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## Geometric scaling in symmetric nucleus-nucleus collisions

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### Abstract

We show that the centrality dependence of the multiplicity is identical in Pb+Pb and Xe+Xe collisions at the LHC, up to a constant factor. This geometric scaling is revealed if one defines centrality according to impact parameter, as opposed to the usual experimental definition, which is in terms of multiplicity. We reconstruct the impact parameter dependence of the multiplicity from experimental data using a recently-developed inversion method, which turns out to describe ALICE Xe+Xe multiplicity data much better than usual Monte Carlo Glauber fits. The multiplicity as function of impact parameter extracted from ALICE data is compared to model calculations.

*Keywords:* heavy ions, nucleus-nucleus collisions, LHC, centrality, bayesian inference

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### 1. Introduction

The impact parameter,  $b$ , of a nucleus-nucleus collision at ultrarelativistic energy is a well-defined quantity, in the sense that its quantum uncertainty is negligible. It is a key input to model calculations, such as hydrodynamic calculations. However,  $b$  cannot be directly measured. Therefore, measuring a quantity as simple as the average multiplicity at fixed  $b$  is not straightforward. This is unfortunate, as such a quantity is actually very useful: It is typically used to fix the initial temperature [1] or entropy [2] in hydrodynamic calculations.

We show that this quantity can be extracted from data under general assumptions. The impact parameter is related to the multiplicity, in the sense that more central collisions produce on average more particles, but the relation between multiplicity and impact parameter is probabilistic, not one to one. We have shown that this relation can be inferred from data [3, 4] without relying on any specific microscopic model. The problem of reconstructing impact parameter from multiplicity is a typical inverse problem, which is solved by a deconvolution method. The main ingredient is the fluctuation kernel which is used to model multiplicity fluctuations at a fixed impact parameter. This kernel is presented in Sec. 2. We then recall briefly the principle of the inversion method and apply it to recent Xe+Xe data at 5.44 TeV in Sec. 3. In Sec. 4, the outcome of our procedure is used to perform a comparison of the variation of the mean multiplicity as a function of the true centrality (defined by the impact parameter) between Xe+Xe and Pb+Pb collisions. The resulting variation of multiplicity is then compared to model calculations.

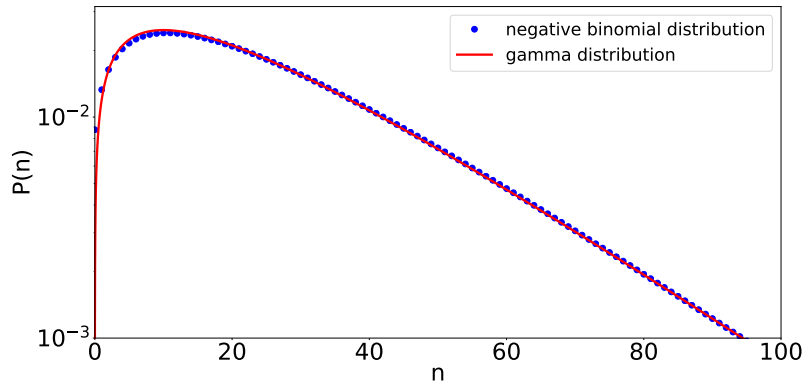


Fig. 1. (Color online) Symbols: Negative binomial distribution used by the ALICE collaboration in their Glauber Monte Carlo fit to Pb+Pb data [5]. Solid line: gamma distribution with the same mean and variance.

## 2. Parametrizing multiplicity fluctuations: negative binomial and gamma distributions

The multiplicity of particles,  $n$ , seen in a detector fluctuates event to event for a fixed parameter,  $b$ . These fluctuations are partly due to dynamical fluctuations, and to statistical fluctuations which depend on the detector acceptance. The principle of our method is to parametrize these fluctuations—or, more precisely, the probability of  $n$  at fixed  $b$ , denoted by  $P(n|b)$ —in the most general way. If fluctuations are small, one expects them to be Gaussian [3]. Since the multiplicity is positive, however, a more accurate parametrization of its fluctuations is provided by the gamma distribution [4] which, unlike the Gaussian, has positive support. The gamma distribution [6, 7] can be viewed as a continuous version of the negative binomial distribution (NBD), which has long been used to fit multiplicity distributions in high-energy collisions [8]. We choose the continuous version because it also applies to the case when one uses an energy, instead of a multiplicity, to estimate the centrality. The similarity between the gamma distribution and the NBD is illustrated in Fig. 1.

## 3. Inversion method and application to Xe+Xe collisions at $\sqrt{s_{NN}} = 5.44$ TeV

The observed multiplicity distribution,  $P(n)$ , is obtained by summing the contributions to multiplicity at all impact parameters:

$$P(n) = \int_0^\infty P(n|b)P(b)db = \int_0^1 P(n|c)dc, \quad (1)$$

where  $P(b)$  is the probability distribution of impact parameter, and  $c$  denotes the centrality, defined as the cumulative distribution of impact parameter:  $c \equiv \int_0^b P(b')db'$ . We write  $P(n|c) = P(n|b)$  the probability of  $n$  at fixed centrality, that is, at fixed  $b$ . As explained above,  $P(n|c)$  is assumed to be a gamma distribution. The gamma distribution has two parameters corresponding to the mean  $\bar{n}$  and to the variance  $\sigma^2$ . Both depend on  $c$ . Our sole assumption is that  $\bar{n}$  is a smooth, monotonously decreasing function of  $c$  (specifically, an exponential of a polynomial of degree 6, where the exponential guarantees positivity). We also assume for simplicity that the ratio of the variance to the mean,  $\sigma^2/\bar{n}$ , is independent of  $c$ , but the results presented below are robust with respect to this assumption [9]. We then fit  $\bar{n}(c)$  and  $\sigma^2(c=0)$  so that  $P(n)$  defined by Eq. (1) matches experimental data.

We now apply this procedure to ALICE data on Xe+Xe collisions at  $\sqrt{s_{NN}} = 5.44$  TeV [10]. In the case of ALICE data,  $n$  is the multiplicity of hits in the V0 detector, which we shall simply denote by V0. We fit the measured  $P(V0)$  using Eq. (1). We exclude 8% of the events with the smallest values of V0 from the fit, for technical reasons explained in Ref. [4]. Our fit is displayed together with ALICE data in the top panel of Fig. 2. The bottom panel displays the ratio fit/data. This ratio is very close to 1 for our fit, but deviates by up to 10% for the fit performed by the ALICE collaboration using a Monte Carlo Glauber model [11]. This

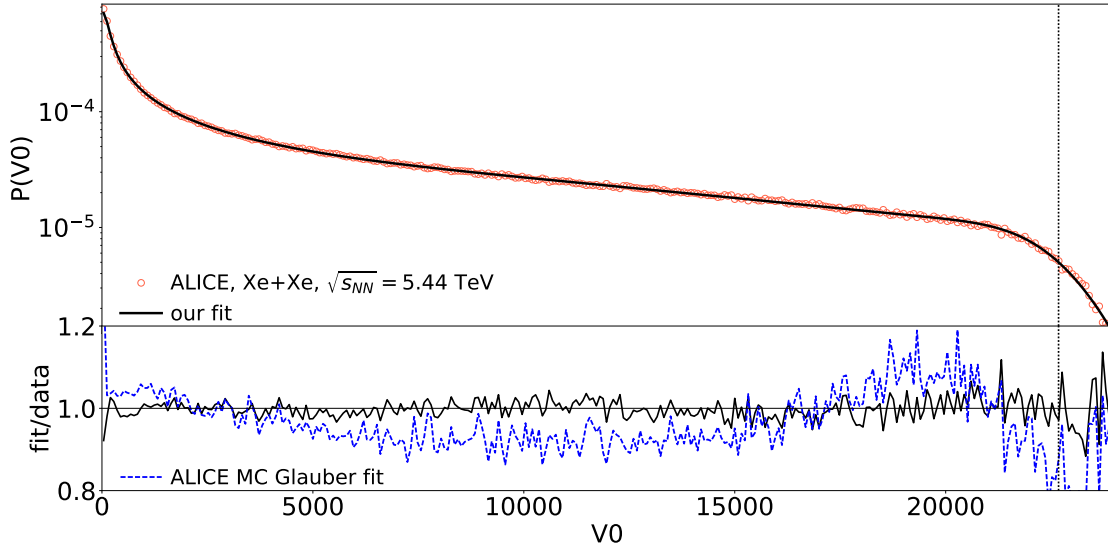


Fig. 2. (Color online) Top panel: circles: ALICE data for the distribution of the multiplicity in the V0 detector [10]. Solid line: fit using Eq. (1) (see text), where  $n$  denotes the V0 amplitude. The vertical line is the position of the knee, defined as the mean value of V0 for  $b = 0$ . Bottom panel: ratio fit over data for our fit (solid line) and for the MC Glauber fit used by ALICE (dashed line).

shows that our inversion method provides a more accurate representation of data than specific models of the collision. Note that our procedure neglects the deformation of the  $^{129}\text{Xe}$  nucleus. Deformation is important for elliptic flow [12, 13], but it is expected to play a modest role for multiplicity distributions [7]. This is confirmed by the excellent quality of our fit.

#### 4. Comparison between Xe+Xe and Pb+Pb collisions

Figure 3 (a) displays the mean multiplicity as a function of centrality percentile, as given by our fit to data. We again emphasize that this is the true centrality, defined from impact parameter (called “ $b$ -centrality”, and denoted by  $c_b$ , in Refs. [3, 4]), which differs from the usual centrality, as defined experimentally from the cumulative distribution of the multiplicity. On the vertical axis, we divide the mean multiplicity by its value at  $b = 0$ . When plotted in this way, Xe+Xe and Pb+Pb collisions essentially fall on the same curve. This means that colliding larger nuclei amounts to multiplying the multiplicity by a constant factor, which is independent of centrality. We call this phenomenon geometric scaling for obvious reasons. (It should not be confused with the geometric scaling observed in deep inelastic electron-proton scattering [14].) It would be interesting to measure the proportionality factor between two different colliding systems at the same energy, and check whether the multiplicity is proportional to the mass number of colliding nuclei.

Finally, Figure 3 (b) displays a comparison with model calculations using the T<sub>R</sub>ENTo model [7], which generates initial conditions for hydrodynamic calculations. We assume that the final multiplicity of an event is, up to a multiplicative constant, the total entropy calculated in T<sub>R</sub>ENTo. The T<sub>R</sub>ENTo model with  $p = 1$  is equivalent to the wounded nucleon model [15], that is, to the Glauber model with participant scaling [16]. The T<sub>R</sub>ENTo model with  $p = 0$ , on the other hand, is known to give results similar to models inspired by high-energy QCD [17, 7]. With both values of  $p$ , geometric scaling is satisfied to a good approximation, in the sense that Xe+Xe and Pb+Pb results are close. This is not surprising as scale invariance is one of the starting assumptions in the T<sub>R</sub>ENTo model. Note that scale invariance, on the other hand, is not satisfied by the two-component Glauber model (it is broken by the term proportional to the number of collisions). The agreement of the T<sub>R</sub>ENTo model with data is better (although not perfect) with  $p = 0$  than with  $p = 1$ , as already noted in the context of flow studies [18].

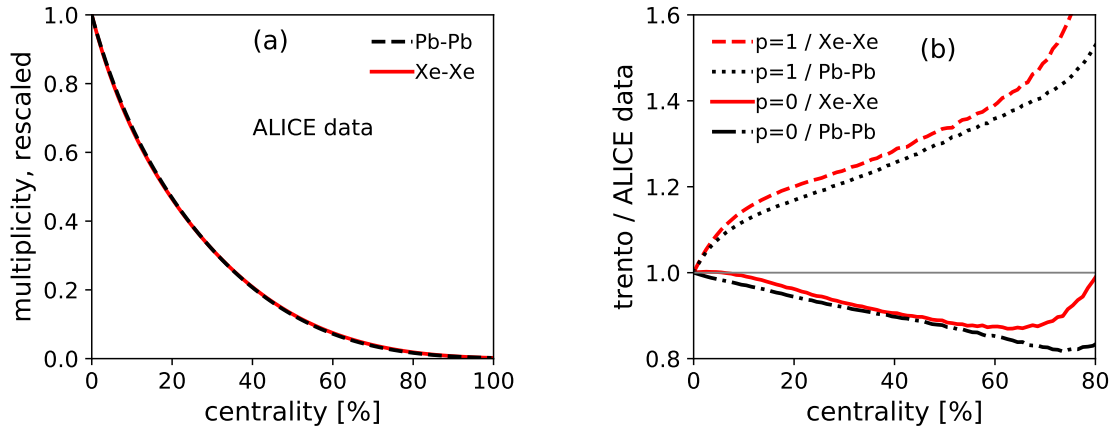


Fig. 3. (Color online) (a) Mean multiplicity versus centrality, rescaled by the value at  $b = 0$ , for Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV (dashed line) and Xe+Xe collisions at  $\sqrt{s_{NN}} = 5.44$  TeV (solid line). (b) Ratios model/data for T<sub>RENTo</sub> with  $p = 1$  and  $p = 0$ .

The T<sub>RENTo</sub> model also implements gamma fluctuations of the multiplicity. These fluctuations are switched off in the calculations presented in Fig. 3 (b), but we have checked explicitly that results are essentially unchanged if fluctuations are included. This is not surprising, since we evaluate the mean multiplicity at a fixed impact parameter, which is averaged over fluctuations.

In conclusion, we have applied a Bayesian inversion method to infer from data the variation of the mean multiplicity as a function of the true centrality, defined as a function of impact parameter. We have shown that this variation is essentially independent of the nuclear size for symmetric collisions between large nuclei, thus revealing a geometrical scaling. This simple observable allows to constrain models of particle production.

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