A pseudo-conformal equation of state in compact-star matter from topology change and hidden symmetries of QCD

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We construct an effective field theory approach to the equation of state (EoS) for nuclear and compact star matter entirely in terms of "effective" hadron degrees of freedom. The putative smooth transition at $n \sim (2-4)n_0$ (where n_0 is the normal nuclear matter density) from hadron degrees of freedom to strongly-coupled quark degrees of freedom, presumably required for a soft-to-hard changeover in the EoS to account for the massive stars observed, is effectuated by the change at $n_{1/2} \gtrsim 2n_0$ from skyrmions to half-skyrmions in a topological description of baryons. The mechanism exploits possible emergence of hidden scale and local symmetries of QCD at high density, leading to a precocious "pseudo-conformal" sound velocity $v_s^2 = 1/3$ (in unit of $c = 1$). It offers an appealing possibility that the topology change density $n_{1/2}$, accessible up to date neither by theory nor by terrestrial experiments, could be determined by the tidal deformability in gravity waves coming from coalescing neutron stars in LIGO/Virgo-type observations.

Introduction— There is a growing indication [1, 2] that certain symmetries, either absent or hidden in QCD, can influence what takes place at low density, say, near the equilibrium nuclear matter density $n_0 \simeq 0.16$ fm^{-3} and "emerge" or get "unhidden" and become crucial in the dynamics at high density in compact-star matter. Although QCD proper has, apparently, neither scale symmetry because it is broken explicitly by the trace anomaly nor local flavor symmetry broken with the non-negligible masses of (light-quark) vector mesons, those symmetries treated as induced by strong correlations in nuclear interactions can play a pivotal role involving a scalar Nambu-Goldstone boson (dilaton) field σ and a hidden local field ρ in the properties of the normal nuclear matter [3] and more importantly in highly dense compact star matter including the $\sim 2M_\odot$ stars observed.

Notable at low density [2], near n_0, are the simple explanation why the g_A, effective in nuclear Gamow-Teller transitions, is seemingly "quenched" to what appears to be the "universal" value $g_A^{\rm eff} \sim 1$ and an effective field theory derivation of the Migdal formula [4] for, and a highly successful calculation [5] of, the anomalous orbital gyromagnetic ratio for the proton in heavy nuclei $\delta g_l^p \simeq 0.23$. What's perhaps more intriguing and striking is that at high compact-star matter density $n \gtrsim 2n_0$, the theory predicts [1] the emergence of parity doubling in conjunction with scale symmetry, giving rise to a precocious conformal sound velocity $v_s^2 = 1/3$ and offers the possibility to confront in a unique way the tidal deformability in gravity waves coming from coalescing neutron stars as recently observed by the LIGO/Virgo observation GW170817 [6].

In this Letter, we reformulate the rather involved V_{lowk} renormalization-group (RG) approach to baryonic matter developed in [1, 2] in a drastically simplified framework that fully captures the essential premises of the nuclear effective field theory obtained and expose novel aspects of high density properties of compact-star matter.

Among the key ingredients that figure in giving the results of [1, 2] is the observation that when baryonic matter is described in terms of skyrmions, there is a topology change at a density $n_{1/2} \gtrsim 2.0n_0$ involving skyrmions fractionzing into half-skyrmions. We suggest this as having the strong-coupling quark degrees of freedom as exploited in, say, [7] in the density range $\sim (2-4)n_0$ traded in for topology as a Cheshire-Cat phenomenon in analogy to the trading-in of the MIT quark bag for a skyrmion [8].

The topology change brings out several important effects which can be summarized in two main observations [1]:

- **A**: The first observation, the most crucial of all, is the cusp at $n_{1/2}$ in the symmetry energy E_{sym} in the EoS of compact-star matter that arises in the rotational quantization of the skyrmion matter [9]. It is of leading order in N_c and robust. This cusp can be incorporated (or translated) into the effective Lagrangian constructed with scale symmetry and hidden local symmetry, dubbed *s*HLS, incorporated into baryonic chiral Lagrangian, called *bs*HLS in [1]. This is done by matching the "bare" parameters of *bs*HLS to QCD at a certain matching scale. Embedded in medium, the Lagrangian is rendered sliding with "intrinsic density dependence (IDD)" encoded in QCD condensates. The cusp exposes the intricate structure of the nuclear tensor force – with the sliding vacuum effect – which decreases as density goes toward $n_{1/2}$ and then increases past $n_{1/2}$. Here the

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vector manifestation (VM) of the ρ meson [10] plays an important role in producing the cusp [9]. The dropping behavior of the tensor force near n_0 is confirmed beautifully in the long life-time of the C14 beta decay [11]. What will turn out to be noteworthy is that this cusp could play an important role in the tidal deformability in gravity waves.

• **B**: Another equally important observation is that the topology change induces the parity doubling in the nucleon structure, with the effective in-medium nucleon mass m_N^* going as the dilaton condensate $\langle\chi\rangle^* \propto m_0$ where $\chi = f_\sigma e^{\sigma/f_\sigma}$ and m_0 is the chirally invariant mass figuring in the parity-doubled linear sigma model [12].[1] As a consequence, the trace of the energy momentum tensor θ_μ^μ of the dense system in $n \gtrsim n_{1/2}$, given in the description entirely as a function of $\langle\chi\rangle^*$, flattens at increasing density and leads to the sound velocity of the star $v_s^2 = 1/3$, usually associated with the conformal sound velocity in the chiral limit. The appearance of $v_s^2 = 1/3$ at non-asymptotic density is arguably thought to be impossible unless there appear non-hadronic degrees of freedom [14]. We are led to attribute to the topology change the mechanism responsible for the requisite change of degrees of freedom. We shall call the resulting object "pseudo-conformal sound velocity" because the trace of the energy momentum tensor is nonzero in the region $n \gtrsim n_{1/2}$ [2]. This pseudo-conformal structure was found to emerge in the skyrmion crystal simulation as a signal for scale symmetry hidden in QCD [15] in the half-skyrmion phase. This can be seen in Fig. 11 of [1].

The pseudo-conformal model— Our objective here is to rephrase the two basic observations **A** and **B** listed above in a form that encapsulates them in a surprisingly simple EoS. We base our reasoning on the bsHLS Lagrangian defined in [1, 2], constructed to suitably encode the QCD vacuum structure sliding with baryonic matter density. This approach can be considered as belonging to the class of what is referred to as "Energy Density Functional" popular in nuclear theory community, in particular in the form of relativistic mean field theory. The potential power of the bsHLS approach over other energy density functional approaches is in the IDD inherited from QCD .

We start with the master formula[2] for the behavior of hadron masses in dense medium formulated in [1] that we assume holds up to the central density of massive stars, i.e., $n_{\rm cen} \approx (5-7)n_0$,

$$\frac{m_N^*}{m_N} \approx \frac{m_\sigma^*}{m_\sigma} \approx \frac{g_V}{g_V^*}\frac{m_V^*}{m_V} \approx \frac{f_\pi^*}{f_\pi} \approx \frac{\langle\chi\rangle^*}{\langle\chi\rangle} \equiv \Phi \qquad (1)$$

where $V = (\rho, \omega)$ and the asterisk stands for intrinsic density dependence. We denote the two regions delineated at $n_{1/2}$ as R(egion)-I for $n < n_{1/2}$ and R(egion-II for $n \geq n_{1/2}$. Since the property of nuclear matter at n_0 is fairly well studied and understood by various ways, phenomenological as well as in effective quantum field theory along the line of Weinberg's Folk Theorem [17], we simply take what's given in [1] for n_0 and extrapolate it to $n_{1/2}$. In R-I, this involves the master formula (1) with $g_V^*/g_V = 1$. There are no unknown parameters once the value f_π^*/f_π is extracted from deeply bound pionic nuclear systems. The outcome of the bulk properties from [1] at n_0 is generally as good as any good standard model available in the literature but with a lot less number of parameters. We therefore adopt the results of [1] as the *input* for the R-I region.

We turn to the region R-II.

Here we encapsulate the observations **A** and **B** described above in a concise form that encompasses the full V_{lowk} calculation. In [1, 2], the emergence of parity-doubling and scale symmetry was a consequence of intricate interplay of Φ and g_V^*/g_V. The latter involved the VM property of the gauge coupling g_ρ which tends to zero at a density $n \gtrsim 20n_0$ and of g_ω with Φ giving rise to the cusp structure in the nuclear symmetry energy predicted by the topology change, and the pseudo-conformal sound velocity. What we wish to do then is to capture this complex structure of R-II embodied in **A** & **B** in a concise EoS for $n > n_{1/2}$ that renders the trace of the energy momentum tensor *non-zero* and *density-independent* up to the central density of star, $n_{\rm cent} \approx (6-7)n_0$. This strategy has been verified to work accurately for the case where $n_{1/2}$ was taken at $2.0n_0$ [1].

To proceed we pick the $n_{1/2}$ such that the topology change takes place at a higher density than the density at which the tidal deformability has reportedly been measured in the LIGO/Virgo observation GW170817 [6]. This is because the dimensionless tidal deformability Λ – as defined by [6] – calculated in the V_{lowk}RG [1] with $n_{1/2} \simeq 2.0n_0$ is found to be ~ 790, close to the reported upper bound 800 for $\sim 1.4M_\odot$ [6]. There is some ambiguity in the result of V_{lowk} [1] that since the central density $n_{\rm cent}$ of the 1.4 $M_\odot$ star turns out to be coincident with the crossover density $n_{1/2}$ in this model, the precise value of the tidal deformability may be subject to the delicate issue of how R-I and R-II are matched. However since Λ is very sensitive to the symmetry energy and with the cusp structure entailing the softening in R-I of the symmetry energy going toward $n_{1/2}$ from below, it is suggestive that the topology change density $n_{1/2}$ be increased to above $2.0n_0$ so as to lower Λ to a value below the given bound. This is clearly feasible since the global properties of massive stars given within the model are found to differ very little between $1.5n_0$

[1] This chiral invariant mass has been associated with the origin of the proton mass, a hitherto unanswered issue of fundamental physics. See [13] where the matter is discussed from the nuclear physics point of view.

[2] In a general scale-chiral effective field theory approach [16], the density-scaling m_σ^*/m_σ should be $\Phi^{\beta'/2+1}$ where β' is the anomalous dimension of $\mathrm{tr}(\mathrm{G}_{\mu\nu})^2$ with $G_{\mu\nu}$ being the field tensor for the gluon field. We will ignore the β' dependence without losing the predictive power [16].

and $2.0n_0$ [18]. Thus we expect it to be feasible to bring $n_{1/2}$ up to $\sim 3n_0$ without disturbing the good global star properties obtained for $n_{1/2} = 2n_0$ in [1].

As a first trial, we pick $n_{1/2} = 2.6n_0$ and explore what transpires with this value.

Now the key idea is to write the EoS in R-II that gives $\langle\theta_\mu^\mu\rangle$ which is nonzero and density-independent so that it gives $v_s^2 = 1/3$. The energy per particle of the system that gives such an EoS in R-II, found in [1] for $n_{1/2} = 2.0n_0$, can be written in the form

$$\frac{E}{A} = A_{II}^\alpha \left(\frac{n}{n_0}\right)^{1/3} + B_{II}^\alpha \left(\frac{n}{n_0}\right)^{-1} - 939\,\mathrm{MeV} \qquad (2)$$

with the coefficients A_{II}^α and B_{II}^α for $\alpha = (N-Z)/(N+Z) = (0,1)$ given by the V_{lowk}. It is easy to verify that $v_s^2 = 1/3$ for *any* values of the coefficients A and B.

We apply the expression (2) to the system given by $n_{1/2} = 2.6n_0$. The coefficients are to be determined by the continuity at $n = n_{1/2}$ between R-I given by [1] and R-II given by (2) for the chemical potential μ and the pressure P (à la "Israel junction" condition)

$$\mu_I = \mu_{II}, \quad P_I = P_{II}. \qquad (3)$$

The pseudo-conformal EoS is the sum of EoSs from R-I given by the V_{lowk} RG of [1] and R-II with (2).

Massive star properties— The energy per particle E/A and the symmetry energy E_{sym} given by the pseudo-conformal model (PCM for short) for $n_{1/2} = 2.6n_0$ are plotted in Fig. 1. The corresponding sound velocity in the PCM is shown in Fig. 2. The anomalous behavior of the sound velocity at $\sim n_{1/2}$ must be an artifact of the sharp matching between R-I and R-II, and hence cannot be taken seriously. The overall results of the EoS, which are found to be fairly consistent with the currently available heavy-ion experimental bounds, are very close to those of V_{lowk} RG of [1]. In particular, the crossover from soft to hard in E_{sym} – which mimics the cusp in the skyrmion description– resembles closely that of the full V_{lowk} RG result [1].

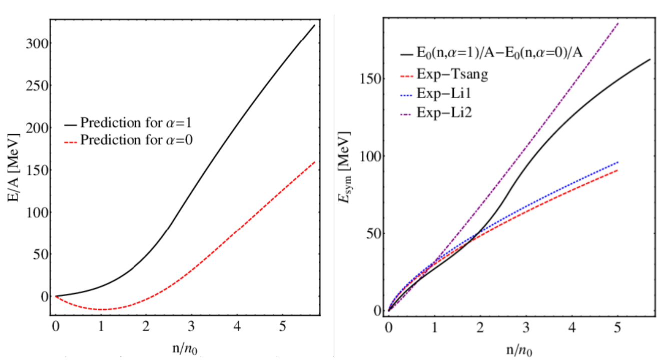

FIG. 1. E/A (left) and E_{sym} (right) vs density n.

We have here a compelling hint that the change-over at $n_{1/2}$ from the Fermi-liquid structure of normal nuclear matter at n_0 [2] to a pseudo-conformal structure, signaling scale symmetry above $n_{1/2}$, captures the topology change from skyrmions to half-skyrmions (seemingly analogous to "pseudo-gaps" in BCS-BEC transitions [19]) that figures in the V_{lowk} RG treatment of [1].

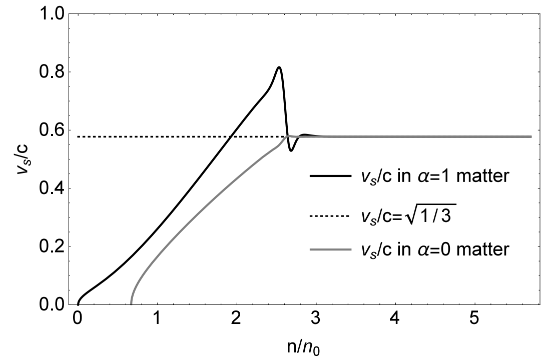

FIG. 2. The sound velocity vs. density.

Now what does the PCM so constructed *predict* for standard star properties?

In Fig. 3 are given M vs. R and the central density of the compact star. The maximum mass comes out at $M_{\max} = 2.02\,M_\odot$ with radius at $R = 11.86$ km. The central density is found to be $n_{\mathrm{cent}} = 5.6\,n_0$. They are to be compared with the V_{lowk} RG results of [1] (for $n_{1/2} = 2n_0$): $M_{\max} = 2.05M_\odot$, $R = 12.19$ km, and $n_{\mathrm{cent}} = 5.1\,n_0$. Note that there is practically no difference – even if no fine-tuning is done – from the case of $n_{1/2} = 2.0n_0$ (and also that of $n_{1/2} = 1.5n_0$ [18]).

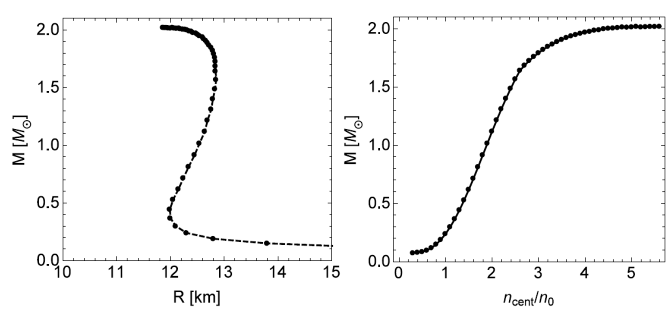

FIG. 3. The mass vs. radius of the neutron star (left panel) in beta equilibrium and the mass vs. the central density of the neutron star (right panel).

Tidal deformability— So far all the standard star properties of the PCM are found to come out more or less the same as the full V_{lowk} approach of [1], both of which are compatible with observations. Although both are equivalent to the Fermi-liquid approach that belongs to the paradigm of energy density functional, it is somewhat surprising that even though $n_{1/2}$ is put at $2.6n_0$, the symmetry energy is nearly the same including the smoothed cusp structure as that found in V_{lowk} RG [1] with $n_{1/2} = 2.0n_0$. This reinforces relative insensitivity of the standard star properties to the location of $n_{1/2}$ found in [18].

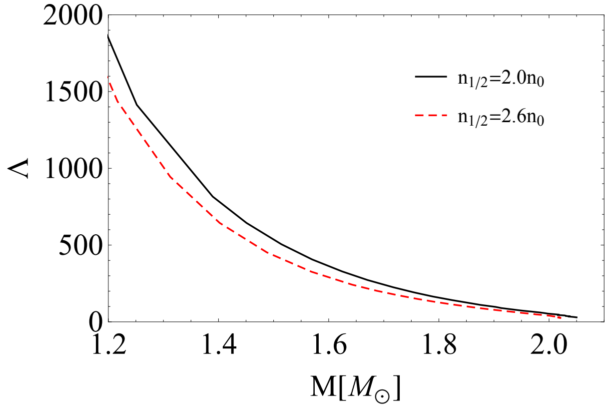

FIG. 4. Dimensionless tidal deformabilities Λ vs. M for $n_{1/2} = 2.0n_0$ and $2.6n_0$.

In a stark contrast, however, the location of the topology change density is found, in the PCM, to have a strong impact on the tidal deformability.[3] This can be seen in Fig. 4.

The results for the Λ given by the PCM for $n_{1/2} = 2.6n_0$ are summarized – and compared with the V_{lowk} RG results for $n_{1/2} = 2.0n_0$ [1] – in Fig. 4 for the range of star masses involved. Given the effective cusp structure as predicted – with the softening symmetry energy toward $n_{1/2}$ – we expected Λ to decrease, when $n_{1/2}$ is increased to $2.6n_0$, from ~ 790 that was found for $n_{1/2} = 2.0n_0$. One indeed obtains a significant drop in Λ. For $1.4\,M_\odot$, we find $\Lambda \sim 600$ with a central density $n_{\rm cent} \simeq 2.3\,n_0$ and a radius $R \simeq 12.8$ km.

It is significant that unlike in the case of [1], $n_{\rm cent}$ in R-I is well separated from the crossover density $n_{1/2} = 2.6n_0$, free from the crossover to R-II at $n_{1/2}$.

What is particularly noteworthy for the topology mechanism in question is two-fold. First, Λ changes by a large amount for small variation of M near $1.4M_\odot$. Thus to identify Λ for a given M, it would require precisely pinning down the value of the M involved. Second, while the tidal deformability decreases by a large amount, say, more than 20% in going from $n_{1/2} = 2n_0$ to $2.6n_0$, the radius gets modified by less than 1.5%. This suggests that in our formulation it probably makes little sense to correlate Λ with R.[4]

[3] This contrasts with other energy density functional models in which Λ could be made to vary arbitrarily by fine-tuning certain parameters available without affecting the global structure of EoS.

[4] It is of course likely that the radius for the low-mass star involved will depend on other details of the star structure – such as the crust etc. – not taken into account in this paper.

Remarks— It is striking that the simple E/A, Eq. (2), efficiently captures the complex structure of **A** & **B** of V_{lowk} RG in R-II and that the appearance of the pseudo-conformal sound velocity $v_s^2 \simeq 1/3$ at $n \gtrsim n_{1/2}$ as found in the V_{lowk} RG calculation and confirmed by the pseudo-conformal model is fully compatible with the observed properties of massive compact stars. If, as suggested in [14], converging to $v_s^2 = 1/3$ is impossible at non-asymptotic density unless there is a change of degrees of freedom, then it must be the half-skyrmions that play the role of non-hadronic degrees of freedom in the range of density in which strongly-coupled quarks, such as quarkyonic phase, are considered to figure [7]. This invites us to conjecture that the pseudo-conformal sound velocity is a signal in dense matter *both* for the emergence of scale symmetry and local flavor symmetry hidden in QCD [15, 20] *and* somewhat intriguingly, for the manifestation of a Cheshire Cat phenomenon. This adds one more to the growing evidence of Cheshire Cat phenomenon in dense hadronic matter, along with the superqualitons [21], vortices [22] etc. in the color-flavor-locked phase.

When the value of Λ is pinned down, it will provide theoretically crucial information on the topology change density $n_{1/2}$, signaling, via Cheshire Cat, for the possible intervention of quark/gluon degrees of freedom. At present there are no known clues, experimental or theoretical, as to how to locate it precisely. Our approach gives $\Lambda \sim 600$ at $n_{1/2} \gtrsim 2.6n_0$. It is likely that the bound will be tightened to a lower value in the future measurements. The PCM has the potential to provide a simple means to probe a range of $n_{1/2}$ with forthcoming tightened bounds, both lower and upper. Should the future tightened Λ go down to near the lower bound, the currently available value of which is ~ 400 [23], then it would require the crossover density $n_{1/2}$ to be raised to even higher. Whether this will not pose a challenge to the PCM in keeping other established star properties unscathed remains to be seen.

Finally, perhaps more fundamental is the possible role of the tidal deformability in elucidating how scale symmetry figures in dense matter [3]. The approach so far employed is anchored on what was referred to as LOSS ("leading order scale symmetry") in [2, 16]. While the LOSS approximation is fairly consistent with nature at low density [2], it is however an open issue at high density. It may be that a precisely determined Λ could provide valuable information on the validity of, or deviation from, LOSS at higher density relevant to compact stars. It would be interesting to see if it gives a clue to the basic infrared structure of the scalar dilaton [3], presently a highly controversial issue in QCD-like gauge theories, both in the Standard Model [15, 20] and for beyond the Standard Model [15, 20, 24]. Specifically, a precise determination of the tidal deformability Λ could lead to information on the anomalous dimension β' – mentioned in the footnote 2 and also in [2] in connection with the puzzling "quenched g_A" in nuclei – which has remained, up to date, totally unknown in QCD. This could be manifested in deviation from the pseudo-conformal sound speed $v_s^2 \simeq 1/3$ at densities $n > n_{1/2}$.

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