

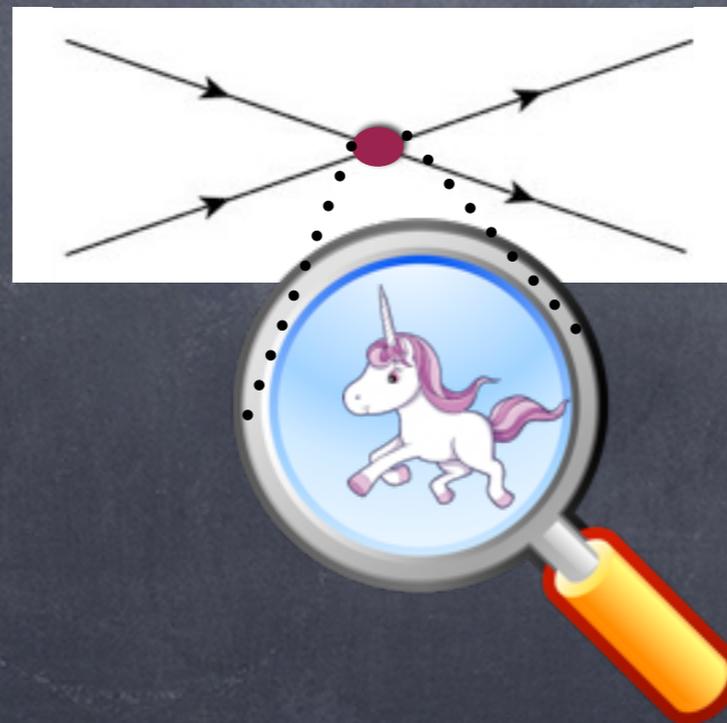


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Saclay-sous-Bois, 21 April 2017



Lectures on Effective Field Theories in Particle Physics



Timetable

- **Part 1:** illustrated philosophy of effective field theories
- **Part 2:** matching simple UV theory to low-energy EFT, with all gory details
- **Part 3:** SM EFT: effective field theory for physics beyond the Standard Model
- **Part 4:** covariant derivative expansion: path integral methods in EFT



Recommended Literature

General education

- Kaplan [[nucl-th/0510023](#)]
- Rothstein [[hep-ph/0308266](#)]
- Manohar [[hep-ph/9606222](#)]

EFT for heavy mesons

- Grinstein [[hep-ph/9411275](#)]

EFT for superconductors

- Polchinski [[hep-th/9210046](#)]

EFT for binary inspirals

- Goldberger [[hep-ph/07101129](#)]

My lecture notes for the 2nd, 3rd, and 4th lecture may or may not be available before that lecture starts



Part 1

Philosophy and Overview of Effective Field Theories

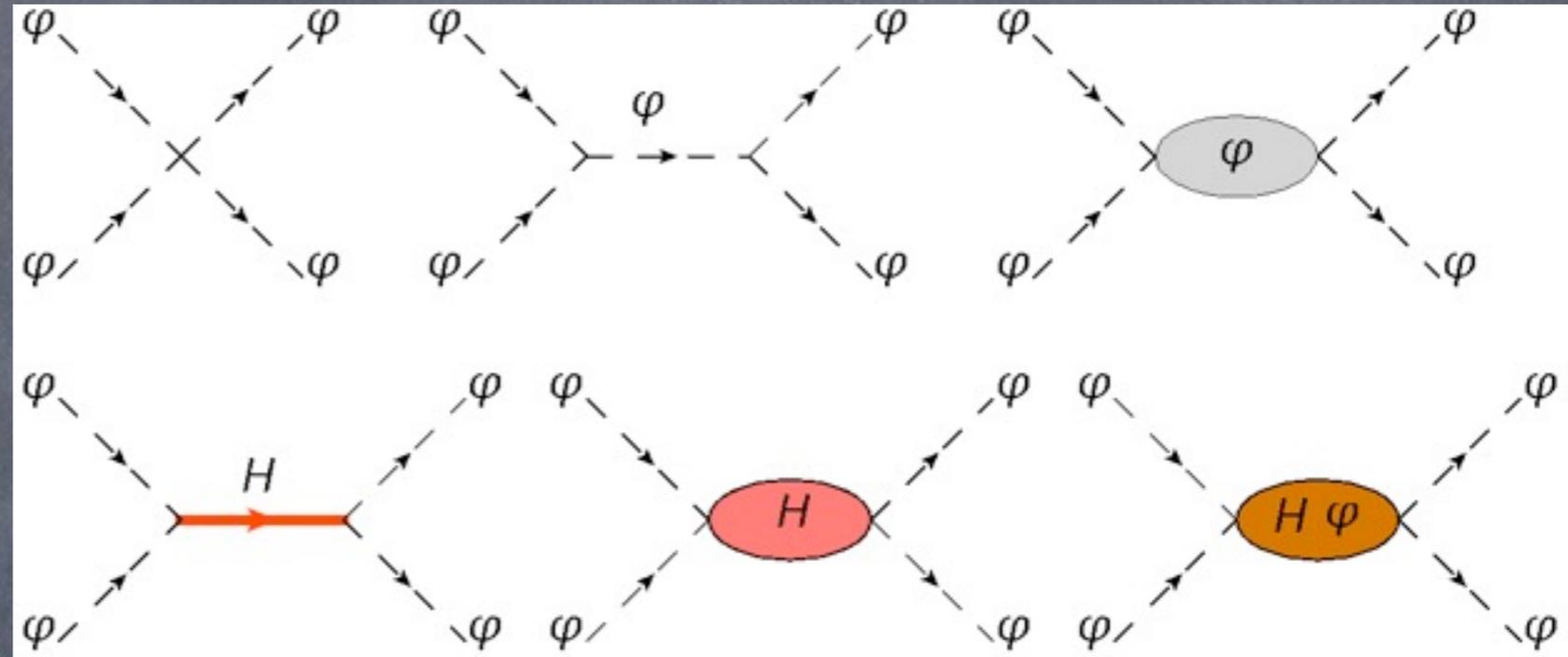
Introduction

Effective Lagrangian in amplitude language

Consider quantum field theory with "light" fields φ and "heavy" fields H $\mathcal{L}(\phi, H)$

We are interested in the scattering amplitudes for "light" fields.

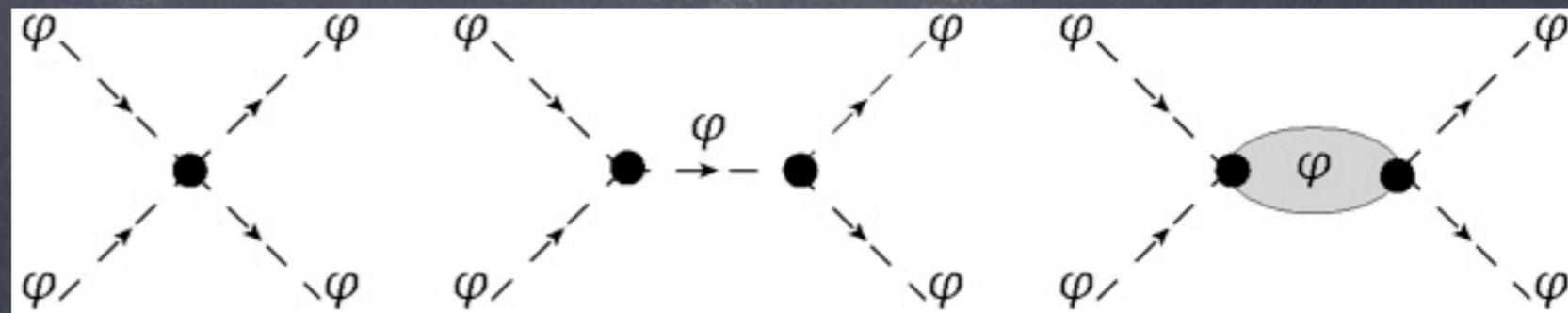
E.g. $2 \rightarrow 2$ amplitude schematically:



The effective theory is a theory containing only "light" fields φ that reproduces all scattering amplitudes of φ of the full theory containing φ and H .

$$\mathcal{L}_{\text{eff}}(\phi)$$

E.g. $2 \rightarrow 2$ amplitude schematically:



Note that: $! \mathcal{L}_{\text{eff}}(\phi) \neq \mathcal{L}(\phi, 0) !$

Crash course of path integrals

Correlation functions in quantum theory can be defined via a path integral

$$\langle \phi(x_1) \dots \phi(x_n) \rangle = \frac{\int [D\phi] \phi(x_1) \dots \phi(x_n) \exp \left[i \int d^4x \mathcal{L}(\phi) \right]}{\int [D\phi] \exp \left[i \int d^4x \mathcal{L}(\phi) \right]}$$

Generating functional for correlation functions

$$Z[J] = \int D\phi \exp \left[i \int d^4x (\mathcal{L}(\phi) + J\phi) \right]$$

$$\langle \phi(x_1) \dots \phi(x_n) \rangle = \left(-i \frac{\delta}{\delta J(x_1)} \right) \dots \left(-i \frac{\delta}{\delta J(x_n)} \right) (\log Z[J])_{J=0}$$

“Quantum action” or generating functional for 1PI correlation functions

$$\Gamma[\phi_c] = \log Z[J] - \int d^4x J(x) \phi_c(x)$$
$$\phi_c(x) = \frac{\delta \log Z[J]}{\delta J(x)}$$

$$\langle \phi(x_1) \dots \phi(x_n) \rangle_{1\text{PI}} = i \frac{\delta^n \Gamma[\phi_c]}{\delta \phi_c(x_1) \dots \delta \phi_c(x_n)}$$

Effective Lagrangian in path integral language

Generating functional for amplitudes of "light" and "heavy" fields

$$Z[J_\phi, J_H] = \int [D\phi][DH] \exp \left[i \int d^4x (\mathcal{L}(\phi, H) + J_\phi\phi + J_H H) \right]$$

Full quantum action:
generates 1PI amplitudes
for both for "light" and
"heavy" fields

$$\Gamma[\phi_c, H_c] = \log Z[J_\phi, J_H] - \int d^4x J_\phi(x)\phi_c(x) - \int d^4x J_H(x)H_c(x)$$
$$\phi_c(x) = \frac{\delta \log Z[J_\phi, J_H]}{\delta J_\phi(x)}, \quad H_c(x) = \frac{\delta \log Z[J_\phi, J_H]}{\delta J_H(x)}$$

Effective quantum action:
generates 1PI amplitudes
for both for "light" fields only

$$\Gamma_{\text{eff}}[\phi_c] = \log Z[J_\phi, 0] - \int d^4x J_\phi(x)\phi_c(x)$$
$$\phi_c(x) = \frac{\delta \log Z[J_\phi, 0]}{\delta J_\phi(x)}$$

$\mathcal{L}_{\text{eff}} = ?$ such that

$$Z_{\text{eff}}[J] = \int [D\phi] \exp \left[i \int d^4x (\mathcal{L}_{\text{eff}} + J\phi) \right]$$

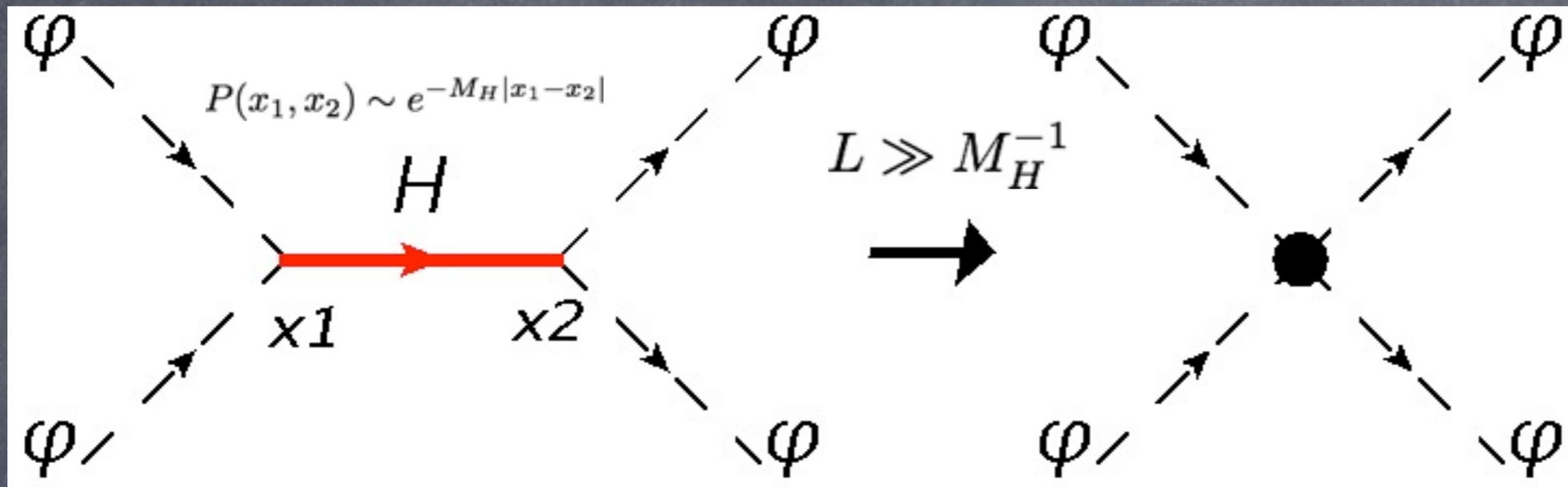
$$\Gamma_{\text{eff}}[\phi_c] = \log Z_{\text{eff}}[J] - \int d^4x J(x)\phi_c(x)$$

$$\phi_c(x) = \frac{\delta \log Z_{\text{eff}}[J]}{\delta J(x)}$$

Effective Lagrangian is a
Lagrangian for a theory containing
"light" fields φ only that leads to
the same quantum action $\Gamma_{\text{eff}}[\varphi]$ as
the full theory with φ and H

$\Gamma[\varphi, H]$ is a complicated non-local (that is non-polynomial) functional.
Hence $\Gamma_{\text{eff}}[\varphi]$ also expected to be non-local in general.

Local effective Lagrangian



- Propagation of heavy particle H with mass M_H is suppressed at distance scale above its inverse mass
- Processes probing distance scales $\gg M_H$, equivalently for energy $\ll M_H$, cannot be resolved
- Then, intuitively, exchange of heavy particle H between light particles φ should be indistinguishable from a contact interaction of φ
- In other words, the effective Lagrangian describing φ interactions should be well approximated by a **local Lagrangian**, that is, by a polynomial in φ and its derivatives

Power Counting

- Effective Lagrangians by construction must contain infinite number of terms. Therefore any useful EFT comes with a set of **power counting** rules which allow one to organize the Lagrangian in a consistent expansion and single out the most relevant terms
- For relativistic theories obtained by integrating out heavy fields H with mass of order M_H , and the inverse of the latter provides a natural expansion parameter to organize the effective Lagrangian. Observables are then expanded in E/M_H where E is the typical energy scale of the experiment
- Sometimes a large mass parameter is not available, and expansion of the effective Lagrangian is in the number of derivatives
- Different power counting rules apply to non-relativistic theories, as then space and time have different scaling dimensions. Still different counting rules apply to relativistic systems with one heavy component (such as atoms or B-mesons)

Scaling in relativistic theories

To isolate UV and IR limits,
consider rescaling of
spacetime coordinates

$$x \rightarrow \xi x'$$

$\xi \rightarrow 0$ is zooming in on small distances (UV limit)
 $\xi \rightarrow \infty$ is zooming in on large distances (IR limit)

$$S = \int d^4x \left((\partial_\mu \phi)^2 - m^2 \phi^2 - \lambda \phi^4 - \frac{c}{\Lambda^{n+d-4}} \phi^n \partial^d \right)$$

$$\rightarrow \int d^4x' \left(\xi^2 (\partial'_\mu \phi)^2 - m^2 \xi^4 \phi^2 - \lambda \xi^4 \phi^4 - \frac{c \xi^{4-d}}{\Lambda^{n+d-4}} \phi^n \partial'^d \right)$$

Since path integral is dominated by kinetic terms
to easily compare the original and rescaled actions

$$\phi(x) \rightarrow \xi^{-1} \phi'(\xi x)$$

it is convenient normalize the kinetic terms canonically

$$S \rightarrow \int d^4x' \left((\partial'_\mu \phi')^2 - m^2 \xi^2 \phi'^2 - \lambda \phi'^4 - \frac{c_{n,d} \xi^{4-d-n}}{\Lambda^{n+d-4}} \phi'^n \partial'^d \right)$$

Scaling in relativistic theories

$$\phi(x) \rightarrow \xi^{-1} \phi'(\xi x)$$

$$S = \int d^4x \left((\partial_\mu \phi)^2 - m^2 \phi^2 - \lambda \phi^4 - \frac{c_{n,d}}{\Lambda^{n+d-4}} \phi^n \partial^d \right)$$
$$\rightarrow \int d^4x' \left((\partial'_\mu \phi')^2 - m'^2 \phi'^2 - \lambda' \phi'^4 - \frac{c'_{n,d}}{\Lambda^{n+d-4}} \phi'^n \partial'^d \right)$$

$$x \rightarrow \xi x'$$

Mass term is **relevant** operator: it gets more important in IR

$$m'^2 = m^2 \xi^2$$

$$\lambda' = \lambda$$

Quartic coupling is **marginal** operator: it is (approximately) the same in UV and in IR

$$c'_{n,d} = c_{n,d} \xi^{4-d-n}$$

Higher dimensional interactions (for $d+n > 4$) are **irrelevant** operators: they get less important in IR

Power counting in relativistic EFT, determining the importance of various interactions, can be organized based on canonical dimension of interactions

Dimensional analysis crash course

Relativistic field theory

$$S = \int d^4x \left[\partial_\mu \phi^\dagger \partial_\mu \phi - \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + i \bar{\psi} \gamma_\mu \partial_\mu \psi + \dots \right]$$

$$[\partial] = \text{mass}^1$$

$$[\phi] = \text{mass}^1$$

$$[A_\mu] = \text{mass}^1$$

$$[\psi] = \text{mass}^{3/2}$$

Scaling in non-relativistic theories

$$S = \int dt d^3x \left[\phi^\dagger \partial_t \phi - \phi^\dagger \frac{\partial_x^2}{2m} \phi - c_4 (\phi^\dagger \phi)^2 + \dots \right]$$

$$x \rightarrow \xi x'$$

$$t \rightarrow \xi^2 t'$$

$$\phi(t, x) \rightarrow \xi^{-3/2} \phi'(\xi^2 t, \xi x)$$

In a non-relativistic theory time and space are on different footing. In order to keep kinetic terms invariant they should be assigned different scaling dimensions

$$S \rightarrow \int dt' d^3x' \left[\phi'^\dagger \partial_{t'} \phi' - \phi'^\dagger \frac{\partial_{x'}^2}{2m} \phi' - c_4 \xi^{-1} (\phi'^\dagger \phi')^2 + \dots \right]$$

$$c'_4 = c_4 \xi^{-1}$$

In usual non-relativistic theory quartic self-interaction is irrelevant!

There are other non-relativistic systems (e.g. $z=3$ fixed point) where scaling is yet different

Power counting in heavy quark effective theory

- In a B-meson system, the bottom quark can be pictured as a heavy ball with mass with mass M_Q moving in a sea of light quarks
- Interactions between the heavy quark and the light ones have a typical momentum exchange of order $\Lambda_{\text{qcd}} \ll M_Q$.
- Thus, the heavy quark moves with a constant 4-velocity v , which is also the B-meson velocity
- One can also prove that spin degrees of freedom decouple at leading order
- Effective theory can be constructed as derivative expansion using heavy meson field ϕ , D , and v

$$k_\mu = M_Q v_\mu + k'_\mu$$

$$v^2 = 1, \quad k' \ll M_Q$$

$$\frac{i}{k^2 - M_Q^2} = \frac{i}{M_Q^2 + 2M_Q v_\mu k'_\mu + k'_\mu k'_\mu - M_Q^2}$$

$$\approx \frac{i}{2M_Q v_\mu k'_\mu}$$

Lorentz-invariant when v transforms as Lorentz vector

$$\mathcal{L}_{\text{HQET}} = i\phi^\dagger v_\mu D_\mu \phi + \mathcal{O}\left(\frac{1}{M_Q}\right)$$

Covariant wrt QCD and/or EM

Irrelevant interactions

Valid for processes with momentum exchange well below M_Q

Why the sky
is blue?

Why the sky is blue...

...or Rayleigh scattering of low-energy photons on neutral atoms

Relevant scales:

$$E_\gamma \ll E_* \ll a_0^{-1} \ll M$$

$$O(2) \text{ eV} \quad O(10) \text{ eV} \quad O(\text{keV}) \quad O(\text{GeV})$$

Lowest order
Lagrangian:

$$\mathcal{L}^{(0)} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + i\phi^\dagger v_\mu \partial_\mu \phi$$

photon

atom

Effective theory
with cutoff of order E^*

Dimensional analysis suggests
magnitude of EFT coefficients
set by atom size \sim Bohr radius

Leading C and P
conserving
interactions:

$$\mathcal{L}^{(1)} = a_0^3 c_1 \phi^\dagger \phi F_{\mu\nu} F_{\mu\nu} + a_0^3 c_2 \phi^\dagger \phi v_\mu v_\nu F_{\mu\alpha} F_{\nu\alpha}$$

Unknown numerical coefficients,
here assumed to be $O(1)$,
which can be computed by
matching to the full theory

Why the sky is blue...

$$\mathcal{L}^{(1)} = a_0^3 c_1 \phi^\dagger \phi F_{\mu\nu} F_{\mu\nu} + a_0^3 c_2 \phi^\dagger \phi v_\mu v_\nu F_{\mu\alpha} F_{\nu\alpha}$$

Elastic scattering amplitude

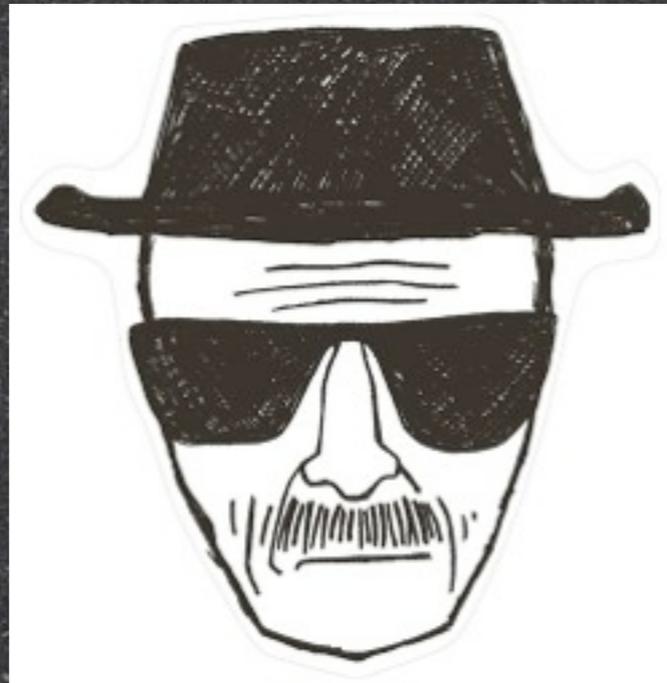
$$\mathcal{M}(\gamma\phi \rightarrow \gamma\phi) \sim E_\gamma^2$$

Elastic scattering cross section

$$\sigma(\gamma\phi \rightarrow \gamma\phi) \sim |\mathcal{M}(\gamma\phi \rightarrow \gamma\phi)|^2 \sim E_\gamma^4$$

Blue light scatters far more strongly than red
(by a factor of $(750/450)^4 \sim 8$)

Heisenberg-Euler effective Lagrangian



Heisenberg-Euler Lagrangian

...or effective theory for very low-energy QED

Relevant scales:

$$E_\gamma \ll 2m_e$$

O(MeV)

Lowest order
Lagrangian:

$$\mathcal{L}^{(0)} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu}$$

Effective theory
with cutoff 2^*m_e

Unknown numerical coefficients,
which can be computed by
matching to full QED

Lowest order
interaction
terms (assuming
parity conserved)

$$\mathcal{L}^{(1)} = \frac{c_1}{32m_e^4}(F_{\mu\nu}F_{\mu\nu})(F_{\rho\sigma}F_{\rho\sigma}) + \frac{c_2}{8m_e^4}(F_{\mu\rho}F_{\nu\rho})(F_{\mu\sigma}F_{\nu\sigma})$$

Dimensional analysis tells us that
interaction terms suppressed by electron mass to 4th power

Amplitude

$$\mathcal{M}(\gamma\gamma \rightarrow \gamma\gamma) \sim \frac{E_\gamma^4}{m_e^4}$$

$$\sigma(\gamma\gamma \rightarrow \gamma\gamma) \sim \frac{E_\gamma^6}{m_e^8}$$

Digression: \hbar counting

- Higher-dimensional operators with dimension D depend on mass scale M in UV theory as $1/M^{D-4}$. But, in general we do not know how their Wilson coefficients depend on couplings of UV theory - that is a model dependent question
- However, there are general rules that in some cases allow us to estimate the coupling dependence
- To this end, a useful trick is to restore explicitly \hbar in action. Then canonically normalized fields carry $\hbar^{1/2}$ dimension, and coupling of operator with n fields has $\hbar^{1-n/2}$ dimension, independently of number of derivatives

$$Z = \int D\phi e^{\frac{iS}{\hbar}}$$

$$S = \int d^4x \left((\partial\phi)^2 - m^2\phi^2 - mc_3\phi^3 - c_4\phi^4 - \frac{c_5}{m}\phi^5 - \frac{c_6}{m^2}\phi^6 + \dots \right)$$

$$[\phi] = \hbar^{1/2}$$

$$[m] = \hbar^0$$

$$[c_3] = \hbar^{-1/2}$$

$$[c_n] = \hbar^{1-n/2}$$

Digression: \hbar counting

$$g(\bar{\psi}\bar{\sigma}_\mu\psi)A_\mu \quad \text{or}$$

- Any gauge coupling has dimension equal to $\hbar^{-1/2}$

$$g(\bar{\psi}\bar{\sigma}_\mu\psi)A_\mu + ig(\phi^\dagger\partial_\mu\phi - \partial_\mu\phi^\dagger\phi)A_\mu + g^2\phi^\dagger\phi A_\mu A_\mu$$

\Rightarrow

$$[g] = \hbar^{-1/2}$$

- Each loop comes with another factor of \hbar (in original variables, each propagator brings \hbar , each vertex brings $1/\hbar$, thus each diagram comes with power of \hbar equal to:

$$N_{\text{propagators}} - N_{\text{vertices}} = N_{\text{loops}} - 1$$

- Leading Wilson coefficients in Heisenberg-Euler Lagrangian have \hbar dimension equal to -1

Tree generated

$$c_i \sim e^2$$

1-loop generated

$$c_i \sim \frac{e^4}{16\pi^2} \sim \alpha^2$$

Heisenberg-Euler Lagrangian

$$\mathcal{L}^{(1)} = a_1 \frac{\alpha^2}{32m_e^4} (F_{\mu\nu}F_{\mu\nu})(F_{\rho\sigma}F_{\rho\sigma}) + a_2 \frac{\alpha^2}{8m_e^4} (F_{\mu\rho}F_{\nu\rho})(F_{\mu\sigma}F_{\nu\sigma})$$

Standard
Form

$$\mathcal{L}^{(1)} = \tilde{a}_1 \frac{\alpha^2}{m_e^4} (F_{\mu\nu}F_{\mu\nu})(F_{\rho\sigma}F_{\rho\sigma}) + \tilde{a}_2 \frac{\alpha^2}{m_e^4} (F_{\mu\nu}\tilde{F}_{\mu\nu})(F_{\rho\sigma}\tilde{F}_{\rho\sigma})$$

$$\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} \partial_\rho A_\sigma, \quad a_1 = 32\tilde{a}_1 - 64\tilde{a}_2, \quad a_2 = 32\tilde{a}_2$$

High-school
form (up to
redundant units)

$$\mathcal{L}^{(1)} = \tilde{a}_1 \frac{4\alpha^2}{m_e^4} \left(\vec{E}^2 - \vec{B}^2 \right)^2 + \tilde{a}_2 \frac{16\alpha^2}{m_e^4} \left(\vec{E} \cdot \vec{B} \right)^2$$

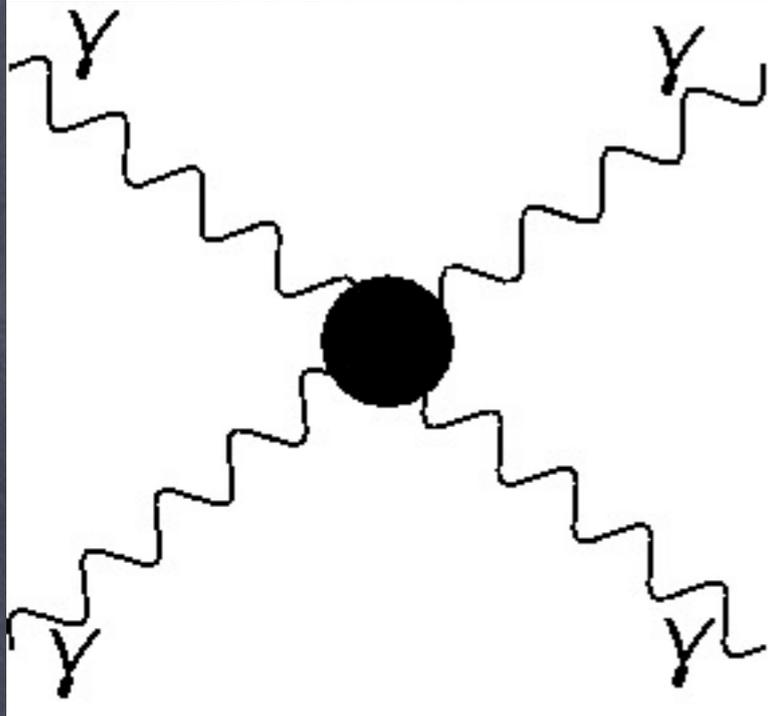
Higher-order interaction terms lead to non-linear field equations for electromagnetic field

One potentially observable effect is the so-called vacuum birefringence:
rotation of light polarization propagating in vacuum in strong magnetic field

Light-by-light scattering has been routinely observed, although typically above the cutoff of the Heisenberg-Euler Lagrangian. For example, Delbruck scattering - photon deflection in nucleus electromagnetic field - was observed in the 50s

Heisenberg-Euler Lagrangian

Calculation in the effective theory



$$\mathcal{M}(\pm\pm; \pm\pm) = \frac{s^2 \alpha^2}{m_e^4} \frac{a_1 + 3a_2}{4}$$

$$\mathcal{M}(\pm\pm; \mp\mp) = \frac{s^2 \alpha^2}{m_e^4} \frac{a_1 + a_2}{8} (3 + \cos^2 \theta)$$

$$\mathcal{M}(\pm\mp; \pm\mp) = \frac{s^2 \alpha^2}{m_e^4} \frac{a_1 + 3a_2}{16} (1 + \cos \theta)^2$$

$$\mathcal{M}(\pm\mp; \mp\pm) = \frac{s^2 \alpha^2}{m_e^4} \frac{a_1 + 3a_2}{16} (1 - \cos \theta)^2$$

Another digression:
EFT coefficients a_1 and a_2
are not completely arbitrary!

Adams et al
hep-th/0602178

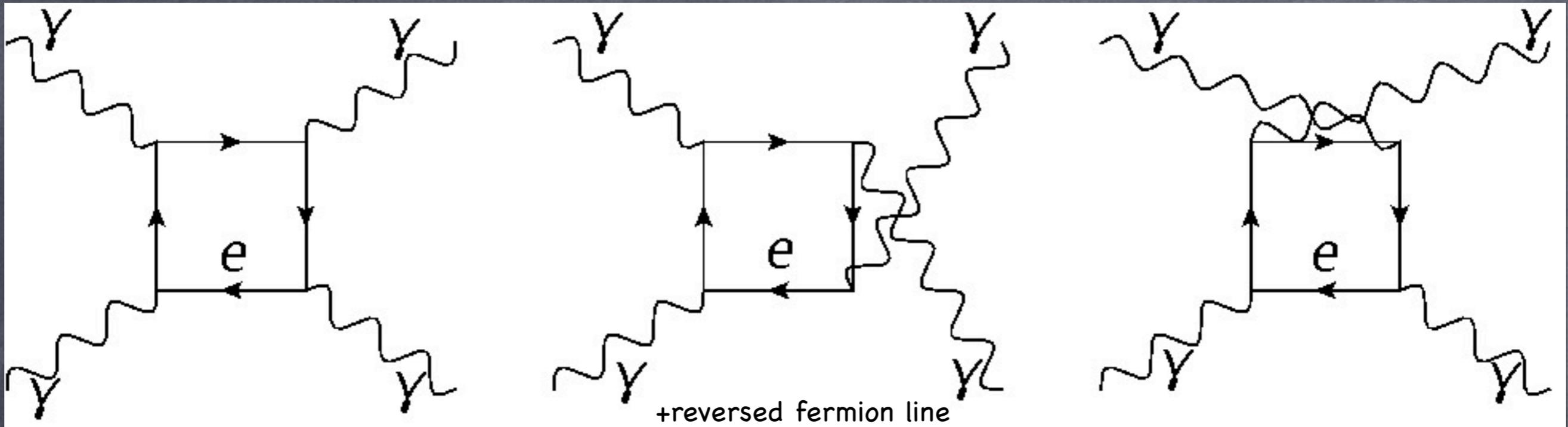
Bellazzini
1605.06111

$$\frac{d^2 \mathcal{M}_{\text{forward}}}{ds^2} \Big|_{s \rightarrow 0} > 0$$

$$\Rightarrow a_1 + 3a_2 > 0$$

Heisenberg-Euler Lagrangian

Calculation in QED



$$\mathcal{M}(\pm\pm; \pm\pm) = \frac{s^2}{m_e^4} \frac{11\alpha^2}{45} + \mathcal{O}(m_e^{-6})$$

$$\mathcal{M}(\pm\pm; \mp\mp) = -\frac{s^2}{m_e^4} \frac{\alpha^2}{30} (3 + \cos^2 \theta) + \mathcal{O}(m_e^{-6})$$

$$\mathcal{M}(\pm\pm; \pm\pm) = \frac{s^2 \alpha^2}{m_e^4} \frac{a_1 + 3a_2}{4}$$

$$\mathcal{M}(\pm\pm; \mp\mp) = \frac{s^2 \alpha^2}{m_e^4} \frac{a_1 + a_2}{8} (3 + \cos^2 \theta)$$

$$\mathcal{M}(\pm\mp; \pm\mp) = \frac{s^2 \alpha^2}{m_e^4} \frac{a_1 + 3a_2}{16} (1 + \cos \theta)^2$$

$$\mathcal{M}(\pm\mp; \mp\pm) = \frac{s^2 \alpha^2}{m_e^4} \frac{a_1 + 3a_2}{16} (1 - \cos \theta)^2$$

Matching HE-EFT and QED

$$a_1 = -\frac{8}{9}$$

$$a_2 = \frac{28}{45}$$

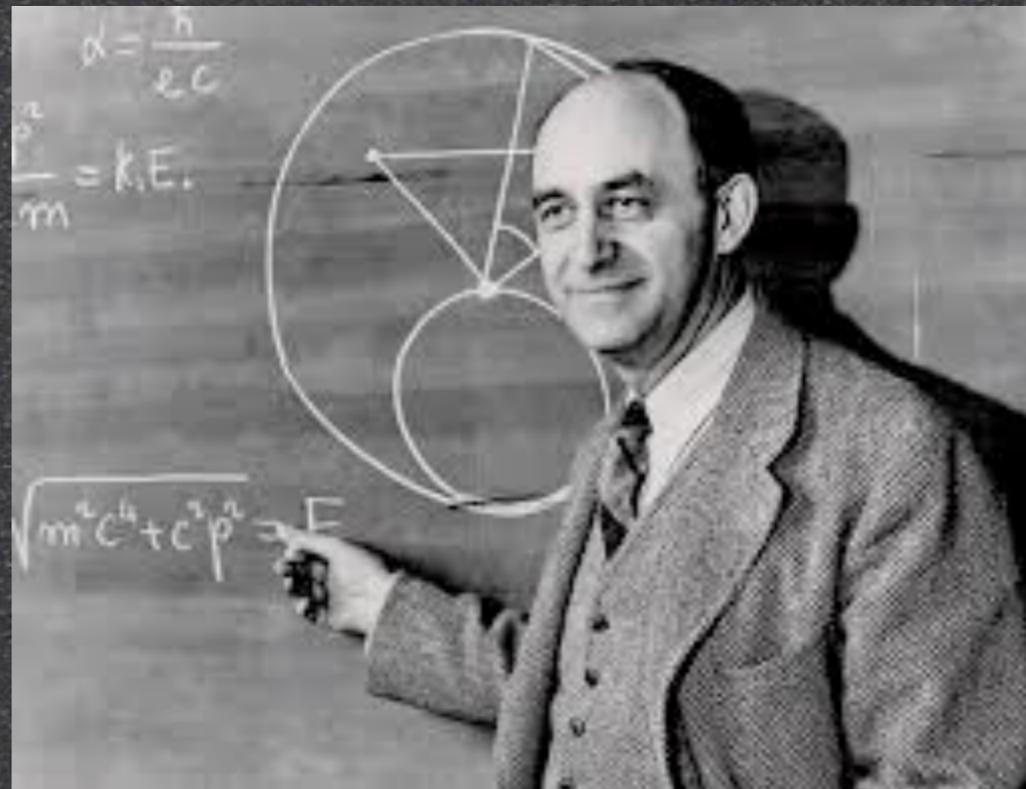
*Exercise: verify this calculation.
Find c_1 and c_2 for scalar QED.*

Heisenberg-Euler Lagrangian

Lessons learned:

- Most of physics of very low-energy photon scattering can be understood by simple power counting. Explicit calculations of the matching the effective theory (HE Lagrangian) to the full theory (QED) are needed only to determine two $O(1)$ numerical coefficients
- The low-energy Lagrangian is a useful tool to work out subtle effects on classical electromagnetic fields
- A future measurement of the Wilson coefficients will be a non-trivial result, as some unknown light particles could in principle contribute to it, along with the electron and other SM charged particles

Fermi effective theory of weak interactions

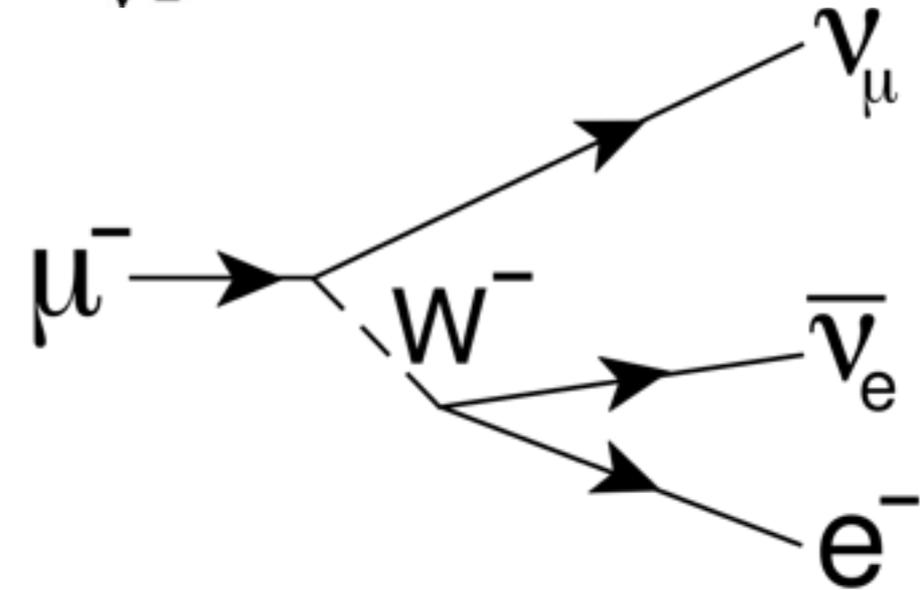


Fermi Theory of weak interactions

See spinor bible 0812.1594
for 2-component notation

- In SM, muon decays to electrons and neutrinos are mediated by W bosons

$$\mathcal{L} = \frac{g_L}{\sqrt{2}} (\bar{\nu}_\mu \bar{\sigma}_\rho \mu + \bar{\nu}_e \bar{\sigma}_\rho e) W_\rho^+ + \text{h.c.}$$



$$\mathcal{M} = \frac{g_L^2}{2} \bar{x}(k_{\nu_\mu}) \bar{\sigma}_\rho x(k_\mu) \frac{1}{q^2 - m_W^2} \bar{x}(k_e) \bar{\sigma}_\rho y(k_{\nu_e})$$

$q = k_\mu - k_{\nu_\mu}$

$$q^2 \leq m_\mu^2 \ll m_W^2$$

$$\frac{d\Gamma(\mu \rightarrow e \nu \bar{\nu})}{dq^2} = \frac{g_L^4 (m_\mu^2 - q^2)^2 (m_\mu^2 + 2q^2)}{3072\pi^3 m_\mu^3 (m_W^2 - q^2)^2}$$

$$\approx \frac{g_L^4 (m_\mu^2 - q^2)^2 (m_\mu^2 + 2q^2)}{3072\pi^3 m_\mu^3 m_W^4} \left(1 + \frac{2q^2}{m_W^2} + \dots \right)$$

Fermi Theory of weak interactions

- In SM, muon decays to electrons and neutrinos are mediated by W bosons

$$\mathcal{L} = \frac{g_L}{\sqrt{2}} (\bar{\nu}_\mu \bar{\sigma}_\rho \mu + \bar{\nu}_e \bar{\sigma}_\rho e) W_\rho^+ + \text{h.c.}$$

$$\mathcal{M} = \frac{g_L^2}{2} \bar{x}(k_{\nu_\mu}) \bar{\sigma}_\rho x(k_\mu) \frac{1}{q^2 - m_W^2} \bar{x}(k_e) \bar{\sigma}_\rho y(k_{\nu_e})$$

$$q = k_\mu - k_{\nu_\mu}$$

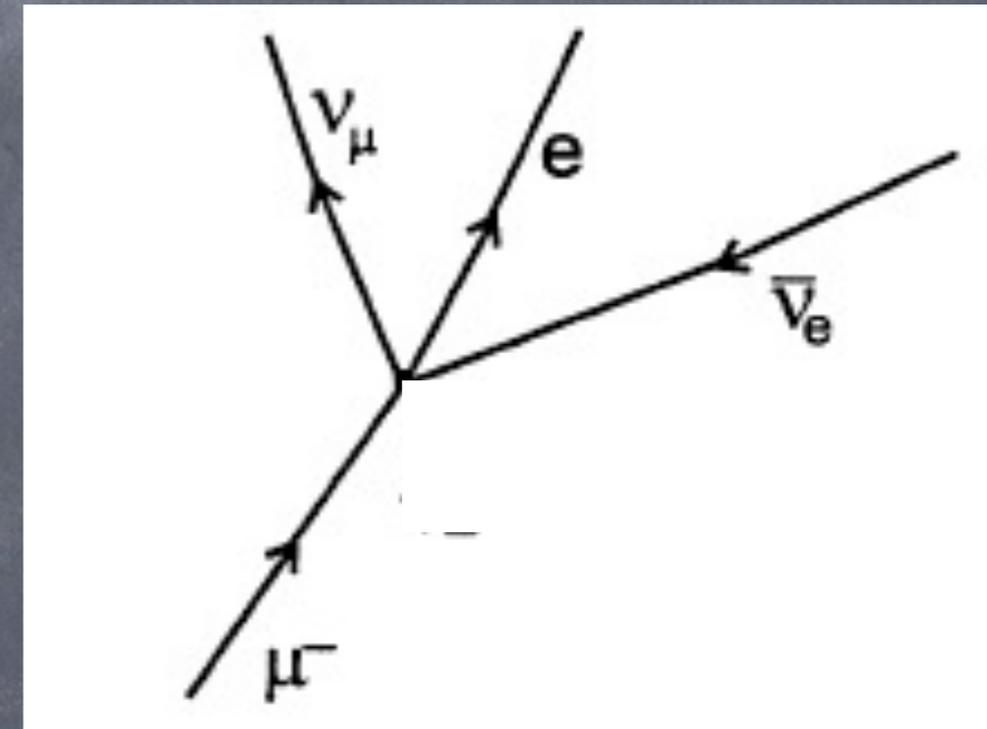
$$q^2 \leq m_\mu^2 \quad \& \quad m_\mu^2 / m_W^2 \sim 10^{-6}$$

$$\frac{d\Gamma(\mu \rightarrow e \nu \bar{\nu})}{dq^2} \approx \frac{g_L^4 (m_\mu^2 - q^2)^2 (m_\mu^2 + 2q^2)}{3072 \pi^3 m_\mu^3 m_W^4}$$

Fermi Theory of weak interactions

$$\mathcal{L}_{\text{eff}} \supset \frac{c}{\Lambda^2} (\bar{\nu}_\mu \bar{\sigma}_\rho \mu) (\bar{e} \bar{\sigma}_\rho \nu_e) + \text{h.c.}$$

- In SM, muon decays to electrons and neutrinos are mediated by W bosons
- Up to 10^{-6} corrections, this process can be approximated by the Fermi theory where W boson is "integrated out" and instead there is a 4-fermion contact interaction between muon, electron, and neutrino



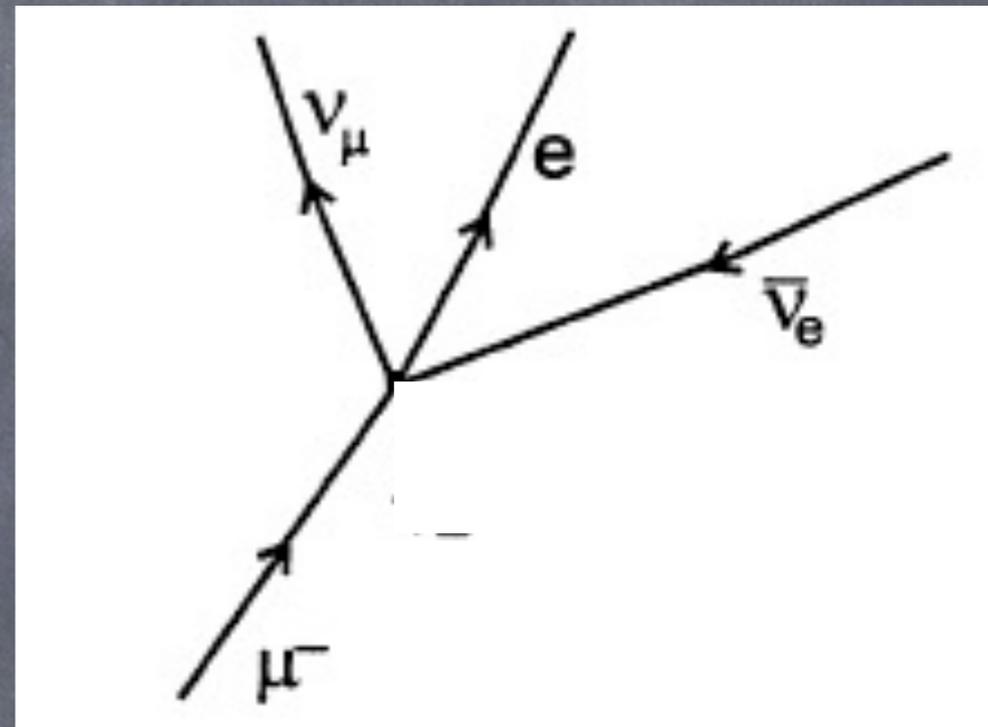
$$\mathcal{M} = \frac{c}{\Lambda^2} \bar{x}(k_{\nu_\mu}) \bar{\sigma}_\rho x(k_\mu) \bar{x}(k_e) \bar{\sigma}_\rho y(k_{\nu_e})$$

$$\frac{d\Gamma(\mu \rightarrow e \nu \nu)}{dq^2} = \frac{c^2 (m_\mu^2 - q^2)^2 (m_\mu^2 + 2q^2)}{768 \pi^3 m_\mu^3 \Lambda^4}$$



Fermi Theory of weak interactions

- In SM, muon decays to electrons and neutrinos are mediated by W bosons
- Up to 10^{-6} corrections, this process can be approximated by the Fermi theory where W boson is "integrated out" and instead there is a 4-fermion contact interaction between muon, electron, and neutrino



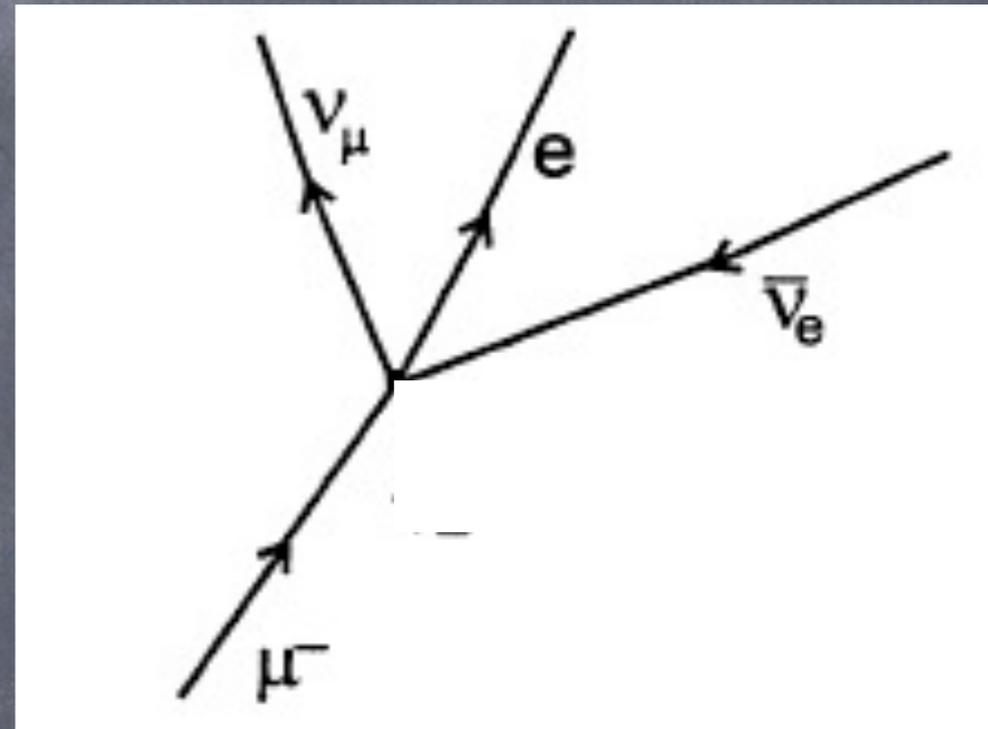
$$\mathcal{L}_{\text{eff}} \supset \frac{c}{\Lambda^2} (\bar{\nu}_\mu \bar{\sigma}_\rho \mu) (\bar{e} \bar{\sigma}_\rho \nu_e) + \text{h.c.}$$

Matching effective theory
amplitude to SM one
at leading order in q^2/m_W^2

$$\begin{aligned} \Lambda &= m_W \\ c &= -\frac{g_L^2}{2} \\ \frac{|c|}{\Lambda^2} &= \frac{2}{v^2} \equiv 2\sqrt{2}G_F \approx \frac{1}{(174\text{GeV})^2} \end{aligned}$$

Fermi Theory of weak interactions

- In SM, muon decays to electrons and neutrinos are mediated by W bosons
- Up to 10^{-6} corrections, this process can be approximated by the Fermi theory where W boson is "integrated out" and instead there is a 4-fermion contact interaction between muon, electron, and neutrino
- One can systematically "improve" Fermi theory by adding higher-order operators suppressed by more powers of Λ

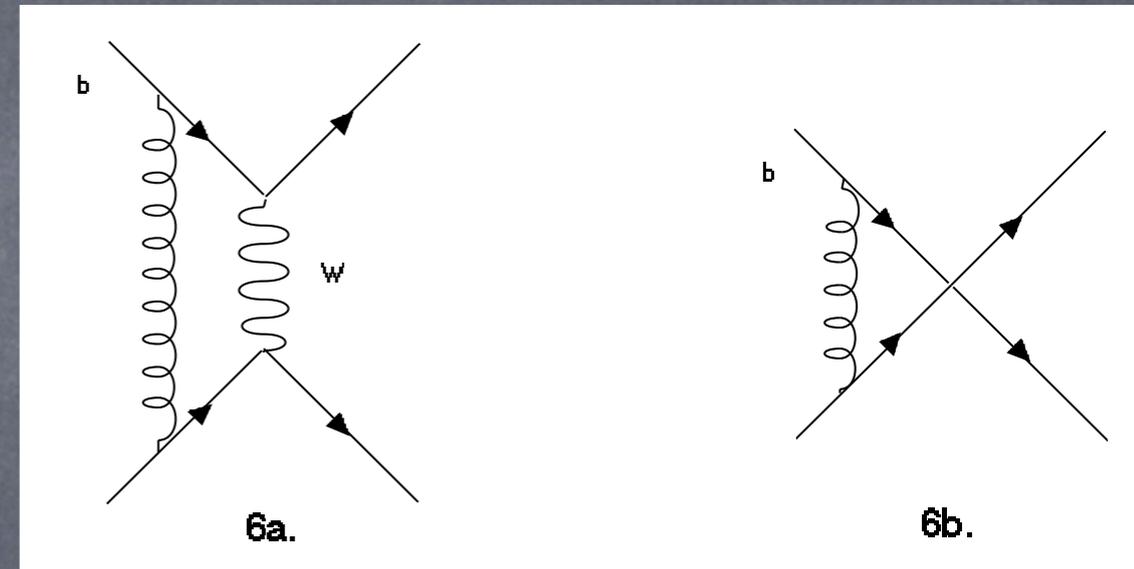


$$\mathcal{L}_{\text{eff}} \supset \frac{c}{\Lambda^2} (\bar{\nu}_\mu \bar{\sigma}_\rho \mu) (\bar{e} \bar{\sigma}_\rho \nu_e) + \frac{c_8}{\Lambda^4} (\bar{\nu}_\mu \bar{\sigma}_\rho \mu) \square (\bar{e} \bar{\sigma}_\rho \nu_e) + \dots + \text{h.c.}$$

$$\begin{aligned} \Lambda &= m_W \\ c &= -\frac{g_L^2}{2} \\ \frac{|c|}{\Lambda^2} &= \frac{2}{v^2} \equiv 2\sqrt{2}G_F \approx \frac{1}{(174\text{GeV})^2} \end{aligned}$$

Fermi Theory of weak interactions

- EFT is not only simpler but often better than original UV theory
- UV theory may suffer from large logs associated with disparate scales in loop diagrams
- In EFT, analogous diagram is log-divergent. Matching UV and EFT absorbs log divergence into counterterms leaving dependence on renormalization scale
- If we match UV and EFT at m_W scale, large log vanishes in matching formula. Instead this log reappears as running of Wilson coefficient in EFT below m_W
- Effectively, large logs are resummed!



$$\text{Diag. 6a} = \lambda_1 \frac{\alpha_s(\mu)}{4\pi} \ln(m_W^2/m_b^2),$$

$$\text{Figure 6b} = \lambda_1 \frac{\alpha_s(\mu)}{4\pi} \left(\frac{1}{\bar{\epsilon}} - \ln(m_b^2/\mu^2) \right) + \kappa.$$

$$C_{4F}(\mu) = G_F \left(1 + \lambda_1 \frac{\alpha_s(\mu)}{4\pi} (\ln(M_W^2/m_b^2) + \ln(m_b^2/\mu^2) + \rho) \right)$$

$$C_{4F} = G_F \left(1 + \lambda_1 \frac{\alpha_s(\mu)}{4\pi} \rho \right).$$

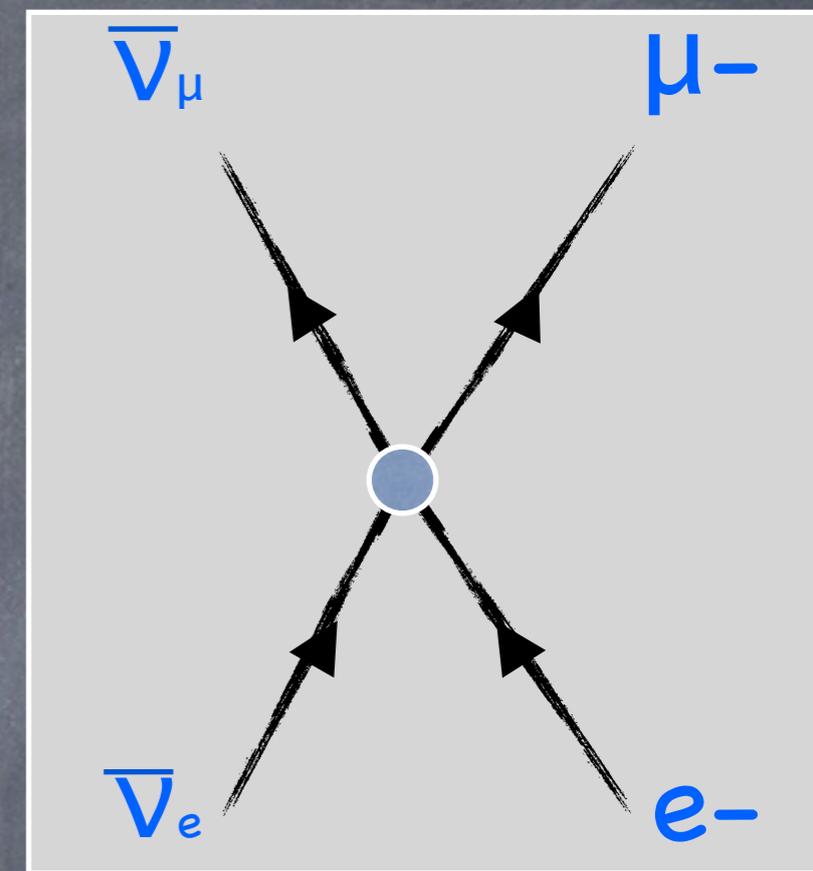
$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} + \gamma_{O_{4F}} \right) O_{4F}^R = 0,$$

Fermi Theory of weak interactions

$$\mathcal{L}_{\text{eff}} \supset -\frac{2}{v^2} (\bar{\nu}_\mu \bar{\sigma}_\rho \mu) (\bar{e} \bar{\sigma}_\rho \nu_e)$$

- Same Fermi theory can be used to describe related processes, e.g. high-energy neutrino inelastic scattering

$$\mathcal{M}(\bar{\nu}_e e^- \rightarrow \bar{\nu}_\mu e^+) = -\frac{2}{v^2} [\bar{x}(k_\mu) \bar{\sigma}_\rho y(k_{\nu_\mu})] [\bar{y}(k_{\nu_e}) \bar{\sigma}_\rho x(k_e)]$$



In the limit all fermions are massless, only 1 helicity amplitude is non-zero:

$$\mathcal{M}(+- \rightarrow +-) = (1 + \cos \theta) \frac{2s}{v^2}$$

In Fermi theory, scattering amplitudes grow indefinitely with scattering energy!

Digression: unitarity constraints

S matrix unitarity

$$S^\dagger S = 1$$

symmetry factor
for n-body final state

Implies relation between forward scattering amplitude,
and elastic and inelastic production cross sections

$$2\text{Im}\mathcal{M}(p_1, p_2 \rightarrow p_1, p_2) = S_2 \int d\Pi_2 |\mathcal{M}_{\text{el.}}(p_1, p_2 \rightarrow k_1, k_2)|^2 + \sum S_n \int d\Pi_n |\mathcal{M}_{\text{inel.}}(p_1, p_2 \rightarrow k_1 \dots k_n)|^2$$

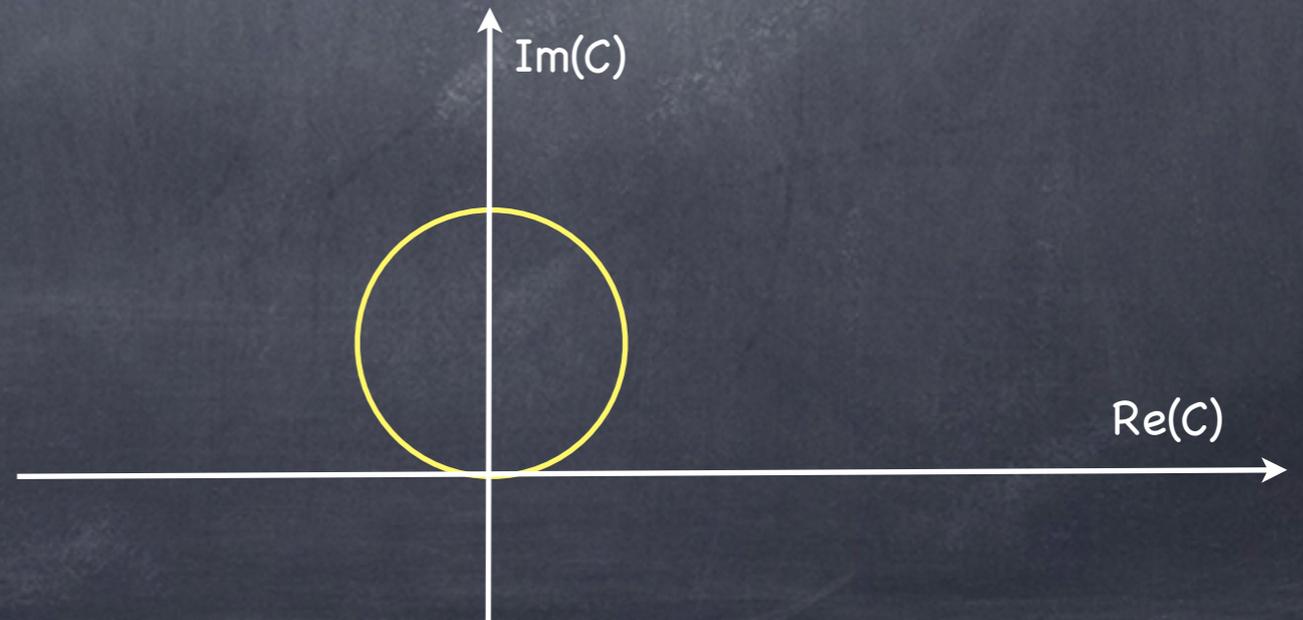
Assuming elastic amplitude approaches s-wave at high energies, $\mathcal{M}_{\text{el.}} \rightarrow C(s)$

$$(\text{Re } C)^2 + (\text{Im } C - 8\pi/S_2)^2 + \frac{8\pi S_n}{S_2} \sum \int d\Pi_n |\mathcal{M}_{\text{inel.}}(p_1, p_2 \rightarrow k_1 \dots k_n)|^2 = \left(\frac{8\pi}{S_2}\right)^2,$$

This implies perturbative unitarity constraints on elastic and inelastic amplitudes

$$|\text{Re } C| \leq \frac{8\pi}{S_2}.$$

$$S_n \sum \int d\Pi_n |\mathcal{M}_{\text{inel.}}|^2 \leq \frac{8\pi}{S_2}.$$

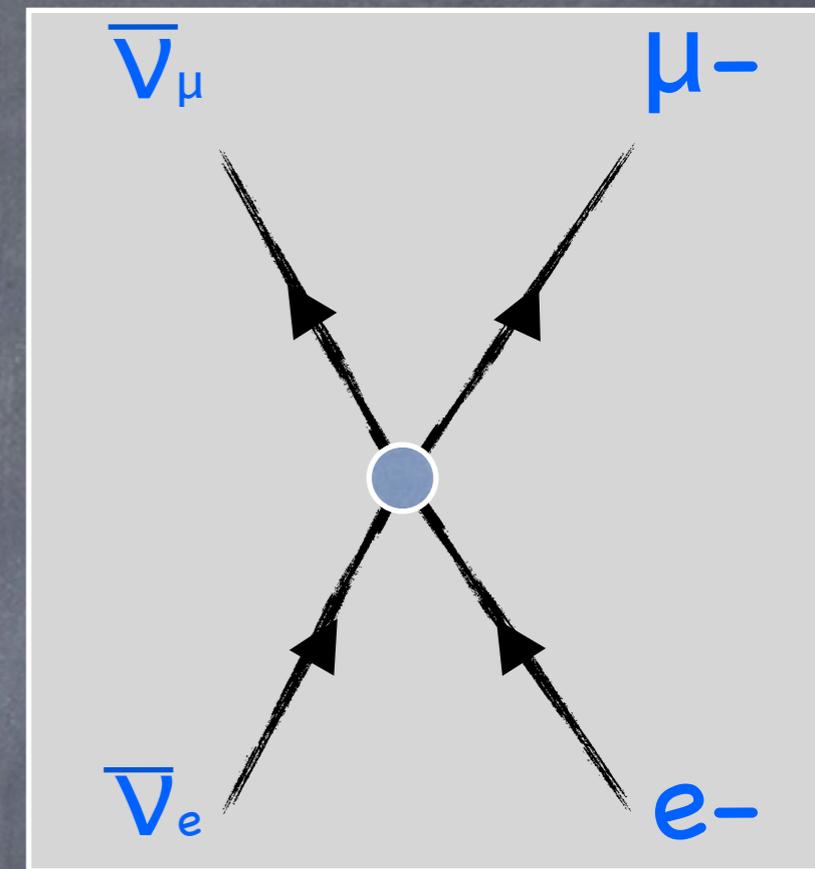


Fermi Theory of weak interactions

$$\mathcal{M}(+- \rightarrow +-) = (1 + \cos \theta) \frac{2s}{v^2}$$

In Fermi theory, amplitudes grow indefinitely with scattering energy!

At some point, s-wave amplitude violates unitarity bound



Fermi theory does not make sense
(at least perturbatively)
for $E \gtrsim 4 \pi v \sim \text{few TeV}$

In reality, Fermi theory ceases to be a correct description of physics well below $4 \pi v$, once we approach the W mass threshold, $m_W = g v/2 \ll 4\pi v$. But a general lesson is that every effective theory has a limited range of validity. For strongly coupled UV completions validity range is close to the maximal one. For weakly coupled UV completions validity range is well below the maximal one.

Fermi Theory of weak interactions

Lessons learned:

- EFT can be a great and simple tool to study low-energy consequences of more complete theories as long as $E \ll \Lambda$, where Λ is the mass scale of the UV theory.
- It predicts correlations between rates of different processes (in our example processes related by crossing symmetry, but it is less trivial in other examples)
- However, EFT has limited validity range. It stops making sense as a perturbative theory for $E \gtrsim 4 \pi \Lambda/g$, where g is the coupling strength in the UV theory
- In reality, as one approaches $E = \Lambda$ from EFT side, higher-dimensional operators become more and more relevant, and expansion in $1/\Lambda$ becomes impractical. For $E \sim \Lambda$ resonance in UV theory can be resolved and EFT description becomes useless

Chiral perturbation theory of low-energy QCD

Chiral perturbation theory

- ChPT describes low energy interactions of pions.
- Underlying theory - QCD - is known, but coefficients of EFT operators cannot be calculated analytically.
- Approximate symmetries inherited from QCD provide a method to write down possible pion interactions in a systematic expansion

Chiral perturbation theory

- QCD has two nearly massless quarks: up and down. In massless limit, QCD Lagrangian has $SU(2)_L \times SU(2)_R$ symmetry corresponding to separate rotations of left-handed and right-handed components
- This symmetry is explicitly and completely broken by quark masses
- There's even larger source of symmetry breaking due to QCD vacuum condensate, $\langle u \hat{u}^c \rangle = \langle d \hat{d}^c \rangle$
- This spontaneously breaks $SU(2)_L \times SU(2)_R$ down to diagonal $SU(2)$ that rotates left-handed and right-handed quarks in the same way
- Therefore, there should be 3 light Goldstone boson states (identified with pions), 1 for each spontaneously broken generator of symmetry

$$\mathcal{L} = i\bar{u}\bar{\sigma}_\mu\partial_\mu u + i\bar{d}\bar{\sigma}_\mu\partial_\mu d + iu^c\sigma_\mu\partial_\mu\bar{u}^c + id^c\sigma_\mu\partial_\mu\bar{d}^c$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow L \begin{pmatrix} u \\ d \end{pmatrix}$$

SU(2)

$$\begin{pmatrix} u_c \\ d_c \end{pmatrix} \rightarrow \begin{pmatrix} u_c \\ d_c \end{pmatrix} R^\dagger$$

SU(2)

$$\mathcal{L}_{mass} = -m_u u u^c - m_d d d^c + \text{h.c.}$$

$$\langle q_i^c q_j \rangle = \delta_{ij} \rightarrow R_{ik}^\dagger L_{kj}$$

Chiral perturbation theory

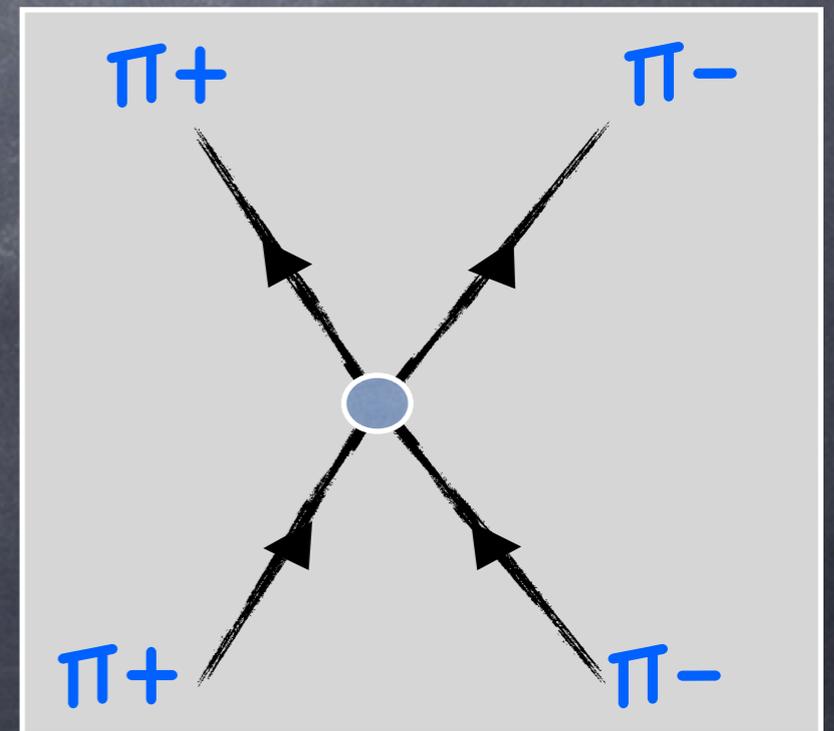
- Low energy theory of pions should inherit symmetries of QCD
- This means the theory should have non-linearly realized $SU(2)_L \times SU(2)_R$ symmetry such that diagonal (vector) part is linearly realized, and under axial part pions transform under shift symmetry
- Effective Lagrangian can then be written in derivative expansion
- Lowest order term that one can write has 2 derivatives. It describes kinetic terms of pions, but also infinite series of 2-derivative pion interaction terms
- These interactions can be tested in pion-pion scattering, which allows one to fit $f \approx 93 \text{ MeV}$

$$U = \exp(i\pi^a \sigma^a / f)$$
$$= \exp \left[\frac{i}{f} \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix} \right]$$

$$U \rightarrow LUR^\dagger, \quad L, R \in SU(2)$$

$$\mathcal{L}_{\text{eff}}^{(2)} = \frac{f^2}{4} \text{Tr}[\partial_\mu U^\dagger \partial_\mu U]$$

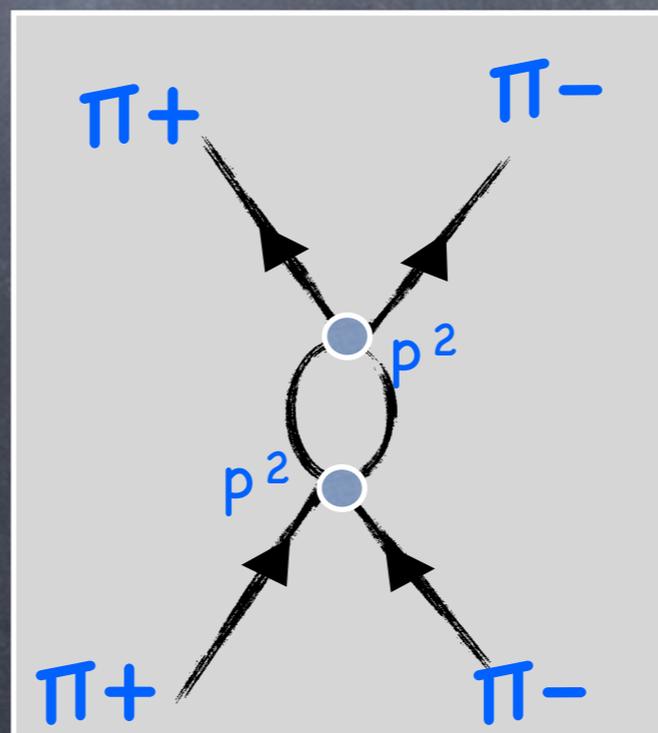
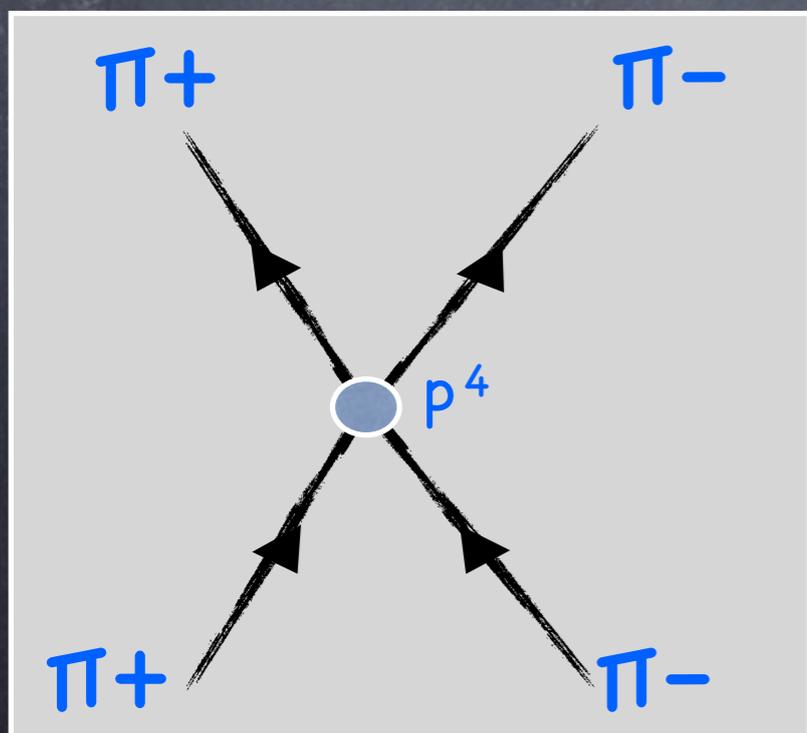
$$\mathcal{L}_{\text{eff}}^{(2)} = \partial_\mu \pi^+ \partial_\mu \pi^- + \frac{1}{2} \partial_\mu \pi^0 \partial_\mu \pi^0$$
$$+ \frac{1}{2f^2} (\partial_\mu \pi^+ \pi^- + \partial_\mu \pi^- \pi^+ + \partial_\mu \pi^0 \pi^0)^2$$
$$+ \dots$$



Chiral perturbation theory

$$\mathcal{L}_{\text{eff}}^{(4)} = L_1 (\text{Tr}[\partial_\mu U^\dagger \partial_\mu U])^2 + L_2 \text{Tr}[\partial_\mu U^\dagger \partial_\nu U] \text{Tr}[\partial_\mu U^\dagger \partial_\nu U] + L_3 \text{Tr}[\partial_\mu U^\dagger \partial_\mu U \partial_\nu U^\dagger \partial_\nu U]$$

- ChPT theory can be extended to 4-derivative level. This produces 4-derivative interactions terms of pions, in addition to 2-derivative ones
- By studying momentum dependence of pion scattering one can fit the parameters L_1 , L_2 , L_3
- Note that in this case 1-loop diagrams with 2-derivative vertices have to be included together with tree-level diagrams with 4-derivative vertices. In ChPT, derivative expansion is intimately tied to loop expansion.



Scherer, hep-ph/0210398

Coefficient	Empirical Value
L_1^r	0.4 ± 0.3
L_2^r	1.35 ± 0.3
L_3^r	-3.5 ± 1.1

(In units of 10^{-3} ,
at scale m_ρ)

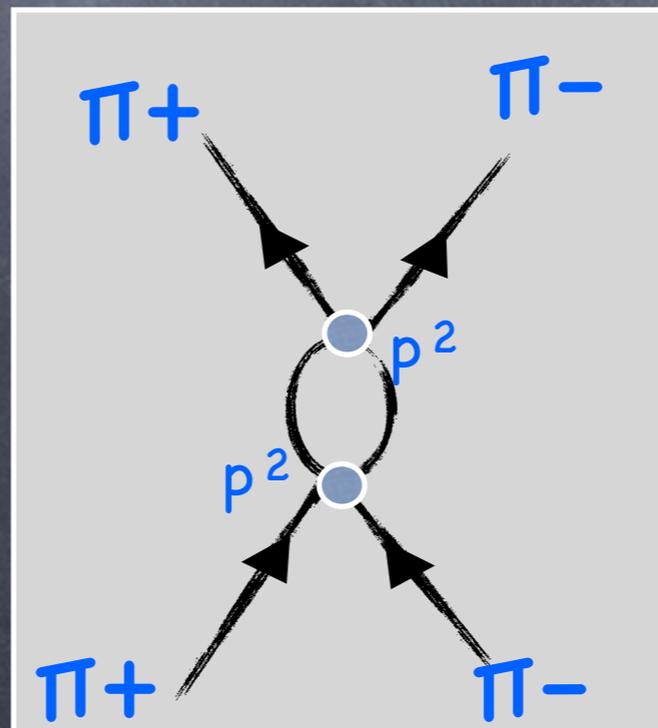
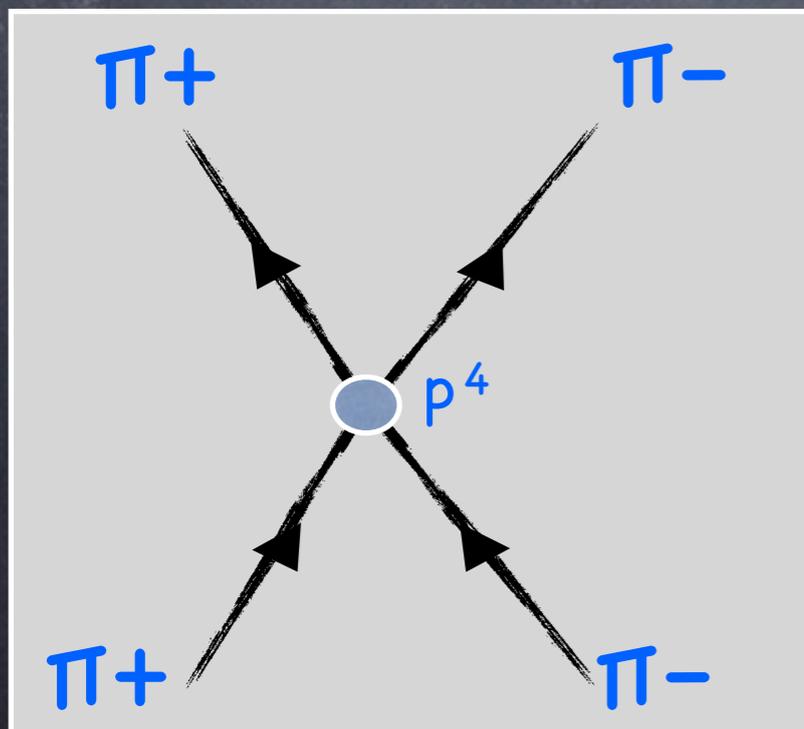
Chiral perturbation theory

?

$$\text{Tr}[\partial^2 U^\dagger \partial^2 U]$$

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{(4)} = & L_1 (\text{Tr}[\partial_\mu U^\dagger \partial_\mu U])^2 \\ & + L_2 \text{Tr}[\partial_\mu U^\dagger \partial_\nu U] \text{Tr}[\partial_\mu U^\dagger \partial_\nu U] \\ & + L_3 \text{Tr}[\partial_\mu U^\dagger \partial_\mu U \partial_\nu U^\dagger \partial_\nu U] \end{aligned}$$

- Operators that can be eliminated or traded for other by equations of motion are not included in effective Lagrangian
- This is because they are redundant - all their effect on on-shell amplitudes can be described by other terms
- In this case, in the limit of massless pions, equation of motion is $\square U = 0$, so the new term above does not contribute to on-shell amplitudes at all



Chiral perturbation theory

Lessons learned:

- It is often advantageous to work with EFT even when matching with UV theory cannot be calculated. Then one needs to write down all possible non-redundant interaction terms consistent with EFT symmetries in some systematic expansion, and determine their coefficients from experiment
- EFT is not renormalizable, therefore it formally has infinite number of parameter. However, at a fixed order in EFT expansion it is renormalizable. As soon as all coefficients are fixed at a given order from experiment, other observables can be predicted at that order

Naturalness in EFT: GIM mechanism

naturalness a-la t'Hooft

Scalar Lagrangian: $\mathcal{L} = \partial_\mu \Phi^\dagger \partial_\mu \Phi - m^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$

$\Phi \rightarrow \Phi + \alpha$ Invariant Not Invariant Not Invariant

Expected scaling: $m^2 \sim \epsilon \Lambda^2, \quad \lambda \sim \epsilon \quad \Rightarrow \quad \Lambda \sim \frac{|m|}{\sqrt{\lambda}}$

Applying this to the Higgs $\frac{|m|}{\sqrt{\lambda}} \approx \frac{88 \text{ GeV}}{\sqrt{0.13}} \approx 250 \text{ GeV}$

This argument suggests new physics should appear below TeV scale, and that in UV completion of SM there's symmetry protecting Higgs mass parameter

In practice, one see t'Hooft scaling argument at one loop as quadratic sensitivity to the cutoff scale



$$\begin{aligned} \delta m^2 &= 4i\lambda \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 - m^2} \\ &= -4\lambda \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{-k_E^2 - m^2} \\ &= \frac{\lambda}{2\pi^2} \int_0^\Lambda dk_E k_E^3 \frac{1}{k_E^2 + m^2} \\ &= \frac{\lambda}{4\pi^2} \left[\Lambda^2 - m^2 \log \left(\frac{\Lambda^2}{m^2} \right) \right] \end{aligned}$$

naturalness a-la t'Hooft

Is t'Hooft naturalness principle applicable to mass parameters in high-energy physics?

Mostly yes!

- Fermion mass terms violate chiral symmetry, thus larger symmetry recovered in limit $m \rightarrow 0$. This makes it natural in t'Hooft sense for fermion masses to span large range
- For composite states of light quarks in QCD, light scalars (pions) are protected by approximate shift symmetry in EFT, while unprotected scalars (e.g. σ) have masses close to EFT cutoff
- GIM mechanism for kaon mixing mass term

kinetic terms symmetry $U(1)_L \times U(1)_R$

$$\psi \rightarrow e^{i\alpha} \psi, \quad \psi^c \rightarrow e^{-i\alpha_c} \psi^c$$

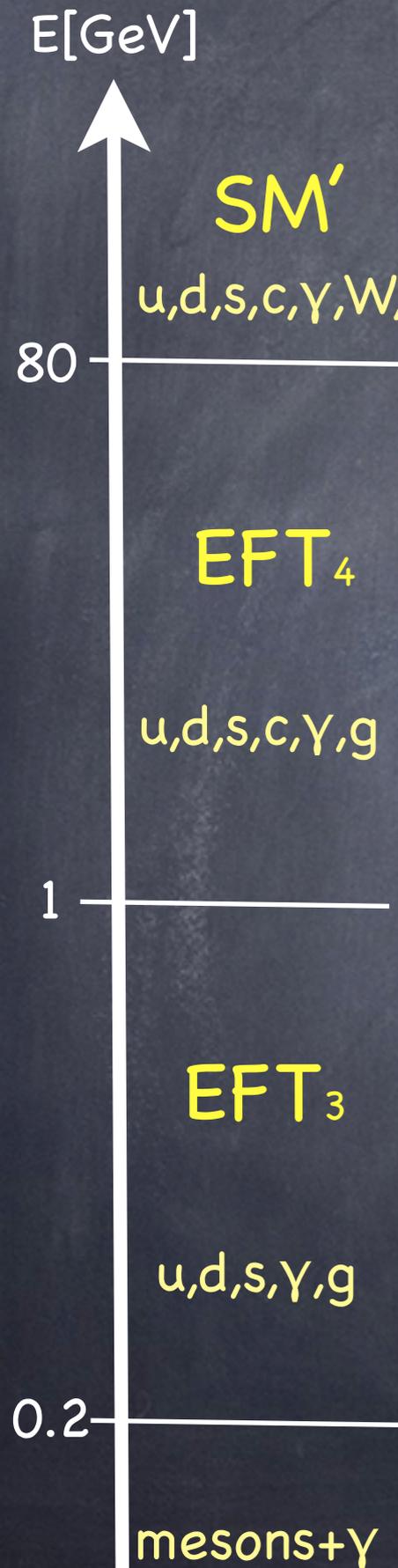
$$\mathcal{L} = i\bar{\psi} \bar{\sigma}_\mu \psi + i\psi^c \sigma_\mu \bar{\psi}^c - m (\psi \psi^c + \bar{\psi} \bar{\psi}^c)$$

$$\psi \rightarrow e^{i\alpha} \psi, \quad \psi^c \rightarrow e^{-i\alpha} \psi^c$$

mass terms symmetry $U(1)_V$

Naturalness works!

GIM mechanism



- Consider alternative universe where SM has two generations of quarks, out of which only charm is massive and up/down/strange are massless

$$\mathcal{L}_{\text{SM}} \supset \frac{g_L}{\sqrt{2}} V_{ik} \bar{u}_i \bar{\sigma}_\mu d_k W_\mu^+ + \text{h.c.}$$

- At tree-level, effective theory at energies below W mass has 4 quarks with effective 4-quark interactions which may violate flavor numbers (S,C) by one unit

$$\mathcal{L}_{\text{EFT}_4} \supset - \sum_{i,j=1,2} \frac{g_L^2 V_{is} V_{jd}^*}{2m_W^2} (\bar{u}_i \bar{\sigma}_\mu s) (\bar{d} \bar{\sigma}_\mu u_j) + \text{h.c.}$$

$$V = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix}$$

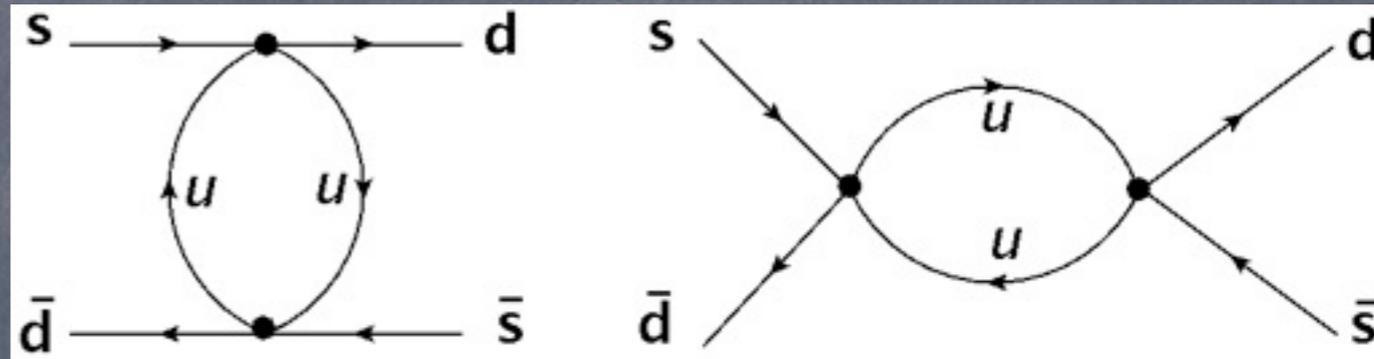
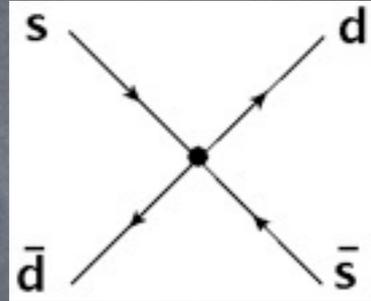
- Effective theory at energies below charm mass has 3 quarks with effective 4-quark interactions which may violate strangeness by one or two units

$$\mathcal{L}_{\text{EFT}_3} \supset - \frac{g_L^2 V_{us} V_{ud}^*}{2m_W^2} (\bar{u} \bar{\sigma}_\mu s) (\bar{d} \bar{\sigma}_\mu u) + \frac{C_{ss}}{\Lambda^2} (\bar{d} \bar{\sigma}_\mu s) (\bar{d} \bar{\sigma}_\mu s) + \text{h.c.}$$

GIM mechanism

- One can describe $\Delta S=2$ kaon mixing using 3-quark EFT
- This process occurs may occur via direct $\Delta S=2$ coupling, or via one loop diagrams with two $\Delta S=1$ vertices
- Loop integral is divergent. This means that kaon mixing is not calculable in 3-quark EFT - it depends on unknown counterterm inside C_{ss}
- One can naively estimate amplitude by putting cutoff Λ in loop integral. Quadratic dependence on Λ is interpreted as dependence of cutoff mixing on physics at scales above cutoff of 3-quark EFT

$$\mathcal{L}_{\text{EFT}_3} \supset -\frac{g_L^2 V_{us} V_{ud}^*}{2m_W^2} (\bar{u} \bar{\sigma}_\mu s) (\bar{d} \bar{\sigma}_\mu u) + \frac{C_{ss}}{\Lambda^2} (\bar{d} \bar{\sigma}_\mu s) (\bar{d} \bar{\sigma}_\mu s) + \text{h.c.}$$



$$\mathcal{M}(s_A^{c_1} \bar{d}_A^{c_2} \rightarrow d_B^{c_3} \bar{s}_B^{c_4}) = \left(\delta^{c_1 c_2} \delta^{c_3 c_4} \bar{\sigma}_\mu^{\dot{A}A} \bar{\sigma}_\mu^{\dot{B}B} - \delta^{c_1 c_3} \delta^{c_2 c_4} \bar{\sigma}_\mu^{\dot{A}B} \bar{\sigma}_\mu^{\dot{B}A} \right) F$$

$$F_{\text{EFT}_3} = \frac{C_{ss}}{\Lambda^2} + \frac{g_L^4}{4m_W^4} (V_{ud} V_{us}^*)^2 \int \frac{d^4 k}{(2\pi)^4} \frac{-i}{k^2}$$

$$F_{\text{EFT}_3} \sim -\frac{\sin^2 \theta_c \cos^2 \theta_c}{4\pi^2 v^4} \Lambda^2$$

GIM mechanism

- Experimentally measured K_{long} - K_{short} mass difference is related to $\Delta S=2$ amplitude we calculated
- If we use amplitude calculated in 3-quark EFT with cutoff Λ , we can get estimate for cutoff of 3-quark EFT
- This naive estimate returns $\Lambda \sim 1 \text{ GeV}$!
- Naturalness suggests 3-quark EFT is replaced by a more complete theory at $E \sim 1 \text{ GeV}$, and in this UV theory $\Delta S=2$ amplitude is calculable and insensitive to high scales
- Note that this conclusion would be lost if we used dimensional regularization to estimate amplitude

$$m_{K_0} = 497.611 \pm 0.013 \text{ MeV}$$

$$\Delta m_{K_0} \approx 3.5 \times 10^{-15} \text{ GeV}$$

$$H = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M_{11} - \frac{i}{2}\Gamma_{11} \end{pmatrix}$$

$$\Delta m_{K_0} = 2|M_{12}|$$

$$M_{12} = \frac{\langle K_0 | O | \bar{K}_0 \rangle}{2m_{K_0}}$$

$$\sim \frac{F}{m_{K_0}} \langle K_0 | (\bar{d}\bar{\sigma}_\mu s)(\bar{d}\bar{\sigma}_\mu s) | \bar{K}_0 \rangle$$

$$\langle K_0 | (\bar{d}\bar{\sigma}_\mu s)(\bar{d}\bar{\sigma}_\mu s) | \bar{K}_0 \rangle \sim f_K^2 m_K^2$$

$$\frac{\Delta m_{K_0}}{m_{K_0}} \sim f_K^2 F \sim \frac{\sin^2 \theta_c \Lambda_{\text{QCD}}^2}{4\pi^2 v^4} \Lambda^2$$

GIM mechanism

$$V = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix}$$

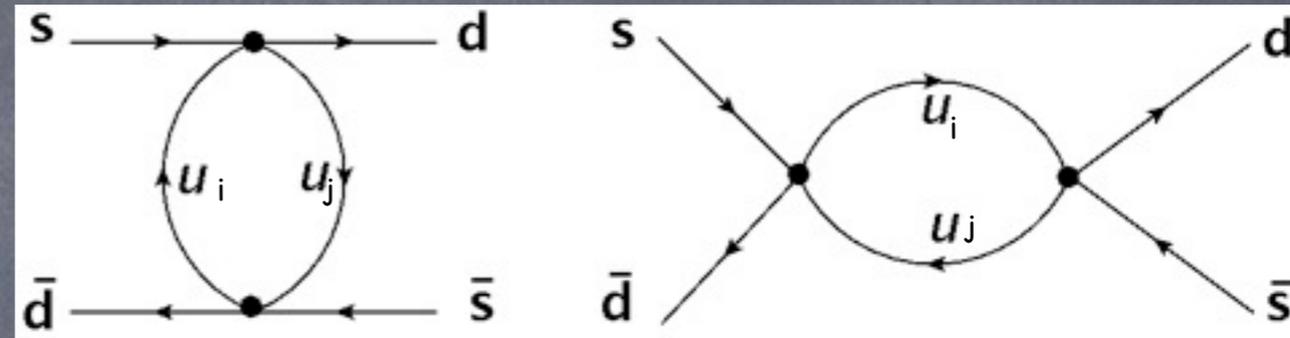
- In our example, UV completion is 4-quark EFT, with 3 massless u/d/s quarks and massive charm quark

- In this set-up, loop mediated $\Delta S=2$ amplitude is finite thanks to GIM mechanism, which follows from unitarity of CKM matrix

- Actually, result in EFT4 is exactly that of EFT3 after replacing Λ with charm mass.

Hence, in this example, looking at hierarchy problem in EFT provides not only qualitative but also quantitative guidance about physics of UV theory

$$\mathcal{L}_{\text{EFT}_4} \supset - \sum_{i,j=1,2} \frac{g_L^2 V_{is} V_{jd}^*}{2m_W^2} (\bar{u}_i \bar{\sigma}_\mu s) (\bar{d} \bar{\sigma}_\mu u_j) + \text{h.c.}$$



$$F_{\text{EFT}_4} = \frac{g_L^4}{4m_W^4} \sum_{i,j=1,2} V_{id} V_{is}^* V_{jd} V_{js}^* \int \frac{d^4 k}{(2\pi)^4} \frac{-ik^2}{(k^2 - m_{u_i}^2)(k^2 - m_{u_j}^2)}$$

$$\text{div} F_{\text{EFT}_4} \sim \frac{1}{4\pi^2 v^4} \sum_{i,j=1,2} V_{id} V_{is}^* V_{jd} V_{js}^* (\Lambda^2 + (m_{u_i}^2 + m_{u_j}^2) \log \Lambda) = 0$$

$$F_{\text{EFT}_4} = - \frac{m_c^2}{4\pi^2 v^4} \sin^2 \theta_c \cos^2 \theta_c$$

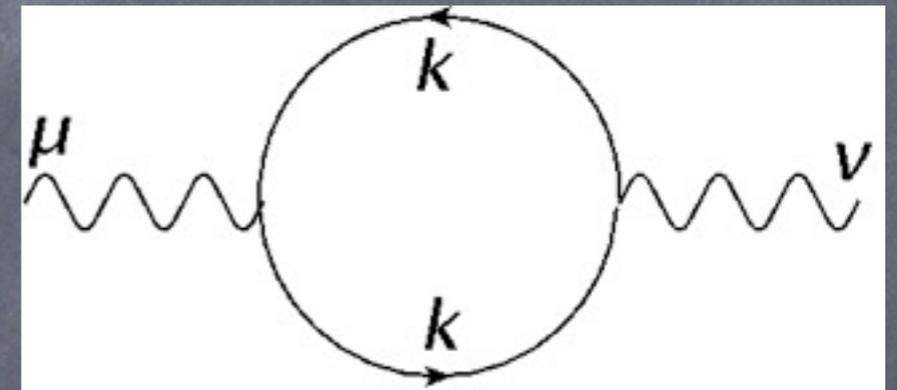
Exercise: In the real world, how does the top quark affects this story, qualitatively and quantitatively

Warning!

In GIM example, quadratic dependence on cutoff in loop integral signifies sensitivity to high scales

But this is not always the case!

Trivial counterexample: photon mass in QED



Calculated with cutoff, photon mass is quadratically divergent at 1 loop:

$$\Delta m_\gamma^2 = -4ie^2 \int \frac{d^4k}{(2\pi)^4} \frac{-\frac{1}{2}k^2 + m_e^2}{(k^2 - m_e^2)^2}$$
$$\sim \frac{e^2 \Lambda^2}{8\pi^2} \quad ?$$

This is just because cutoff is not consistent with gauge invariance.

With gauge invariant regulator, photon mass correction will always vanish

$$\Delta m_\gamma^2 = -4ie^2 \int \frac{d^d k}{(2\pi)^d} \frac{\frac{2-d}{d}k^2 + m_e^2}{(k^2 - m_e^2)^2}$$
$$= 0$$

Many more EFT examples

- **Heavy quark effective theory.** Describes spectrum and dynamics of bound states with 1 heavy quark (bottom or charm)
- **Non-relativistic QED/QCD.** Describes bound states of electrons, positrons, muons, quarks, etc.
- **Nuclear effective theory.** Describes low-energy interactions of protons, neutrons, deuterons, etc.
- and much more...



EFT in Particle Physics and Cosmology

3-28 juillet 2017
Ecole de Physique des Houches

Europe/Paris timezone

- Overview
- Scientific Programme**
- Timetable
- Les Houches webpage
- Application
- Getting to Les Houches

to contact the school or organisers:

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- ✉ s.davidson@ipn...
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Scientific Programme

The underlying idea of EFT, is that at each scale, the relevant physics can be parametrised with appropriate variables, which may change with the scale. EFTs are essential tools both for precision analyses within known theories, and for a concise parameterization of hypothetical models. The school aims to bring together a group of experts, who can give pedagogical and profound introductions to various EFTs that are in use today, presenting the concepts such that attendees can adapt some of the latest developments in other fields to their own problems.

A characteristic feature of EFTs is that they describe systems containing many different scales. These scales could be masses (for instance, the light masses of the particles of the Standard Model versus the heavy masses of yet undiscovered particles), momenta (for instance, the hard, collinear and soft momenta playing a role in jets produced in high-energy collisions), or length scales (for instance, the lattice spacing appearing in numerical simulations versus the pion Compton wavelength of interest to low-energy hadronic interactions). Various of these possibilities are presented in the lectures:

Lectures:

- Introduction to EFT (M. Neubert, 4 lectures)
- EFT: basic concepts and electroweak applications (A. Manohar, 6 lectures)
- EFT for quark flavour (L. Silvestrini, 4 lectures)
- χ PT and electroweak symmetry breaking (A. Pich, 6 lectures)
- Soft Collinear Effective Theory (T. Becher, 4 lectures)
- Heavy Quark Effective Theory & NRQCD (T. Mannel, 4 lectures)
- EFT for large-scale structure formation (T. Baldauf, 4 lectures)
- EFT for thermal systems (S. Caron-Huot, 4 lectures)
- EFT on the lattice, applied to HQET (R. Sommer, 2 lectures)
- EFT for inflation (C. Burgess, 3 lectures)
- EFT for the direct detection of dark matter (J. Hisano, 3 lectures)
- EFT in nuclear physics (U. van Kolck, 2 lectures)
- EFT for post-Newtonian gravity (P. VanHove, 2 lectures)

Summary

- EFTs emerge naturally in particle physics and elsewhere, at vastly different scales and kinematical regimes
- Even when UV theory is known, and matching to IR EFT is calculable, EFT is important tool for calculations (simplicity, quick estimates, resummation of large logs)
- When low energy Lagrangian is unknown a priori because it cannot be calculated or UV theory is not known, EFT framework is important tool to organize physics description