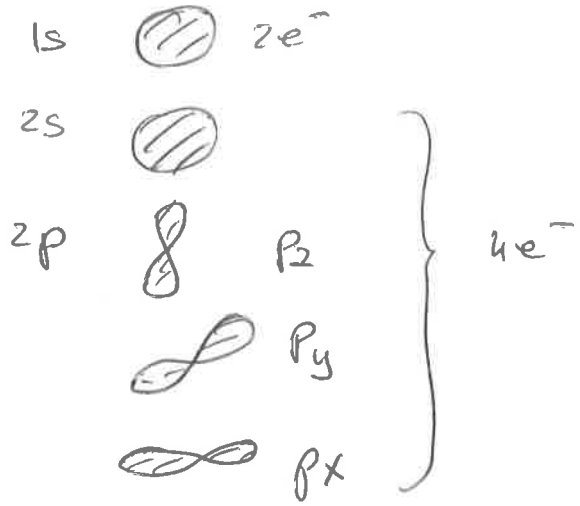
Tight-binding model.

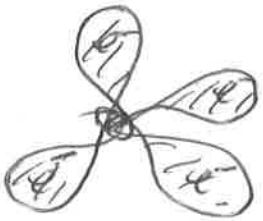
→ atomic orbitals

C_6^{12} $1s^2\ 2s^2\ 2p^2$



hybridisation sp^3

$2s + 2p_x + 2p_y + 2p_z \rightarrow 4\,sp^3$



diamond.

1e⁻/orbital sp^3

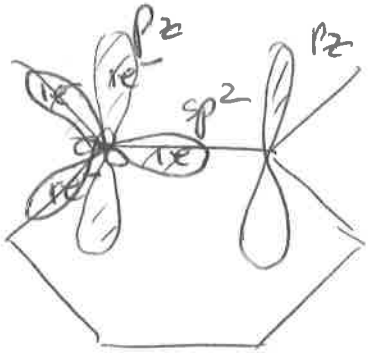
covalent bonds

tetrahedral structure, very stable.

hybridization sp^2

$\underbrace{2s + 2p_x + 2p_y}_{3sp^2} + \underbrace{p_z}_{1e^-}$

$3e^-$



↓

covalent bonding forming hexagonal graphene lattice.

$1e^-$ in $1\,p_z$ orbital → free to move.

p_z e^- → Kinetic term → hopping

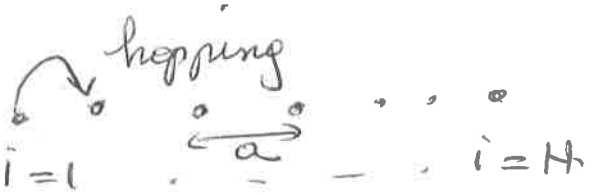
amplitude of hopping ∝ p_z overlap between two atoms.

quantum tunnelling

hopping – second quantization formalism.

c_i^+ / c_i creates / annihilates e^- on site i

Simplest hopping H. → chain



$c_i^+ c_{i+1}$.

$$H = -t \sum_i c_i^+ c_{i+1} + h.c.$$

$$c_i^+ = \int \frac{d\vec{k}}{2\pi} c_k^+ e^{ikR_i} \qquad k \in BZ = \left(-\frac{\pi}{a}, \frac{\pi}{a}\right)$$

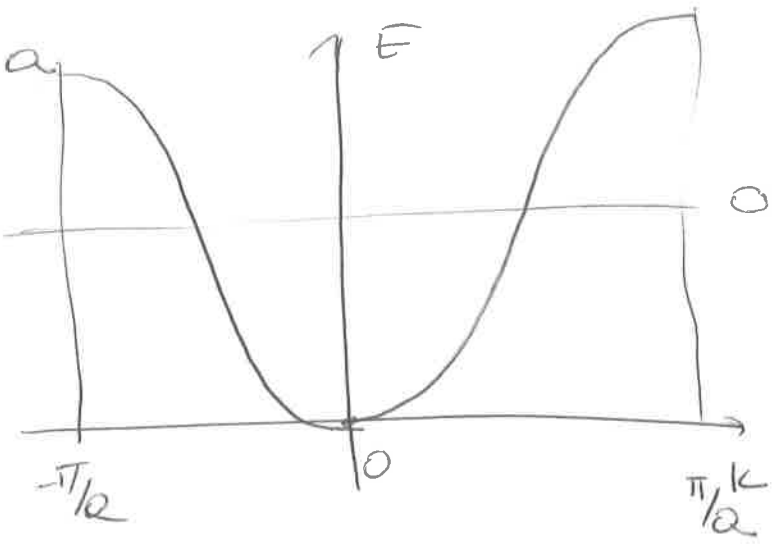
$$H = -t \sum_i \int_{k_1,k_2} e^{ik_1 R_i} e^{-ik_2(R_i+a)} c_{k_1}^+ c_{k_2} + h.c$$

$$\delta(k_1 - k_2)$$

$$= -t \int_k c_k^+ c_k \left(e^{-ika} + h.c\right)$$

$$= -t \int_k 2\cos ka \; c_k^+ c_k.$$

$\Rightarrow \varepsilon_k = -2t \cos ka$



$k \approx 0 \Rightarrow$

$\varepsilon_k = -2t\left(1 + \frac{k^2a^2}{2}\right)$

$= -2t + tk^2a^2$

↓

quadratic dispersion → kinetic term.

Same mechanism in 2D

$$\varepsilon_k = -2t(\cos k_x a + \cos k_y a)$$

graphene – slides 4 – 8.

Majorana states

$$c^+ = \frac{1}{2}(\gamma_1 + i\gamma_2)$$
$$c = \frac{1}{2}(\gamma_1 - i\gamma_2)$$
$$\Rightarrow \gamma_1 = c^+ + c \qquad \gamma_2 = \frac{c^+ - c}{i}$$

$\gamma_1 = \gamma_1^+$ $\gamma_2 = \gamma_2^+$ self adjoint

more generally $\gamma = (c^+ + e^{i\theta} c)\, e^{ie}$

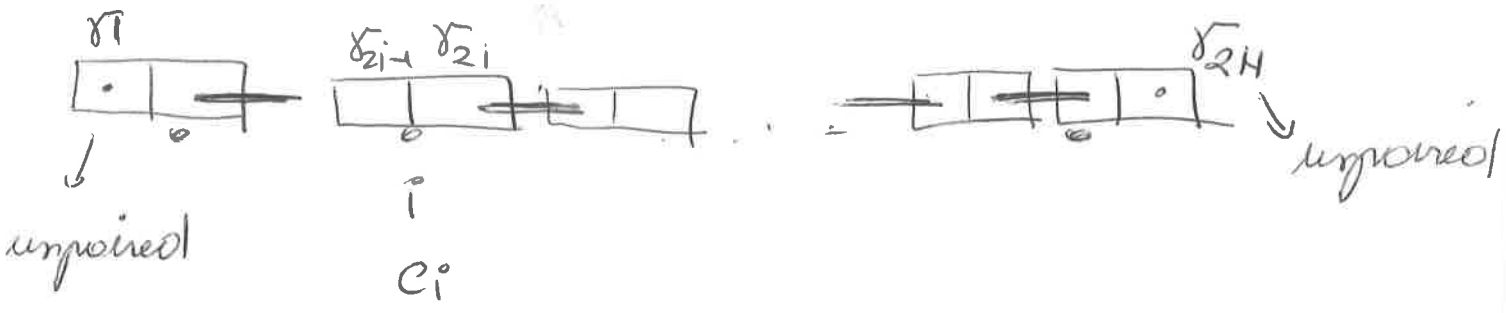
is a Majorana $\gamma^+ = e^{-2ie}\, e^{-i\theta}\, \gamma$

phase factor permitted in $\gamma^+ = \gamma$ condition.

Kitaev modell

$$H = \underbrace{-\mu \sum_i c_i^+ c_i}_{H1} \underbrace{- t \sum_i (c_{i+1}^+ c_i + h.c.)}_{H2}$$
$$+ \Delta \sum_i \underbrace{c_i c_{i+1}}_{H3} + h.c.$$

$\Delta = t$, $\mu = 0 \Rightarrow H = it \sum_i \gamma_{2i}\, \gamma_{2i+1}$



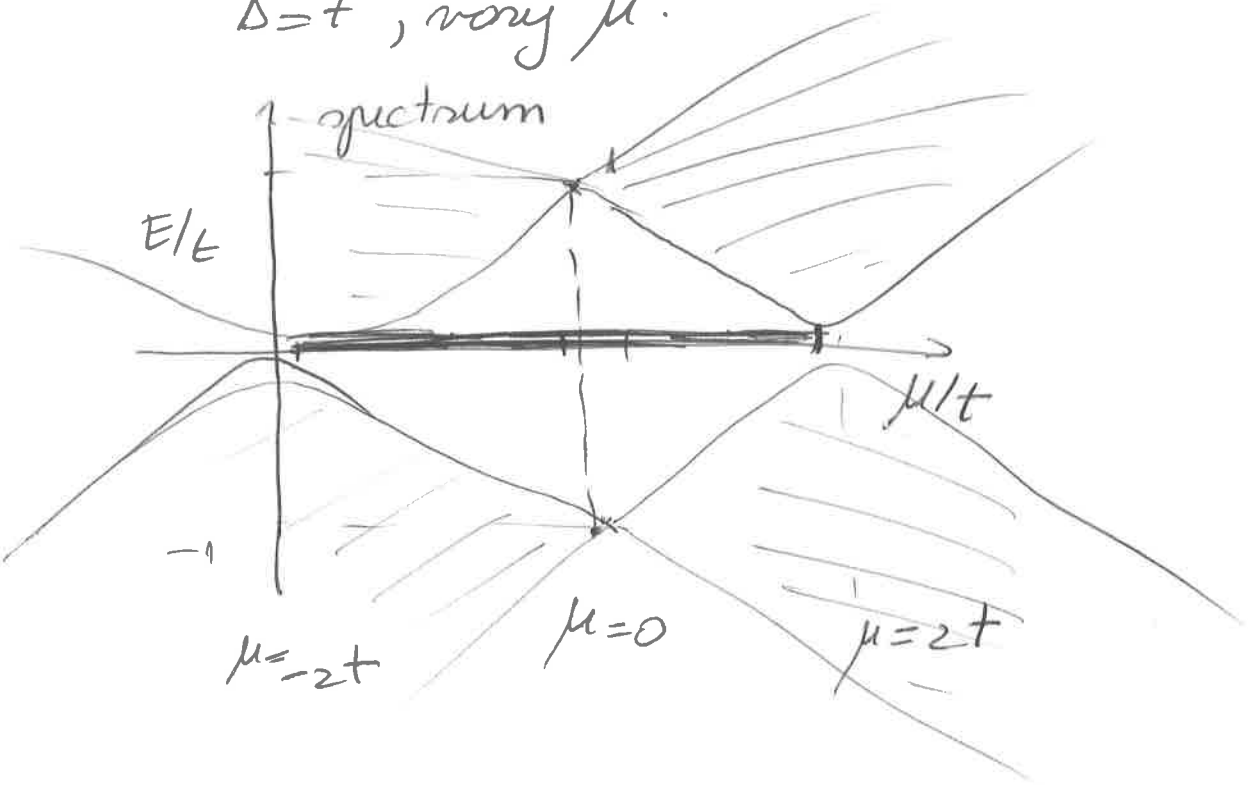
$\gamma_{2i-1} = c_i^+ + c_i \qquad \gamma_{2i} = -i(c_i^+ - c_i)$

Hamiltonian with free Majorana solutions at the ends.

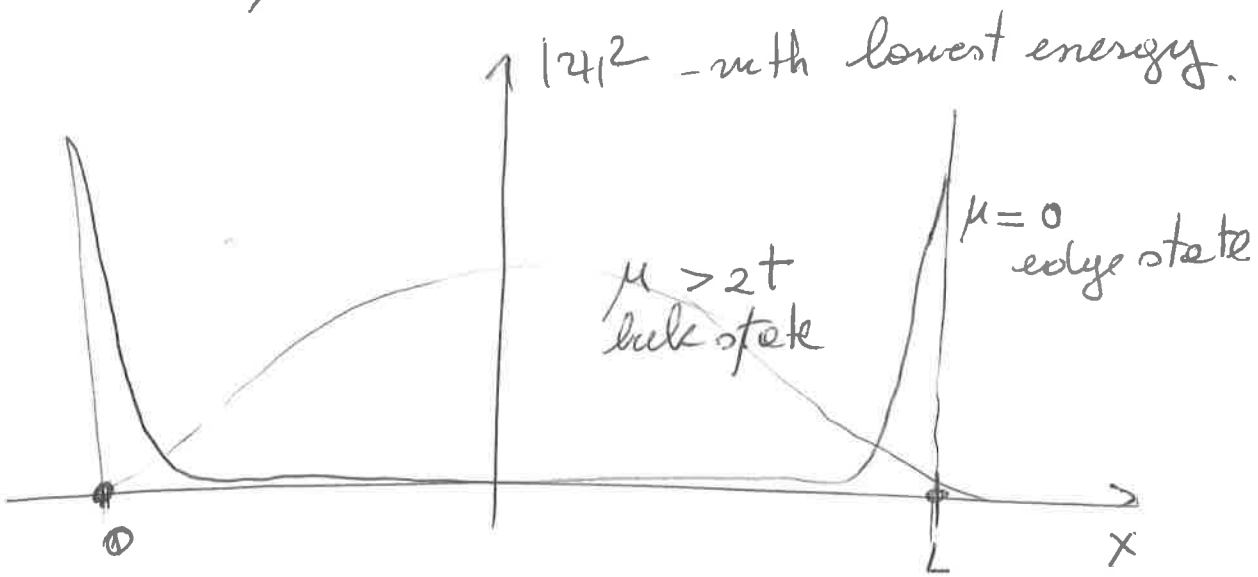
How can we see this?

1) Real space tight-binding numerical analysis

$\Delta = t$, vary μ.



$-2t < \mu < 2t \Rightarrow$ zero energy solutions.



Bulk (infinite system) momentum space analysis

- zero energy states appear in finite size systems
- in infinite systems they are not present
  → bulk band structure., k good quantum number.

$$c_k = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{-ikR_j} c_j$$

$$c_j = \int_{-\pi/a}^{\pi/a} \frac{dk}{2\pi} c_k e^{ikR_j}$$

$$\Rightarrow H = \int \frac{dk}{2\pi} (-2t \cos k - \mu) c_k^+ c_k \qquad (a=1)$$

$$+ 2\Delta i \sin k \; c_k^+ c_{-k}^+ + h.c.$$

rewrite H in a double basis, e^-/hole

$$\begin{pmatrix} c_k^+ & c_{-k} \end{pmatrix} \begin{pmatrix} H \end{pmatrix} \begin{pmatrix} c_k \\ c_{-k}^+ \end{pmatrix}$$

BdG → Bogoliubov de Gennes

$$H_1 = -\mu \sum_j \int_{k_1,k_2} c_{k_1}^+ c_{k_2} \underbrace{e^{-ik_1R_j} e^{ik_2R_j}}_{\delta_{k_1,k_2}} = -\mu \int_k c_k^+ c_k$$

$$2H_1 = -\mu \int_k c_k^+ c_k \; -\mu \int_k c_{-k}^+ c_{-k} =$$

$$-\mu \int_k c_k^+ c_k + \mu \int_k c_{-k} c_{-k}^+ \quad = (c_k^+ c_{-k}) \begin{pmatrix} -\mu & 0 \\ 0 & \mu \end{pmatrix} \begin{pmatrix} c_k \\ c_{-k}^+ \end{pmatrix}$$

$$H_2 = -t \sum_j \int_{k_1 k_2} c_{k_1}^+ c_{k_2} \underbrace{e^{-ik_1(R_j+1)} e^{ik_2 R_j}}_{\delta_{k_1,k_2}} \cdot$$

$$= -2t \int_k c_k^+ c_k \cos k_x \, .$$

$$2H_2 = (c_k^+ \; c_{-k}) \begin{pmatrix} -2t & 0 \\ 0 & 2t \end{pmatrix} \begin{pmatrix} c_k \\ c_{-k}^+ \end{pmatrix}$$

$$H_3 = \Delta \sum_j \int_{k_1 k_2} c_{k_1} e^{ik_1 R_j} \underbrace{c_{k_2} e^{ik_2(R_j+1)}}_{\delta(k_1,-k_2)} + h.c$$

$$= \Delta \int_k c_k c_{-k} e^{ik_x} + c_{-k}^+ c_k^+ e^{-ik_x}$$

$$2H_3 = \Delta \int_k c_k c_{-k} e^{ik_x} + \Delta \int_k c_{-k} c_k e^{-ik_x}$$

$$+ \Delta \int_k c_k^+ c_{-k}^+ e^{-ik_x} + \Delta \int_k c_{k}^+ c_{-k}^+ e^{ik_x}$$

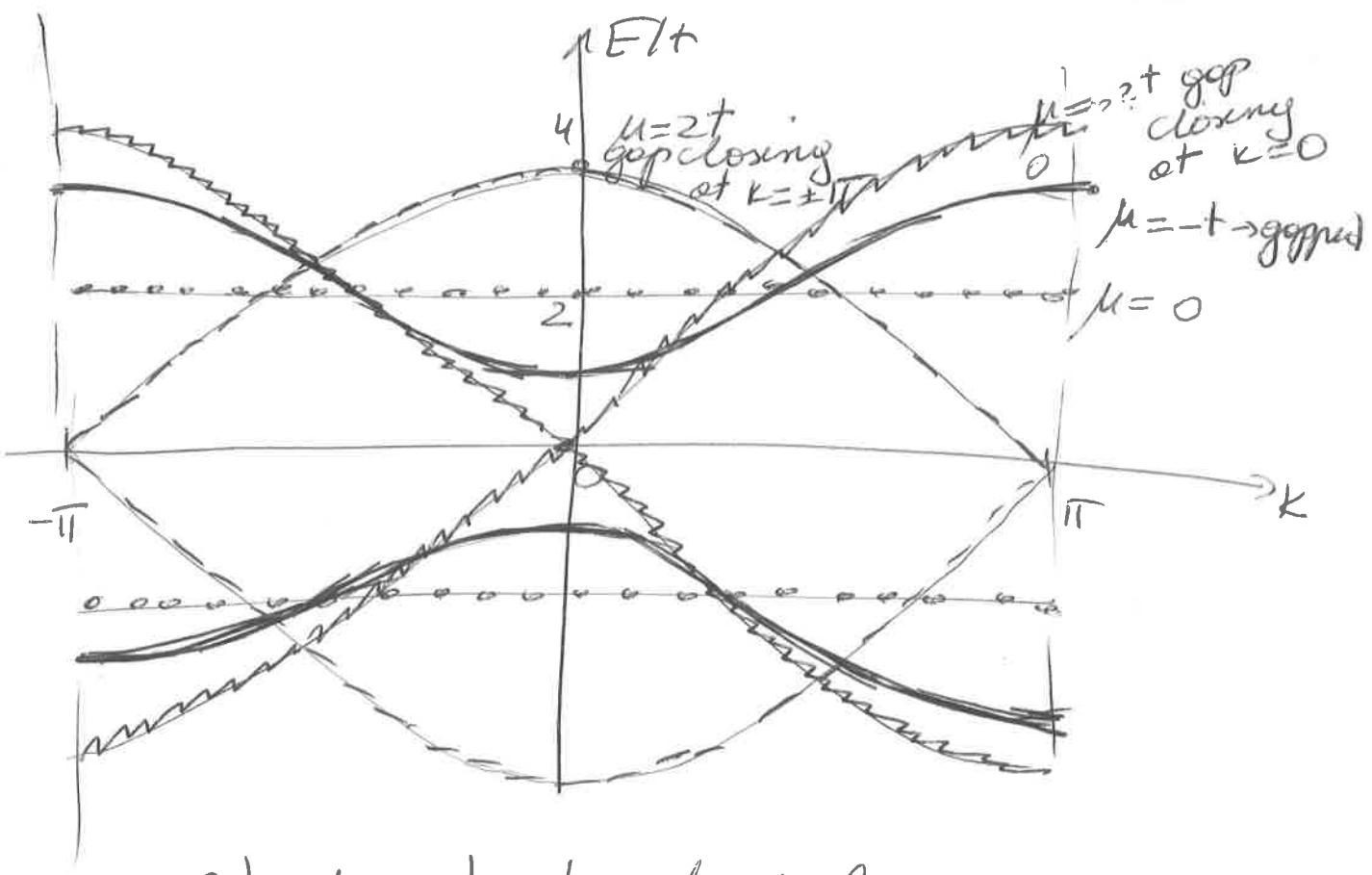
$$= \Delta \int_k c_{-k} c_k (-2i \sin k) + \Delta \int_k 2i \sin k \, c_k^+ c_{-k}^+$$

$$= (c_k^+ \quad c_{-k}) \begin{pmatrix} 0 & 2i \sin k \\ -2i \sin k & 0 \end{pmatrix} \begin{pmatrix} c_k \\ c_{-k}^+ \end{pmatrix}$$

$$\Rightarrow 2H(k) = (-2t \cos k - \mu)\tau_z + 2\Delta \sin k \, \tau_y.$$

Bond structure

$$E(k) = \pm \sqrt{(2t \cos k + \mu)^2 + 4\Delta^2 \sin^2 k}.$$

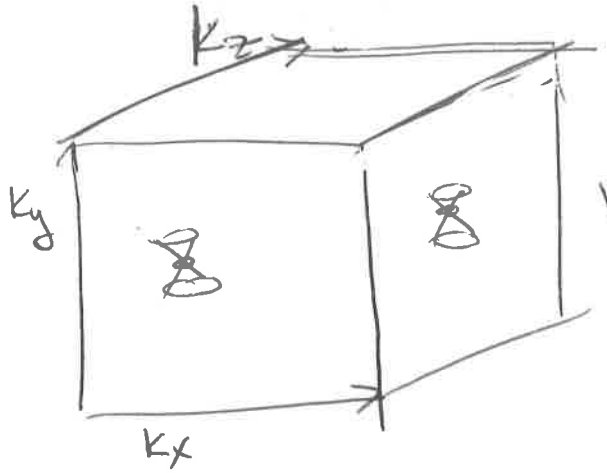


$-2t < \mu < 2t$ topological gap

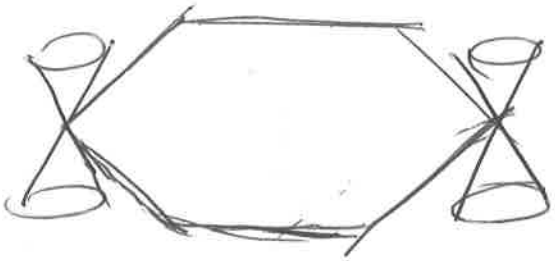
$|\mu| > 2t$ → trivial gap.

Weyl semimetals

- bulk Hamiltonian → analog to Dirac graphene.

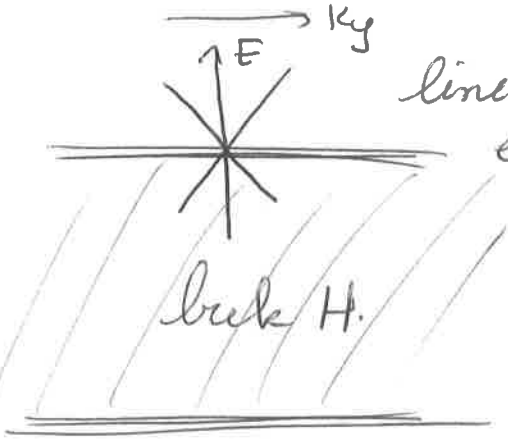


Weyl.
↓
Dirac-like points
Weyl → 3D
linear dispersion



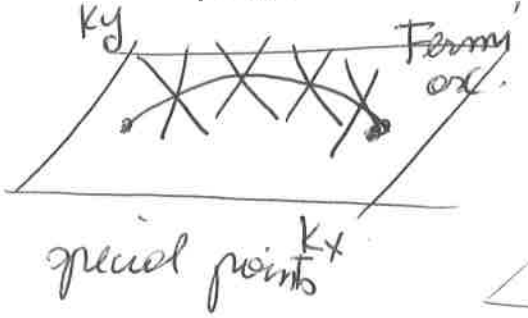
→ graphene
Dirac points → 2D

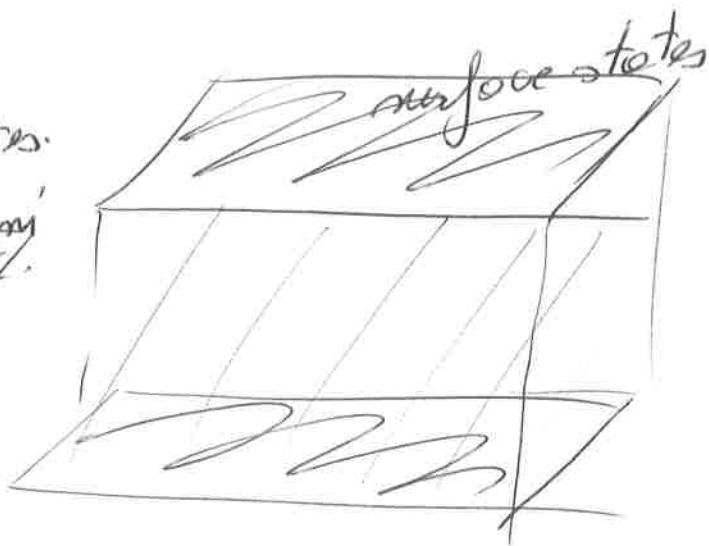
2D edge states.



linear dispersion
edge states.

3D edge states
Fermi arc states.





Weyl.

· Edge states derivation techniques.

1. Tight-binding model → numerical approach.

$$H = -\sum_{\langle ij \rangle} t_{ij}\, c_i^+ c_j + h.c. \rightarrow \text{matrix form.}$$

basis $\quad c_1^+ \; c_2^+ \; \ldots \; c_N^+$

chain with open BC.

sites $\quad \overset{1}{\cdot} \quad \overset{2}{\cdot} \quad \ldots \quad \overset{N}{\cdot}$

$\qquad c_1^+ \quad c_2^+ \quad \ldots \quad c_N^+$

$$H = \begin{pmatrix} c_1^+ & \ldots & c_N^+ \end{pmatrix} \begin{pmatrix} & & \\ & H_{N\times N} & \\ & & \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_N \end{pmatrix}$$

$$= \psi^+ \cdot H \cdot \psi$$

ψ^+ = N-dim vector.

H = $N \times N$ matrix.

$$H = \sum_{i=1}^{N} \mu_i c_i^+ c_i - \sum_{\langle ij \rangle} t_{ij}\, c_i^+ c_j + h.c.$$

NN hopping

$\Rightarrow$

$H_{H\times H}$ (matrix form) =

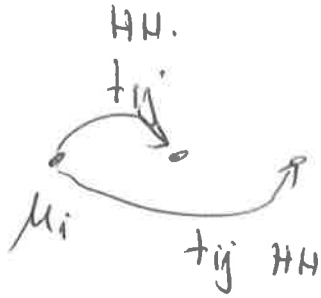
$$\begin{pmatrix} \mu_1 & -t_{12} & & & \\ -t_{12} & \mu_2 & -t_{23} & & 0 \\ & \ddots & \ddots & \ddots & \\ & & -t_{H-2,H-1} & \mu_{H-1} & -t_{H-1,H} \\ 0 & & & -t_{H-1,H} & -\mu_H \end{pmatrix}$$

H_{ij} element = coefficient of $c_i^+ c_j$ term in Hamiltonian

oftentimes $\mu_i = \mu$ (constant)

$t_{ij} = t$ → nearest neighbors.

we can easily add next to nearest neighbors HHH.



$\Rightarrow H_{13} = t_{HHH.}$, etc.

We can also add internal degrees of freedom

→ spin, e^-/hole, etc.

spin $c_i^+ \Rightarrow c_{i\uparrow}^+ \; c_{i\downarrow}^+ \Rightarrow H = H_{2N \times 2N}$ matrix

$$H_{ij} \Rightarrow \begin{matrix} H_{ij}^{\uparrow\uparrow} & H_{ij}^{\uparrow\downarrow} \\ H_{ij}^{\downarrow\uparrow} & H_{ij}^{\downarrow\downarrow} \end{matrix}$$

basis $\Rightarrow (c_{1\uparrow}^+ \; c_{1\downarrow}^+ \; c_{2\uparrow}^+ \; c_{2\downarrow}^+ \; \ldots \; c_{N\uparrow}^+ \; c_{N\downarrow}^+)$

expl: local magnetic field term.

$$H_m = \sum_i V_i \left(c_{i\uparrow}^+ c_{i\uparrow} - c_{i\downarrow}^+ c_{i\downarrow} \right)$$

$$\Rightarrow H_{ii} \xrightarrow{(c_{i\uparrow}^+ \; c_{i\downarrow})} \begin{pmatrix} \mu_i + V_i & 0 \\ 0 & \mu_i - V_i \end{pmatrix} \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{pmatrix}$$

exp2: e^- + hole components → necessary for superconductivity

$$H_\Delta = \sum_i \Delta_i \left(c_{i\uparrow}^+ c_{i\downarrow}^+ + c_{i\downarrow} c_{i\uparrow} \right)$$

$$H_{ii} \rightarrow (c_{i\uparrow}^+ \; c_{i\downarrow}^+ \; c_{i\uparrow} \; c_{i\downarrow}) \begin{pmatrix} \mu_i + V_i & 0 & 0 & \Delta_i \\ 0 & \mu_i - V_i & \Delta_i & 0 \\ 0 & \Delta_i & \mu_i - V_i & 0 \\ \Delta_i & 0 & 0 & \mu_i + V_i \end{pmatrix} \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \\ c_{i\uparrow}^+ \\ c_{i\downarrow}^+ \end{pmatrix}$$

$H \rightarrow H_{4N \times 4N}$ matrix.

exp 3: consider other terms → Rashba

$H_{i,i+1} \sim i\alpha_R c^\dagger_{i\uparrow} c_{i+1\downarrow} + h.c.$

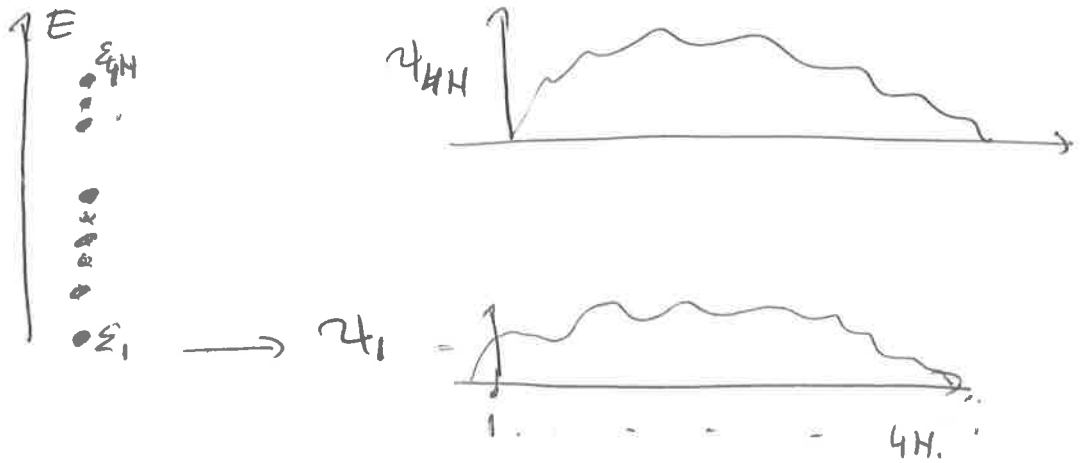
p-wave SC (Kitaev) $H_{i,i+1} \sim i\Delta c^\dagger_{i\uparrow} c^\dagger_{i+1\uparrow} + h.c$

How can we solve this ?

Numerical diagonalization of $H_{N\times N}$, $H_{2N\times 2N}$, $H_{4N\times 4N}$ matrix.

$\Rightarrow$ eigenvalues $\varepsilon_1, \varepsilon_2 \ldots \varepsilon_{4N}$.

eigenstates $\psi_1, \psi_2 \ldots \psi_{4N}$



$\psi_i = \alpha_1, \alpha_2 \ldots \alpha_{4N}$.

$\alpha_{4j}, \alpha_{4j+1}, \alpha_{4j+2}, \alpha_{4j+3}$ $\Rightarrow$ coefficients of.

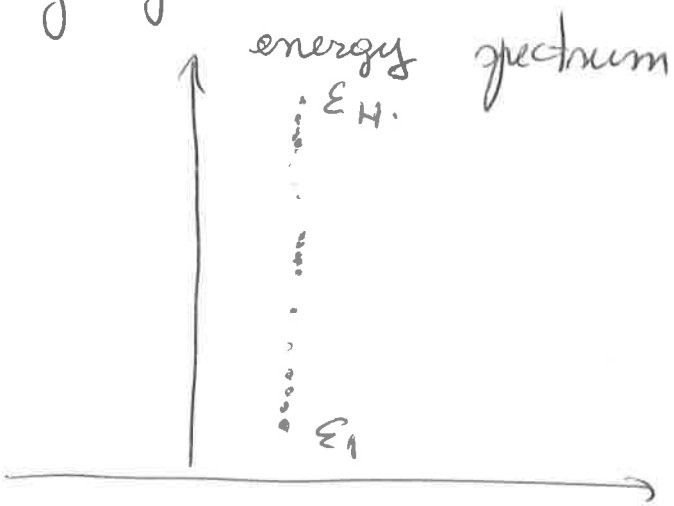
$c^\dagger_{j\uparrow}\, c^\dagger_{j\downarrow}\, c_{j\uparrow}\, c_{j\downarrow}$

=> coefficients of $e_\uparrow$, $e_\downarrow$, $h_\uparrow$, $h_\downarrow$ components on site j

=> we know at each energy how much of $e_\uparrow$, $e_\downarrow$, $h_\uparrow$, $h_\downarrow$ we have at each j

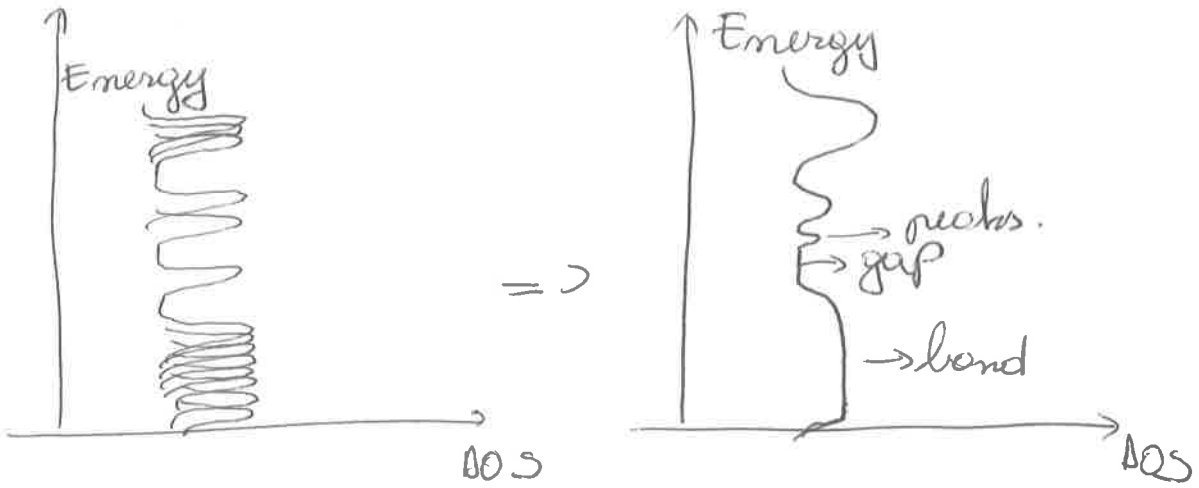
How do we go from eigenvalues / eigenstates to physical properties ?

1. Density of states :



$DOS(\varepsilon) = \sum_{i=1,H.} \delta(\varepsilon - \varepsilon_i)$ in practice each state introduces a broadening

level broadening => inverse quasiparticle lifetime.

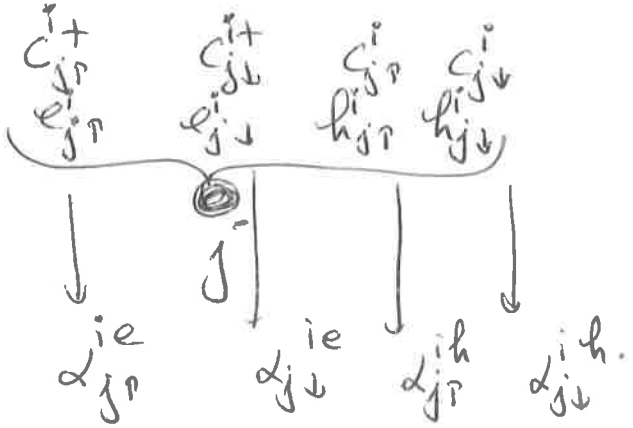


2) local Density of states <u>of electrons</u>

$$LDOS^j(\varepsilon) = \sum_{i=1,4N} \left(|\alpha^{ie}_{j\uparrow}|^2 + |\alpha^{ie}_{j\downarrow}|^2 \right) \delta(\varepsilon - \varepsilon_i)$$

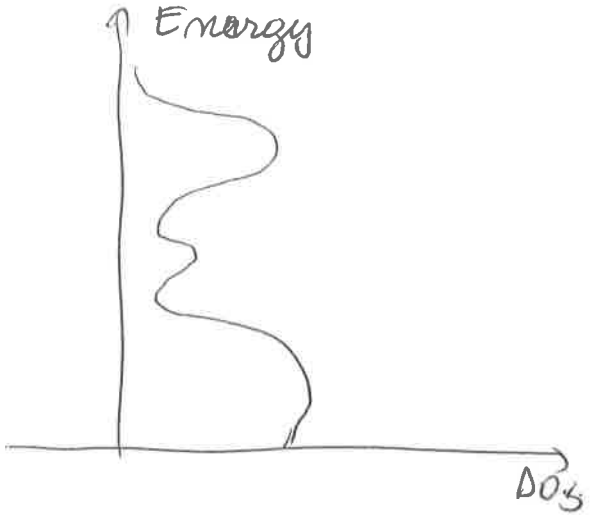
site j → $4N$ eigenstates contributing.

$\varepsilon_1, \ldots \varepsilon_{4N}$ $\quad \psi_i$, $i = 1, \ldots 4N$.

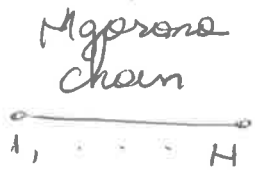


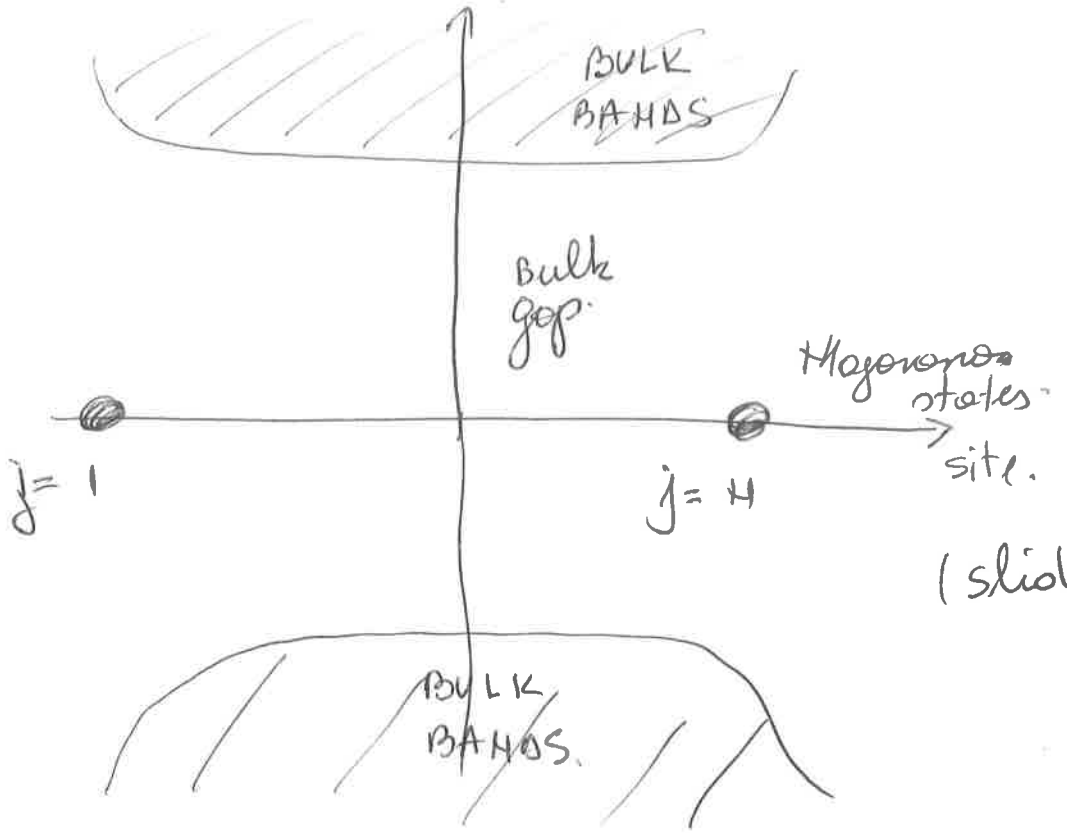
$|\alpha^{ie}_{j\uparrow}|^2$ → density of electrons with spin ↑ at site j coming from the i^{th} energy ε_i

=> at each site j



LDOS → 2D plot in energy and position





(slide 20)

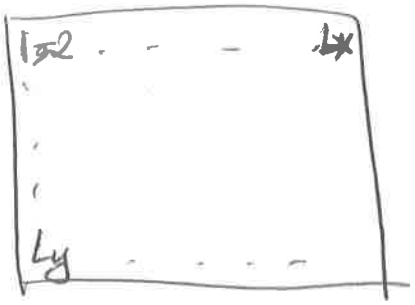
can generalize to any thing

$$\text{Spin}_z = |\psi_\uparrow|^2 - |\psi_\downarrow|^2$$

$$\Rightarrow S_z^j(\varepsilon) = \sum_{i<1,\dots} \left(|\psi_{j\uparrow}^{ie}|^2 - |\psi_{j\downarrow}^{ie}|^2 \right) \delta(\varepsilon - E_i)$$

Tight-binding numerics in 2D

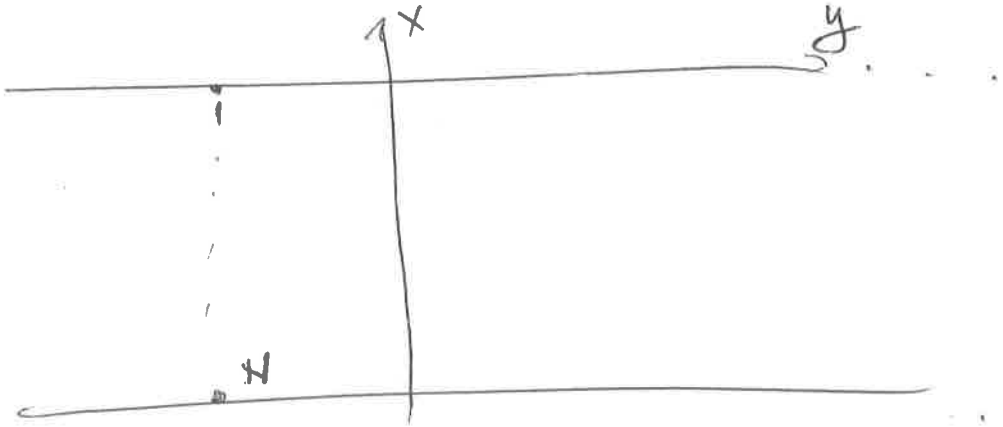
1. open BC



sites $L_x \times L_y$ $\Rightarrow$ Hamiltonian is a $4L_xL_y \times 4L_xL_y$ matrix $\rightarrow$ same procedure as before for chain

$\rightarrow$ time restraints $\rightarrow$ number of atoms

2. periodic BC $\rightarrow$ infinite system.



solution along y $\rightarrow$ plane waves

$$c_{xy} = \int \frac{dk_y}{2\pi} \; c_{x,k_y} \; e^{-ik_y y}$$

partial Fourier transform.

$$H = -t\left[\sum_{x,y} c_{xy}^{+} c_{x+1,y} + c_{xy}^{+} c_{x,y+1} + h.c.\right]$$

$$= -t \sum_{x,y} \int_{k_y} \int_{k_y'} c_{x\,k_y}^{\dagger} c_{x+1\,k_y'} e^{i(k_y - k_y')y}$$

$$+ c_{x\,k_y}^{+} c_{x\,k_y'} e^{i(k_y - k_y')y} e^{-ik_y' a} + h.c.$$

$$= -t \sum_{x} \int_{k_y} \left[c_{x\,k_y}^{\dagger} c_{x+1\,k_y} + c_{x\,k_y}^{\dagger} c_{x\,k_y} \, 2\cos k_y a\right]$$

decompose the system at each k_y.

$$\Rightarrow H_{k_y} = -t \sum_{x} \left[c_{x}^{+} c_{x+1} + c_x^{\dagger} c_x \, 2\cos k_y a\right]$$

$\Downarrow$

1D finite size TB Hamiltonian $\Rightarrow$ apply the same procedure as before.

each k_y $\Rightarrow$ $\begin{cases} E_{k_y}^{i} & i = 1, \ldots 4N. \\ \psi_{k_y}^{i}(x) \end{cases}$ $\Rightarrow$ band structure (Slide 21)

each k_y $\Rightarrow$ spectrum, LDOS, DOS, etc.

② Ma+chung

Kitaev model

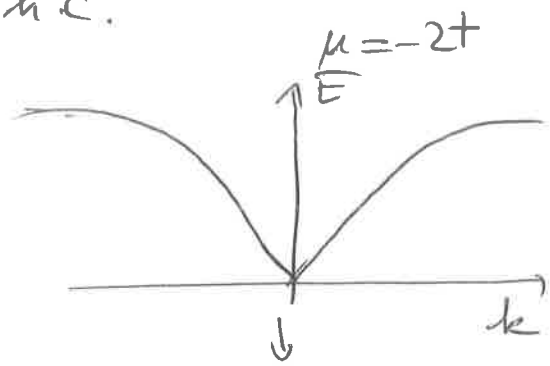
$$H = -\mu \sum_i c_i^+ c_i - t \sum_i c_{i+1}^+ c_i + h.c.$$

$$+ \Delta \sum_i c_i c_{i+1} + h.c.$$

take $\mu = -2t$ $k \approx 0$

expand around $k \approx 0$

low-energy expansion.



linear dispersion close to $k \approx 0$.

$$H(k) = \underbrace{(-2t-\mu)}_{m} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \underbrace{2\Delta}_{v} \begin{pmatrix} 0 & ik \\ -ik & 0 \end{pmatrix}$$

‼

Real space $H = m \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}_{\sigma_z} - v i \partial_x \underbrace{\begin{pmatrix} 0 & 1 \\ -i & 0 \end{pmatrix}}_{\sigma_y}$

in $(c_i^+ c_i)$ basis

Assum $E=0$ solution.

$\Rightarrow H\psi = 0$

$\Rightarrow m\sigma_z\psi - v i \partial_x \sigma_y \psi = 0 \,|\, \sigma_y \Rightarrow m i \sigma_x \psi - v \tfrac{i}{2} \partial_x \psi = 0$

Ansatz $\psi = A e^{zx}$ $\quad A = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

$\Rightarrow m \sigma_x \psi - v I_2 \partial_x \psi = 0$

$m \sigma_x A e^{zx} - v I_2 z A e^{zx} = 0 \Rightarrow$

$$\begin{pmatrix} -vz & m \\ m & -vz \end{pmatrix} A e^{zx} = 0 \quad \Rightarrow$$

possible for $\begin{vmatrix} -vz & m \\ m & -vz \end{vmatrix} = 0 \qquad v^2 z^2 = m^2$

$\Rightarrow z = \pm \frac{m}{v} \qquad x > 0 \quad$ take $z = -\frac{m}{v}$

$$\psi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} e^{-\frac{m}{v}x}$$

$$\begin{pmatrix} m & m \\ m & m \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 0 \quad \Rightarrow \quad \alpha = -\beta$$

Normalization $\int_0^\infty dx |\psi|^2 = 1$

$$= 2|\alpha|^2 \int_0^\infty e^{-\frac{2mx}{v}} = -\frac{v}{2m}\left(e^{-\infty} - 1\right) 2|\alpha|^2$$

$$= 2|\alpha|^2 \frac{v}{2m} = 1 \Rightarrow |\alpha| = \sqrt{\frac{m}{v}}$$

$$\psi = \sqrt{\frac{m}{v}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-\frac{m}{v}x} \rightarrow \psi^+ = \psi : \text{Majorana!}$$

(11)

Some procedure for domain walls $m(x)$, other HS

(3). Bulk topological invariant.

- not fully justified method in the literature.

defn $Q = \prod_{k_i} [\text{Pf } H(k_i)]$

k_i - special points of the BZ, gap closing points

for Kitaev model $0, \pi$

$$H(0) = \begin{pmatrix} -2t-\mu & 0 \\ 0 & 2t+\mu \end{pmatrix}$$

$$H(\pi) = \begin{pmatrix} 2t-\mu & 0 \\ 0 & -2t+\mu \end{pmatrix}$$

to calculate Pfaffian $\propto \sqrt{\text{Det}}$

antisymmetrize.

$$\tilde{H}(0) = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} H(0) \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} = -2i \begin{pmatrix} 0 & -2t-\mu \\ 2t+\mu & 0 \end{pmatrix}$$

$$\tilde{H}(\pi) = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} H(\pi) \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} = -2i \begin{pmatrix} 0 & 2t-\mu \\ -2t+\mu & 0 \end{pmatrix}$$

Pf $\tilde{H}(0) = -2i(-2t-\mu)$

Pf $\tilde{H}(\pi) = -2i(2t-\mu)$

$$\mathrm{Pf}[BAB^T] = \det B \cdot \mathrm{Pf}\, A$$

$$\Rightarrow \mathrm{Pf}\,\tilde{H}(0) = \det \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \mathrm{Pf}\, H(0) \quad = -2i\, \mathrm{Pf}\, H(0)$$

$$\Rightarrow \mathrm{Pf}\, H(0) = -2t - \mu$$

$$\mathrm{Pf}\, \tilde{H}(\pi) = 2t - \mu$$

$Q = (-2t-\mu)(2t-\mu)$ → changes sign at

$\mu = -2t$ and $\mu = 2t$

$Q = 1$ for $\mu < -2t$, $\mu > 2t$ → trivial

$Q = -1$ for $-2t < \mu < 2t$ → topological.

Hard to justify a priori.

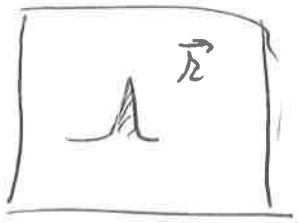
4. T-matrix approximation.

Slides 22, 23

Solve exactly the problem of infinite system + localized impurity

δ-function potential

H_0 = unperturbed H. (e.g. 2D)

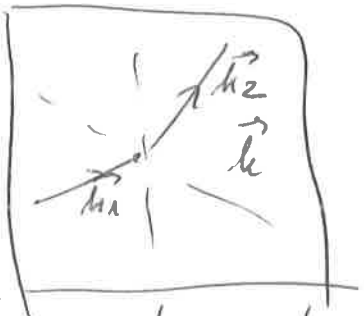


$$H_{imp} = \int d^2\vec{r}\ \bar{V}(\vec{r})\, c^+(r)\, c(r)$$

$\bar{V}(\vec{r}) = \bar{V}\,\delta(\vec{r})$ $\quad \vec{r}=0 \rightarrow$ localized imp

$$H_{imp} = V \int d^2\vec{r} \sum_{k_1,k_2} \delta(\vec{r})\, c^+(k_1)\, c(k_2)\, e^{ik_1 r}\, e^{-ik_2 r}$$

$$= \bar{V} \sum_{k_1,k_2} c^+(k_1)\, c(k_2)$$



all scattering processes are equally possible, scattering is independent of momentum (due to $\delta(\vec{r})$ potential)

Perturbation theory for Green's function, expand all order in $\bar{V}$ term.

imaginary time Green's functions (Matsubara)

$$G(k_1, k_2, i\omega_n) = \langle \mathcal{T}\, c_{k_1}(\omega_n),\, c^+_{k_2}(\omega_n) \rangle$$

Matsubara $\Rightarrow$ retarded Green's function.
(all physical properties)

$$G^M(k_1, k_2, i\omega_m) \xrightarrow{i\omega_m \to \omega + i\sigma} G^R(k_1, k_2, \omega + i\sigma)$$

spectral function $A(k,\omega)$ = number of particles with k, ω

$$A(k,\omega) = -\frac{1}{\pi} \operatorname{Im}\left[G^R(k, k, \omega + i\sigma)\right]$$

DOS = number of particles with ω

$$n(\omega) = \int_k A(k,\omega)$$

For $H(k) = \varepsilon_k c_k^+ c_k \Rightarrow S \approx \frac{\partial}{\partial t} - H \sim (i\omega_n - \varepsilon_k) c_k^+ c_k$

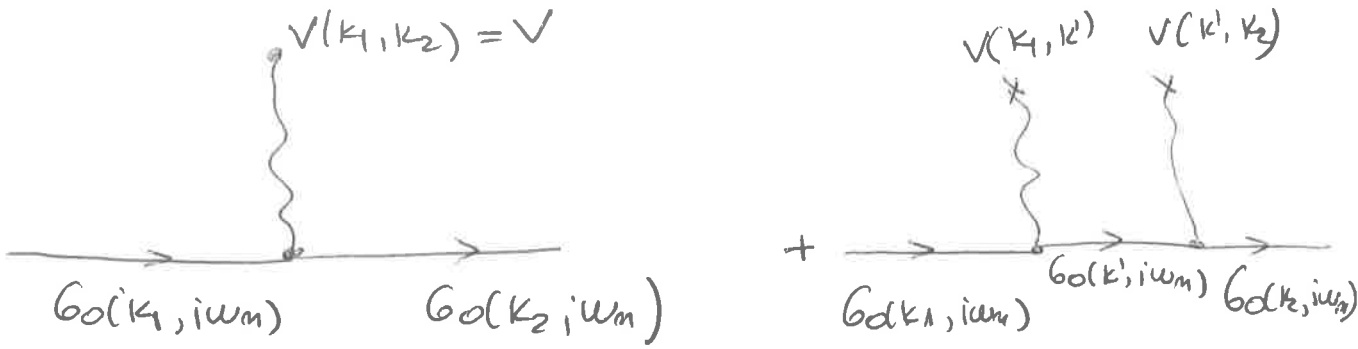
$$G^0(k, i\omega_n) = G^0(k, k, i\omega_n) = \frac{1}{i\omega_n - \varepsilon_k}$$

$k_1 = k_2$ – translational invariance
momentum conservation

$$\xrightarrow{G_0(k, i\omega_n)}$$

Scattering

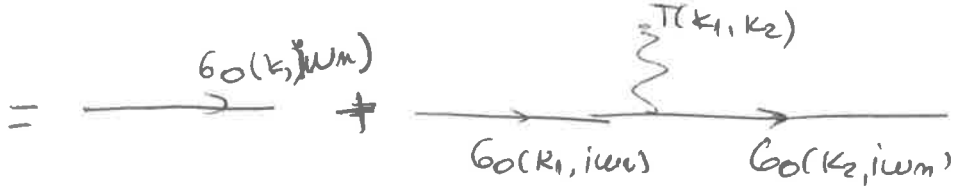
$$\xrightarrow[G(k_1, k_2, i\omega_n)]{} = \xrightarrow[k_1 = k_2]{G_0(k_1, k_2, i\omega_n)} +$$



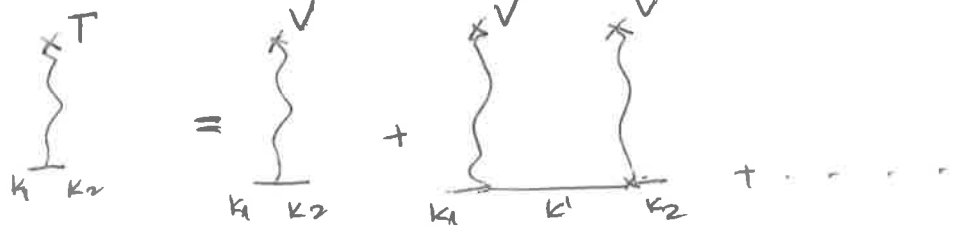
$$G_0(k_1, i\omega_m)\,\bar{V}(k_1, k_2)\,G_0(k_2, i\omega_m) \quad + \quad G_0(k_1, i\omega_m) \int_{k'} G_0(k', i\omega_m)\, V(k_1, k')\, \bar{V}(k', k_2)\, G_0(k_2, i\omega_m)$$

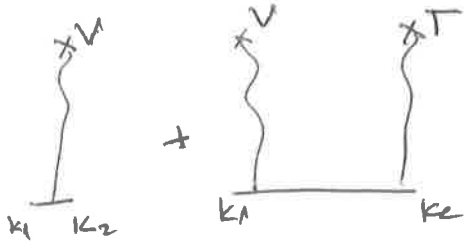
$+ \cdots$

$$G(k_1, k_2, i\omega_m) = G_0(k_1 = k_2 = k, i\omega_m) + G_0(k_1, i\omega_m)\, T(k_1, k_2)\, G_0(k_2, i\omega_m)$$



rewrite then





self consistent equation.

first term = Born approximation.

if V independent of k_1, k_2 $\Rightarrow$ 0th function potential.

$\Rightarrow$ T independent of k_1, k_2.

and $T(k_1, k_2, i\omega_M) = \bar{V}(k_1, k_2) + \sum_{k'} V(k_1, k') \cdot G_0(k', i\omega_M) \cdot T(k'k_2; i\omega_M)$

becomes

$$T(i\omega_M) = \bar{V} + \int_{k'} V \, G_0(k', i\omega_M) \, T(i\omega_M)$$

$$T(i\omega_M) \left[1 - \int_{k'} V \, G_0(k', i\omega_M) \right] = \bar{V}$$

$$\boxed{T(i\omega_M) = \frac{\bar{V}}{1 - \int_{k'} V \, G_0(k', i\omega_M)}}$$

$$G(k_1, k_2 i\omega_M) = G_0(k_2, i\omega_M) + G_0(k_1, i\omega_M) \, T(i\omega_M) \cdot G_0(k_2, i\omega_M)$$

FT of momentum space $\Rightarrow$ real space.

$$G(\vec{R}_1, \vec{R}_2, i\omega_M) = G_0(\vec{R}_1 - \vec{R}_2, i\omega_M) + \int_{\vec{R}} G_0(\vec{R} - \vec{R}_1, i\omega_M) \, T(\vec{R}, i\omega_M) \, G_0(\vec{R}_2 - \vec{R}, i\omega_M)$$

for $\delta(r)$ potential

$$= G_0(\vec{R}_1 - \vec{R}_2, i\omega_M) + G_0(-\vec{R}_1, i\omega_M) \, T(i\omega_M) \, G_0(\vec{R}_2, i\omega_M)$$

$$LDOS(\vec{R}, E) = -\frac{1}{\pi} \operatorname{Im} \left[G(\vec{R}, \vec{R}, E + i0^+) \right]$$

→ exp 23-26 slides