Symmetric cumulants and event-plane correlations

Giuliano Giacalone,[1] Li Yan,[1] Jacquelyn Noronha-Hostler,[2] and Jean-Yves Ollitrault[1]
[1]*Institut de physique théorique, Université Paris Saclay, CNRS, CEA, F-91191 Gif-sur-Yvette, France*
[2]*Department of Physics, University of Houston, Houston TX 77204, USA*

(Dated: May 27, 2016)

The ALICE Collaboration has recently measured the correlations between amplitudes of anisotropic flow in different Fourier harmonics, referred to as symmetric cumulants. We derive approximate relations between symmetric cumulants involving v_4 and v_5 and the event-plane correlations measured by ATLAS. The validity of these relations is tested using event-by-event hydrodynamic calculations. The corresponding results are in better agreement with ALICE data than existing hydrodynamic predictions. We make quantitative predictions for three symmetric cumulants which are not yet measured.

Anisotropic flow is the key observable showing that the matter produced in an ultrarelativistic nucleus-nucleus collision behaves collectively as a fluid [1]. Following the discovery of flow fluctuations [2] and triangular flow [3], a "flow paradigm" has emerged, which states that particles are emitted independently (up to short-range correlations) but with a momentum distribution that fluctuates event to event [4]. The azimuthal (φ) distribution in a given event is written as a Fourier series:

$$P(\varphi) = \frac{1}{2\pi}\sum_{n=-\infty}^{+\infty} V_n e^{-in\varphi}, \tag{1}$$

where $V_n = v_n \exp(in\Psi_n)$ is the (complex) anisotropic flow coefficient in the nth harmonic, and $V_{-n} = V_n^*$. Both the magnitude [5] and phase [2, 6] of V_n fluctuate event to event. In the last five years or so, an extremely rich phenomenology has emerged from this simple paradigm. RMS values of v_n have been measured up to $n = 6$ [7–10], and more recently, the full probability distribution of v_n [11]. An even wider variety of new observables can be constructed by combining different Fourier harmonics [12–14]. This new direction was pioneered by the ATLAS collaboration which has measured fourteen mixed correlations involving relative phases between Fourier harmonics, dubbed event-plane correlations [15].

Recently, the ALICE collaboration has taken a new step in this direction [16] by measuring the correlation between the magnitudes of different Fourier harmonics using a cumulant analysis [17]. We define the symmetric cumulant $SC(n,m)$ [1] with $n \neq m$ by

$$SC(n,m) \equiv \frac{\langle v_n^2 v_m^2\rangle - \langle v_n^2\rangle\langle v_m^2\rangle}{\langle v_n^2\rangle\langle v_m^2\rangle}. \tag{2}$$

ALICE has measured $SC(3,2)$ and $SC(4,2)$ as a function of centrality. While these two quantities are formally similar, the hydrodynamic mechanisms giving rise to these correlations differ. Elliptic flow, v_2, and triangular flow, v_3, are both determined to a good approximation by linear response to the anisotropies of the initial density profile in the corresponding harmonics [18, 19]. Therefore, $SC(3,2)$ directly reflects correlations present in the initial spatial density profile, which are preserved by the hydrodynamic evolution as the spatial anisotropy is converted into a momentum anisotropy. Standard models for the initial density indeed reproduce the negative sign and overall (small) magnitude of the measured $SC(3,2)$ for all centralities [16]. By contrast, V_4 gets a significant nonlinear contribution proportional to V_2^2 generated by the hydrodynamic evolution [20–22] in addition to the linear contribution from the initial anisotropy in the fourth harmonic [23, 24]. The nonlinear response explains [25] the large event-plane correlation between V_2 and V_4. It also explains qualitatively why $SC(4,2)$ is positive.

In this paper, we derive a proportionality relation between $SC(4,2)$ and the corresponding event-plane correlation, where the proportionality constant involves the fluctuations of v_2. Using this, we are able to relate recent ALICE measurements with previously measured quantities, which circumvents the most typical limitation of hydrodynamic predictions that depend on initial conditions or medium properties [26–32]. The sole assumption underlying our derivation is that the linear and nonlinear contributions to V_4 are independent. The validity of this assumption is tested using hydrodynamic calculations. The value of $SC(4,2)$ derived using our relation and previous ATLAS measurements is compared with the recent direct measurement by ALICE. We make predictions along the same lines for $SC(5,2)$, $SC(5,3)$ and $SC(4,3)$, which are not yet measured.

We decompose V_4 and V_5 into linear and non-linear parts [21]

$$\begin{aligned} V_4 &= V_{4L} + \chi_4 (V_2)^2 \\ V_5 &= V_{5L} + \chi_5 V_2 V_3. \end{aligned} \tag{3}$$

We define χ_4 and χ_5 in such a way that the linear correlations between linear and nonlinear parts vanish, that is, $\langle V_{4L}(V_2)^{*2}\rangle = \langle V_{5L}V_2^* V_3^*\rangle = 0$. We now introduce a measure of the relative magnitude of the linear and nonlinear parts via the Pearson correlation coefficients

[1] Note the ALICE collaboration uses the same notation for the numerator only.

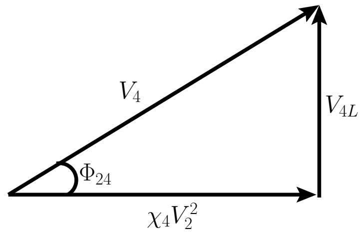

FIG. 1. (Color online) Schematic picture of the relation between the event-plane angle Φ_{24} in Eq. (4) and the decomposition Eq. (3). The legs of the triangle correspond to the rms values of the linear and nonlinear parts, and the hypothenuse is the rms v_4. A similar figure can be drawn for V_5.

between V_4, or V_5, and their nonlinear parts:

$$\cos\Phi_{24} \equiv \frac{\mathrm{Re}\langle V_4 (V_2^*)^2\rangle}{\sqrt{\langle v_4^2\rangle\langle v_2^4\rangle}}$$
$$\cos\Phi_{235} \equiv \frac{\mathrm{Re}\langle V_5 V_2^* V_3^*\rangle}{\sqrt{\langle v_5^2\rangle\langle v_2^2 v_3^2\rangle}}, \qquad (4)$$

where Φ_{24} and Φ_{235} lie between 0 and π. The first angle Φ_{24} corresponds precisely to the event-plane correlation measured by ATLAS [15] and denoted by $\langle\cos(4(\Phi_2 - \Phi_4))\rangle_w$.[2] The second angle Φ_{235} almost corresponds to the quantity denoted by $\langle\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)\rangle_w$. The only difference is that the latter has $\langle v_2^2\rangle\langle v_3^2\rangle$ in the denominator, instead of $\langle v_2^2 v_3^2\rangle$ [21]. Therefore the precise relation is

$$\cos\Phi_{235} = \frac{\langle\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)\rangle_w}{\sqrt{1 + SC(3,2)}}, \qquad (5)$$

where $SC(3,2)$ is defined in Eq. (2).

Inserting Eq. (3) into Eq. (4), one obtains

$$\chi_4^2\langle v_2^4\rangle = \langle v_4^2\rangle\cos^2\Phi_{24}$$
$$\chi_5^2\langle v_2^2 v_3^2\rangle = \langle v_5^2\rangle\cos^2\Phi_{235}. \qquad (6)$$

These equations are exact and simply follow from the definition of χ_4 and χ_5. They are depicted in Fig. 1.

We now assume that the linear parts V_{4L} and V_{5L} are statistically independent of V_2 and V_3. This is a stronger statement than just assuming that the linear correlation vanishes. As will be shown below, it is a reasonable approximation in hydrodynamics. Then, only the nonlinear response contributes to the correlation between v_4 and v_2, and Eq. (3) gives:

$$\langle v_4^2 v_2^2\rangle - \langle v_4^2\rangle\langle v_2^2\rangle = \chi_4^2\left(\langle v_2^6\rangle - \langle v_2^4\rangle\langle v_2^2\rangle\right). \qquad (7)$$

[2] We only consider the event-plane correlations measured using the scalar-product method, which are denoted by the subscript "w" in the ATLAS paper and have a clear interpretation in terms of V_n, in contrast to the results obtained using the event-plane method [33].

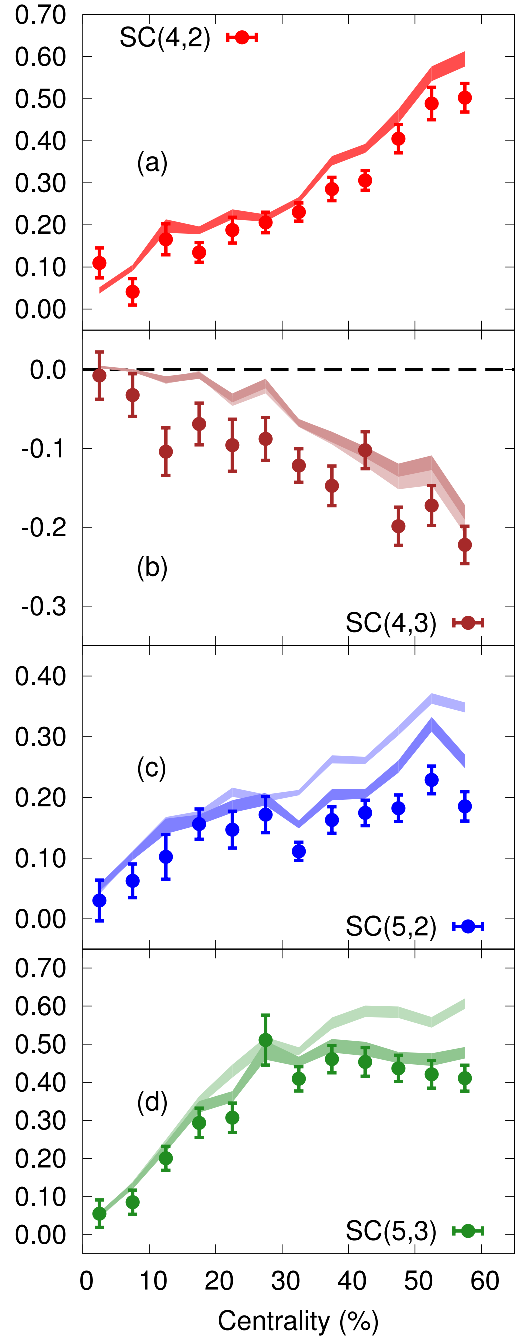

FIG. 2. (Color online) Test of Eqs.(8) using hydro calculations. Symbols correspond to the left-hand sides of Eqs. (8), dark shaded bands to the right-hand sides. Light-shaded bands correspond to Eqs. (9) and (12). Errors are statistical and estimated via jackknife resampling.

Similar relations can be written for the correlations between v_4^2 and v_3^2, v_5^2 and v_2^2 or v_3^2. Substituting in χ_4 and χ_5 extracted from Eqs. (6), one obtains

$$SC(4,2) = \left(\frac{\langle v_2^6\rangle}{\langle v_2^4\rangle\langle v_2^2\rangle} - 1\right)\cos^2\Phi_{24}$$

$$SC(4,3) = \left(\frac{\langle v_2^4 v_3^2\rangle}{\langle v_2^4\rangle\langle v_3^2\rangle} - 1\right)\cos^2\Phi_{24}$$
$$SC(5,2) = \left(\frac{\langle v_2^4 v_3^2\rangle}{\langle v_2^2 v_3^2\rangle\langle v_2^2\rangle} - 1\right)\cos^2\Phi_{235}$$
$$SC(5,3) = \left(\frac{\langle v_2^2 v_3^4\rangle}{\langle v_2^2 v_3^2\rangle\langle v_3^2\rangle} - 1\right)\cos^2\Phi_{235} \qquad (8)$$

These equations express symmetric cumulants in terms of event-plane correlations and moments of v_2 and v_3. Based on these equations, one expects symmetric cumulants involving v_4 or v_5 to increase with viscosity, in the same way as event-plane correlations [34, 35].

In order to test Eqs. (8), we carry out event-by-event hydrodynamic calculations using the same setup as in Ref. [36]: initial conditions are given by the Monte-Carlo Glauber model [37], the shear viscosity over entropy ratio is $\eta/s = 0.08$ [38] within the viscous relativistic hydrodynamical model v-USPhydro [39, 40], and V_n is calculated at freeze-out [41] for pions. Note, however, that the particular setup used, and whether or not it quantitatively reproduces experimental data, is irrelevant in this context, since the statement is that Eqs. (8) should hold to a good approximation for *any* hydrodynamic calculation. In hydrodynamics, V_n can be computed exactly from the one-particle momentum distribution for each event [42–44]. Therefore, reasonable accuracy is obtained with fewer events than in an actual experiment. We generate 1000 events for each 5% centrality bin. Figure 2 displays the comparison between the left-hand side (symbols) and the right-hand side (dark shaded bands) of Eqs. (8). Agreement is good for all four quantities and all centralities, in the sense that the absolute difference is typically a few 10^{-2}. The values of $SC(n,m)$ derived using Eqs. (8) tend to be above the actual values. This shows that the magnitude of of V_{4L} (or V_{5L}) and that of v_2 (or v_3) are not quite independent in hydrodynamics, but have a slight negative correlation. However, Eqs. (8) correctly capture the sign, magnitude and centrality dependence of symmetric cumulants.

The equation for $SC(4,2)$ can also be tested against existing data. The moments of v_2 are not directly measured but they can be expressed [21] as a function of cumulants, which have also been measured by ATLAS [45]. Figure 3 displays the comparison between the left-hand side of Eq. (8) measured by ALICE [16] and the right-hand side using ATLAS data. Agreement is reasonable for all centralities. In particular, our data-driven approach gives a better result for $SC(4,2)$ than existing hydrodynamic predictions [16, 35]. Based on the hydrodynamic calculation of Fig. 2, one would expect that the right-hand side of Eq. (8) is larger than the left-hand side. However, it is the other way around above 30% centrality. One reason may be that the event-plane correlation for ATLAS uses a much larger pseudorapidity window ($|\eta| < 4.8$) than ALICE ($|\eta| < 0.8$). Now, the phase of V_n depends slightly on rapidity [46–48], which induces a decoherence of azimuthal correlations for larger $\Delta\eta$ [49, 50]. Due to these longitudinal flow fluctuations, the event-plane correlation measured by ATLAS is smaller than what ALICE would measure in a more central rapidity window. Ideally, the comparison between the two sides of Eq. (8) should be done in the exact same rapidity window.

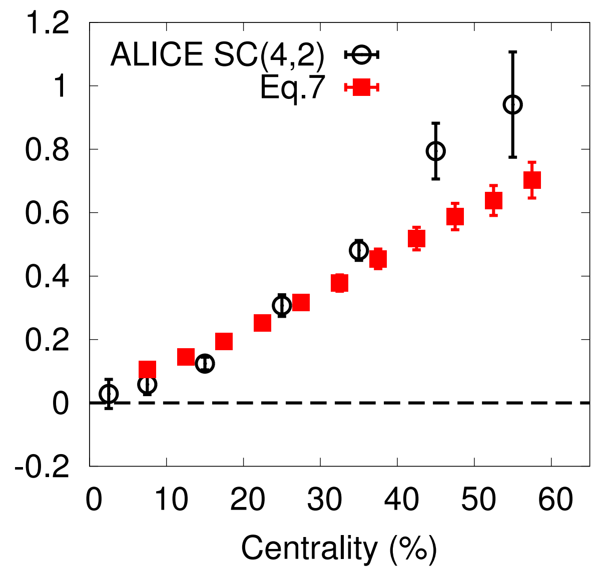

FIG. 3. (Color online) Open symbols: ALICE data for $SC(4,2)$ [16]. Closed symbols: value obtained using the right-hand side of Eq. (8) using ATLAS data for the moments of v_2 [45] and the event-plane correlation [15].

We now make predictions for $SC(4,3)$, $SC(5,2)$ and $SC(5,3)$ using Eqs. (8). The right-hand sides involve the mixed moments $\langle v_2^4 v_3^2\rangle$ and $\langle v_2^2 v_3^4\rangle$ which could be measured directly [14] but are not yet measured. However, the ALICE collaboration measures $|SC(3,2)| \ll 1$ for all centralities [16], which implies $\langle v_2^2 v_3^2\rangle \approx \langle v_2^2\rangle\langle v_3^2\rangle$. Therefore, one can assume, as a first approximation, that v_2^2 and v_3^2 are independent. Out of curiosity's sake, we also neglect the correlation in evaluating Φ_{235}, i.e., we make the approximation $\cos\Phi_{235} \approx \langle\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)\rangle_w$ (see Eq. (5)). Eqs. (8) then give

$$SC(5,2) \approx \left(\frac{\langle v_2^4\rangle}{\langle v_2^2\rangle^2} - 1\right)\langle\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)\rangle_w^2$$
$$SC(5,3) \approx \left(\frac{\langle v_3^4\rangle}{\langle v_3^2\rangle^2} - 1\right)\langle\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)\rangle_w^2 . (9)$$

The validity of Eqs. (9) can again be tested using event-by-event hydrodynamics. The right-hand sides are shown as light-shaded bands in Figs. 2 (c) and (d). Agreement is excellent for central collisions but becomes worse as the centrality percentile increases, as expected since we have neglected $SC(3,2)$ which becomes sizable for peripheral collisions.

If one assumes that v_2^2 and v_3^2 are independent, the second line of Eqs. (8) gives $SC(4,3) = 0$. In order to obtain a non-trivial prediction for $SC(4,3)$, we need to take into account the small correlation between v_2^2 and v_3^2. We do this by assuming that v_3^2 can be decomposed

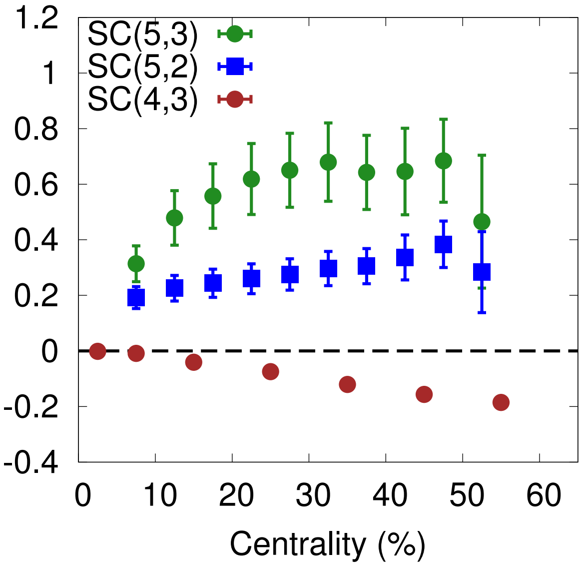

FIG. 4. (Color online) Predictions using the right-hand sides of Eqs. (9) and (12), using ATLAS data for the moments of v_2 and v_3 [45] and the event-plane correlations [15], and ALICE data for $SC(3,2)$ [16].

as

$$v_3^2 = c v_2^2 + \beta, \tag{10}$$

where c is the same for all events in a centrality class, and β is independent of v_2^2. Using Eq. (10), the correlation between an arbitrary moment of v_2 and v_3^2 is given in terms of moments of v_2:

$$\begin{aligned}\langle v_2^2 v_3^2\rangle - \langle v_2^2\rangle\langle v_3^2\rangle &= c\left(\langle v_2^4\rangle - \langle v_2^2\rangle^2\right)\\ \langle v_2^4 v_3^2\rangle - \langle v_2^4\rangle\langle v_3^2\rangle &= c\left(\langle v_2^6\rangle - \langle v_2^4\rangle\langle v_2^2\rangle\right).\end{aligned} \tag{11}$$

The first equation relates c with $SC(3,2)$ through Eq. (2). Taking the ratio of Eqs. (11) and inserting into Eq. (8), one obtains

$$SC(4,3) \approx \frac{\langle v_2^2\rangle\left(\langle v_2^6\rangle - \langle v_2^4\rangle\langle v_2^2\rangle\right)}{\langle v_2^4\rangle\left(\langle v_2^4\rangle - \langle v_2^2\rangle^2\right)} SC(3,2)\cos^2\Phi_{24}. \tag{12}$$

The right-hand side of this equation is shown as a light-shaded band in Fig. 2 (b). It is very close to the dark-shaded banded for all centralities, thus showing that the decomposition in Eq. (10) appropriately takes into account the correlation between v_2 and v_3.

Figure 4 displays our predictions for $SC(5,3)$, $SC(5,2)$ and $SC(4,3)$ using Eqs. (9) and (12), where we use ATLAS data for the quantities in the right-hand side. Since $\langle v_3^4\rangle$ is not measured below 15% centrality, we assume $\langle v_3^4\rangle \approx 2\langle v_3^2\rangle^2$, i.e., Gaussian fluctuations [51] for $SC(5,3)$ in the most central bins.[3] For $SC(4,3)$, we use ALICE data for $SC(3,2)$, and the other quantities in the right-hand side of Eq. (12) (moments of v_2 and $\cos\Phi_{24}$) are interpolated from ATLAS data, since ALICE and ATLAS use different centrality bins.

We have derived proportionality relations between symmetric cumulants involving v_4 or v_5 and event-plane correlations. These relations link correlations of different orders (symmetric cumulants are 4-particle correlations, while event-plane correlations are 3-particle correlations) and are fully non trivial. They are satisfied to a good approximation in event-by-event hydrodynamics, and thus offer a direct test of hydrodynamic behavior, which does not rely on a specific model of initial conditions and medium properties. The recent measurement of $SC(4,2)$ by ALICE passes the test. We have made predictions for $SC(5,2)$, $SC(5,3)$ and $SC(4,3)$ which can be measured in the near future. These new observables will allow to test hydrodynamic behavior directly, provided that one also measures higher-order correlations between v_2 and v_3 such as $\langle v_2^4 v_3^2\rangle$.

ACKNOWLEDGMENTS

This work is supported by the European Research Council under the Advanced Investigator Grant ERC-AD-267258. JNH acknowledges the use of the Maxwell Cluster and the advanced support from the Center of Advanced Computing and Data Systems at the University of Houston to carry out the research presented here.

[3] This is actually a good approximation for *all* centralities.

[1] U. Heinz and R. Snellings, Ann. Rev. Nucl. Part. Sci. **63**, 123 (2013) doi:10.1146/annurev-nucl-102212-170540 [arXiv:1301.2826 [nucl-th]].
[2] B. Alver *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **98**, 242302 (2007) doi:10.1103/PhysRevLett.98.242302 [nucl-ex/0610037].
[3] B. Alver and G. Roland, Phys. Rev. C **81**, 054905 (2010) Erratum: [Phys. Rev. C **82**, 039903 (2010)] doi:10.1103/PhysRevC.82.039903, 10.1103/PhysRevC.81.054905 [arXiv:1003.0194 [nucl-th]].
[4] M. Luzum, J. Phys. G **38**, 124026 (2011) doi:10.1088/0954-3899/38/12/124026 [arXiv:1107.0592 [nucl-th]].
[5] M. Miller and R. Snellings, nucl-ex/0312008.
[6] R. Andrade, F. Grassi, Y. Hama, T. Kodama and O. Socolowski, Jr., Phys. Rev. Lett. **97**, 202302 (2006) doi:10.1103/PhysRevLett.97.202302 [nucl-th/0608067].
[7] A. Adare *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **107**, 252301 (2011)

doi:10.1103/PhysRevLett.107.252301 [arXiv:1105.3928 [nucl-ex]].

[8] K. Aamodt *et al.* [ALICE Collaboration], Phys. Rev. Lett. **107**, 032301 (2011) doi:10.1103/PhysRevLett.107.032301 [arXiv:1105.3865 [nucl-ex]].

[9] G. Aad *et al.* [ATLAS Collaboration], Phys. Rev. C **86**, 014907 (2012) doi:10.1103/PhysRevC.86.014907 [arXiv:1203.3087 [hep-ex]].

[10] S. Chatrchyan *et al.* [CMS Collaboration], Phys. Rev. C **89**, no. 4, 044906 (2014) doi:10.1103/PhysRevC.89.044906 [arXiv:1310.8651 [nucl-ex]].

[11] G. Aad *et al.* [ATLAS Collaboration], JHEP **1311**, 183 (2013) doi:10.1007/JHEP11(2013)183 [arXiv:1305.2942 [hep-ex]].

[12] D. Teaney and L. Yan, Phys. Rev. C **83**, 064904 (2011) doi:10.1103/PhysRevC.83.064904 [arXiv:1010.1876 [nucl-th]].

[13] R. S. Bhalerao, M. Luzum and J. Y. Ollitrault, Phys. Rev. C **84**, 034910 (2011) doi:10.1103/PhysRevC.84.034910 [arXiv:1104.4740 [nucl-th]].

[14] R. S. Bhalerao, J. Y. Ollitrault and S. Pal, Phys. Lett. B **742**, 94 (2015) doi:10.1016/j.physletb.2015.01.019 [arXiv:1411.5160 [nucl-th]].

[15] G. Aad *et al.* [ATLAS Collaboration], Phys. Rev. C **90**, no. 2, 024905 (2014) doi:10.1103/PhysRevC.90.024905 [arXiv:1403.0489 [hep-ex]].

[16] J. Adam *et al.* [ALICE Collaboration], arXiv:1604.07663 [nucl-ex].

[17] A. Bilandzic, C. H. Christensen, K. Gulbrandsen, A. Hansen and Y. Zhou, Phys. Rev. C **89**, no. 6, 064904 (2014) doi:10.1103/PhysRevC.89.064904 [arXiv:1312.3572 [nucl-ex]].

[18] H. Niemi, G. S. Denicol, H. Holopainen and P. Huovinen, Phys. Rev. C **87**, no. 5, 054901 (2013) doi:10.1103/PhysRevC.87.054901 [arXiv:1212.1008 [nucl-th]].

[19] F. G. Gardim, J. Noronha-Hostler, M. Luzum and F. Grassi, Phys. Rev. C **91**, no. 3, 034902 (2015) doi:10.1103/PhysRevC.91.034902 [arXiv:1411.2574 [nucl-th]].

[20] N. Borghini and J. Y. Ollitrault, Phys. Lett. B **642**, 227 (2006) doi:10.1016/j.physletb.2006.09.062 [nucl-th/0506045].

[21] L. Yan and J. Y. Ollitrault, Phys. Lett. B **744**, 82 (2015) doi:10.1016/j.physletb.2015.03.040 [arXiv:1502.02502 [nucl-th]].

[22] J. Qian, U. W. Heinz and J. Liu, arXiv:1602.02813 [nucl-th].

[23] F. G. Gardim, F. Grassi, M. Luzum and J. Y. Ollitrault, Phys. Rev. C **85**, 024908 (2012) doi:10.1103/PhysRevC.85.024908 [arXiv:1111.6538 [nucl-th]].

[24] D. Teaney and L. Yan, Phys. Rev. C **86**, 044908 (2012) doi:10.1103/PhysRevC.86.044908 [arXiv:1206.1905 [nucl-th]].

[25] D. Teaney and L. Yan, Phys. Rev. C **90**, no. 2, 024902 (2014) doi:10.1103/PhysRevC.90.024902 [arXiv:1312.3689 [nucl-th]].

[26] M. Luzum and P. Romatschke, Phys. Rev. C **78**, 034915 (2008) Erratum: [Phys. Rev. C **79**, 039903 (2009)] doi:10.1103/PhysRevC.78.034915, 10.1103/PhysRevC.79.039903 [arXiv:0804.4015 [nucl-th]].

[27] C. Gale, S. Jeon, B. Schenke, P. Tribedy and R. Venugopalan, Phys. Rev. Lett. **110**, no. 1, 012302 (2013) doi:10.1103/PhysRevLett.110.012302 [arXiv:1209.6330 [nucl-th]].

[28] Z. Qiu and U. Heinz, Phys. Lett. B **717**, 261 (2012) doi:10.1016/j.physletb.2012.09.030 [arXiv:1208.1200 [nucl-th]].

[29] U. Heinz, Z. Qiu and C. Shen, Phys. Rev. C **87**, no. 3, 034913 (2013) doi:10.1103/PhysRevC.87.034913 [arXiv:1302.3535 [nucl-th]].

[30] P. Bozek and W. Broniowski, Phys. Rev. C **88**, no. 1, 014903 (2013) doi:10.1103/PhysRevC.88.014903 [arXiv:1304.3044 [nucl-th]].

[31] W. van der Schee, P. Romatschke and S. Pratt, Phys. Rev. Lett. **111**, no. 22, 222302 (2013) doi:10.1103/PhysRevLett.111.222302 [arXiv:1307.2539].

[32] E. Retinskaya, M. Luzum and J. Y. Ollitrault, Phys. Rev. C **89**, no. 1, 014902 (2014) doi:10.1103/PhysRevC.89.014902 [arXiv:1311.5339 [nucl-th]].

[33] M. Luzum and J. Y. Ollitrault, Phys. Rev. C **87**, no. 4, 044907 (2013) doi:10.1103/PhysRevC.87.044907 [arXiv:1209.2323 [nucl-ex]].

[34] D. Teaney and L. Yan, Nucl. Phys. A **904-905**, 365c (2013) doi:10.1016/j.nuclphysa.2013.02.025 [arXiv:1210.5026 [nucl-th]].

[35] H. Niemi, K. J. Eskola and R. Paatelainen, Phys. Rev. C **93**, no. 2, 024907 (2016) doi:10.1103/PhysRevC.93.024907 [arXiv:1505.02677 [hep-ph]].

[36] J. Noronha-Hostler, L. Yan, F. G. Gardim and J. Y. Ollitrault, Phys. Rev. C **93**, no. 1, 014909 (2016) doi:10.1103/PhysRevC.93.014909 [arXiv:1511.03896 [nucl-th]].

[37] B. Alver, M. Baker, C. Loizides and P. Steinberg, arXiv:0805.4411 [nucl-ex].

[38] G. Policastro, D. T. Son and A. O. Starinets, Phys. Rev. Lett. **87**, 081601 (2001) doi:10.1103/PhysRevLett.87.081601 [hep-th/0104066].

[39] J. Noronha-Hostler, G. S. Denicol, J. Noronha, R. P. G. Andrade and F. Grassi, Phys. Rev. C **88**, 044916 (2013) doi:10.1103/PhysRevC.88.044916 [arXiv:1305.1981 [nucl-th]].

[40] J. Noronha-Hostler, J. Noronha and F. Grassi, Phys. Rev. C **90**, no. 3, 034907 (2014) doi:10.1103/PhysRevC.90.034907 [arXiv:1406.3333 [nucl-th]].

[41] D. Teaney, Phys. Rev. C **68**, 034913 (2003) doi:10.1103/PhysRevC.68.034913 [nucl-th/0301099].

[42] B. Schenke, S. Jeon and C. Gale, Phys. Rev. Lett. **106**, 042301 (2011) doi:10.1103/PhysRevLett.106.042301 [arXiv:1009.3244 [hep-ph]].

[43] F. G. Gardim, F. Grassi, Y. Hama, M. Luzum and J. Y. Ollitrault, Phys. Rev. C **83**, 064901 (2011) doi:10.1103/PhysRevC.83.064901 [arXiv:1103.4605 [nucl-th]].

[44] Z. Qiu and U. W. Heinz, Phys. Rev. C **84**, 024911 (2011) doi:10.1103/PhysRevC.84.024911 [arXiv:1104.0650 [nucl-th]].

[45] G. Aad *et al.* [ATLAS Collaboration], Eur. Phys. J. C **74**, no. 11, 3157 (2014) doi:10.1140/epjc/s10052-014-3157-z [arXiv:1408.4342 [hep-ex]].

[46] P. Bozek, W. Broniowski and J. Moreira, Phys. Rev. C **83**, 034911 (2011) doi:10.1103/PhysRevC.83.034911 [arXiv:1011.3354 [nucl-th]].

[47] L. G. Pang, G. Y. Qin, V. Roy, X. N. Wang and G. L. Ma, Phys. Rev. C **91**, no. 4, 044904 (2015) doi:10.1103/PhysRevC.91.044904 [arXiv:1410.8690 [nucl-th]].

[48] J. Jia and P. Huo, Phys. Rev. C **90**, no. 3, 034915 (2014) doi:10.1103/PhysRevC.90.034915 [arXiv:1403.6077 [nucl-th]].

[49] L. Adamczyk *et al.* [STAR Collaboration], Phys. Rev. C **88**, no. 1, 014904 (2013) doi:10.1103/PhysRevC.88.014904 [arXiv:1301.2187 [nucl-ex]].

[50] V. Khachatryan *et al.* [CMS Collaboration], Phys. Rev. C **92**, no. 3, 034911 (2015) doi:10.1103/PhysRevC.92.034911 [arXiv:1503.01692 [nucl-ex]].

[51] S. A. Voloshin, A. M. Poskanzer, A. Tang and G. Wang, Phys. Lett. B **659**, 537 (2008) doi:10.1016/j.physletb.2007.11.043 [arXiv:0708.0800 [nucl-th]].