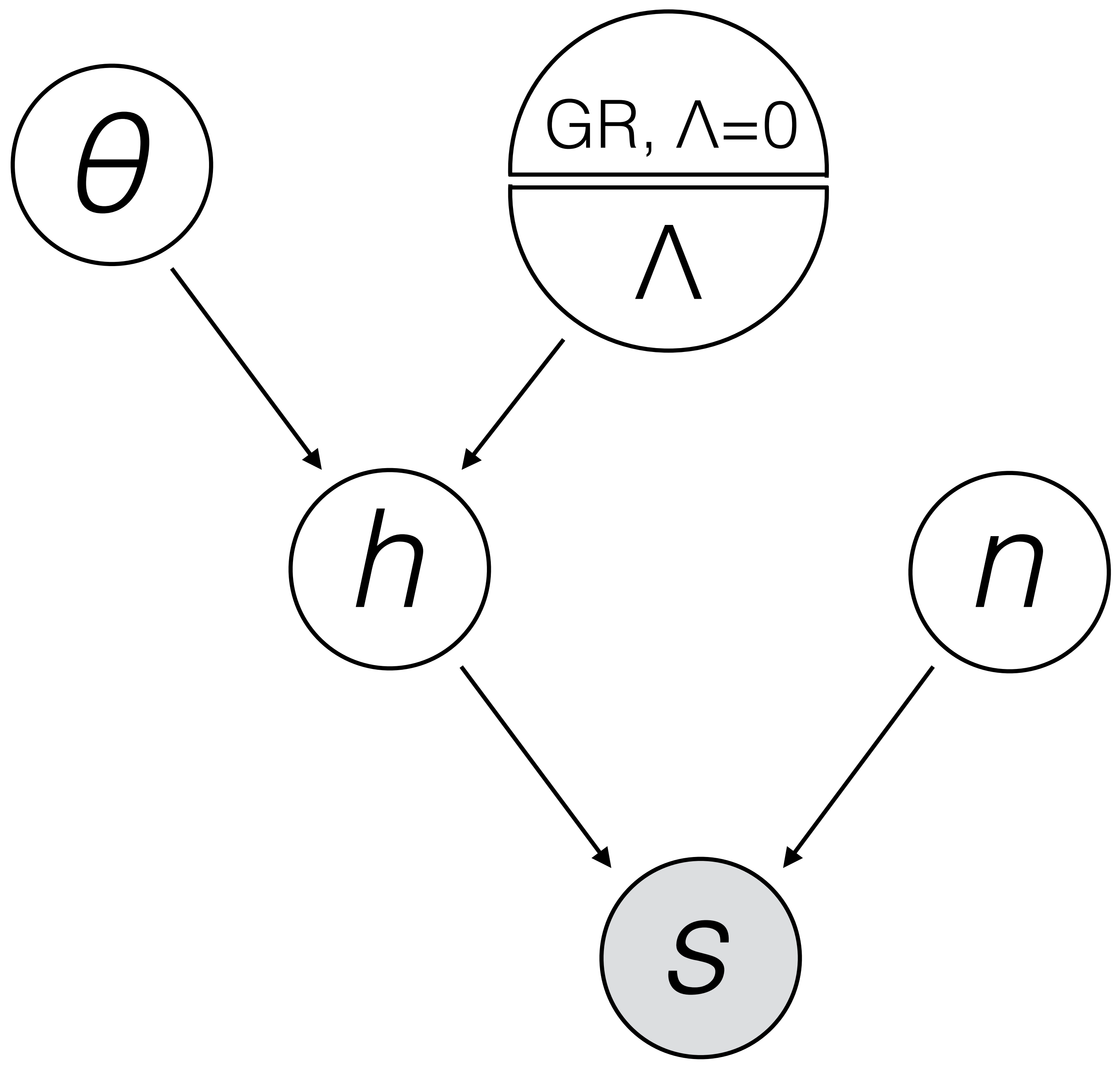
- **Size of effects**: detection SNR determines the magnitude of detectable waveform anomalies. 1% for LIGO–Virgo, up to 10^{-5} for LISA and future ground-based detectors.

How well can we do? [MV PRD 86, 2021]

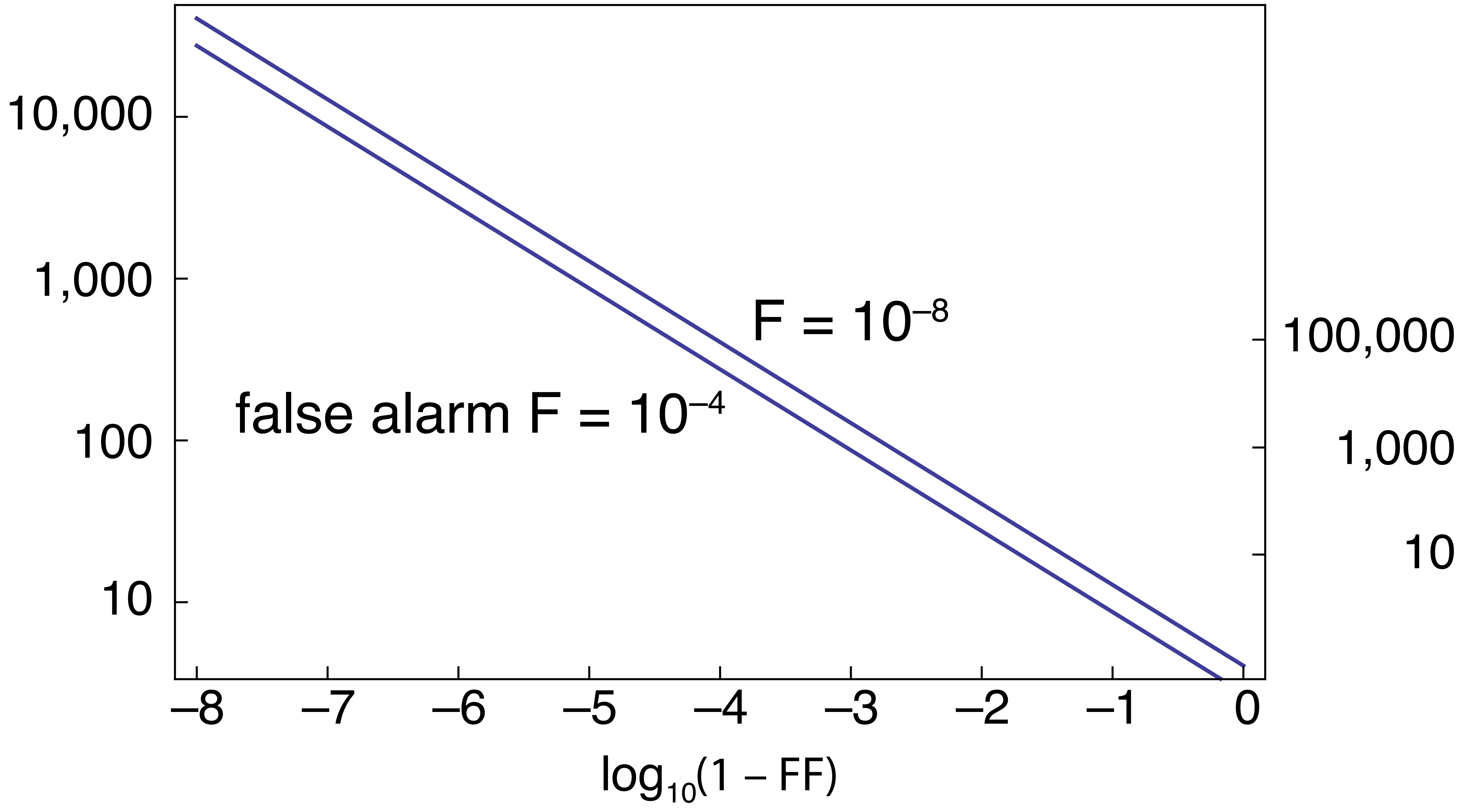


new physics follows from establishing an **anomaly**: we need to obtain convincing evidence that the data prefers an alternative theory of gravity over GR

How well can we do? [MV PRD 86, 2021]

- For a fixed false-alarm rate, we ask what **SNR** is needed to detect AG with 50% probability as a function of **fitting factor FF**, using the Bayesian odds ratio as “detection” statistic.

SNR
to detect AG



events
to yield SNR

Testing GR with GW observations

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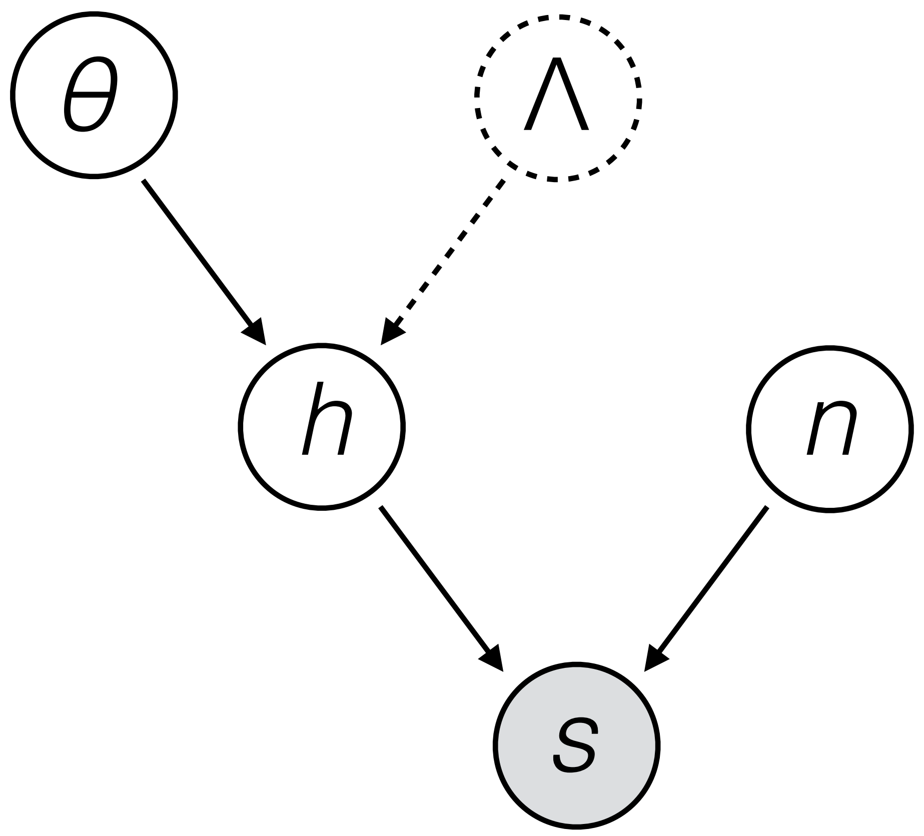
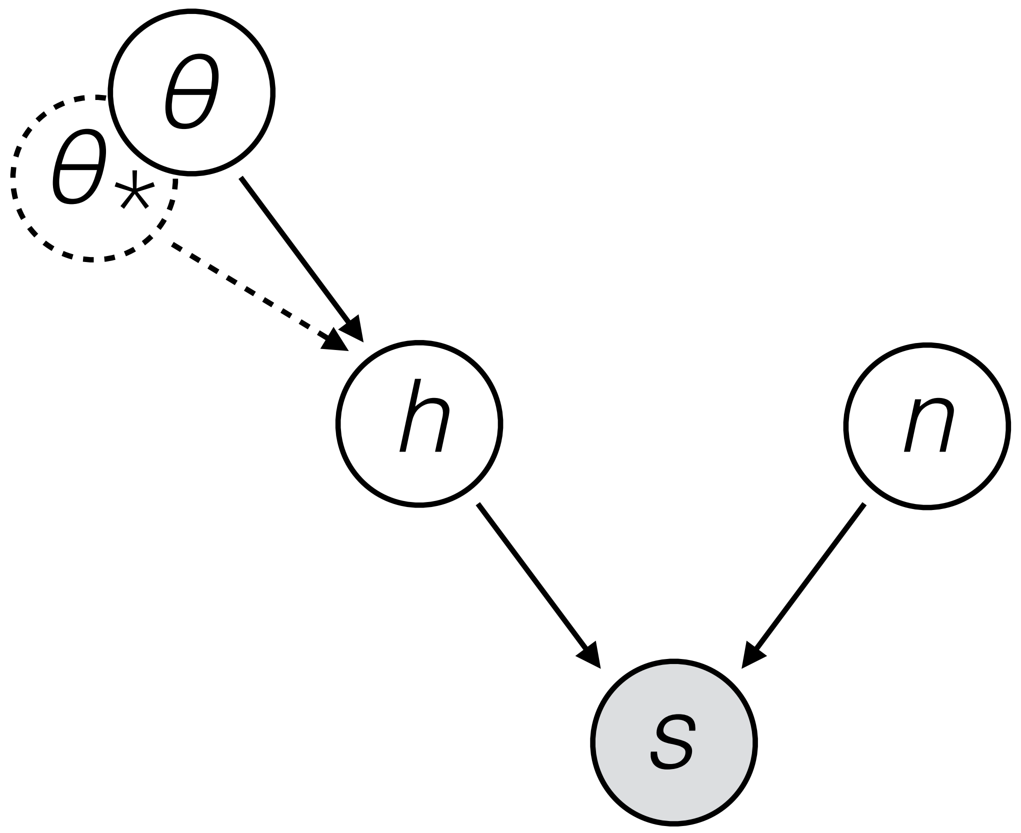
- **Systematics**: beyond statistical-significance arguments, we will need a solid chain of evidence before we claim GR is violated.

Testing GR with GW observations

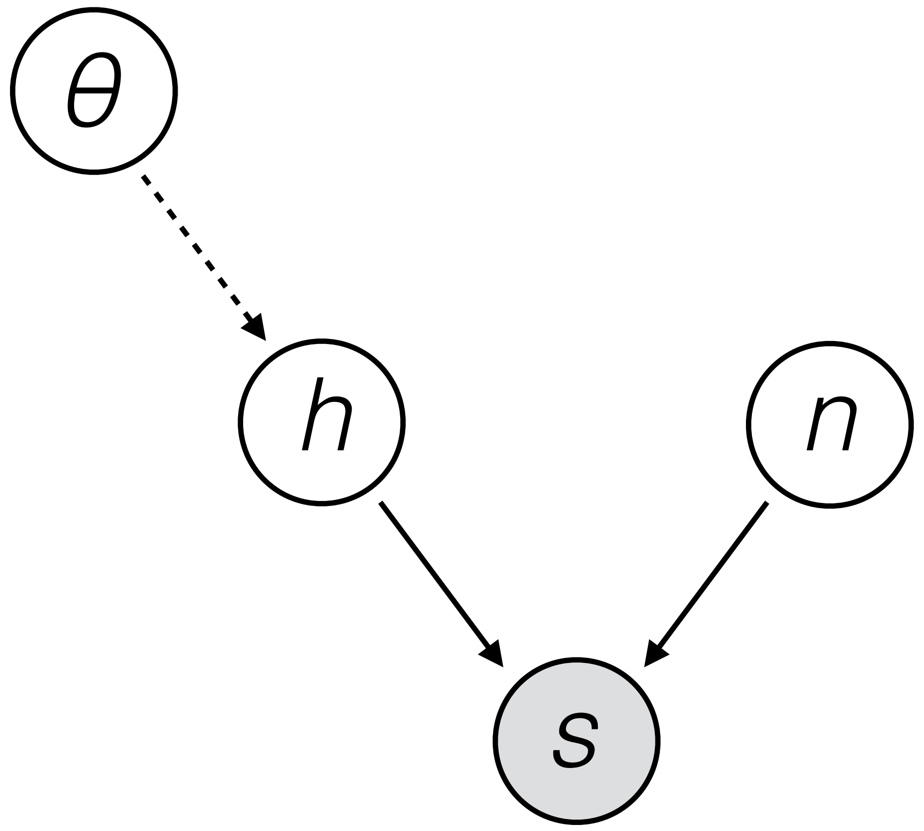
fundamental bias (Yunes)



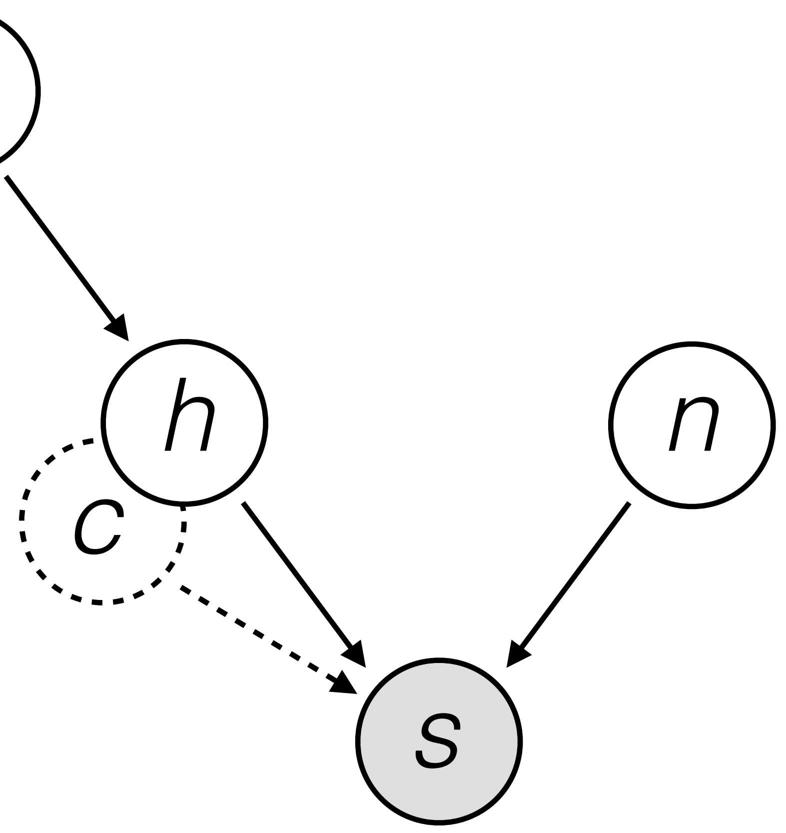
astrophysical bias



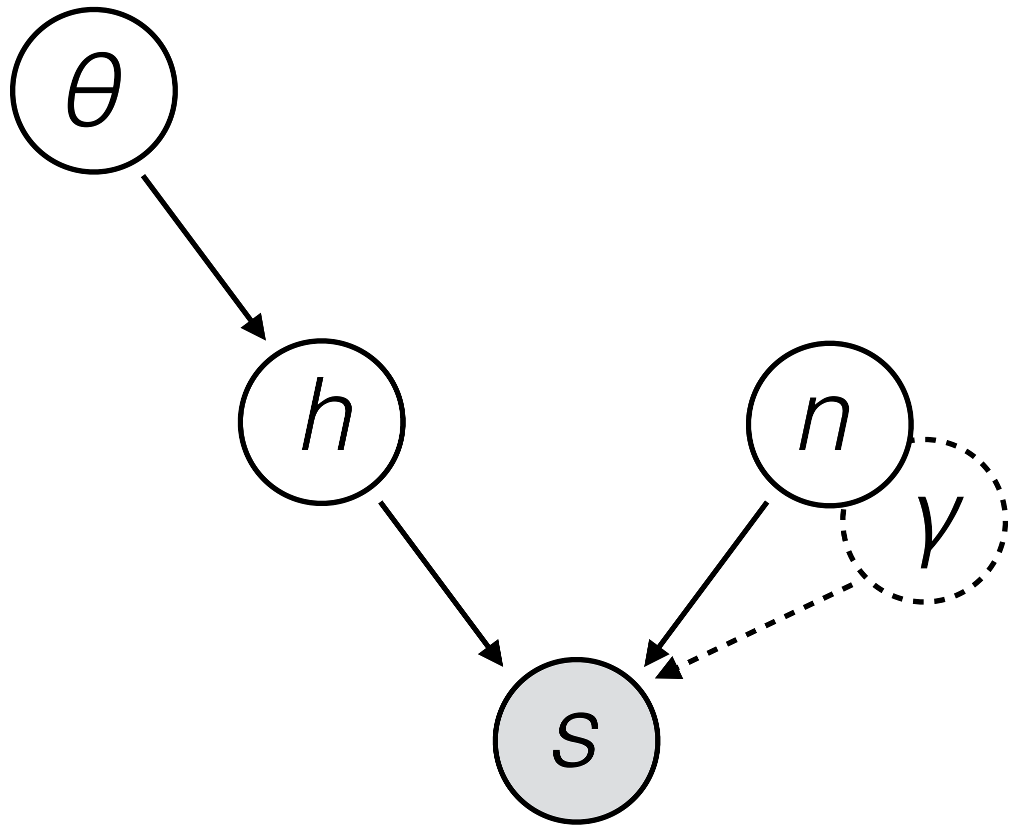
mismodeling



calibration



glitches



Gravitational waves in a nutshell—4

PN/NRGR theory. Tests of GR.

Michele Vallisneri, Jet Propulsion Laboratory

CEA/IPhT, June 17 2022