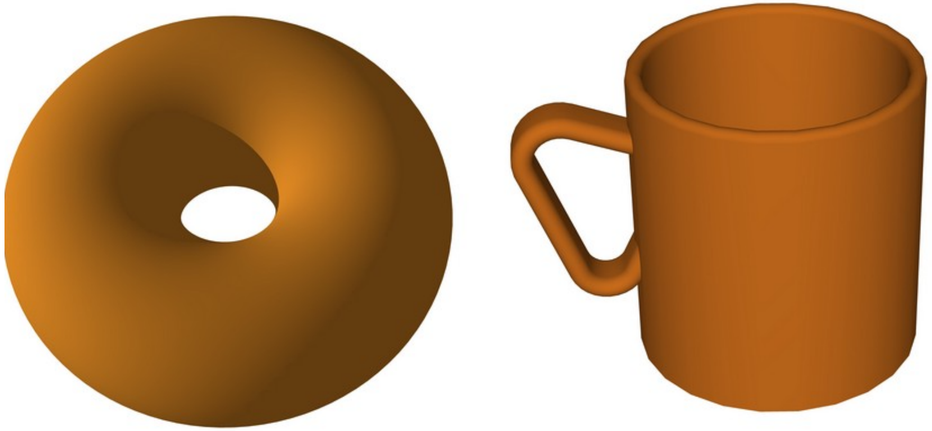
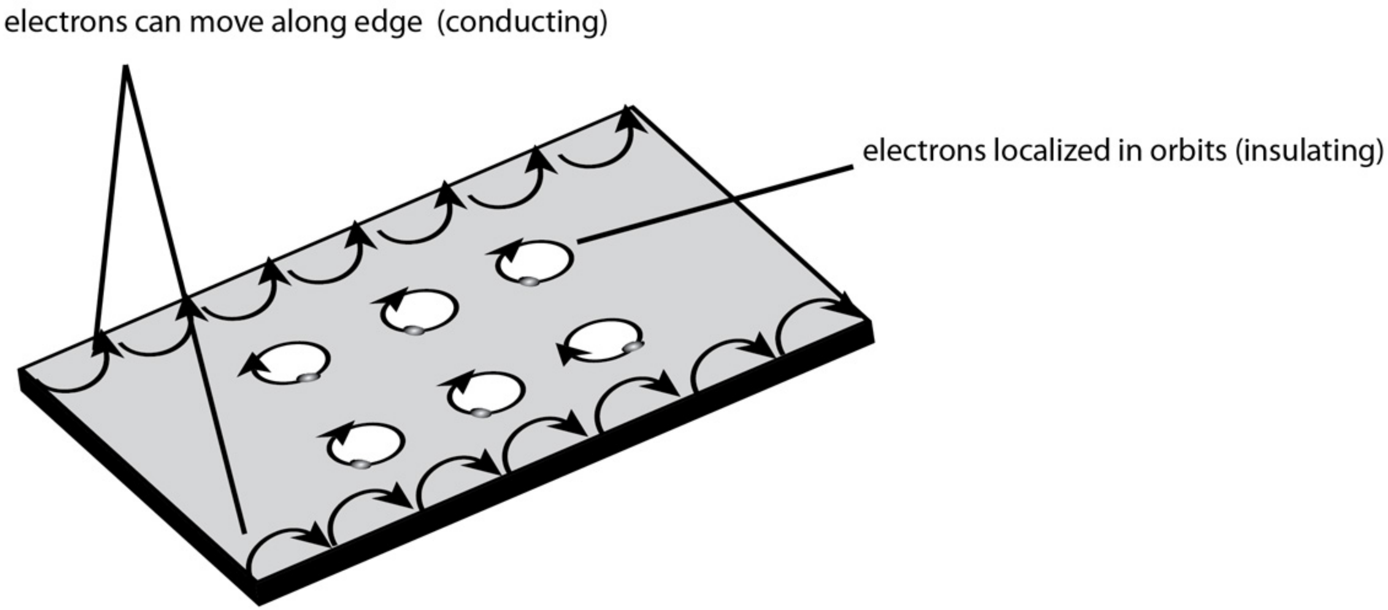
Topology in condensed matter



- stability to deformations: continuous modification between the two shapes possible

Topology in condensed matter

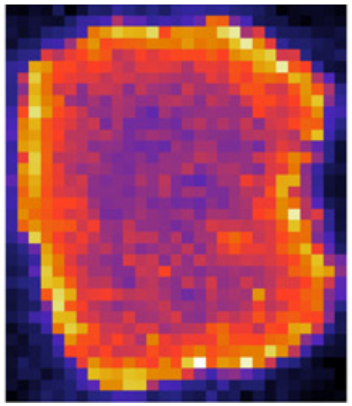
- mathematics: properties of spaces that are invariant under smooth deformations
- physics: two different Hamiltonians can smoothly be deformed into each other
- Examples: edge states

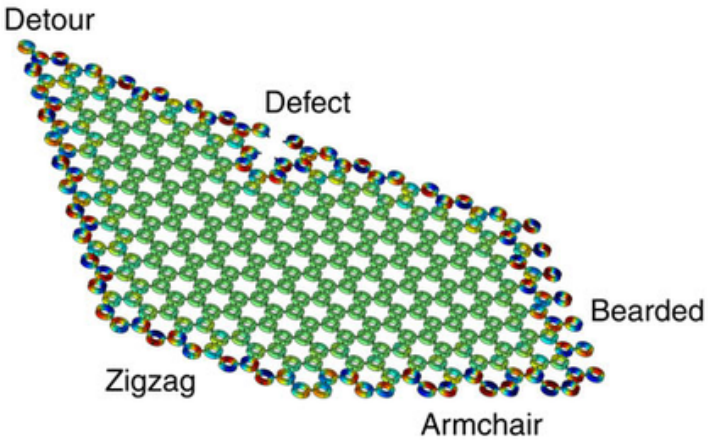


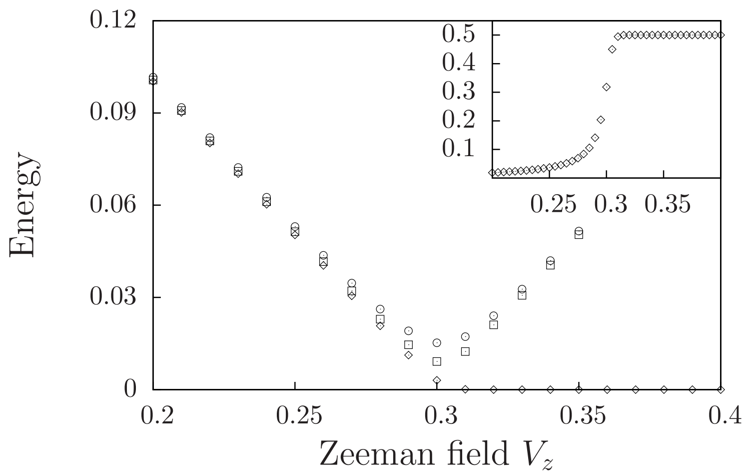
Topology in condensed matter

Topological edge states

Stable with respect to:
- disorder
- shape of boundary
- modifications of the parameters in the Hamiltonian







Stability: usefulness for quantum computation

Non-topological edges states

Graphene

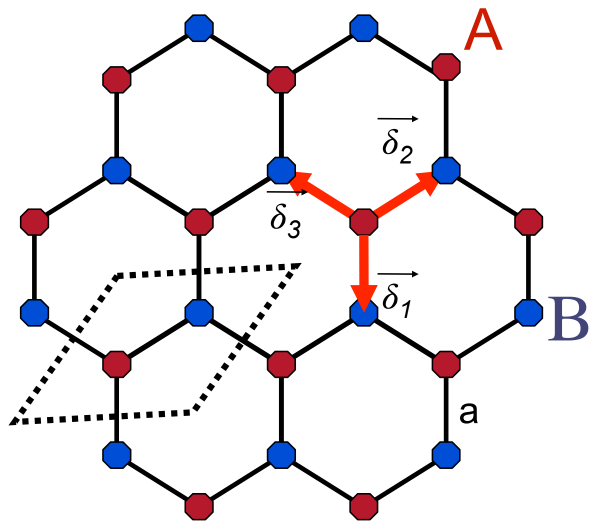
Tight-binding Hamiltonian: kinetic terms - ‘hopping’ between nearest neighbors

$$\hat{H} = -t\sum_{\langle ij\rangle}(\hat{a}_i^\dagger \hat{b}_j + \hat{b}_j^\dagger \hat{a}_i)$$

$$\hat{a}_i^\dagger = \frac{1}{\sqrt{N/2}}\sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}_i}\hat{a}_{\mathbf{k}}^\dagger$$

a,b: annihilation/creation operators for electrons on sites A and B

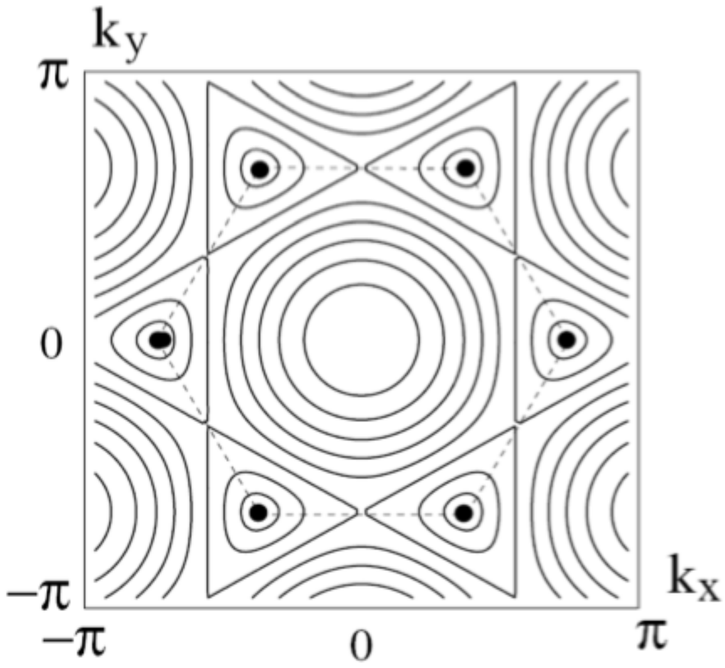
$$\hat{H} = \sum_{\mathbf{k}} \mathbf{\Psi}^\dagger \mathbf{h}(\mathbf{k})\mathbf{\Psi} \qquad \mathbf{\Psi} \equiv \begin{pmatrix} \hat{a}_{\mathbf{k}} \\ \hat{b}_{\mathbf{k}} \end{pmatrix} \qquad \mathbf{\Psi}^\dagger = \begin{pmatrix} \hat{a}_{\mathbf{k}}^\dagger & \hat{b}_{\mathbf{k}}^\dagger \end{pmatrix}$$

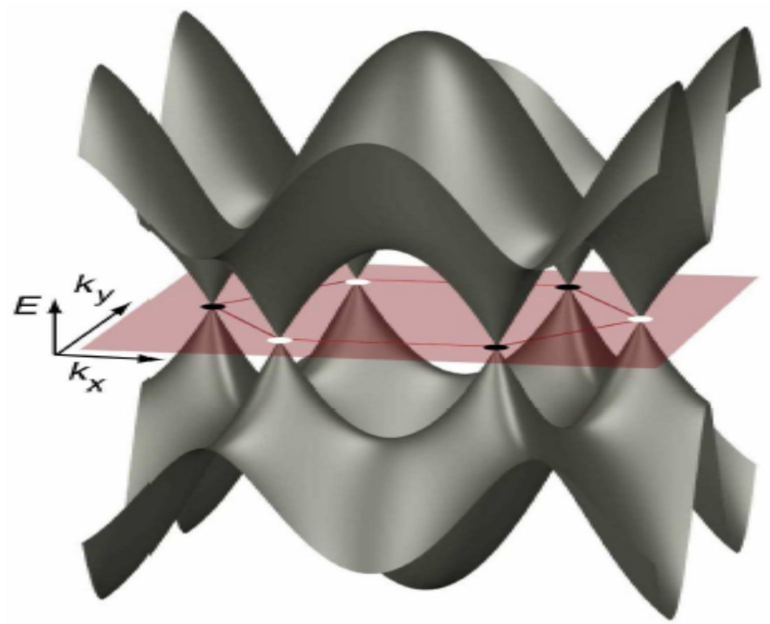


$$\mathbf{h}(\mathbf{k}) \equiv -t\begin{pmatrix} 0 & \Delta_{\mathbf{k}} \\ \Delta_{\mathbf{k}}^* & 0 \end{pmatrix} \qquad \Delta_{\mathbf{k}} = e^{i\mathbf{k}\cdot\boldsymbol{\delta}_1} + e^{i\mathbf{k}\cdot\boldsymbol{\delta}_2} + e^{i\mathbf{k}\cdot\boldsymbol{\delta}_3}$$

$$E_\pm(\mathbf{k}) = \pm t\sqrt{1 + 4\cos\left(\frac{3}{2}k_x a\right)\cos\left(\frac{\sqrt{3}}{2}k_y a\right) + 4\cos^2\left(\frac{\sqrt{3}}{2}k_y a\right)}$$

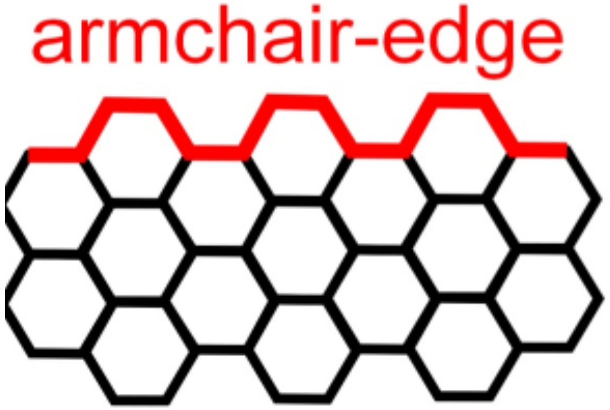
Graphene band structure

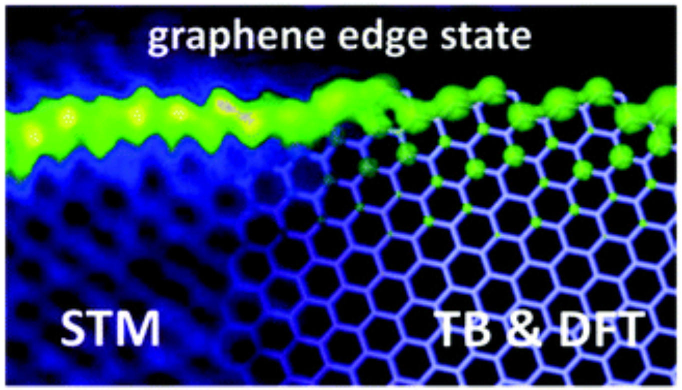




- Hexagonal Brillouin zone
- Zero energy = corners of BZ: Dirac points
- Dirac points → nodal quasiparticles with linear dispersion

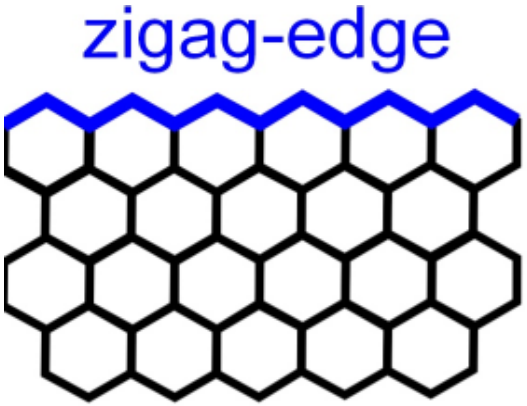
Graphene edges states

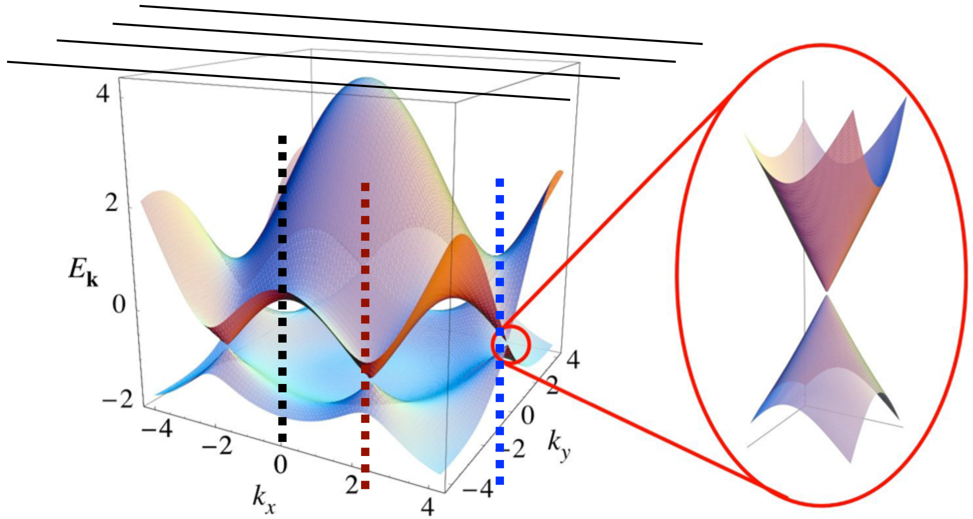




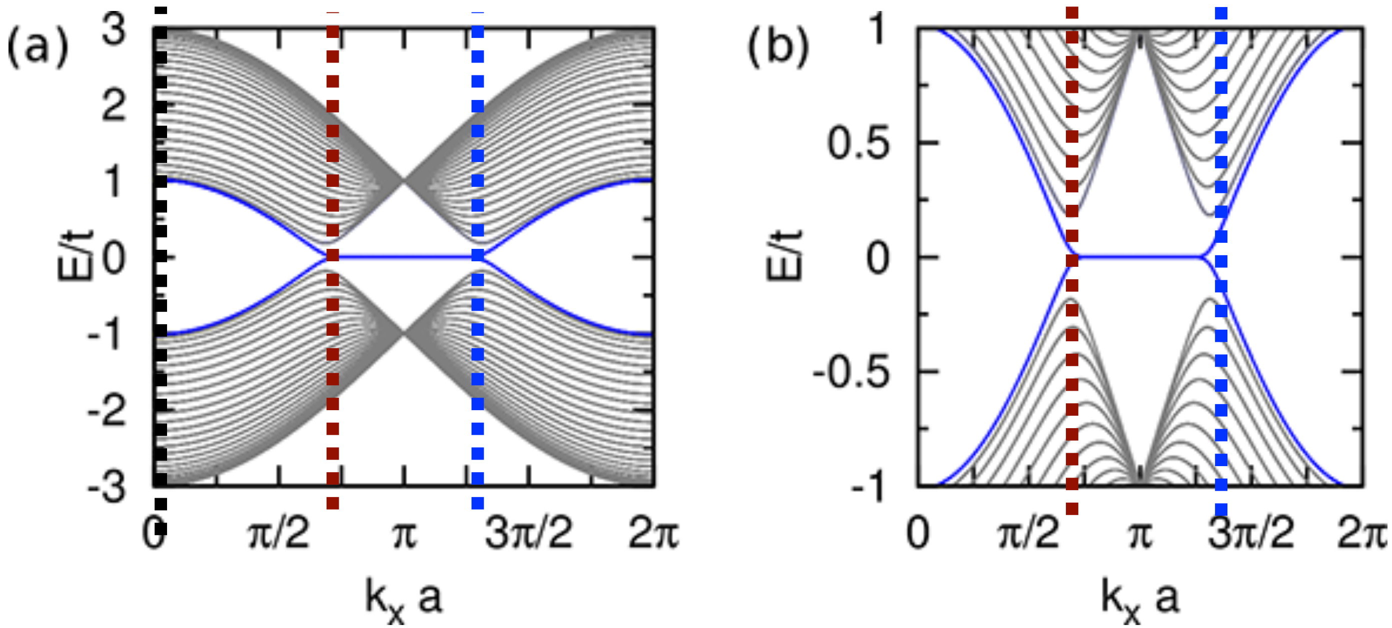
Edge states: wavefunctions localized on the edges, increased density of states

Graphene edges states

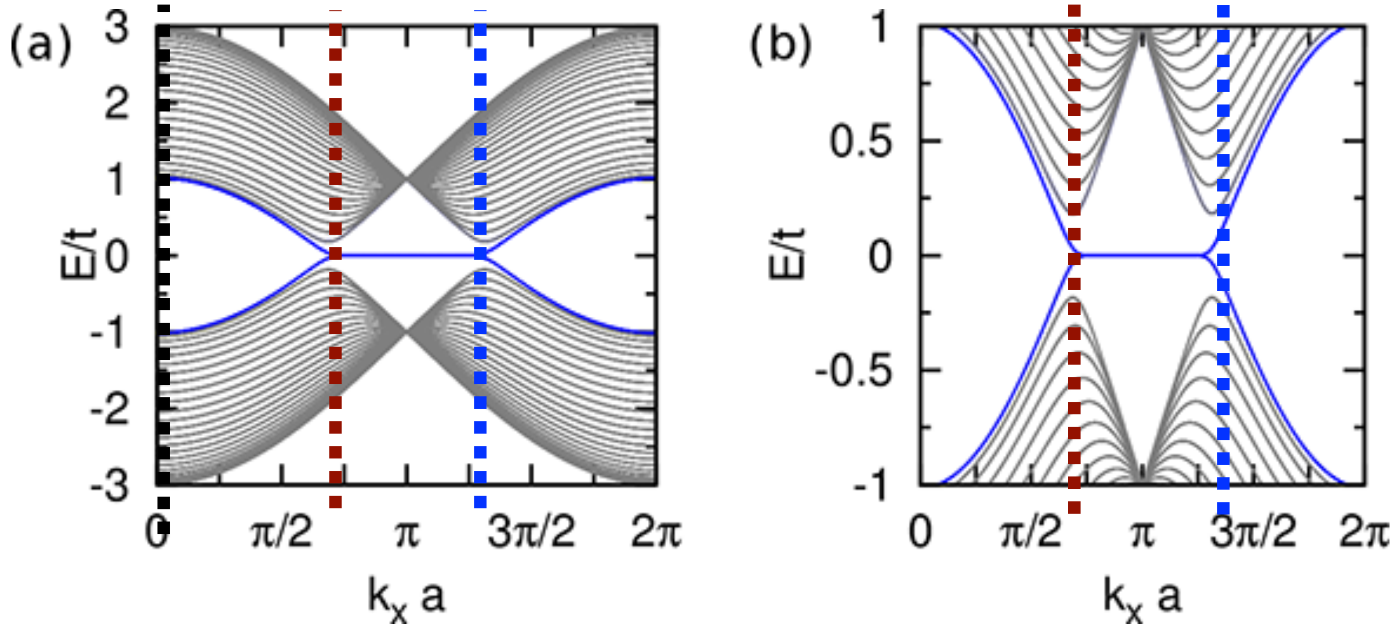


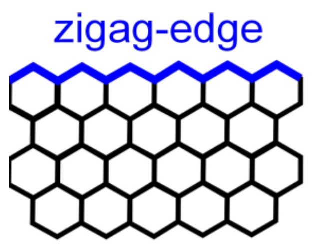


- finite dimension in the *y* direction: quantization of k_y
- band structure of the ribbon obtained from projection of cuts of the bulk band structure for all allowed values of k_y
- one good quantum number: k_x



Graphene edges states

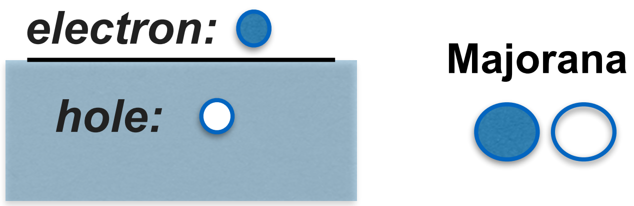




- in black: the bulk band structure
- new states denoted in blue: edge states
- flat band: zero energy for a large set of momenta: each momentum corresponds to one edge state
- non-topological: unstable to perturbations
- armchair edges - no edge states

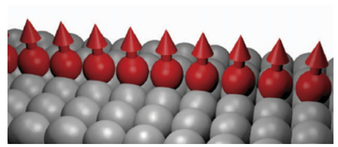
Topological superconductivity and Majorana states

- Majorana states: condensed matter version of Majorana fermions (their own antiparticle)
  neutrinos ?
- equal combination of electrons and holes → self-adjoint



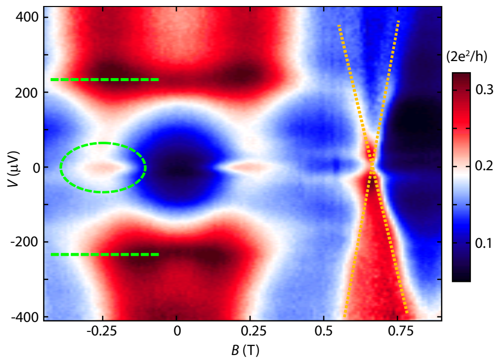
- Statistics
  - Majorana fermions (particle physics): fermionic statistics
  - Majorana states (condensed matter): non-Abelian statistics
- Topological
  - do not depend on the details of the system, disorder, etc.
  - protected and more stable than classical cubits
- Important factors for quantum computation
- Can be used as qubits for quantum computers

- **Materials proposes to host Majorana states**
  - p-wave superconductors: Sr_2RuO_4
  - semi-conductor wires: strong spin-orbit coupling, Zeeman, SC proximity
  - magnetic atom chains

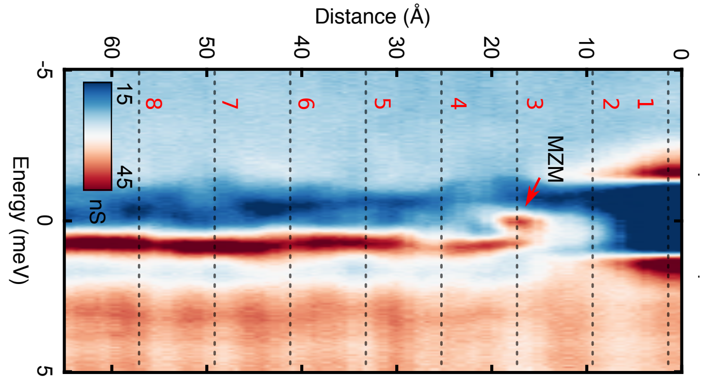


Majorana states

- Do they exist? How to probe experimentally?
- Observed signatures (zero-bias peaks) are controversial
- Can come from non-Majorana states *(ABS Pillet, Quay, Morfin, **Bena**, Levy Yeyatti, Nat. Phys. 2010,* others *Lutchyn & al. Nat. Rev. Mat. 2018)*



Mourik, Kouwenhoven & al. Science 2012



Jeon, Yazdani & al. Science 2017

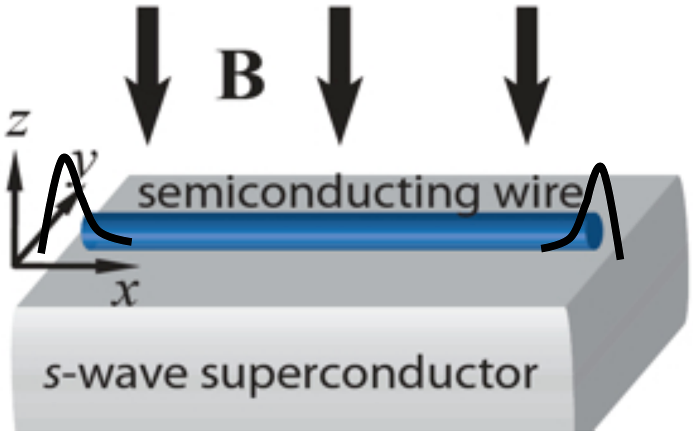
Majorana fermions in quantum wires

$$H = \sum_j \Psi_j^\dagger[(\mu - t)\tau_z + V_x\sigma_x - \Delta\tau_x]\Psi_j$$
$$- \frac{1}{2}\left[\Psi_j^\dagger(t + i\alpha\sigma_y + i\beta\sigma_x)\tau_z\Psi_{j+1} + \text{h.c.}\right]$$

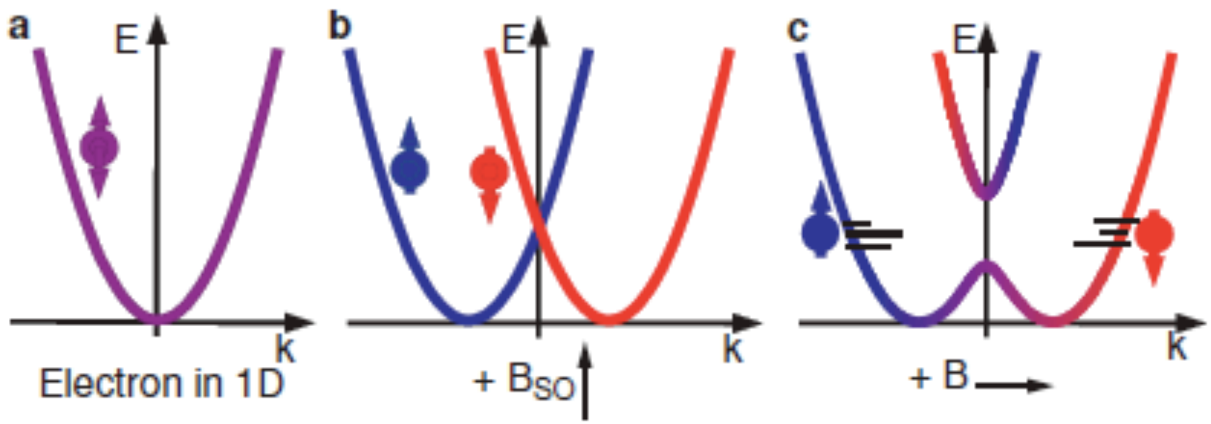
Proximity with s-wave SC

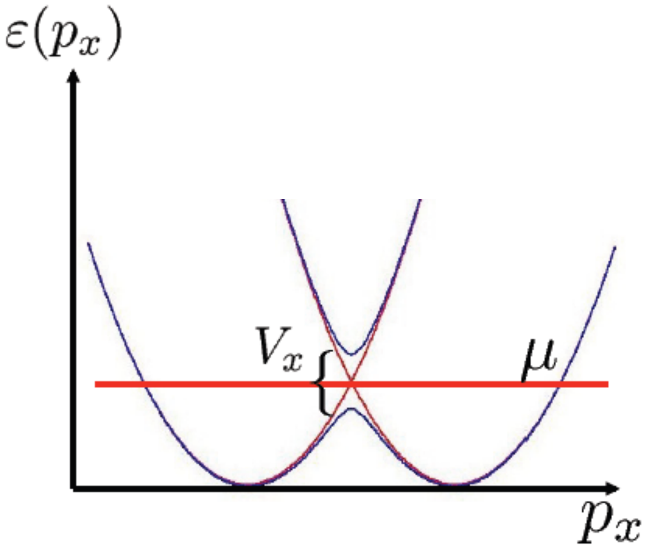
Zeeman

Rashba spin-orbit



Rashba spin-orbit coupling: nearest neighbour hopping with spin-flip





Effect on band structure: doubling and lateral shifting of the two original dispersions

Majorana fermions in quantum wires

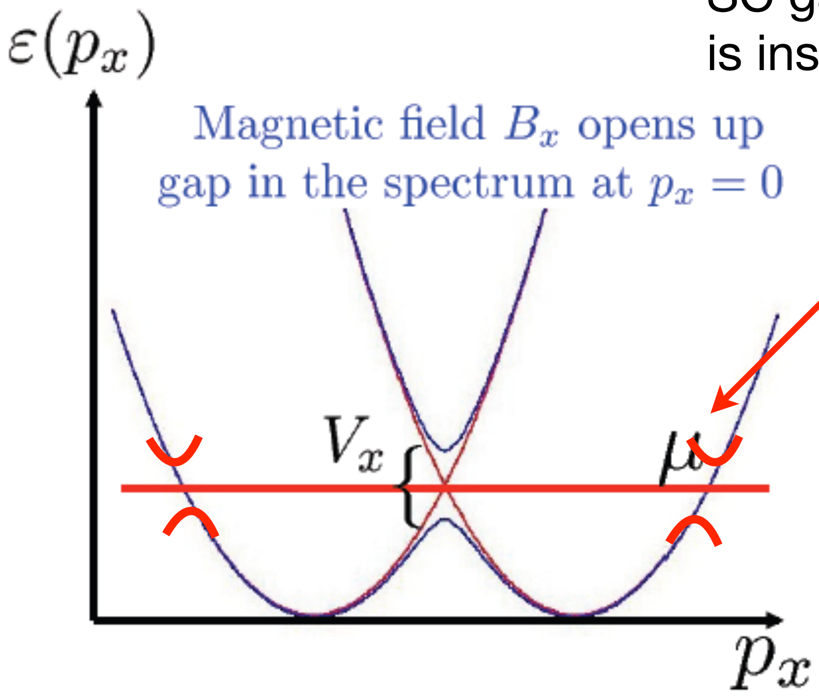
$$H = \sum_j \Psi_j^\dagger[(\mu - t)\tau_z + V_x\sigma_x - \Delta\tau_x]\Psi_j - \frac{1}{2}\left[\Psi_j^\dagger(t + i\alpha\sigma_y + i\beta\sigma_x)\tau_z\Psi_{j+1} + \mathrm{h.c.}\right]$$

Proximity with s-wave SC

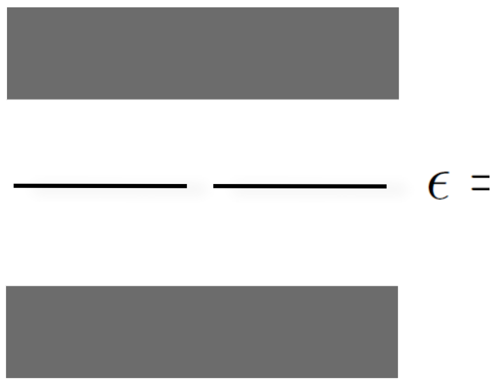
Spin-orbit

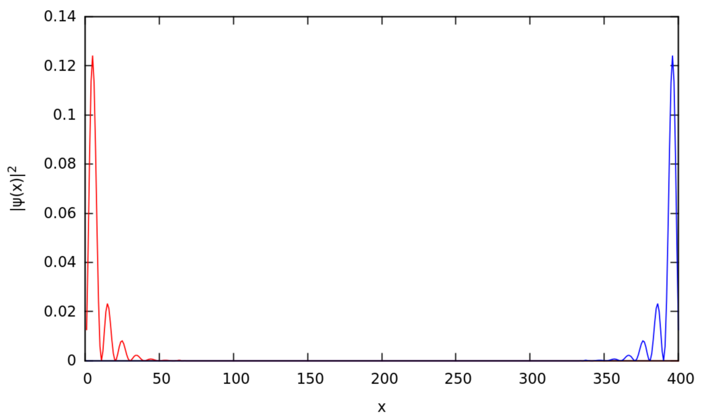
Zeeman

SC gap opening: p-wave type if μ is inside the gap opened by B



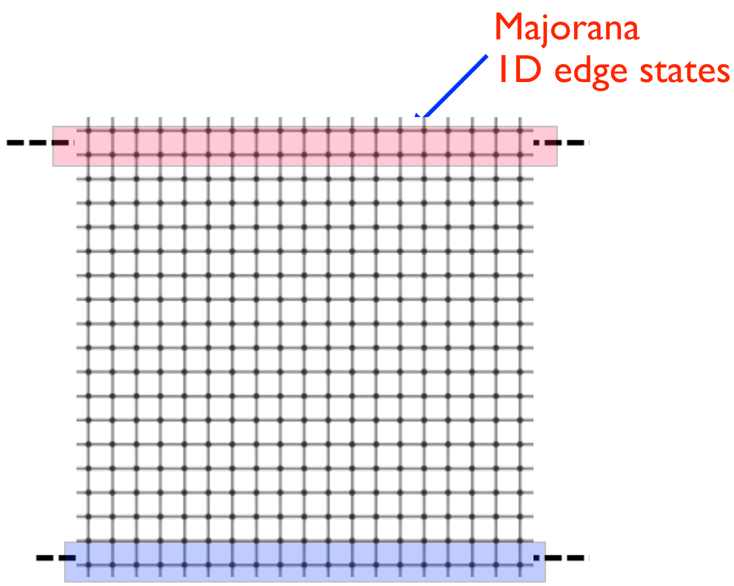
Majorana states

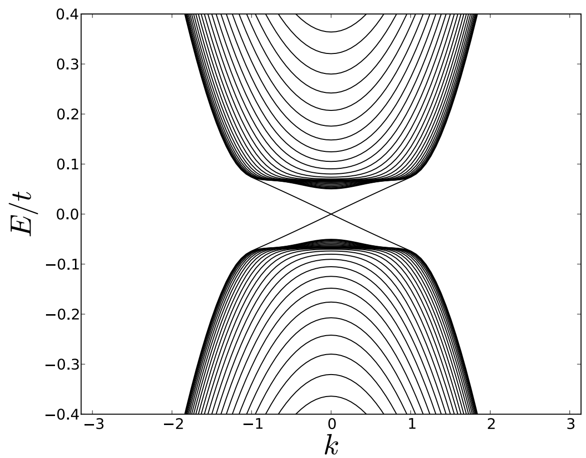


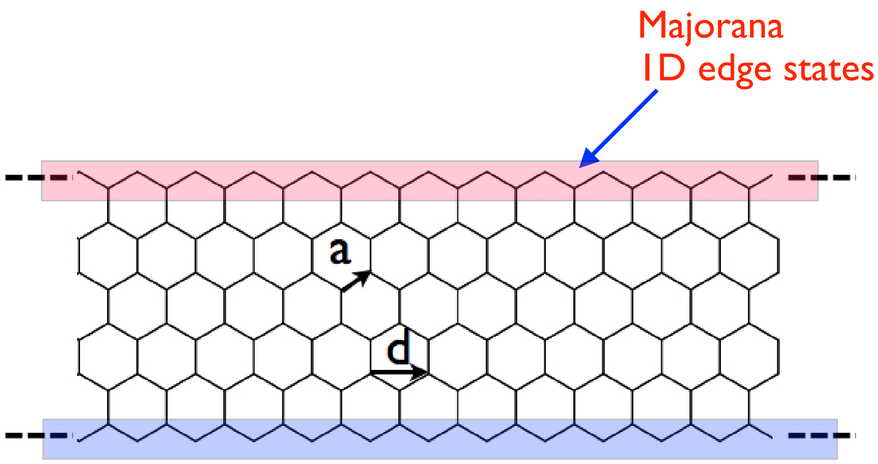


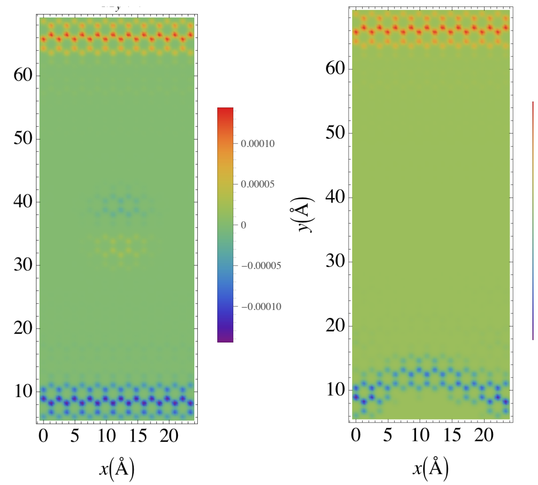
Majorana fermions in 2D models

- Spinless p-wave SC: 2D Kitaev model
- Rashba + Zeeman + superconductivity
- graphene + Rashba + Zeeman
- Rashba can be substituted by an inhomogeneous magnetic field







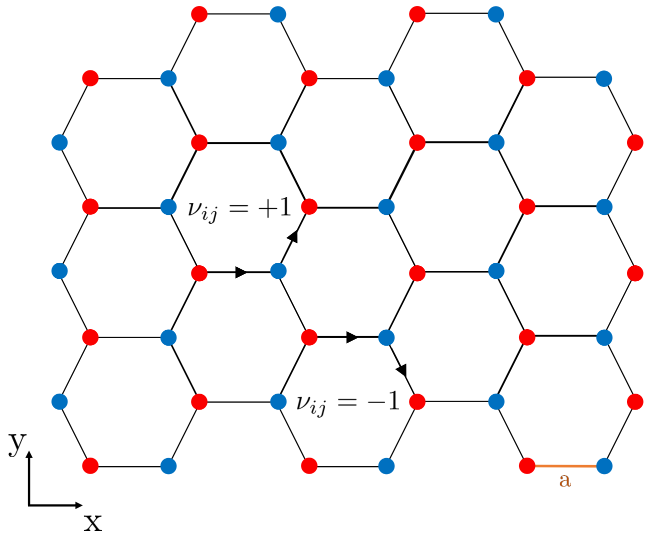


Kane-Mele model
Topological insulators

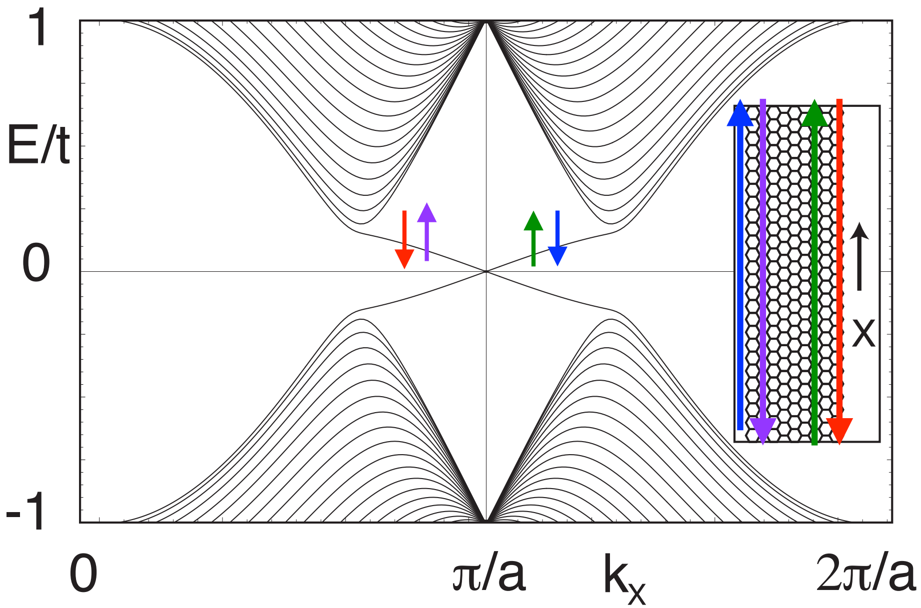
$$\mathcal{H} = \sum_{\langle ij\rangle\alpha} t c^\dagger_{i\alpha} c_{j\alpha} + \sum_{\langle\langle ij\rangle\rangle\alpha\beta} i t_2 \nu_{ij} s^z_{\alpha\beta} c^\dagger_{i\alpha} c_{j\beta}$$

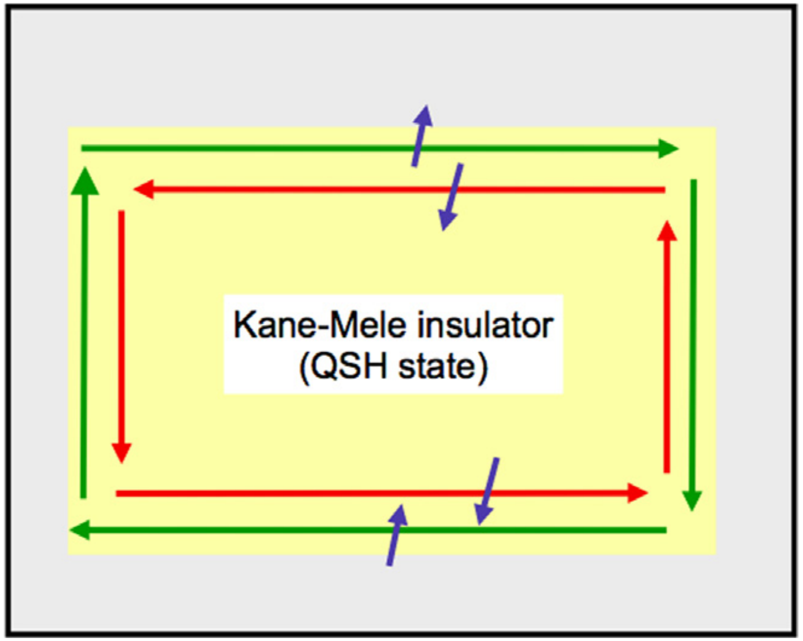
Regular hopping

Spin-orbit coupling
next to nearest neighbour
spin-dependent hopping



Band structure: bulk + edge states





Bulk - gapped: insulator

Helical spin-polarised edge modes

Conducting edges

↓↓ spin down

↑↑ spin up

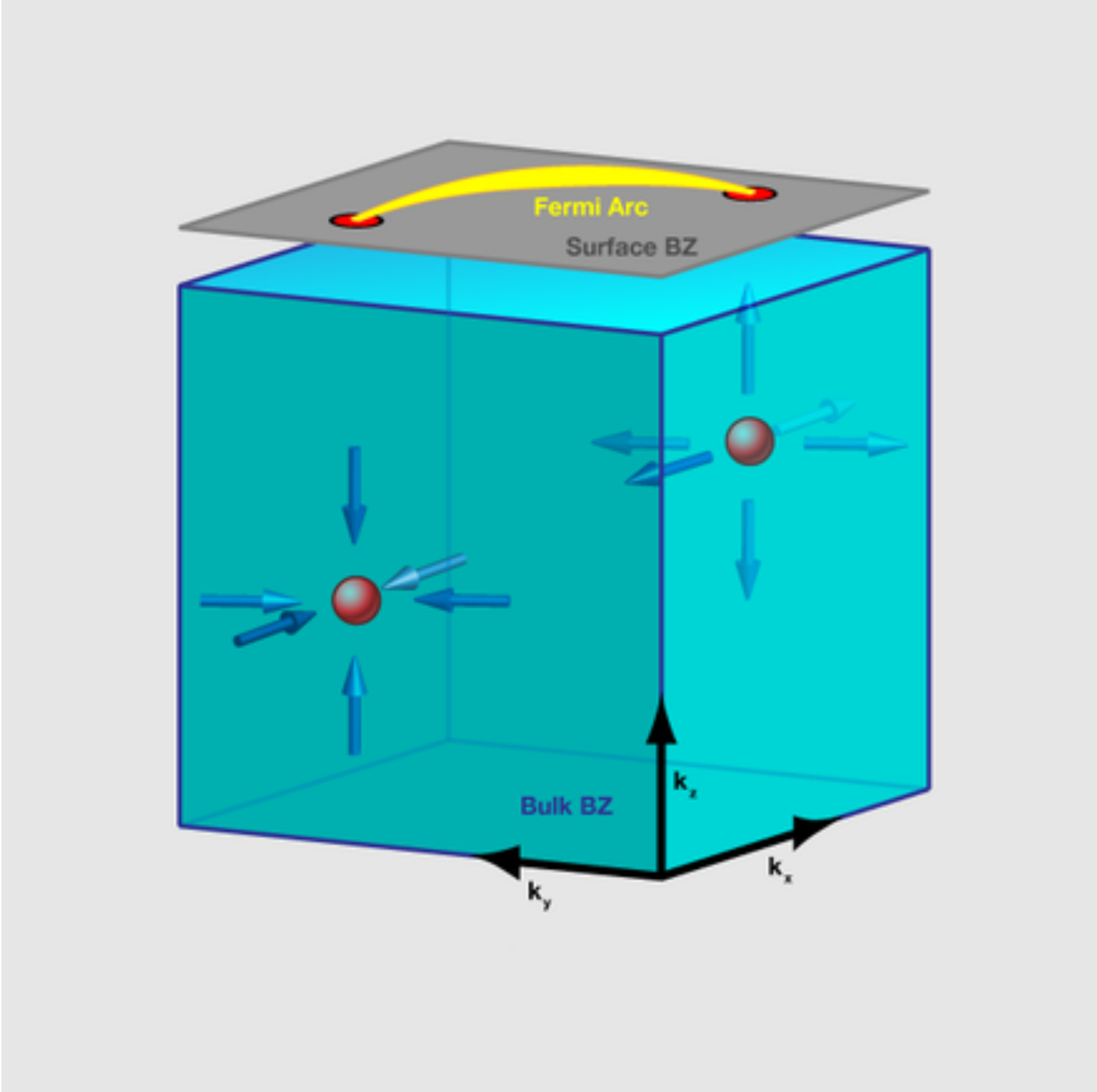
↑↓ right edge

↑↓ left edge

Quantum Hall Effect

To be described by Thierry in the 3-6 lectures

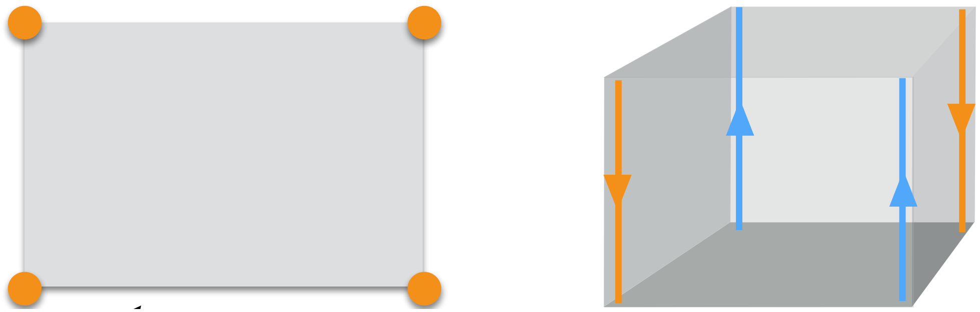
Weyl semimetals



Higher order topological insulators

- 1st order (regular) d_{sys}-d_{edge}=1

- 2nd order d_{sys}-d_{edge}=2



- 3rd order d_{sys}-d_{edge}=1