Higgs Physics in the SM and Beyond

within the "Cours de Physique Théorique" à l'IPhT, CEA-SACLAY
(June-July /2014 ; 4 lectures , 2h each.)

Suggested Readings:

QFT Textbooks:
- QFT and the SM , M. Schwartz
- Modern QFT , T. Banks
- The quantum th. of fields, S. Weinberg

EW sector :
- The physics of EWSB , J. Santiago (2009)
- The Higgs hunter's guide , Gunion, Ha...
- Ten Lectures on EW int. , R. Barbieri
- Weak inter. and moder particle ph., Georgi
- The Higgs as a PNGB , TASI 2009 Contino 1005.4269 arXiv
- Composite Higgs , B.B., C. Csaki, J. Serra (arXiv: 1401.2457)

Summary of the course

- Review of SM (from Fermi t. of EW-interactions to the SM lagrangian)
- Custodial sym. ; Goldstone Bosons & the Higgs mechanism
  The equivalence theorem and Higgs pheno
- Higgs low-energy theorem , Higgs as a dilaton ;
  Higgs coupling to nucleons and the Higgs portal
- Higgs sum rules , Higgs potential & vacuum stability
  Hierarchy problem , composite Higgs models

From Fermi to Yang-Mills

3 energy scales are considered fundamental: $\Lambda_{QCD} \ll v_{EW} \ll M_{Pl}$

$\Lambda_{QCD} \sim$ few 100's MeV, $v_{EW} \simeq 246\,\text{GeV}$, $M_{Pl} \sim 10^{19}\,\text{GeV}$

This course is about $E \gtrsim v_{EW}$

I will thus focus mostly on EW interactions

First Lagrangian for EW interactions (to explain β-decay $n \to p + e + \bar{\nu}_e$)
by Fermi (1934): all started w/ an Effective Lagrangian and the need of UV completion

$$\mathcal{L}_{fermi} = -\frac{4 G_F}{\sqrt{2}} J_\mu^+ J_\mu^- \qquad J_\mu^- = \bar{e}_L \gamma_\mu \nu_L + \bar{d}_L \gamma_\mu u_L + \dots$$

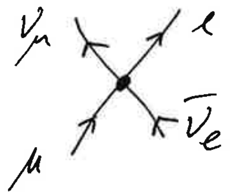
$G_F \simeq 1.17 \times 10^{-5}\,\text{GeV}^{-2}$
Fermi Constant

$$\frac{2 G_F}{\sqrt{2}} = \frac{1}{v_{EW}^2} = \frac{1}{(246\,\text{GeV})^2}$$

$P_L = \frac{1-\gamma_5}{2}$, $e_L = P_L e$

left-handed only
Break Parity (and C)
(Wu 1957)

Contact interaction, that is ultra-local



$\mu \to e + \bar{\nu}_e + \nu_\mu$

$$\Gamma(\mu \to e + \bar{\nu}_e + \nu_\mu) = \frac{G_F^2 M_\mu^5}{192\,\pi^3}$$

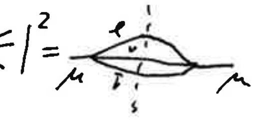
measuring this we know G_F

by Dimensional Analysis we get M_μ^5 (neglect m_e, m_ν)

counting the π's: $\frac{2\pi}{(16\pi^2)^{n-1}}$ for decay into n-body

by unitarity $\text{—O—} = \frac{1}{p^2 - m^2 - i m \Gamma}$

$\text{Im}\, A(\mu \to \mu)$ $\quad |\!\!-\!\!\!<|^2 =$



it breaks parity ✓
it is short distance ✓
it is well verified experimentally ✓

BUT ...

2 problems as a fundamental theory

(a) $\mu\nu \xrightarrow[\text{elastic}]{} \mu\nu$ $\quad A \sim G_F E^2$ by dim. analysis

$E \gg m_\mu$

it grows w/ energy, eventually breaks down

__Unitarity:__ $\mathrm{Im}\, A_{elast} = s\,\sigma_{TOT} \geq s\,\sigma_{elast} = \cancel{s}\int \frac{|A_{el}|^2 d\Omega}{64\pi^2 \cancel{s}} \underset{\text{s-wave}}{=} \frac{|A|^2_{el.}}{16\pi}$

$\|$

$|A_{el}| \geq |A|_{el} \sin\theta$

$$\Longrightarrow |A_{el}| > \frac{|A_{el}|^2}{16\pi}$$

$$|A_{el}| < 16\pi \Longrightarrow \underbrace{\boxed{E^2 \lesssim \frac{16\pi}{G_F}}}_{}$$

unitarity bound

it's actually an euristic argument.

(b) The theory is __not-renormalizable__

 $\sim G_F^2 \int \frac{d^4 k}{\cancel{k}\cancel{k}} \sim G_F^2 (b_0 \Lambda^2 + b_1 s + b_2 s \ln(\frac{\Lambda^2}{s}))$

it generate an infinite tower of new operators because __$[G_F] = -2$__

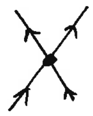
 $\sim \bar{\psi}\psi \Box \bar{\psi}\psi\, G_F^2 a_1$

schematically

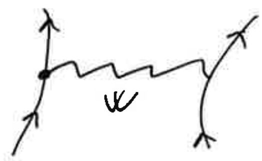
Both (a) and (b) tell us that $\mathcal{L}_{fermi}$ is an effective valid at $E \ll \Lambda_{cut\,off}$, The cut-off here is provided by $\Lambda_{cut\text{-}off} \sim G_F^{-1/2}$ (parametrically)

$\hookrightarrow$ __At High Energy we need an UV completion__

UV-complet Fermi with a gauge theory



$\longrightarrow$ by analogy with Electromagnetism $\longrightarrow$



too many fields in a point
bad short-distance behavior
$[G_F] = -2$; $V = \delta^3(x)$

W massive: short range
but finite $r \sim m_W^{-1}$
$V = \frac{e^{-m_W r}}{r}$

$$\mathcal{L}_{Fermi} \longrightarrow \mathcal{L}_{int} = \frac{g}{\sqrt{2}} W_\mu^+ J^- + h.c.$$

$$\left(\frac{1}{2}\frac{g^2}{m_W^2} = \frac{2}{V_{EW}^2} = \frac{G_F 4}{\sqrt{2}}\right)$$ ok

$$J_\mu^- = \bar{\nu}\,\gamma^\mu\left(\frac{1-\gamma^5}{2}\right) l + \bar{u}\,\gamma^\mu\left(\frac{1-\gamma^5}{2}\right) d + \dots$$
$$= \bar{\nu}_L \gamma^\mu l_L + \bar{u}_L \gamma^\mu d_L + \dots$$

How this interaction comes about?

<u>Basics of gauge theory:</u>

$$\psi(x) \xrightarrow{G\text{-local}} U(x)\,\psi(x)$$
gauge redundancy

gauge fields or connection $\quad A_\mu(x) = A_\mu^a T^a \longrightarrow U A_\mu U^{-1} - i\,U^{-1}\partial_\mu U$

(or, infinitesimally $U = e^{i\Lambda^a(x) T^a}$
$$A_\mu \to A_\mu - i[A_\mu \Lambda] + \partial_\mu \Lambda \;)$$

$\nabla_\mu = \partial_\mu - i A_\mu$ covariant derivative $\qquad \nabla_\mu \psi \longrightarrow U(x)(\nabla_\mu \psi)$ transforms covariantly $\Rightarrow$ $[\nabla_\mu, \nabla_\nu] \equiv -i F_{\mu\nu}$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - [A_\mu, A_\nu]$$

$$\mathcal{L}_{gauge} = -\frac{1}{2g^2}\,\mathrm{Tr}\left[F_{\mu\nu}F_{\mu\nu}\right] + \mathcal{L}_{matter}(\psi, \nabla\psi)$$

$$\mathcal{L}_{matter} = \bar{\psi}_L\, i\,\not{\nabla}\, \psi_L = \underbrace{\bar{\psi}_L\, i\,\not{\partial}\, \psi_L}_{\text{kinetic term}} + \underbrace{\bar{\psi}_L \gamma^\mu T^a \psi_L}_{\text{current}}\,\underbrace{A_\mu^a}_{\text{gauge field}}$$

~~Also~~ What is the gauge group for the EW int.?

$\mathcal{L} = \frac{g}{\sqrt{2}} W_\mu^+ J_\mu^-$ $\qquad J_\mu^- = \bar{\nu}_L \gamma^\mu \ell_L + \bar{u}_L \gamma^\mu d_L + \dots$

$L_L = \begin{pmatrix}\nu\\ e\end{pmatrix}_L \qquad Q_L = \begin{pmatrix}u\\ d\end{pmatrix}_L \qquad J_\mu^\pm = \bar{L}_L \gamma^\mu T^\pm L_L + \bar{Q}_L \gamma^\mu T^\pm Q_L$

with $T^+ = \begin{pmatrix}0&1\\0&0\end{pmatrix} \quad T^- = \begin{pmatrix}0&0\\1&0\end{pmatrix} \Rightarrow [T^+, T^-] = 2T^3 = \begin{pmatrix}1&\\&-1\end{pmatrix}$

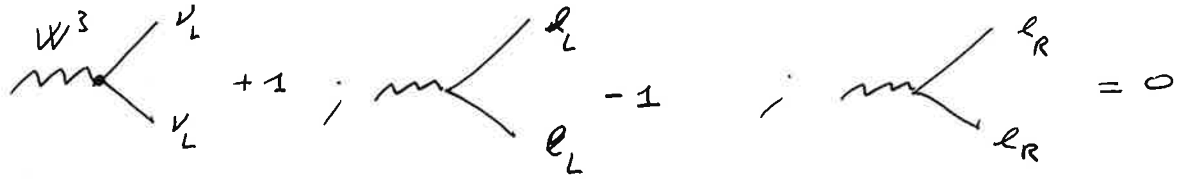
$\underline{SU(2) \text{ gauge}}$

$$\begin{cases} \frac{T^+ + T^-}{2} = T^1 \\ \frac{T^+ - T^-}{2i} = T^2 \end{cases} \qquad [T^a, T^b] = i\epsilon^{abc} T^c$$

$G \supset SU_L(2) \Rightarrow$ There is another gauge boson W_μ^3 coupled to T^3

new neutral currents

$$\mathcal{L}_{int} = \left(\bar{L}_L \gamma^\mu T_L^a L_L + \bar{Q}_L \gamma^\mu T_L^a Q_L\right) W_\mu^a$$



it can't be the photon (W^3 couples to ν and not to e_R, moreover it is massive)

But $\begin{cases} Q = T_L^3 + Y \\ [Y, T_L^a] = 0 \end{cases}$ $\qquad (T_L^3 + Y\mathbb{1}_2) L = \begin{pmatrix}1/2 + Y & 0\\ 0 & -1/2 + Y\end{pmatrix} L = \begin{pmatrix}0&0\\0&-1\end{pmatrix} L$

$\Rightarrow \boxed{Y_L = -1/2}$; $(T_L^3 + Y\mathbb{1}) e_R^c = Y e_R^c \Rightarrow \boxed{Y_{e_R} = 1}$

$(T_L^3 + Y\mathbb{1}_2) Q_L = \begin{pmatrix}2/3 & 0\\ 0 & -1/3\end{pmatrix} Q_L \Rightarrow \boxed{Y_{Q_L} = 1/6}$

$\Rightarrow G_{EW} = SU_L(2) \times U_Y(1) \qquad Q = T^3 + Y$

Including color $\boxed{G = SU(3)_{color} \times SU(2)_L \times U(1)_Y}$

	$SU(3)$	$SU(2)$	$U_Y(1)$
Leptons L_L^i	1	2	$-1/2$
Leptons e_R^i	1	1	1
quarks Q_L^i	3	2	$1/6$
quarks $u_R^{c\,i}$	$\bar{3}$	1	$-2/3$
quarks $d_R^{c\,i}$	$\bar{3}$	1	$1/3$

$$\mathcal{L}_{gauge} = -\frac{1}{4g_i^2} F^A_{\mu\nu\,i} F^A_{\mu\nu\,i} + \sum_i \bar{\Psi}_i \, i \not{D} \, \Psi_i$$

$\begin{cases} u^{i=1,2,3} = u, c, t \\ d^{i=1,2,3} = d, s, b \end{cases}$

$\leftarrow$ flavor index : 3 copies $i = 1, 2, 3$

This fix the interactions $\uparrow \nearrow$

The non abelian structure of G predicts also self-couplings experimentally verified at $\sim 1\%$, or $\sim 0.1\%$ level (LEP) coupling to matter

Changing Basis: $Q = T^3 + Y$

(or g_2) g : $SU(2)_L$ coupling

(or g_1) g' : $U(1)_Y$ coupling

$\longrightarrow \frac{1}{g^2} + \frac{1}{g'^2} = \frac{1}{e^2}$ or

$e = \frac{g g'}{\sqrt{g^2 + g'^2}}$ electric charge

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \underbrace{\begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix}}_{\text{rotation by Weinberg angle } \theta_W} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \Rightarrow g W_\mu^3 T^3 + g' B_\mu Y = \overbrace{e}^{\frac{gg'}{\sqrt{\ldots}}} \overbrace{(T^3 + Y)}^{Q} A_\mu + \ldots$$

$$\Rightarrow \frac{g}{\sqrt{g^2 + g'^2}} = c_W, \quad \frac{g'}{\sqrt{g^2 + g'^2}} = s_W \;\; (s_W^2 \simeq 0.23) \quad \frac{g'}{g} = t_W$$

$$\boxed{g W_\mu^3 T^3 + g' B_\mu Y = e Q A_\mu + \frac{e}{s_W c_W}\left(T^3 - \sin^2\theta_W Q\right) Z_\mu}$$

NEUTRAL Interactions

$$\mathcal{L}_{gauge} = -\frac{1}{4}F^2 + \sum_i \bar{\psi}_{(i)} \not{D}\, \psi_{(i)}$$

- Interactions are $\underline{G = SU(3) \times SU(2) \times U(1) \text{ symmetric}}$
- They are also <u>universal</u> (no flavor dependence)

<u>How the rich structure of our World emerge?</u>

The gauge theory is in a $\boxed{\text{Higgs phase}}$: the vacuum is not invariant under G but only under $\underline{H = U(1)_{em} \times SU(3) \subset G}$

$\Rightarrow$ <u>The spectrum does not respect G</u> : $\boxed{\text{IR deformation}}$

$$\mathcal{L}^{IR}_{masses} = m_W^2 W_\mu^2 + \frac{1}{2} m_Z^2 Z_\mu^2$$
$$+ m^u_{ij}\, \bar{u}_L^{(i)} u_R^{(j)} + m^d_{ij}\, \bar{d}_L^{(i)} d_R^{(j)} + h.c.$$
$$+ m^\ell_{ij}\, \bar{\ell}_L^{(i)} \ell_R^{(j)} \left(+ m^\nu_{ij}\, \bar{\nu}_L^{i} \nu_R^{j}\right) + h.c.$$

<u>SOME NUMBERS</u>

$m_W \simeq 80$ GeV ($\pm$ 15 MeV) $\quad \Gamma_W \simeq 2.1$ GeV $\pm$ 2%

$m_Z \simeq 91$ GeV ($\pm$ 2 MeV) $\quad \Gamma_Z = 2.5$ GeV $\pm$ 0.1%

$m_t \simeq 173$ GeV ($\pm$ 1 GeV)
$m_b \simeq 4$ GeV
$m_c \simeq 1.3$ GeV
} heavy quarks

$m_s \simeq 95$ MeV
$m_{avg\,d} \simeq 5$ MeV
$m_u \simeq 2.5$ MeV
} Light quarks $m \ll \Lambda_{QCD}$

ch. leptons {
$m_\tau \simeq 1.7$ GeV
$m_\mu \simeq 105$ MeV
$m_e \simeq 0.5$ MeV

neutrinos {
m_ν sub eV $\quad \Delta m^2_{21} \simeq 10^{-4} - 10^{-5}$
$\Rightarrow m_\nu \gtrsim 0.04$ eV $\quad \Delta m^2_{32} \simeq 10^{-3}$

- <u>The masses break Flavor Symmetry</u>

$$\mathcal{L}^{\text{quark}}_{\text{masses}} = m^u_{ij}\, \bar{u}^{(i)}_L u^{(j)}_R + m^d_{ij}\, \bar{d}^{(i)}_L d^{(j)}_R$$

$$\mathcal{L}_{\text{gauge}} = \sum_i \bar{Q}^{(i)}_L i\not{D} Q^{(i)}_L + \bar{u}^c_{(i)} i\not{D} u^c_{(i)} + \bar{d}^{(i)}_c i\not{D} d^{(i)}_c$$

$$U_Q(3) \times U_u(3) \times U_d(3)$$

<u>Flavor</u>: $U^3(3)$ (one $U(1)$ is Hypercharge $\Rightarrow SU^3(3) \times U(1)^2$)

(adding lepton is similar)

$$U^3(3) \xrightarrow[\text{explicitly}]{} U_B(1) \times U_{em}(1) \quad \text{Baryon number} \times \text{Electric charge}$$

$$\left.\begin{array}{l} \bar{u}_L \to \bar{u}_L V^\dagger_{u_L} \\ u_R \to V_{u_R} u_R \end{array}\right\} \quad m^u \longrightarrow V^\dagger_{u_L} m^u V_{u_R} = m^u_{\text{diag.}} = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}$$

$$\left.\begin{array}{l} \bar{d}_L \to \bar{d}_L V^\dagger_{d_L} \\ d_R \to V_{d_R} d_R \end{array}\right\} \quad m^d \longrightarrow V^\dagger_{d_L} m^d V_{d_R} = m^d_{\text{diag}} = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}$$

- Right-handed couple only to (γ, B_μ) $\Rightarrow V_{u_R}$ & V_{d_R} have no impact
- Left-handed couple to A_μ & Z_μ in a flavor universal way: no impact $\quad A_\mu \bar{u}^i_L u^i_L + \dots$
- Left-handed couple to W_μ which mix up & down

$$\Rightarrow \bar{u}^i_L \gamma^\mu d^i_L W_\mu \longrightarrow \bar{u}_L \underbrace{(V^\dagger_{u_L} V_{d_L})}_{V_{CKM}} \gamma^\mu d_L W_\mu$$

only charged currents give flavor transitions

<u>No flavor changing neutral at tree-level in the SM</u>

- Masses are needed and the flavor structure is well tested within the CKM model

- Masses break gauge invariance: by itself not a problem
  Gauge is a redundancy

- The breaking must be spontaneous $G \to H$

$$G_{EW} = SU(2)_L \times U(1)$$
$$\downarrow$$
$$H = U(1)_{em} = U(1)_{Q=T_3+Y}$$ because the photon remains massless ($A_\mu \to A_\mu - \partial_\mu \Lambda$ forbids a mass)

and masses are compatible with $U(1)_{em}$

- $G \to H$ gives rise to $\begin{cases} G/H & \text{Goldstone Bosons if } G \text{ global} \\ G/H & \text{Longitudinal dofs of } A_\mu\text{'s} \end{cases}$

In any case at least $\dim\left(SU(2)_L \times U(1)/_{U(1)}\right) = 3$ new dofs

They are provided by the field(s) that picks vev:

Take e.g. $m^{\ell}_{ij}\, \bar{\ell}^{(i)}_L\, \ell^{(j)}_R \to m^{(\ell)}$ must be a doublet under $SU(2)_L$ to build an invariant

$$\bar{\ell}_L \subset L = (1,2)_{-1/2} \qquad \ell_R: (1,1)_1 = e^c_R$$

let's promote $m^{\ell} \xrightarrow[\text{to a field}]{} H$ that gets vev $\to H = (1,2)_{1/2}$ indeed

$$\bar{L}^{(i)} H\, \ell^{(j)}_R\, Y^{(\ell)}_{ij}$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$+1/2 \quad 1/2 \quad -1$$

it's G-invariant if H transforms as
and give mass $m^{(\ell)}_{ij} = Y^{(\ell)}_{ij} \frac{v}{\sqrt{2}}$ when $\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

moreover, it doesn't break $U(1)_{em}$ since $Q = T_3 + Y$ $\quad Q\begin{pmatrix} 0 \\ v \end{pmatrix} = 0$

now, let's look at the quarks

$$m^d_{ij}\ \bar{d}_L^{(i)}\, d_R^{(j)} \quad \Rightarrow \quad m^d \text{ transforms as } (1,2)_{1/2} \ \underline{\underline{\text{again}}}$$

$$\bar{d}_L \subset Q_L = (3,2)_{1/6} \qquad d_R^c = (\bar{3},1)_{1/3} \qquad\qquad 2\otimes 2 = \underline{\underline{1}} \oplus 3$$

$\underline{m^d \propto H \text{ too}}$

$$m^d\, \bar{d}_L\, d_R \longrightarrow Y^d_{ij}\ \underset{-1/6}{\bar{Q}_L}\ \underset{1/2}{H}\ \underset{-1/3}{d_R} \quad \text{is } \underline{\mathcal{G}\text{-invariant}}$$

$$m^d_{ij} = \frac{Y^d_{ij}}{\sqrt{2}}\, v \qquad\qquad \text{H-vev } \langle H\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \ \underline{\text{breaks}}\ \underline{\text{spontaneously}}$$

what about the up-quark?

$$m^u_{ij}\ \bar{u}_L^{(i)}\, u_R^{(j)}$$

$$\bar{u}_L \subset Q_L = (3,2)_{1/6} \qquad u_R^c = (\bar{3},1)_{-2/3}$$

We need a doublet again, but the ~~hy~~ hypercharge doesn't match with H

$$\tilde{H} = (1,2)_{-1/2} \qquad \underset{-1/6}{\bar{Q}_L^{(i)}}\ \underset{-1/2}{\tilde{H}}\ \underset{2/3}{u_R^{(j)}}\ Y^u_{ij} \qquad m^u_{ij} = Y^u_{ij}\frac{v}{\sqrt{2}}$$

$$\text{where } \langle \tilde{H}\rangle = \begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix} \quad \& \quad Q\langle \tilde{H}\rangle = 0 \ \ \underline{\text{ok}}$$

$\tilde{H}$ can be an independent field than H: it is a two-Higgs model

but we can still use H: indeed $\boxed{i\sigma^2 H^* = \tilde{H} = (1,2)_{-1/2}}$

because $\bar{2} = 2$ in $SU(2)$

$$\to \mathcal{L}_{fermions} = y^u \bar{Q}_L \tilde{H} u_R + y^d \bar{Q}_L H d_R + y^\ell \bar{L} H e_R$$

what about the W & Z masses?

$$\mathcal{L}_K = |D_\mu H|^2 = \left|\left(\partial_\mu - i g W_\mu^a T_L^a - i g' B_\mu Y\right)\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}\right|^2$$

→ photon is massless

→ $T^\pm$ are raising/lowering operators $\left|\frac{g}{\sqrt{2}}(W^+T^- + W^-T^+)\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}\right|^2 =$

$$= \left|\frac{gv}{2} W^-\right|^2 \Rightarrow \boxed{\frac{g^2v^2}{4} = m_W^2}$$

$$\to \quad (g W^3 T^3 + g' B Y) = e A_\mu Q + \frac{e}{s_W c_W}(T^3 - s_W^2 Q) Z_\mu$$

$$\Rightarrow \left|(\quad\quad)\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}\right|^2 = \left|\frac{e Z_\mu}{s_W c_W} T^3 \frac{v}{\sqrt{2}}\right|^2 = \left(\frac{e^2 v^2}{s_W^2 c_W^2 4}\right)\frac{Z_\mu^2}{2}$$

$$\Rightarrow \boxed{m_Z^2 = \frac{e^2 v^2}{s_W^2 c_W^2 4}} \quad \text{but} \quad e = g s_W \Rightarrow m_Z^2 = \frac{m_W^2}{c_W^2}$$

$$\boxed{\rho \equiv \frac{m_W^2}{m_Z^2 c_W^2} = 1 \Big|_{\text{at tree-level}}}$$

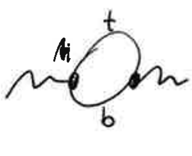
top-loop corrections are 1%

the fact that = 1 is due to a symmetry called custodial sym we will review next lecture

global unbroken $SU(2)_c$ $SO_c(3)$

$$\rho = 1 + \frac{3 G_F m_t^2}{8\sqrt{2}\pi^2} + \text{higgs contribution}$$

$(\rho - 1) = \hat{T}$ calculable in terms of vacuum polarizations



If there is time... $\left\{ \begin{array}{l} \text{Anomaly Cancellation \& GUT} \\ \text{Neutrino Masses} \end{array} \right.$

Anomaly

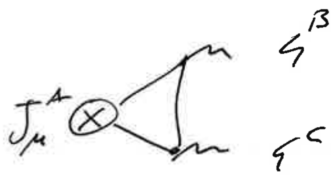
- $\psi = (3,2)_{1/6} \oplus (\bar{3},1)_{-2/3} \oplus (\bar{3},1)_{1/3} \oplus (1,2)_{-1/2} \oplus (1,1)_1$

$$= Q_L \oplus u_R^c \oplus d_R^c \oplus L \oplus e_R^c$$

it's a 15-dim. Weyl spinor, it provides a representation of $G = SU(3)_c \times SU(2)_L \times U(1)_Y$ that is 5-times reducible

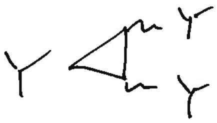
- This matter content is anomaly free

$$\partial_\mu J^\mu_A \propto \mathrm{Tr}[T^A\{T^B T^C\}]\, G^B_{\mu\nu} G^C_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}$$



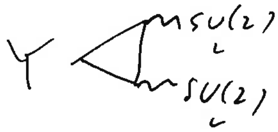
- $SU^3_L(2)$ vanishes since have only real and pseudo-real rep.
- $SU^3(3)$ vanishes since QCD is vector like (real rep.)
- ~~[illegible]~~ only 1 $SU_L(2)$ or 1 $SU_c(3)$ vanishes because $\mathrm{Tr}[T_{3,2}] = 0$

But there are 3 non-trivial conditions:

1) 

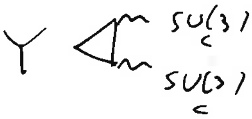
$\mathrm{Tr}[Y^3] = 6\cdot\left(\frac{1}{6}\right)^3 + 3\left(-\frac{2}{3}\right)^3 + 3\left(\frac{1}{3}\right)^3 + 2\left(-\frac{1}{2}\right)^3 + 1^3 = 0$

amazing cancellation among quarks & leptons

2) 

$\mathrm{Tr}[Y\{T_L^a, T_L^b\}] \propto \delta^{ab}\,\mathrm{Tr}[Y]_L$ = Y sum over doublet

$= 3\cdot\frac{1}{6} + \cancel{2}\left(-\frac{1}{2}\right) = 0$ again, Q_L cancels L_L

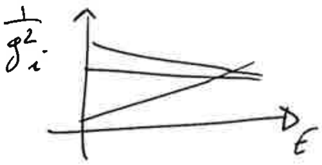
3) 

$\mathrm{Tr}[Y\{T^a_{su(3)}, T^b_{su(3)}\}] \propto \delta^{ab}\,\mathrm{Tr}[Y]_{\text{color states}} = 2\cdot\frac{1}{6} + \left(-\frac{2}{3}\right) + \frac{1}{3} =$

This Quark – Lepton connection can be explained within GUT

$\boxed{\text{GUT}}$ ψ is 15-dim. rep. of $SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y$

- We have seen that Y must be quantized to explain anomaly c. between quarks & leptons
- $g_{1,2,3}$ runs and roughly meet at $M_{GUT} \sim 10^{15}$ GeV



- $SU(5) \supset SU(3) \times SU(2) \times U(1)$

$$T_{su(5)} = \left(\begin{array}{c|c} su(3) & \\ \hline & su(2) \end{array}\right) \begin{array}{l} \updownarrow 3\times 3 \\ \updownarrow 2 \end{array} \rightarrow Y = \alpha \left(\begin{array}{c|c} 2\mathbb{1}_3 & 0 \\ \hline 0 & -3\mathbb{1}_2 \end{array}\right)^{\in\, su(5) \text{ algebra}}$$

$Tr[Y] = 0$ automatically

$su(5)$ has rank 4 like $SU(3) \times SU(2) \times U(1)$ and therefore is the smallest that contains it.

- Let's try to fit together leptons & quarks

$5 = \square \in SU(5)$ decomposes as $(\boxed{3} \cdot 1) + (1 \cdot \boxed{2})$
$\qquad (3,1)_{2\alpha} \quad (1,2)_{-3\alpha}$

$\Rightarrow \bar{5} = (\bar{3}, 1)_{-2\alpha} \oplus (1, 2)_{3\alpha}$

- choosing $\alpha = -1/6$ we fit d_R^c and L_L $\boxed{\bar{5} = d_R^c \oplus L_L} = \begin{pmatrix} d_R^{c\;i=1,2,3} \\ \ell \\ \nu \end{pmatrix}$
- out of 15 Weyl spinors, I'm left with $15 - 5 = 10$ dof's to fix

let's try with $10 \in SU(5)$ $\quad \underset{5}{\square} \times \underset{5}{\square} = \underset{10}{\boxminus} + \underset{15}{\square\square}$

$\Rightarrow$ 10 is antisymmetric $\boxminus = \left(\begin{smallmatrix}\boxed{3}\\\boxed{2}\end{smallmatrix} \cdot 1\right) \oplus (\boxed{3} \cdot \boxed{2}) \oplus \left(1 \cdot \begin{smallmatrix}\boxed{2}\\\boxed{1}\end{smallmatrix}\right)$
$\qquad (\bar{3}, 1)_{4\alpha} \quad (3, 2)_{-\alpha} \quad (1, 1)_{-6\alpha}$

with $\alpha = -1/6$ as above $\boxed{10 = u_R^c \oplus Q_L \oplus e_R^c}$ =

Neutrino Masses

add a right-handed neutrino $\nu_R = (1,1)_0$
completely neutral

$$Y^\nu \bar{L} \underset{1/2}{} \tilde{H} \underset{-1/2}{} \nu_R \underset{0}{} + \frac{1}{2} \nu_R^T i\sigma^2 \nu_R M_\nu + h.c.$$

integrating out ν_R, if M_ν is very large we generate a $D=5$ (in fact the only $D=5$ operator in the SM that's gauge inv.)

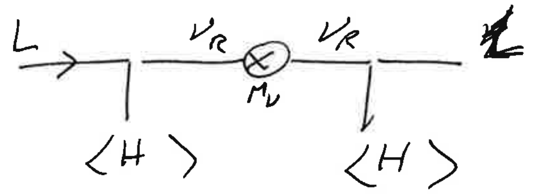
$$\mathcal{L} = \frac{Y^{\nu 2}}{M_\nu} (\bar{L}\tilde{H})(i\sigma^2)(\bar{L}\tilde{H})^T$$

$$\Rightarrow m_\nu \sim \frac{Y_\nu^2 v^2}{M_\nu} \Rightarrow \boxed{\text{see-saw mechanism}}$$

M_ν very large gives m_ν very small

$M_\nu \gg v \Rightarrow m_\nu \ll v$

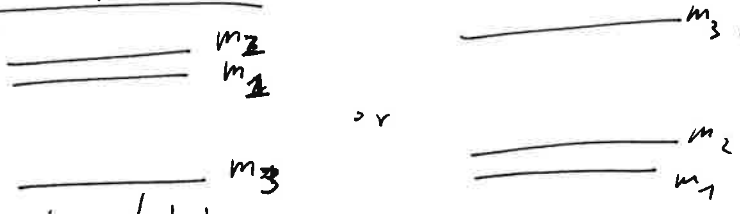
Diagrammatically



$$\sim Y_\nu^2 \frac{M_\nu}{M_\nu^2} v^2 = m_\nu$$

From neutrino oscillations we know that $\left\{ \begin{array}{l} \Delta m_{21}^2 \sim 7.5\, 10^{-5}\, eV^2 \\ \Delta m_{32}^2 \sim 2 \cdot 10^{-3}\, eV^2 \end{array} \right.$

so either



inverted h.

in any case $m_\nu \gtrsim \sqrt{\Delta m_{32}^2} \simeq$ few 10^{-2} eV.