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Z boson production in bottom-quark fusion: a study of b-mass effects beyond leading order

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Abstract

We compute the total cross-section for Z boson production in bottom-quark fusion, applying to this case the method we previously used for Higgs production in bottom fusion. Namely, we match, through the FONLL procedure, the next-to-next-to-leading-log five-flavor scheme result, in which the b quark is treated as a massless parton, with the next-to-leading-order $\mathcal{O}(\alpha_s^3)$ four-flavor scheme computation in which bottom is treated as a massive final-state particle. Our computation provides a test-case for the discussion of issues of scale dependence and treatment of heavy quarks, which we discuss in light of our results.

The production of a Z boson is one of the main standard candles at the LHC, and is now measured at the sub-percent level. The main production mode is through quark-anti-quark fusion, of which the bottom-initiated contribution accounts to $\mathcal{O}(4\%)$ of the total cross section. Being that a small but non negligible fraction of the total cross section, its contribution affects both the normalisation and the shape of the kinematic distributions and therefore a precise estimate of the bottom-initiated contribution is important for precision physics, for example in the determination of the W mass [1]. This process is thus an ideal test case for matched computations, recently applied to Higgs production in bottom quark fusion [2–5]. As we shall show here, it provides a theoretically transparent setting for the discussion of issues of choice of scheme and scale in the treatment of heavy quark contributions.

Like any process involving bottom quarks at the matrix-element level, the bottom-initiated Z production process may be computed using two different factorisation schemes, which we refer to, as usual, as four- and five-flavour schemes for short. In the four-flavour scheme (4FS), the b quark is treated as a massive object, which decouples from QCD perturbative evolution. Calculations in this scheme are thus performed by only including the four lightest flavour together with the gluon in evolution equations for parton distributions (PDFs), and in the running of α_s, so $n_f = 4$ in the QCD β function. In the five-flavour scheme (5FS), instead, the b quark is treated on the same footing as other light quark flavours, there is a b PDF, and $n_f = 5$ in evolution equations for PDFs and in the QCD β function.

In matched calculations, both scheme are combined, in such a way that the result differs by that of each of the two schemes by terms which are sub-leading with respect to the accuracy of either of them. Whereas several matching schemes have been proposed and used in the past in the context of deep-inelastic (DIS) or hadronic processes, the FONLL scheme, first proposed for heavy quark production in hadronic collisions [6] has the advantage of being universally applicable; also, it allows for the matching of four- and five-flavour computations performed at any combination of individual perturbative orders. It has been extended to deep-inelastic scattering in Ref. [7] (also including [8, 9] the case in which the heavy quark PDF is independently parametrised) and, as mentioned, it has been used in Refs. [2, 3] for the computation of the total cross-section for Higgs production in bottom quark fusion.

Here, the methodology of Refs. [2, 3] is applied to Z production. In the 5FS, the Z-production cross section has been known up to next-to-next-to leading order (NNLO) (i.e. $\mathcal{O}(\alpha_s^2)$) for almost three decades [10] and the heavy-quark initiated contribution has been specifically discussed in several papers [11–13]. The next-to-leading order (NLO) ($\mathcal{O}(\alpha_s^3)$) four-flavour scheme $Zb\bar{b}$ production cross section was originally computed in Ref. [14] for exclusive 2-jet final states, neglecting the b quark mass. The b-quark mass was subsequently fully included in Refs. [15, 16].

In this work we combine these two results following the procedure we presented in Refs. [2,3] for the closely related case of Higgs production: indeed, the counting of perturbative orders for these two processes is the same, and many of the Feynman diagrams are identical, with the only replacement of Higgs Yukawa couplings with gauge couplings. Following the nomenclature introduced in Ref. [2, 3] (and originally in Ref. [7] for DIS) we have constructed an FONLL-A result, which combines the NNLO 5FS with the LO $\mathcal{O}(\alpha_s^2)$ 4FS fully massive computation, and an FONLL-B, where instead the NNLO 5FS is matched to the full NLO $\mathcal{O}(\alpha_s^3)$ massive results.

Our construction is essentially identical to that of Refs. [2, 3], to which we refer for details: it can be obtained from it by simply replacing the matrix elements for Higgs production with those for gauge boson production. Specifically, we have computed the 5FS NNLO cross-section using the code of Ref, [13], which we cross-checked both at LO and NLO against MG5_aMC@NLO [17]. For the massive 4FS LO and NLO we have also used MG5_aMC@NLO. The construction of the FONLL matched results requires the computation of the massless limit of the massive result: we have implemented this in the public code [18] used in [3], in an updated version soon to be made public. All predictions are obtained using the NNLO NNPDF3.1 PDF set [19]. In order to be consistent with the PDF set used we take, in the 4F scheme, the b pole mass to be $m_b = 4.92$ GeV, while the strong coupling is run at NNLO, with $\alpha_s(m_Z) = 0.118$.

New in comparison to Refs. [2, 3], we have now implemented the possibility of varying the scale μ_b at which the 4FS and 5FS schemes are matched. This scale was taken to coincide with the bottom mass $\mu_b = m_b$ in previous FONLL implementations, but there is no fundamental reason for this choice. The reason why results depend on a matching scale is that in the 5FS the b PDF is not independently parametrised. Rather, it is assumed that it is radiatively generated by the gluon. The matching scale is then the scale at which the b PDF is determined from the gluon.

The matching condition itself depends on the matching scale in such a way that, at any given order, results are independent of it up to sub-leading corrections. This dependence persists in the FONLL matched results, but it is alleviated at scales which are not too far from the bottom production threshold, where the FONLL results almost reduces to the exact mass-dependent result in which the physical threshold is implemented exactly. It reappears at high-enough scales, where the FONLL result reduces to the 5FS, and it only goes away when computing the matching condition to increasingly high perturbative order, or by independently parametrising the heavy quark PDF (indeed, this is the main motivation for independently parametrising charm [19, 20]).

The generalisation of the FONLL matching formulae of Refs. [2, 3] for a generic choice of matching scale is given in the Appendix. Dependence on this matching scale for Higgs in bottom fusion was studied explicitly in Ref. [4, 5]. The matching scheme of Ref. [4, 5], based on a EFT approach, was benchmarked in Ref. [21] to that of Refs. [2, 3] and found to agree with it at the percent level, hence a very similar dependence is expected for FONLL.

Our results are summarised in Figs. 1-3, where matched results in the FONLL-A and FONLL-B scheme are compared to each other and to the 4FS and 5FS scheme computations. In the three plots we study respectively the renormalisation, factorisation, and matching scale dependence of the results. In each case, renormalisation and factorisation scales are fixed, and then varied about, either a high value $\mu = m_Z$, or a low value $\mu_{R,F} = \frac{m_Z+2m_b}{3}$. While the higher scale choice is standard in inclusive W and Z production, the lower choice was advocated in Refs. [22, 23] based on arguments that it is closer to the physical hard scale of the process and thus leads to faster perturbative convergence.

Furthermore, in the case of our preferred FONLL-B result, scale variation is performed in each case in two different ways which differ by sub-leading terms, and the result is shown as a band between these two choices with the central prediction constructed as the mid-point. The two possibilities correspond to the observation (see e.g. [24]) that scale variation by a factor k

of a quantity $F(\mu)$ which is scale-independent up to NLO but has a NNLO scale dependence can be performed by either letting

$$F(\mu_0; k) = F(\mu_0 + \ln k) - \ln k \frac{d}{d \ln \mu} F(\mu)\Big|_{\mu_0 = \mu_0 + \ln k}, \tag{1}$$

or

$$F(\mu_0; k) = F(\mu_0 + \ln k) - \ln k \frac{d}{d \ln \mu} F(\mu)\Big|_{\mu = \mu_0}, \tag{2}$$

where the first term on the r.h.s. is computed up to NLO, while the second term may be computed up to LO, and thus the two expressions differ by NNLO terms (and similarly for higher orders). The two options define the two extremes of the band. The width of the band can be viewed as the ambiguity, i.e. the scale uncertainty, on the scale uncertainty itself. Finally, matching scale variation is performed by varying it between the default $\mu_b = m_b$ and $\mu_b = 2m_b$.

We first describe and comment our results, then discuss their interpretation, also in view of various approximations which have been suggested in the literature. A first observation is that comparison of Figs. 1-2 to the corresponding plots for Higgs production in bottom fusion (Figs. 2-3 of Ref. [3]) show that they are qualitatively almost indistinguishable: this is not unexpected given the similarity between Higgs and Z production which we already repeatedly emphasised.

Coming now to these qualitative features we note that:

- The factorisation scale dependence is generally very slight, while the renormalisation scale dependence is, instead, stronger.
- The scale dependence is quite large in the 4FS scheme, even at NLO though it is reduced in comparison to the LO case. It is much weaker in the 5FS and FONLL cases which all have similar and similarly weak scale dependence, except for very low values $\mu_R \sim \frac{m_Z}{10}$ where however the ambiguity on the scale uncertainty blows up.
- The perturbative expansion is very unstable in the 4FS, with the LO and NLO results differing by a factor two or more. This instability is completely removed when the 4FS is matched to the 5FS: indeed, the FONLL-A and FONLL-B are quite close to each other.
- The 4FS and 5FS results are quite far from each other, with the 4FS NLO significantly closer to the 5FS than the LO. The FONLL results are in turn quite close to the 5FS.
- The perturbative expansion is indeed more stable for a lower choice of factorisation and renormalisation scale. For very low scales $\mu \sim \frac{m_Z}{10}$ the 4FS and 5FS results become similar, but the scale dependence and its uncertainty (i.e. the slope of the curve, and the band about it) become very large.
- A change of matching scale has essentially the same effect on the 5FS and the FONLL results, and it has the effect of moving both towards the 45S, though by a moderate amount.

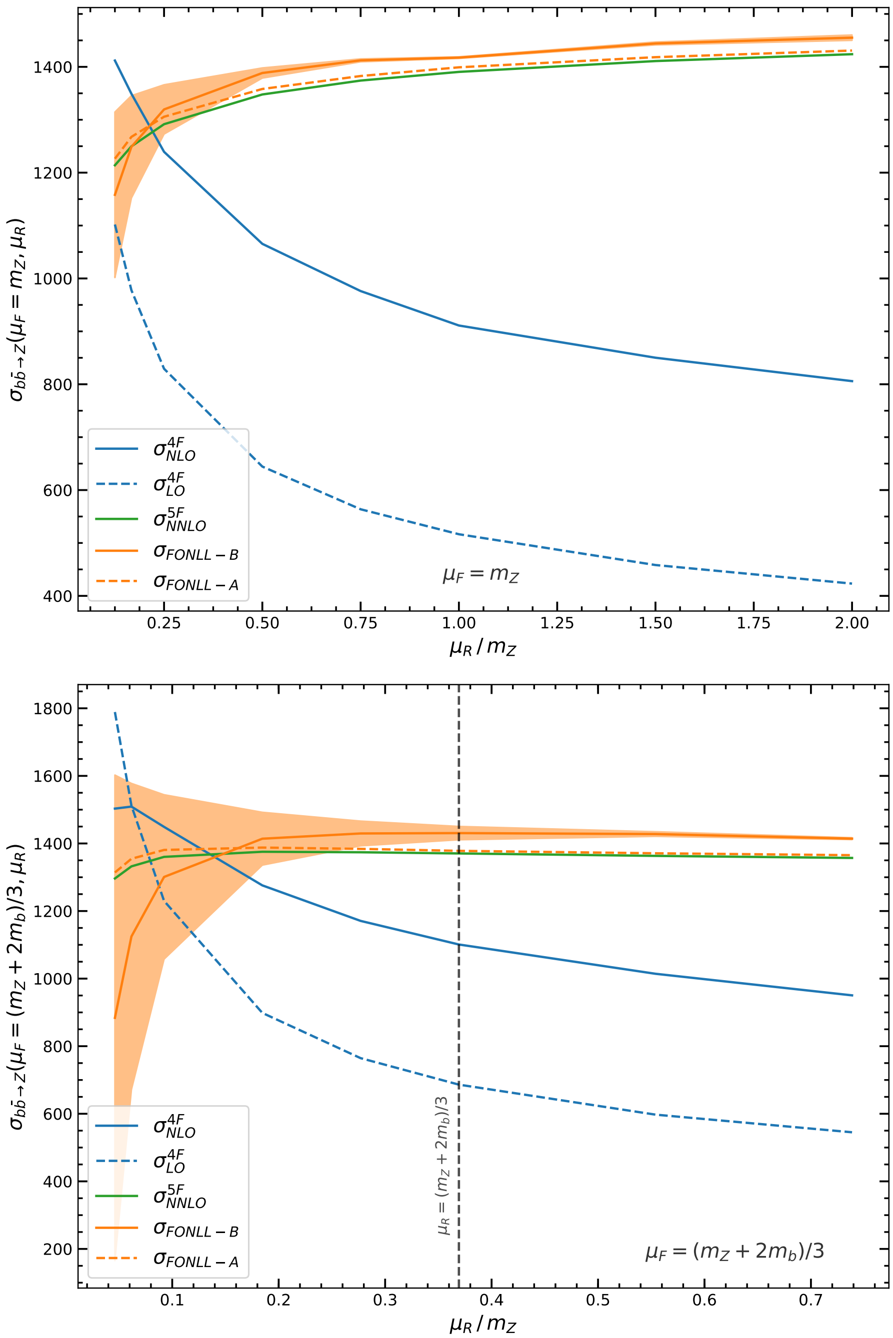

Figure 1: Comparison of the FONLL-A and FONLL-B matched results to each other, to the 4FS LO ($\mathcal{O}(\alpha_s^2)$) and NLO ($\mathcal{O}(\alpha_s^3)$), and to the 5FS NNLO. Results are shown as a function of the renormalisation scale, with the factorisation scale fixed at a high value $\mu_F = m_Z$ (top) or a low value $\mu_F = \frac{(m_Z+2m_b)}{3}$ (bottom). The band about the FONLL-B result is obtained from two different implementations of NLO scale variation that differ by NNLO terms (see text) and is thus an estimate on the ambiguity of the scale variation itself.

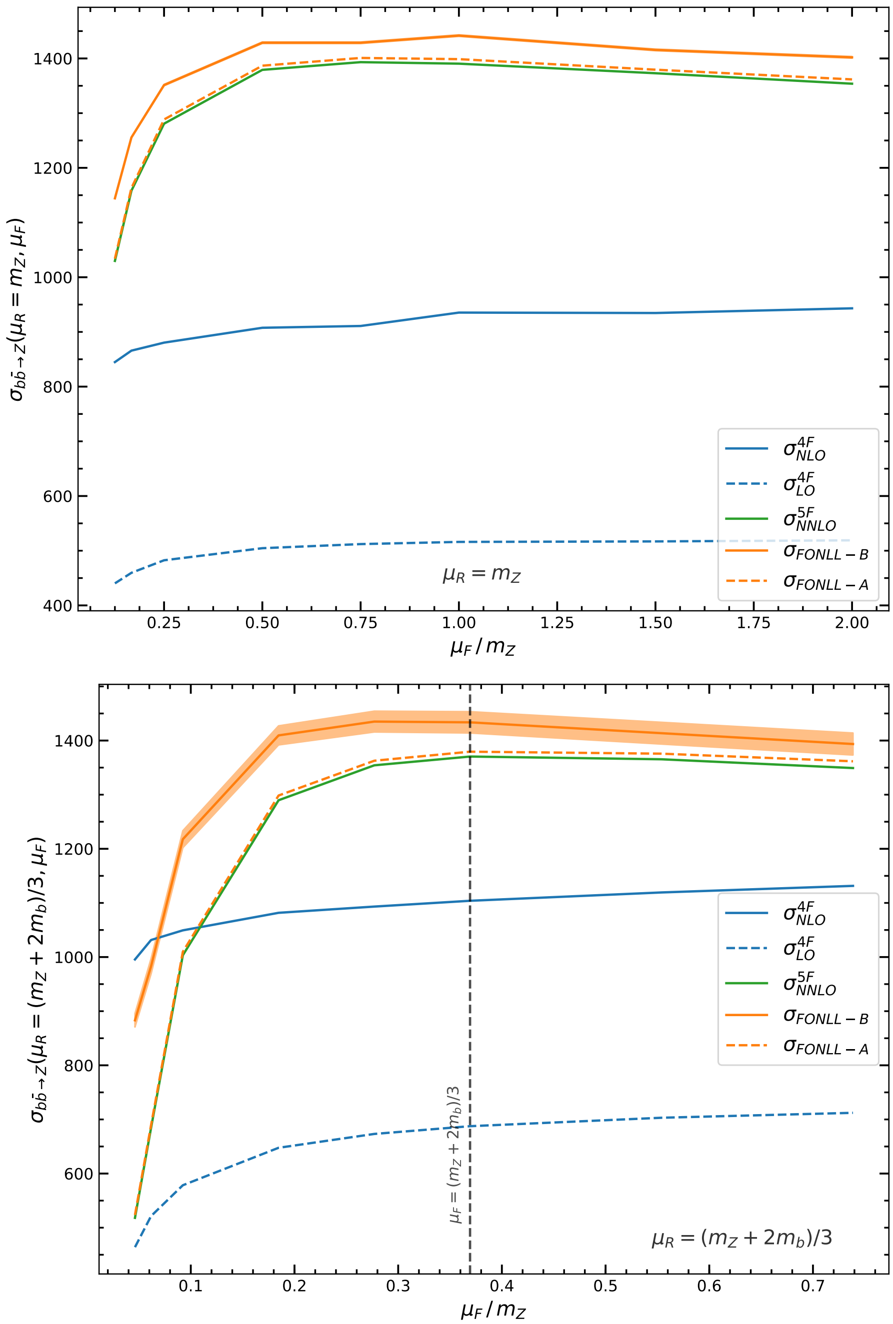

Figure 2: Same as Fig. 1, but now with the factorisation scale varied with the renormalisation scale kept fixed at a high value $\mu_R = m_Z$ (top) or a low value $\mu_R = \frac{(m_Z+2m_b)}{3}$ (bottom) .

These qualitative features have a simple theoretical interpretation. To this purpose, note that the cross-section for this process contains collinear logarithms regulated by the heavy quark mass, i.e. powers of $\ln \frac{\mu_Z}{m_b^2}$, one at each perturbative order. These logs arise from a transverse momentum integration, whose the upper limit is the maximum value of the transverse momentum, i.e. the hard scale of the process, which is proportional to but not equal to m_Z, and the lower limit is the physical production threshold, which is proportional to but not equal to m_b. Of course, one can always rewrite the ensuing logarithm as $\ln \frac{\mu_Z^2}{m_b^2}$, plus constants (i.e. terms which only depend on the dimensionless ratio $\tau = \frac{\mu_z^2}{s}$), and mass corrections (i.e. terms suppressed by powers of $\frac{\mu_b^2}{m_Z^2}$).

In the 4FS the result is exact, so whatever is not included in the log is included in the constants or in the mass corrections; on the other hand at N^{k}LO only the first $k+1$ logs are included. In the 5FS the logs are rewritten as $\ln \frac{\mu_Z^2}{m_b^2} = \ln \frac{\mu_Z^2}{\mu_F^2} + \ln \frac{\mu_F^2}{\mu_b^2} + \ln \frac{\mu_b^2}{m_b^2}$, where μ_F is the factorisation scale and μ_b is the matching scale. The logs of the factorisation scale $\ln \frac{\mu_F^2}{\mu_b^2}$ are then resummed to all orders into the evolution of the PDF, while the logs of the hard scale $\ln \frac{\mu_Z^2}{\mu_F^2}$ are included to finite order in the hard partonic cross-section and logs the matching scale, $\ln \frac{\mu_b^2}{m_b^2}$, are included to finite order in the matching condition, which expresses the initial b PDF in terms of the gluon (they would be implicitly included in the initial PDF if the b PDF were independently parametrised). When varying the factorisation scale, logs at the upper end of the evolution are reshuffled between the resummed PDF and the fixed-order but exact hard cross-section. When varying the matching scale, logs at the bottom end of the evolution are reshuffled between the resummed PDF and the fixed-order but exact hard matching condition.

Note that both the hard coefficient and the matching condition contains logs and constants, but not mass-suppressed terms: so in the 5FS constants and logs of the matching scale, as well as constants and logs in the hard coefficient, are treated exactly but to fixed order, while logs of the factorisation scale are resummed to all orders, but neglecting constants. When the 5FS and the 4FS are matched into FONLL, also mass-suppressed terms, on top of constants and logs of the matching scale, are treated exactly.

The fact that the 4FS is perturbatively unstable while the 5FS is not then is easily explained as a manifestation of the fact that the 4FS contains large logs which are resummed in the 5FS. This is confirmed by the fact that the large difference between the 4FS LO and NLO is of the same order of the scale variation of the LO: indeed the scale variation by construction captures the size of logarithmic contribution. So the sizeable difference which persists between the 5FS and NLO 4FS results is explained as being due to the higher order (NNLO and beyond) logs which are missing in the 4FS NLO, their size being quantitatively estimated in [23]. This is confirmed by the observation that the FONLL-A and FONLL-B include both the large log resummation, and the full constants and mass suppressed terms, up to LO and NLO respectively. The difference between the FONLL-A and FONLL-B is thus the size of the constant and mass-suppressed contributions to the difference between the 4FS LO and NLO. This is seen to be much smaller than the total difference between 4FS LO and NLO, which must therefore be due to the log.

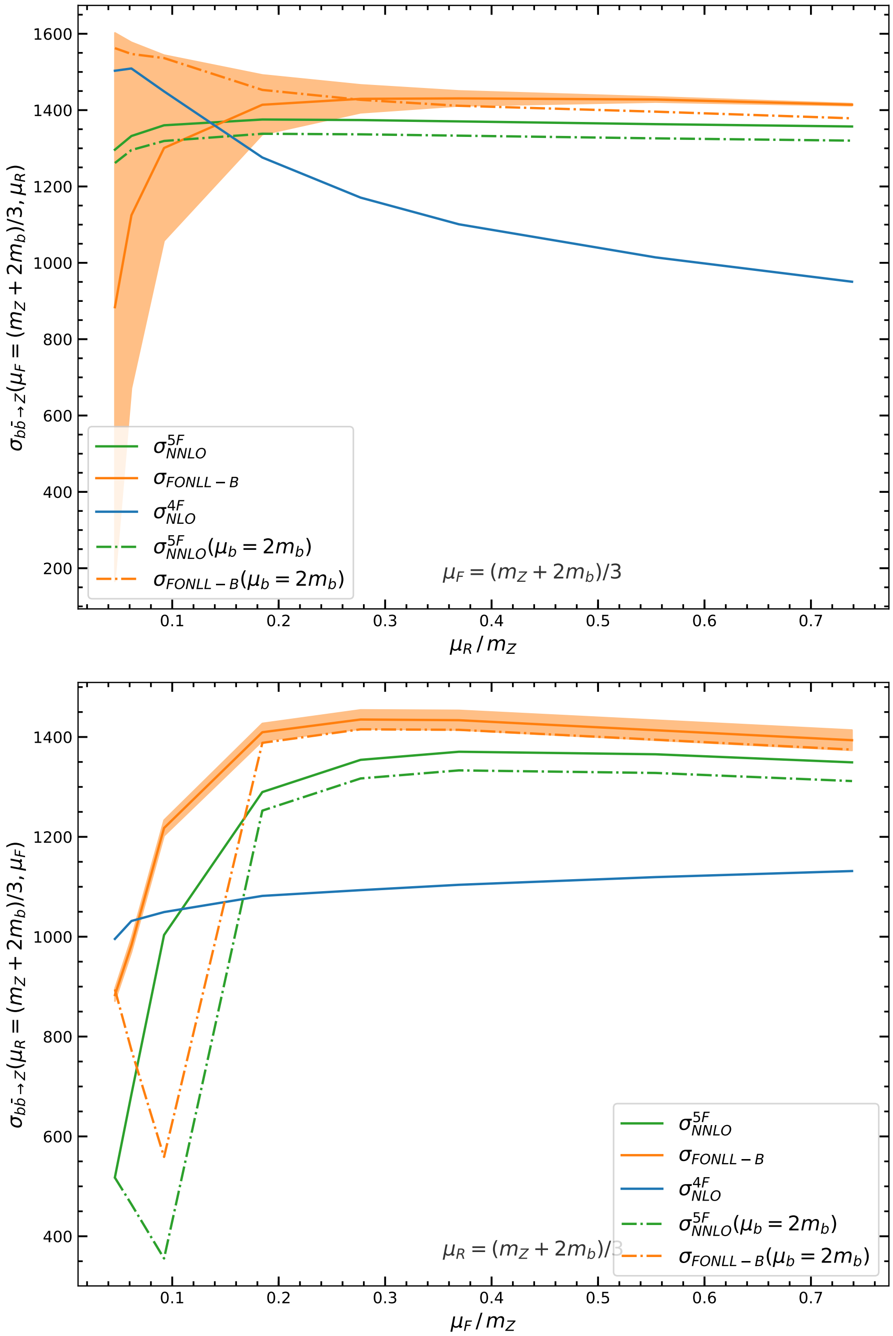

Figure 3: Comparison of the FONLL-B and of the 5FS NNLO results for two different values of the matching scale $\mu_b = m_b$ (same as in Figs. 1-2) to each other and to the 4FS NLO. Results are shown as a function of the renormalisation scale for fixed factorisation scale (top) or as a function of the factorisation scale for fixed renormalisation scale (bottom), in each case with the fixed scale chosen as $\mu = \frac{(m_Z+2m_b)}{3}$.

In order to further disentangle, within this small contribution, the constant from mass-suppressed term, one would have to vary the hard scale, i.e. the Z mass. This was done in Ref. [3] for Higgs production: variation of the Higgs mass left the difference between FONLL-A and FONLL-B essentially unchanged, thus showing that mass corrections are negligible and the bulk of the difference between FONLL-A and FONLL-B is due to a constant. Given the similarity between the two processes we expect the same to be the case here. Given the small size of this contribution the issue is largely academic anyway.

The qualitative form of the renormalisation scale dependence of the 4FS result is also easy to understand: as the scale is decreased, the value of α_s multiplying the large collinear log increases, and both the LO and NLO predictions grow; this growth is only partly reduced by the higher-order compensating term, at least down to scales $\mu_R \sim 0.2\mu_Z$ where the ambiguity on the scale variation itself becomes very large. The fact that the 5FS (and FONLL) result have almost no renormalisation scale dependence shows that this scale dependence is coming from the b quark term which is treated differently between 4FS and 5FS.

The factorisation scale dependence is particularly intriguing. The fact that this dependence is very slight in the 4FS is again consistent with the observation that scale dependence is driven by the heavy quark terms: in this scheme, in the absence of a b PDF, the factorisation scale dependence is related to perturbative evolution of the light quarks and gluons, which is moderate at NNLO. On the other hand, in the 5FS (and in FONLL) collinear logs are resummed in the evolution of the b-PDF up to μ_F, and then expanded out in the partonic cross-section from μ_F to the physical hard scale of the process. We therefore expect the factorisation scale dependence in this scheme to be approximately stationary around this physical hard scale, very slight above it (where α_s is small) and to only become significant when μ_F is lower than the physical hard scale itself. This behaviour is clearly seen in Fig. 2, with the stationary point close to the low scale advocated in Ref. [22, 23] that indeed this scale of the hard process, and it nicely explains the very weak factorisation scale dependence also seen in the 5FS unless $\mu_F \lesssim 0.2 m_Z$ or so.

Finally, the fact that when increasing the matching scale μ_b the 5FS and FONLL-B result decrease and get closer to the 4FS is understood as a consequence of the fact that the resummed log becomes smaller: clearly, if one were to choose $\mu_b = m_Z \sim 10 m_b$ the 5FS would reduce to the 4FS one because the resummed logs then vanish. With the choice $\mu_b = 2m_b$ the FONLL and 5FS move towards the the 4FS by an accordingly smaller amount.

We can finally discuss, in light of all this, the two related issues of choosing the various scales, μ_F, μ_R and μ_b, and of the validity of various approximations. As discussed, the scale dependence of this process is driven by the collinear logs in the b quark contribution, and thus the bulk of it comes from the choice of argument in these logs.

In a fully massive 4FS calculation, these collinear logs are treated exactly, so the scale dependence comes purely from the choice of argument in the strong coupling. It then turns out that reducing the renormalisation scale increases the 4FS unresummed results up to the point where it agrees with the 5FS resummed one. This is however accidental: the lack of resummation is made up by artificially increasing α_s, and indeed at low scale the scale dependence of the 4FS result is not improved: if anything, it increases. Hence, the 4FS appears to be a poor approximation to this process and its improvement by lowering the renormalisation scale is unreliable.

In a 5FS calculation, instead, as mentioned, the exact upper and lower limits of the transverse momentum integration are replaced by μ_F and μ_b, respectively. As also mentioned, it has been argued [22, 23] that the exact, kinematics-dependent upper limit of integration is on average close to a scale $\frac{m_Z+2m_b}{3} \sim 0.35m_Z$. This is borne out by our results: for all $\mu_F \gtrsim 0.3m_Z$ the factorisation scale dependence of the 5FS result is flat, and with this choice of μ_R the 5FS scale dependence is visibly flatter. Given the smallness of mass corrections, in practice a 5FS with low factorisation and renormalisation scales appears to be a good approximation of the full FONLL result.

On the other hand, it has been recently argued [25] that a higher choice of matching scale may provide a better approximation. Clearly, this is a process-dependent statement that should be checked on a case-by-case basis: as discussed raising the matching scale improves the accuracy of the starting, dynamically generated PDF, as it matches it at a scale where perturbation theory is more reliable, but it reduces the size of the logs which are resummed. In the present case, the resummed logs are a large effect and the constants a small correction, so raising the matching scale does not appear to be advantageous: indeed, the renormalisation and factorisation scale dependence is the same with $\mu_b = m_b$ or $\mu_b = 2m_b$, with no obvious improvement.

In fact, when raising μ_b the 5FS result decreases, and moves towards the low 4FS, but with no improvement in perturbative stability of the latter. This is to be contrasted to the case in which μ_R and μ_F are lowered, which also brings the 4FS and the 5FS closer but now towards a high value, and with a visible increase in perturbative stability. In fact, the FONLL result shows that exact inclusion of the mass corrections (most likely the constant) increases the pure 5FS, by a small amount. On the contrary, raising the matching scale lowers it: this means that the deterioration of the log resummation is a larger effect than the improvement made by starting the PDF at a scale at which perturbation theory is more reliable. So a 5FS with large μ_b does not appear to be a better approximation in our case: it is likely to be a worse approximation if μ_b is raised by a moderate amount, and it definitely appears to be a poor approximation if μ_b is raised up to the point at which the 5FS result reduces to the 4FS one. On the other hand, a variation of μ_b by perhaps a factor two, as shown in Fig. 3, might well be a reasonable estimate of the uncertainty due to the use of a fixed-order matching condition and should be included in the theoretical uncertainty, as was done in Refs. [5, 21]. This theoretical uncertainty can only be removed by parametrising the b PDF, in which case it is traded for a PDF uncertainty.

In summary, we have determined the total cross-section for Z production in bottom quark fusion at the highest available accuracy in a matched FONLL scheme, and we have used our results as a test case for the discussion of issues of scale dependence and heavy quark treatment, by generalising our previous results for Higgs production, and studying not only renormalisation and factorisation scale, but also matching scale dependence.

Our main phenomenological conclusion is that, similarly to the case of Higgs production, mass effects are small, but non-negligible in comparison to the high experimental accuracy to which this process is measured. However, the contribution due to the resummation of collinear logs of the heavy quark is sizeable, thereby making a five-flavour scheme in which the b quark is endowed with a PDF a better approximation to the full FONLL result than the fixed-order

4FS calculation with massive b, which falls short of the full prediction and displays large scale uncertainties.

A low choice of renormalisation and factorisation scale reduces the scale dependence of both the full FONLL and pure 5FS result and is likely to improve their accuracy, though in practice this makes little difference as the scale dependence of both these results is very slight. However, it does suggest that the hard physical scale for this process is lower than the final-state mass, as previously advocated. All in all, our results support the conclusion that a fully matched treatment of heavy quarks with a proper inclusion of mass effects is necessary for LHC phenomenology at the percent level, either through its direct use, or as a guide to construct efficient and accurate approximations.

A public implementation of our NNLL+NLO FONLL-B matched computation will be added to our code for Higgs production [3], publicly available from

`http://bbhfonll.hepforge.org/`.

Acknowledgments

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A FONLL expressions with μ_b different from m_b

We give for completeness the FONLL expressions by using m_b different from μ_b. Note that the only difference with respect to the formulae presented in [3], is in the logarithm obtained from the expansion of the b PDF, where $L = \log(Q^2/m_b^2)$, becomes $L = \log(Q^2/\mu_b^2)$.

With this modification in place we get, for the 4F scheme, B coefficients:

$$B_{gg}^{(2)}\left(y, \frac{Q^2}{m_b^2}\right) = \hat{\sigma}_{gg}^{(2)}\left(y, \frac{Q^2}{m_b^2}\right) \tag{A.1}$$

$$B_{q\bar{q}}^{(2)}\left(y, \frac{Q^2}{m_b^2}\right) = \hat{\sigma}_{q\bar{q}}^{(2)}\left(y, \frac{Q^2}{m_b^2}\right) \tag{A.2}$$

while at $\mathcal{O}(\alpha_s^3)$ the redefinition of α_s contributes:

$$B_{gg}^{(3)}\left(y, \frac{Q^2}{m_b^2}, \frac{\mu_R^2}{\mu_b^2}, \frac{\mu_F^2}{\mu_b^2}\right) = \hat{\sigma}_{gg}^{(3)}\left(y, \frac{Q^2}{m_b^2}\right) - \frac{2T_R}{3\pi}\ln\frac{\mu_R^2}{\mu_F^2}\hat{\sigma}_{gg}^{(2)}\left(y, \frac{Q^2}{m_b^2}\right) \tag{A.3}$$

$$B_{q\bar{q}}^{(3)}\left(y, \frac{Q^2}{m_b^2}, \frac{\mu_R^2}{\mu_b^2}, \frac{\mu_F^2}{\mu_b^2}\right) = \hat{\sigma}_{q\bar{q}}^{(3)}\left(y, \frac{Q^2}{m_b^2}\right) - \frac{2T_R}{3\pi}\ln\frac{\mu_R^2}{\mu_b^2}\hat{\sigma}_{q\bar{q}}^{(2)}\left(y, \frac{Q^2}{m_b^2}\right) \tag{A.4}$$

$$B_{gq}^{(3)}\left(y, \frac{Q^2}{m_b^2}\right) = \hat{\sigma}_{gq}^{(3)}\left(y, \frac{Q^2}{m_b^2}\right) \tag{A.5}$$

$$B_{qg}^{(3)}\left(y, \frac{Q^2}{m_b^2}\right) = \hat{\sigma}_{qg}^{(3)}\left(y, \frac{Q^2}{m_b^2}\right). \tag{A.6}$$

The massless limit of the 4F scheme coefficients, $B^{(0)}$, are, in this case, given by

$$\sigma^{(4),(0)}\left(\alpha_s(Q^2), L\right) = \int_{\tau_H}^1 \frac{dx}{x}\int_{\frac{\tau_H}{x}}^1 \frac{dy}{y^2}\sum_{ij=q,g} f_i(x,Q^2) f_j\left(\frac{\tau_H}{xy}, Q^2\right) B_{ij}^{(0)}\left(y, L, \alpha_s(Q^2)\right), \tag{A.7}$$

with

$$B_{ij}^{(0)}\left(y, L, \alpha_s(Q^2)\right) = \sum_{p=2}^{N}\left(\alpha_s(Q^2)\right)^p B_{ij}^{(0),(p)}(y, L), \tag{A.8}$$

and

$$B_{gg}^{(0)(2)}(y,L) = y\int_y^1 \frac{dz}{z}\left[2\mathcal{A}_{gb}^{(1)}(z,L)\,\mathcal{A}_{gb}^{(1)}\left(\frac{y}{z},L\right) + 4\mathcal{A}_{gb}^{(1)}\left(\frac{y}{z},L\right)\hat{\sigma}_{gb}^{(1)}(z)\right] + \hat{\sigma}_{gg}^{(2)}(y), \tag{A.9}$$

$$B_{q\bar{q}}^{(0)(2)}(y,L) = \hat{\sigma}_{q\bar{q}}^{(2)}(y); \tag{A.10}$$

while the new contributions to $\mathcal{O}(\alpha_s^3)$ are

$$\begin{aligned} B_{gg}^{(0)(3)}(y,L) = y\int_y^1 \frac{dz}{z}\Bigg[&4\mathcal{A}_{gb}^{(2)}(z,L)\,\mathcal{A}_{gb}^{(1)}\left(\frac{y}{z},L\right) + 2\mathcal{A}_{gb}^{(1)}(z,L)\,\mathcal{A}_{gb}^{(2)}\left(\frac{y}{z},L\right)\hat{\sigma}_{b\bar{b}}^{(1)}(z) \\ &+ 4\mathcal{A}_{gb}^{(2)}\left(\frac{y}{z},L\right)\hat{\sigma}_{gb}^{(1)}(z) + 4\mathcal{A}_{gb}^{(1)}\left(\frac{y}{z},L\right)\hat{\sigma}_{gb}^{(2)}(z)\Bigg], \end{aligned} \tag{A.11}$$

$$\begin{aligned} B_{gq}^{(0)(3)}(y,L) = y\int_y^1 \frac{dz}{z}\Bigg[&2\mathcal{A}_{\Sigma b}^{(2)}(z,L)\,\mathcal{A}_{gb}^{(1)}\left(\frac{y}{z},L\right) + 2\mathcal{A}_{\Sigma b}^{(2)}\left(\frac{y}{z},L\right)\hat{\sigma}_{gb}^{(1)}(z) \\ &+ 2\mathcal{A}_{gb}^{(1)}\left(\frac{y}{z},L\right)\hat{\sigma}_{qb}^{(2)}(z)\Bigg], \end{aligned} \tag{A.12}$$

which completes our result in the case in which $\mu_b \neq m_b$.

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