Higgs Physics in the SM and Beyond: 2° lecture

Today:
- Recap
- Custodial sym.
- GB's and CCWZ $\leftrightarrow$ Higgs mech.

Recap: • Interactions are invariant under gauge transform.

$$\begin{cases} \mathcal{G} = SU(3)_c \times SU(2)_L \times U(1)_Y \\ \mathcal{L}_{gauge} = -\frac{1}{2g_\alpha^2} Tr[F_{\mu\nu}^{(\alpha)} F_{\mu\nu}^{(\alpha)}] + \bar{\psi}_{(i)} i \not{D} \psi_{(i)} \\ D_\mu = \partial_\mu - i g_\alpha A_\mu^{a(\alpha)} T^a \\ \psi = L \oplus e_R^c \oplus Q_L \oplus u_R^c \oplus d_R^c \end{cases}$$

i flavor $i=1,2,3$

$L \to (1,2)_{-1/2}$, $e_R^c \to (1,1)_1$, $Q_L \to (3,2)_{1/6}$, $u_R^c \to (\bar{3},1)_{-2/3}$, $d_R^c \to (\bar{3},1)_{+1/3}$

• The spectrum is not invariant, masses respect only $\mathcal{H} = SU(3)_c \times U(1)_{em}$

$\Rightarrow \mathcal{G}$ is broken spontaneously by the vacuum $\longrightarrow \mathcal{H}$

• We can restore the full symmetry by using Higgs field

$$\begin{cases} H = (1,2)_{1/2} \quad \langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad Q\langle H \rangle = (T_3 + Y)\langle H \rangle = 0 \\ \tilde{H} = i\sigma^2 H^* = (1,2)_{-1/2} \end{cases}$$

$$\mathcal{L}_{masses} \Rightarrow \begin{cases} \mathcal{L}_{yukawa} = Y_{ij}^u \bar{Q}^{(i)} \tilde{H} u_R^{(j)} + Y_{ij}^d \bar{Q}^{(i)} H d_R^{(j)} + Y_{ij}^l \bar{L}^{(i)} H e_R^{(j)} + h.c. \\ \mathcal{L}_K = |D_\mu H|^2 \xrightarrow{vev} \left| \left[\frac{g}{\sqrt{2}}(W_\mu^- T^+ + h.c.) + eA_\mu Q + \frac{g Z_\mu}{c_W}(T_3 - s_W^2 Q) \right] \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \right|^2 \end{cases}$$

$\Rightarrow m_W^2 = g^2 v^2/4 \qquad m_Z^2 = m_W^2 / c_W^2$

H is a complex doublet, $H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$

$\Rightarrow$ 4 degrees of freedom

- 3 are needed to give mass to $W^\pm_\mu$ and Z_μ $\rightarrow$ longitud. polarizations
  - $\hookrightarrow$ they are the Goldstone Bosons of G/H
- 1 is a bonus, an extra, that is about the UV completion: the Higgs boson h

– They describe very different excitations of the vacuum

$H \rightarrow \pi^a, h \qquad H = e^{i\pi(x)} \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix}$

$\pi(x) \in G/H$ ~~...~~ and do not change the order parameter $\langle H^2 \rangle = v^2$

h moves instead the vev: $\langle |H|^2 \rangle = \frac{(v+h)^2}{2}$

With $\pi = \text{const}$ we are just choosing a physically equivalent vacuum

$\Rightarrow E_\pi(k) \xrightarrow[k\to 0]{} 0$ they are massless

The Lagrangian of the SM reads:

$$
\begin{aligned}
\mathcal{L}_{SM} = & -\frac{1}{2g_\alpha^2} \mathrm{Tr}[F_{\mu\nu}^{(\alpha)} F_{\mu\nu}^{(\alpha)}] + \bar{\psi} i \not{D} \psi && \text{gauge sect.} \\
& + y \bar{\psi}_L H \psi_R && \text{yukawa sect.} \\
& + |D_\mu H|^2 && \text{gauge boson masses} \\
& - V(|H|^2) && \text{Higgs potential}
\end{aligned}
$$

We have π's and h; and W_μ and fermions.
We can remove the π from the spectrum by a gauge transformation:

$$D_\mu \underbrace{e^{i\pi}\langle\ \rangle}_{H} = (\partial_\mu - iA_\mu)e^{i\pi} = e^{i\pi}(\partial_\mu - iA'_\mu)$$

with $A'_\mu = \underbrace{e^{-i\pi}A_\mu e^{i\pi} + i e^{-i\pi}\partial_\mu e^{i\pi}}$

This is a gauge tr. with $U = e^{i\pi} = e^{i\pi^a T^{\hat{a}}}$
(parameter $\alpha^a = \pi^a$)

$$\Rightarrow \begin{cases} F'_{\mu\nu} \equiv F_{\mu\nu}(A') = e^{-i\pi} F_{\mu\nu} e^{i\pi} \\ \mathrm{Tr}(F'_{\mu\nu}F'_{\mu\nu}) = \mathrm{Tr}(F_{\mu\nu}F_{\mu\nu}) \end{cases} \Rightarrow \underline{\mathcal{L}_{gauge} \text{ invariant}}$$

$$\boxed{\mathcal{L}_{SM} \overset{gauge}{\Longleftrightarrow} \mathcal{L}_{gauge} + \mathcal{L}_{masses} \oplus \underbrace{\text{interaction w/ } h}}$$

but the h can be made very heavy with $V(|H|^2) = V(h^2)$

→ The "Higgs mechanism", by itself, does not need the Higgs boson. The <u>Higgs boson "h" is needed to UV complete the theory</u>, to make it valid to arbitrary high-energy (but other states could do that too, so <u>h is about UV</u>)

→ For the Higgs mechanism to work we need <u>π's of G/H</u> and <u>gauge (part of) G</u>.

- With a gauge transf. we can remove the GB π from $\mathcal{L}$
- Or, vice versa, with a gauge transf. we can put π back in $\mathcal{L}$

Example: Abelian $U(1)$ $\quad \mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu}^2 + \frac{m^2}{2g^2} A_\mu^2$ massive spin-1

- We can restore gauge invariance by adding back the π:

$$A_\mu \longrightarrow A_\mu - \partial_\mu \pi \implies \mathcal{L}_{mass} \longrightarrow \boxed{\mathcal{L}_{gauge} = -\frac{1}{4g^2} F_{\mu\nu}^2 + \frac{m^2}{2g^2}(\partial_\mu \pi - A_\mu)^2} \circledast$$

- gauge invariance is now manifest:

$$\begin{cases} \pi(x) \longrightarrow \pi(x) + \alpha(x) \\ A_\mu \longrightarrow A_\mu + \partial_\mu \alpha(x) \end{cases}$$

- Notice that we may interpret $\circledast$ as "gauging" a global group broken spontaneously; indeed

$\mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu \phi)^2$: theory of $\underbrace{YM}_{2\,dofs} + \underbrace{\phi}_{1\,dofs}$ with $\phi \to \phi + c$ global sym. as for GB's

gauging $\phi \to \phi + c$ with A_μ:

$$\begin{cases} D_\mu \phi \equiv \partial_\mu \phi - A_\mu M, & M \text{ is some scale } [M] = 1 \text{ to match the dim.} \\ A_\mu \to A_\mu + \partial_\mu c \frac{1}{M} \end{cases}$$

$$\boxed{\begin{gathered} \mathcal{L}_{gauge} = -\frac{1}{4g^2} F_{\mu\nu}^2 + \frac{1}{2}(D_\mu \phi)^2 = -\frac{1}{4g^2} F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu \phi - A_\mu M)^2 \\ M^2 = \frac{m^2}{g^2} \qquad \frac{\phi}{M} = \pi \end{gathered}}$$

This is the general pattern we will see in more depth later: $G \longrightarrow H$ spontaneously give rise to $\dim(G/H)$ GB's some of which are the longitudinal dof needed to give mass to $A_\mu^{\hat{a}}$ associated with the broken generators $T^{\hat{a}}$ that have been gauged.

Example : Non-abelian massive spin-1

$$\mathcal{L} = -\frac{1}{4g^2}\,\mathrm{Tr}\left[F_{\mu\nu}^2\right] + \frac{m^2}{2g^2} A_\mu^{\hat{a}} A_\mu^{\hat{a}} \rightarrow \frac{m^2}{2g^2}\,\mathrm{Tr}\left[A_\mu T^{\hat{a}}\right]^2$$

$$\boxed{\begin{array}{l} \mathrm{Tr}\left[T^A, T^B\right] = \delta^{AB} \\ T^A = \{a, \hat{a}\} \quad \left\{\begin{array}{l} a: \text{unbroken generators} \\ \hat{a}: \text{broken generators} \end{array}\right. \end{array}}$$

- We can add back the GB's with a gauge transformation

$$\left\{\begin{array}{l} A_\mu \rightarrow U^{-1} A_\mu U - i U \partial_\mu U^{-1} = U^{-1} A_\mu U + i U^{-1}\partial_\mu U = U^{-1} i(\partial_\mu - iA_\mu) U \\ U = e^{i\pi} \qquad \pi = \pi^{\hat{a}}_{(x)} T^{\hat{a}} \end{array}\right.$$

$$\rightarrow \mathcal{L} = -\frac{1}{4g^2}\,\mathrm{Tr}\underbrace{\left[F_{\mu\nu}^2\right]}_{\text{invariant}} + \frac{m^2}{2g^2}\,\mathrm{Tr}\left[T^{\hat{a}}\, \underbrace{U^{-1} i(\partial_\mu - iA_\mu) U}_{\text{gauged version of the Maurer-Cartan 1-form}}\right]^2$$

$$\mathrm{Tr}\left[T^{\hat{a}} U^{-1} i(\partial_\mu - iA_\mu) U\right] = \left(-\partial_\mu \pi^{\hat{a}} + A_\mu^{\hat{a}} + \ldots\right)$$

$$\Rightarrow \mathcal{L} = -\frac{1}{4g^2}\,\mathrm{Tr}\left(F_{\mu\nu}^2\right) + \frac{m^2}{2g^2}\left(\partial_\mu \pi^{\hat{a}} - A_\mu^{\hat{a}} + \ldots\right)^2$$

- Now gauge invariance is manifest: $\left\{\begin{array}{l} U \rightarrow g U = g e^{i\pi} \\ A_\mu \rightarrow g A_\mu g^{-1} - i g^{-1}\partial_\mu g \end{array}\right.$

The ratio $\frac{m^2}{g^2}$ defines the pion- or Goldstone-decay constant $\frac{m^2}{g^2} = f^2$

The canonical normalization for π is $f\pi = \pi_{\text{canonical}}$

General Effective field Theory for GB's à la CCWZ

2 seminal papers by (Callan), Coleman, Wess, Zumino PRD 1969 177

- $\mathcal{G} \longrightarrow \mathcal{H}$ by a field vev $\langle\phi\rangle \neq 0$ — see also chap. 19 in Weinberg II.

with $\mathcal{G}$ global for now

$T^{A=a,\hat{a}}$ $\begin{cases} a: \text{ unbroken gen. } & T^a\langle\phi\rangle = 0 \\ \hat{a}: \text{ broken generators } & T^{\hat{a}}\langle\phi\rangle \neq 0 \end{cases}$ and w/ normalization $\mathrm{Tr}(T^A T^B) = \delta^{AB}$

- GB's matrix $U = e^{i\pi(x)}$ but in fact $U \in \mathcal{G}/H$

$\pi^{\hat{a}}T^{\hat{a}} \in$ Algebra$(T^{\hat{a}})$

indeed $U\langle\phi\rangle = U h(x)\langle\phi\rangle$ for any $h(x) \in H \Rightarrow U$ has a gauge redundancy

- Under action of the group: $U(x) \longrightarrow \underset{\mathcal{G}\text{ global}}{g}U(x) = U'(x)\underset{H\text{ local}}{h(x)}$

any element of $\mathcal{G}$ can be written as $g_0 = e^{i\xi^{\hat{a}}T^{\hat{a}}} \cdot e^{i\Omega^a T^a} = (\text{broken}) \times (\text{unbroken})$

$$\rightarrow \boxed{U(x) \longrightarrow U'(x) = \underset{\text{global}}{g}\,U(x)\,\underset{\text{local}}{h^+(x)}} \leftarrow \text{compatible with } U \in \mathcal{G}/H$$

We need to write invariants under such a transformation $U \rightarrow U'$

$U^+U = \mathbb{1}$ is trivial; it must contain at least one derivative

$$\Rightarrow \boxed{\underset{\text{Algebra}(\mathcal{G})}{U^{-1}\partial_\mu U} = i\,\underset{d_\mu^{\hat{a}}T^{\hat{a}}}{d_\mu} + i\,\underset{E_\mu^a T^a}{E_\mu} \quad \text{Maurer-Cartan 1-form}}$$

- invariant under g
- as a gauge field under $h(x)$: $\boxed{U^{-1}\partial_\mu U \rightarrow h(U^{-1}\partial_\mu U)h^+ + h\partial_\mu h^+}$

(for spacetime symmetries broken spont. $U = e^{ix^\mu P_\mu} e^{i\pi^a T^a_{a\neq 0}}$)

Actually, only E_μ part of $U^{-1}\partial_\mu U$ transforms as a gauge field

$$\begin{cases} i d_\mu^{\hat a} = \mathrm{Tr}(T^{\hat a} U^{-1}\partial_\mu U) \\ i E_\mu^{a} = \mathrm{Tr}(T^{a} U^{-1}\partial_\mu U) \end{cases}$$

but $\underline{h\partial_\mu h^+ \in \text{Algebra}(T^a) \text{ since } h \in \mathcal{G}}$

$\hookrightarrow \mathrm{Tr}(T^{\hat a} h\partial_\mu h^+) = 0$

TRANSFORMATIONS RULES:

$d_\mu \to h\, d_\mu h^+$ covariantly

$E_\mu \to h E_\mu h^+ - i h\partial_\mu h^+$ as gauge f.

$E_{\mu\nu} \to h E_{\mu\nu} h^+$ as field strenght

$E_{\mu\nu} = \partial_\mu E_\nu - \partial_\nu E_\mu + i[E_\mu, E_\nu]$

note: $h(x) = h(\pi(x))$

$U(x) = e^{i\pi(x)} \longrightarrow U'(x) = g\,U(x)\,h^+$

I can use $E_\mu(\pi)$ to define a covariant derivative $\Rightarrow$ $U^+\partial_\mu U$ get's replaced by $U^+\nabla_\mu U = U^+(\partial_\mu - i U E_\mu)$ which transforms covariantly, and indeed is nothing but id_μ: $\underline{U^+\nabla_\mu U = i d_\mu}$

The transformations on GB's matrix U acts linearly for $g \in H$ and non-linearly otherwise:

1) $g_0 \in H$: $g_0 U(x) = e^{i\alpha^a T^a} U = e^{i\alpha^a T^a} U e^{-i\alpha^a T^a} g_0$

$$\begin{cases} U' = g_0 U g_0^{-1} \\ h = g_0 \end{cases}$$ $\underline{\text{linear rep.}}$ $\underline{\pi \to \pi + [\alpha T, \pi]}$

2) $\hat g\, U(x) = e^{i\beta^{\hat a} T^{\hat a}} U = U' h(x)$ $\quad \hat g \notin H$

$\pi \to \pi' = \pi + \beta + \dots$ $\underline{\text{non-linear rep}}$

The GB's can be used to promote a linearly realized sym H to $\mathcal{G} \supset H$ with $\mathcal{G}$ non-linearly realized.

Take e.g. $\mathcal{L} = \mathcal{L}(\psi, \partial_\mu \psi)$ where $\psi \longrightarrow D(h)\psi$ $h \in H$ (linear rep. of H)

is a sym. $\Rightarrow \hat{\mathcal{L}} = \mathcal{L}(\psi, \nabla_\mu \psi)$ with $\boxed{\nabla_\mu \psi = (\partial_\mu - i E_\mu(\pi))\psi}$

is invariant under $\mathcal{G}$ because $E_\mu(\pi)$ transforms as a gauge field, although it is not elementary but made of π's.

The lowest order Lagrangian invariant under $\mathcal{G}$ that realizes only $\mathcal{H}$ linearly for GB's is built with d_μ:

$$\mathcal{L}^{(2)} = \frac{f_\pi^2}{2} \underbrace{\mathrm{Tr}(d_\mu d_\mu)}_{\text{2 derivative and invariant under } \mathcal{G}} = \frac{f_\pi^2}{2} d_\mu^{\hat{a}} d_\mu^{\hat{a}} = \frac{f_\pi^2}{2}(\partial_\mu \pi^{\hat{a}} + \dots)^2$$

, see previous page ($d_\mu \to h\, d_\mu h^+$)

For example, for symmetric cosets $[T^{\hat{a}}, T^{\hat{b}}] \supset T^c \Rightarrow$

$\Rightarrow d_\mu \sim \partial_\mu \pi + \frac{1}{2}[[\pi, \partial_\mu \pi], \pi] + \dots \quad \mathcal{L} = (\partial_\mu \pi + o(\pi^2 \partial \pi))^2 f_\pi^2$

it's an expansion in $\left(\frac{\partial}{f_\pi}\right)$ (after canonically normalize $(\partial\pi)^2$)

Higher orders $o(p^4)$ contain other terms: $\mathrm{Tr}[E_{\mu\nu} E^{\mu\nu}]$, $\mathrm{Tr}(d_\mu d_\mu)^2$, $\mathrm{Tr}(d_\mu d_\nu)^2$

Higgs Mechanism: $\mathcal{G} \to \mathcal{H}$ with $\underline{\mathcal{H}' \subset \mathcal{G} \text{ local}}$

maurer-cartan: $\begin{cases} U^{-1}\partial_\mu U \to U^{-1}(\partial_\mu - iA_\mu)U = i d_\mu + i E_\mu \\ U \to g(x)\, U\, h^+(x) \end{cases}$

$\Rightarrow \begin{cases} i d_\mu(A, \pi) = i d_\mu(\pi) - i\,\mathrm{Tr}(U^{-1}A_\mu U T^{\hat{a}})T^{\hat{a}} = i(\partial_\mu \pi^{\hat{a}} - A_\mu^{\hat{a}} + \dots) \\ i E_\mu(A, \pi) = i E_\mu(\pi) - i\,\mathrm{Tr}(U^{-1}A_\mu U T^a)T^a \end{cases}$

this vanish if $A_\mu \in$ Algebra $T^{\hat{a}}$ only

$\Rightarrow \begin{cases} \mathcal{L}^{(2)}_{\pi} \to \mathcal{L}^{(2)}_{\text{mass}} = \frac{f_\pi^2}{2}(\partial_\mu \pi^{\hat{a}} - A_\mu^{\hat{a}} + \dots)^2 - \frac{1}{4g^2} F_{\mu\nu}^2 + (A_\mu^{\hat{a}} - \partial\pi) J^{\hat{a}\mu}_{\text{matter}} \\ \underline{m_A^2 = f_\pi^2 g^2} \end{cases}$

after canonic. normalizat.

$\Rightarrow$ Goldberger-Treiman relation:
($J_\mu^{\hat{a}}$ is the current that couples to $A_\mu^{\hat{a}}$)

$$\boxed{\begin{aligned} J_\mu^{\hat{a}} &= -f_\pi \partial_\mu \pi^{\hat{a}} + J_{\mu\,\text{matter}}^{\hat{a}} \\ \langle 0 | J_\mu^{\hat{a}}(x) | \pi^{\hat{b}} \rangle &= \frac{p_\mu e^{-ipx}}{\sqrt{2E}} f_\pi \delta^{\hat{a}\hat{b}} \end{aligned}}$$

by consistency w/ $\partial_\mu J^\mu =$

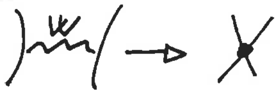
Custodial Symmetry: (or G/H pattern of SM?)

from $H = e^{i\pi}\begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$ and $|D_\mu H|^2$ we get $\boxed{\dfrac{m_W^2}{m_Z^2 c_W^2} = \rho = 1}$ at tree-level

This is non trivial since $\sin\theta_W$ controls coupling strength, whereas m_W and m_Z are IR deformations

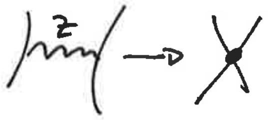
Moreover, $\delta\rho = \rho - 1$ is measure to 0.1% accuracy!!

ρ-parameter measure the relative strength of charged-currents and neutral (weak, non-$U(1)_{em}$) currents:



$\mathcal{L} = \frac{g}{\sqrt{2}} W_\mu^+ J^- \longrightarrow \frac{4G_F}{\sqrt{2}} J_\mu^+ J_\mu^-$

$\frac{1}{p^2 - m_W^2}$ $\qquad \frac{g^2}{2M_W^2} = \frac{4G_F}{\sqrt{2}}$



$\mathcal{L} = \frac{g}{c_W} Z_\mu (T^3 - s_W^2 Q)$ $\downarrow$ $J_Z^\mu J_Z^\mu G_Z \frac{2}{\sqrt{2}}$

$\frac{1}{p^2 - m_Z^2}$ $\qquad \frac{g^2}{M_Z^2 c_W^2}$

$$\mathcal{L}_{Fermi} = \frac{4G_F}{\sqrt{2}}\left(J_+^\mu J_-^\mu + \rho\, J_Z^{\mu\,2}\right) = \underline{\frac{4G_F}{\sqrt{2}}\left(J_+^\mu J_-^\mu + \rho\,(J_\mu^3 - s^2 J_\mu^{em})^2\right)}$$

at one-loop expect deviations:

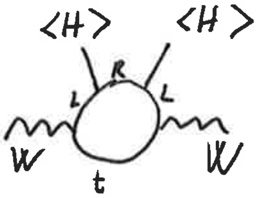
$\sim \dfrac{-g^{\mu\nu} + (p^\mu p^\nu\text{-terms})}{p^2 - m_W^2 - \Pi_{+-}(p^2)}$

$$\underset{\text{low } E,p}{\sim} + \frac{g^{\mu\nu}}{M_W^2}\left(1 - \frac{\Pi_{+-}(0)}{m_W^2}\right)$$ and analogous for Z_μ or W_μ^3

$$\Longrightarrow G^{+-} = \hat{G}^{+-}\left(1 - \frac{\Pi_{+-}(0)}{m_W^2}\right) \qquad G^{33} = \hat{G}^{+-}\left(1 - \frac{\Pi_{33}(0)}{m_W^2}\right) = \hat{G}^{+-}\left(1 - \frac{\Pi_{ZZ}(0)}{m_Z^2}\right)$$

$$\Longrightarrow \boxed{\delta\rho = \rho - 1 = \frac{\Pi_{33}(0) - \Pi_{+-}(0)}{m_W^2}}$$

Largest contribustion in the SM is from top-loop:



$$\delta\cancel{\Pi}\big|_{top} \sim \frac{y_t^2 v^2 g_4^2 N_c}{16\pi^2} \Longrightarrow \boxed{\delta\rho\big|_{top} \sim \frac{m_t^2 \cancel{g^2} N_c}{16\pi^2 v^2}}$$

(homework: calculate $\delta\rho\big|_{top} = \dfrac{N_c G_F m_t^2}{8\sqrt{2}\pi^2} \sim 1\%$) $\uparrow$ huge

There is a sym. that sets $\rho = 1$: CUSTODIAL SYM.

Define
$$\begin{cases} \Sigma = \frac{1}{v/\sqrt{2}} \underbrace{\left(\tilde{H} \quad H \right)}_{2\times 2 \text{ matrix}} = \frac{1}{v/\sqrt{2}} \begin{pmatrix} \phi_0^* & \phi_+ \\ \phi_- & \phi_0 \end{pmatrix} = e^{i\pi} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ H = e^{i\pi} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \qquad \tilde{H} = i\sigma^2 H^* \qquad \pi = \pi^a \sigma^a \\ \text{(higgs boson frozen, e.g. } m_h \to \infty) \end{cases}$$

$\boxed{\Sigma = e^{i\pi}}$ is just a change of variables, of parametrization of the coset of GB's.

Under $\mathcal{G} = SU(2)_L \times U(1)$: $\boxed{\Sigma \longrightarrow L \, \Sigma \, R_3^+}$ w/ $\begin{cases} L = e^{i\frac{\sigma^i}{2}\alpha^i} \in SU(2)_L \\ R_3 = e^{i\frac{\sigma^3}{2}\beta} \in U(1)_Y \end{cases}$

indeed $\Sigma = (\tilde{H} \; H)$ and $H = (1,2)_{1/2}$ $\tilde{H} = (1,2)_{-1/2}$

- There are two invariants: $\boxed{(a)\; Tr(D_\mu \Sigma^+ D_\mu \Sigma) \quad \text{and} \quad (b)\; Tr(\Sigma^+ D_\mu \Sigma \sigma^3)^2}$

where $D_\mu \Sigma = \partial_\mu \Sigma + i g W_\mu^i \frac{\sigma^i}{2} \Sigma - i g' B_\mu \Sigma \frac{\sigma_3}{2}$

$$\Rightarrow \begin{cases} \mathcal{L}_{eff}^{(2)} = \frac{v^2}{4} Tr(D_\mu \Sigma^+ D_\mu \Sigma) + \frac{v^2}{4} c_T \, Tr(\Sigma^+ D_\mu \Sigma \sigma^3)^2 \\ \text{going to } \Sigma = \langle \Sigma \rangle = \mathbb{1} \text{ (unitary gauge) we see that} \end{cases}$$

$Tr(\sigma^i \sigma^j) = 2\delta^{ij}$

$$\begin{cases} \frac{v^2}{4} Tr(D_\mu \Sigma^+ D_\mu \Sigma) = \frac{v^2}{4} g^2 |W|^2 + \frac{v^2}{4} (g W_\mu^3 - g' B_\mu)^2 Tr\left(\frac{\sigma_3^2}{4}\right) = m_W^2 |W_\mu|^2 + \frac{(g^2 + g'^2) v^2}{8} Z_\mu^2 \\ \frac{v^2}{4} c_T \, Tr(\Sigma^+ D_\mu \Sigma \sigma^3)^2 = \frac{v^2}{4} c_T (g W_\mu^3 - g' B_\mu)^2 = \frac{(g^2 + g'^2)}{4} v^2 c_T (Z_\mu)^2 \end{cases}$$

$$\Rightarrow \mathcal{L}_{eff}^{(2)} \xrightarrow[\Sigma = 1]{} m_W^2 |W_\mu|^2 + \frac{1}{2} m_Z^2 Z_\mu^2 \quad \text{with} \begin{cases} m_W^2 = \frac{g^2 v^2}{4} \\ m_Z^2 = \frac{(g^2 + g'^2) v^2}{4} (1 + 2c_T) \end{cases}$$

$$\Rightarrow \boxed{\rho = \frac{m_W^2}{m_Z^2 c_W^2} = \frac{1}{1 + 2c_T} \simeq 1 - 2c_T \quad \hookrightarrow \boxed{c_T \lesssim 10^{-3}} \text{ experim.}}$$

c_T is allowed but experimentally very small: is there a symmetry?

YES $\longrightarrow$ CUSTODIAL SYM.

• $\mathrm{Tr}[D_\mu\Sigma^+ D_\mu\Sigma]$ has a larger symmetry: $SO_c(3)$-custodial when $g'\to 0$ $\quad SU^2_c(2)$

indeed $\mathrm{Tr}[D_\mu\Sigma^+ D_\mu\Sigma] \supset \left(\partial_\mu\pi^{i=1,2} - g\frac{v}{2}W^i\right)^2 + \left(\partial\pi^3 - \frac{\sqrt{g^2+g'^2}}{2}v\,Z_\mu\right)^2$

$\xrightarrow[g'\to 0]{}$ $SO_c(3)$-symmetry where π^i and W^i are triplets

$$\begin{cases}\Sigma \to R\Sigma R^+ & R\in SU_c(2)\\ W_\mu \to R W_\mu R^+\end{cases}$$

This symmetry is respected by the vacuum: $\langle\Sigma\rangle = 1 \xrightarrow{R} 1$

notice: h-boson is a singlet of $SU_c(2)$

and the Goldstone Bosons π, indeed, transf. linearly $\pi^i \to \pi^i + \frac{i}{2}\epsilon^{ijk}\pi^j\theta^k + \dots$

• On the other hand, instead, $\mathrm{Tr}[\Sigma^+ D_\mu\Sigma\,\sigma] \supset \left(\partial_\mu\pi^3 - \frac{\sqrt{g^2+g'^2}}{2}v\,Z\right)^2$ break explicitly $SO_c(3)$

$\Rightarrow$ We can forbid c_T by demanding $SO_c(3)$

Moreover, when $g, g'\to 0$ Σ has an even larger symmetry

$$SU_L(2)\times SU_R(2) \sim SO(4) \xrightarrow[\Sigma=\mathbb{1}]{} SO_c(3) \sim SU_{L+R=c}(2)$$

$$\Sigma \to L\Sigma R^+$$

Under this custodial $SU_L(2)\times SU_R(2)$, π^i transf. non-linearly

$$\begin{cases}\pi^i \to \pi^i + \alpha^i + \dots\\ \pi^i \to \pi^i - \beta^i + \dots\end{cases} \quad \text{where } L = e^{i\alpha^i\sigma^i/2};\ R = e^{i\beta^i\sigma^i/2}$$

$\Rightarrow$ The natural pattern is then $G = SU_L(2)\times SU_R(2) \to SU_c(2)$

where $H' = SU_L(2)\times U(1)_{Y=T^3_R}$ is gauged $\quad H'\subset G$

Yukawas: $y^t\bar{Q}_L\tilde{H}t_R + y^b\bar{Q}_L H b_R = \bar{Q}_L\Sigma\begin{pmatrix}y^t t_R\\ y^b b_R\end{pmatrix}$ is also $SU_c(2)$ invariant

when $y^t = y^b \Rightarrow \delta\rho^{\mathrm{top}} \simeq (y_t^2 - y_b^2)$