Breaking of factorization of two-particle correlations in hydrodynamics

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The system formed in ultrarelativistic heavy-ion collisions behaves as a nearly-perfect fluid. This collective behavior is probed experimentally by two-particle azimuthal correlations, which are typically averaged over the properties of one particle in each pair. In this Letter, we argue that much additional information is contained in the detailed structure of the correlation. In particular, the correlation matrix exhibits an approximate factorization in transverse momentum, which is taken as a strong evidence for the hydrodynamic picture, while deviations from the factorized form are taken as a signal of intrinsic, "nonflow" correlations. We show that hydrodynamics in fact predicts factorization breaking as a natural consequence of initial state fluctuations and averaging over events. We derive the general inequality relations that hold if flow dominates, and which are saturated if the matrix factorizes. For transverse momenta up to 5 GeV, these inequalities are satisfied in data, but not saturated. We find at least as large factorization breaking in event-by-event ideal hydrodynamic calculations as in data, and argue that this phenomenon opens a new window on the study of initial fluctuations.

I. INTRODUCTION

In relativistic heavy-ion collision experiments a large second Fourier harmonic is observed in two-particle correlations as a function of relative azimuthal angle [1–4]. This has long been considered a sign of significant collective behavior [5], or "elliptic flow", indicating the existence of a strongly-interacting, low-viscosity fluid [6]. However, only recently has it been realized that *all* such correlations observed between particles separated by a large relative pseudorapidity could be explained by this collective behavior [7, 8], at least for the bulk of the system.

One significant piece of evidence for this view was the recent observation of the factorization [9–12] of two-particle correlations into a product of a function of properties of only one of the particles times a function of the properties of the second. Specifically, for pairs of particles in various bins of transverse momentum p_T, factorization of each Fourier harmonic was tested as [9]:

$$V_{n\Delta}(p_T^a, p_T^b) \equiv \left\langle \cos n(\phi^a - \phi^b) \right\rangle \stackrel{?}{=} v_n(p_T^a) \times v_n(p_T^b), \quad (1)$$

where the brackets indicate an average over pairs of particles (a and b) coming from the same event as well as an average over a set of collision events, and $\phi^a(\phi^b)$ is the azimuthal angle of particle $a(b)$. The left-hand side is a (symmetric) function of two variables, p_T^a and p_T^b, and in general may not factorize into a product of a function v_n of each variable individually. The fact that this factorization holds at least approximately, then, is a non-trivial observation about the structure of the correlation.

While most known sources of non-flow correlations do not factorize at low p_T [13], a type of factorization comes naturally in a pure hydrodynamic picture where particles are emitted independently. They thus have no intrinsic correlations with other particles, carrying only information about their orientation with respect to the system as a whole. This causes the two-particle probability distribution in a single collision event to factorize [14] into a product of one-particle distributions,

$$\frac{dN_{pairs}}{d^3p^a d^3p^b} \stackrel{\text{(flow)}}{=} \frac{dN}{d^3p^a} \times \frac{dN}{d^3p^b}. \quad (2)$$

Inspired by this fact, it has often been stated [12, 15, 16] that the factorization test in Eq. (1) should work perfectly in hydrodynamics. The observed approximate factorization was hailed as a success for the flow interpretation of correlations, while small deviations from the factorized form was interpreted as a gradual breakdown of the hydrodynamic description with increasing transverse momentum, and of increasing contribution from other sources of correlations.

In this work, we show that factorization as in Eq. (1) is not necessarily present even in an ideal hydrodynamic system governed by Eq. (2). We show that the correlation matrix satisfies general inequalities, which are saturated by Eq. (1). We test these inequalities on ALICE data and point out where breaking of factorization occurs. We then illustrate with a full event-by-event hydrodynamic calculation that the same deviation seen in experiment is also present in ideal hydrodynamics.

II. HYDRODYNAMICS AND TWO-PARTICLE CORRELATIONS

We begin by recalling the discussion originally found in Ref. [17]. In a pure hydrodynamic picture, particles are emitted independently from the fluid at the end of the system evolution according to some underlying one-particle probability distribution. One can write any such distribution as a Fourier series in the azimuthal angle ϕ

of the particles

$$\frac{2\pi}{N}\frac{dN}{d\phi} = \sum_{n=-\infty}^{\infty} V_n(p_T,\eta)e^{-in\phi}, \quad (3)$$

where $V_n = \{e^{in\phi}\}$ is the nth complex Fourier flow coefficient, and curly brackets indicate an average over the probability density in a single event. Writing $V_n = v_n e^{in\Psi_n}$, where v_n is the (real) anisotropic flow coefficient and Ψ_n the corresponding phase, and using $V_{-n} = V_n^*$ (where V_n^* is the complex conjugate of V_n), this can be rewritten as

$$\frac{2\pi}{N}\frac{dN}{d\phi} = 1 + 2\sum_{n=1}^{\infty} v_n(p_T,\eta)\cos n\left(\phi - \Psi_n(p_T,\eta)\right). \quad (4)$$

Note that, for this form to describe an arbitrary distribution, both v_n and Ψ_n may depend on transverse momentum p_T and pseudorapidity η.

In this picture, the relation in Eq. (2) holds, and a complex Fourier harmonic of the two-particle correlation factorizes in each event as:

$$\begin{aligned}\left\{e^{in(\phi^a-\phi^b)}\right\} &= \left\{e^{in\phi^a}\right\}\left\{e^{-in\phi^b}\right\} \\ &= V_n^a V_n^{b*} = v_n^a v_n^b e^{in(\Psi_n^a-\Psi_n^b)}. \quad (5)\end{aligned}$$

The experimental quantity, Eq. (1), is then obtained by averaging over events:

$$V_{n\Delta}(p_T^a,p_T^b) = \left\langle V_n^a V_n^{b*}\right\rangle = \left\langle v_n^a v_n^b e^{in(\Psi_n^a-\Psi_n^b)}\right\rangle \quad (6)$$

Due to parity symmetry, only the real part remains after this average, hence the cosine in Eq. (1).

From this relation alone, one can make the following general statements about the event-averaged correlation matrix when flow is dominant: the diagonal elements must be positive, and the off-diagonal elements must satisfy a Cauchy-Schwarz inequality,

$$V_{n\Delta}(p_T^a,p_T^a) \geq 0, \quad (7)$$

$$V_{n\Delta}(p_T^a,p_T^b)^2 \leq V_{n\Delta}(p_T^a,p_T^a)V_{n\Delta}(p_T^b,p_T^b). \quad (8)$$

Factorization, Eq. (1), implies that the second inequality is saturated, i.e., equality is achieved. While flow does not necessarily imply factorization, any violation of these inequalities is an unambiguous indication of the presence of non-flow correlations.

An inspection of published data from the ALICE Collaboration [9] shows that these inequalities are indeed violated in certain regimes [18]. For $n = 3$, diagonal elements $V_{3\Delta}(p_T^a,p_T^a)$ are negative above 5 GeV for 0–10% centrality, and above 4 GeV for 40–50% centrality. This is a clear indication that there are nonflow correlations at high p_T. For instance, the correlation between back-to-back jets typically yields a relative angle $\Delta\phi \sim \pi$, thus producing a negative $V_{3\Delta}$ at high p_T. For $n = 1$, diagonal elements are negative not only at high p_T (with a slightly higher threshold than for $n = 3$), but also for p_T between 1 and 1.5 GeV. This is believed to be caused by the correlation from global momentum conservation [12, 19], but it is interesting to note that its effect can be noticed by a simple inspection of elements.

In order to check the validity of the second inequality (8), we introduce the ratio

$$r_n \equiv \frac{V_{n\Delta}(p_T^a,p_T^b)}{\sqrt{V_{n\Delta}(p_T^a,p_T^a)V_{n\Delta}(p_T^b,p_T^b)}}, \quad (9)$$

which is defined when diagonal elements $V_{n\Delta}(p_T^a,p_T^a)$ and $V_{n\Delta}(p_T^b,p_T^b)$ are both positive, and lies between -1 and $+1$ if Eq. (8) holds. Factorization corresponds to the limit $r_n = \pm 1$. Figure 1 displays r_2 and r_3 as a function of p_T^a and p_T^b for Pb-Pb collisions at 2.76 TeV, 0–10% centrality. ALICE results for r_2 satisfy the inequalities (8) at all p_T. When both particles are below 1.5 GeV, the inequality is saturated, $r_2 = 1$, within errors. As soon as one of the particles is above 1.5 GeV, however, r_2 is smaller than unity, and the difference with unity increases with the difference $p_T^a - p_T^b$. Results for r_3 are qualitatively similar below 5 GeV, with larger error bars. However, r_3 is closer to 1 than r_2 between 2 and 3 GeV. The values of r_n for mid-central collisions (40-50% centrality, not shown) are comparable to the values for central collisions, although r_2 is slightly closer to 1.

The ALICE collaboration concluded from their analysis that factorization holds approximately for $n > 1$ and p_T below 4 GeV. However, their results actually show evidence for a slight breaking of factorization for $n = 2$, as soon as one of the particles has $p_T > 1.5$ GeV. Even though factorization is broken, the general inequalities implied by flow are satisfied for $n = 2$ and $n = 3$ below 5 GeV for central collisions. It is therefore worth investigating in more detail to what extent the breaking of factorization which is seen experimentally can be understood within hydrodynamics.

First, we recall under which conditions factorization holds in hydrodynamics. It implies that the Cauchy-Schwarz inequality (8) is saturated. By inspection of Eq. (6), this in turn implies that the complex flow vectors V_n^a and V_n^b are linearly dependent. This is true only under the following assumptions:

1. By parity symmetry, $\Psi_n^a - \Psi_n^b = 0$ in each event. I.e., Ψ_n does not depend on p_T, which removes the exponential from the right-hand side of Eq. (5).

2. $v_n(p_T)$ changes from event to event by only a global factor, with no p_T-dependent fluctuations. $v_n(p_T)$ in the right-hand side of Eq. (1) then represents the rms value over events.

In general, fluctuations ensure that these conditions are not met exactly, and the factorization of Eq. (1) will not be perfect. Within hydrodynamics, the ratio r_n in Eq. (9) has a simple interpretation. Inserting Eq. (6) into Eq. (9), one obtains

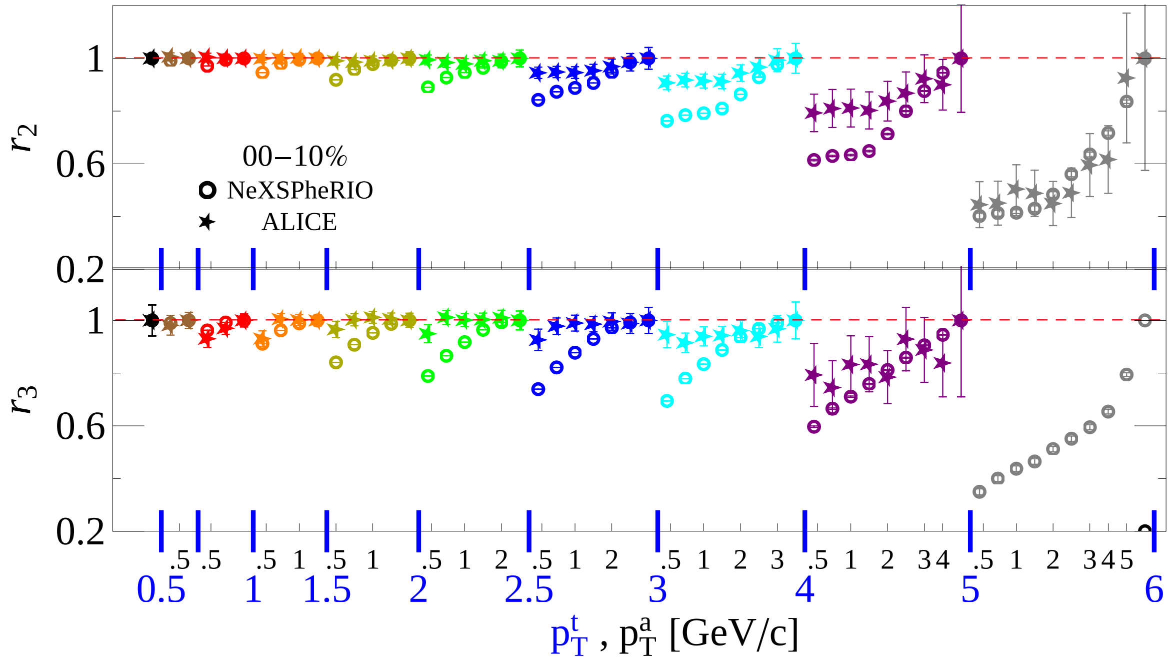

FIG. 1. (Color online) Ratio of nondiagonal to diagonal correlations, defined by Eq. (9) versus p_T^a and p_T^b. Filled stars: ALICE data for 0-10% central Pb-Pb collisions at 2.76 TeV [9]. Open cicles: ideal hydrodynamic calculations for 0-10% central Au-Au collisions at 200 GeV.

$$r_n = \frac{\langle V_n^{a*} V_n^b \rangle}{\sqrt{\langle |V_n^a|^2 \rangle \langle |V_n^b|^2 \rangle}}, \tag{10}$$

The ratio r_n thus represents the linear correlation between the complex flow vectors at momenta p_T^a and p_T^b. Since in each event, V_n^a is a smooth function of p_T^a, one expects that the correlation is stronger when $p_T^a \simeq p_T^b$, and decreases as the difference between p_T^a and p_T^b increases, as a result of the decoherence induced by initial fluctuations. ALICE data confirm this qualitative expectation.

Note that even in a single hydrodynamic event, factorization holds in the complex form Eq. (5), but is broken if one takes the real part before averaging over particle pairs, as in Eq. (1). The ratio r_n in Eq. (10) is then $\cos n(\Psi_n^a - \Psi_n^b)$, which is smaller than unity as soon as the flow angle Ψ_n depends on p_T.

The question then becomes: how large are factorization-breaking effects in hydrodynamics, and do they have the same properties as seen in data? If purely hydrodynamic calculations give the same result as experiment, then the observed breaking of factorization may not indicate the presence of non-flow correlations.

III. IDEAL HYDRODYNAMIC CALCULATIONS

To illustrate these concepts we perform calculations using the NeXSpheRIO model [20]. This model solves the equations of relativistic ideal hydrodynamics with fluctuating initial conditions given by the NeXuS event generator [21]. It has proven succesful in reproducing RHIC results, in particular the structure of two-particle angular correlations in Au-Au collisions at the top RHIC energy [22]. It has recently been shown to reproduce the whole set of measured anisotropic flow data [23–25]. Our calculations are therefore performed for Au-Au collisions at the top RHIC energy, not for Pb-Pb collisions at LHC energy, as would be appropriate for a direct quantitative comparison with ALICE data. Our results are merely meant as a proof of concept, and as a prediction for measurements at RHIC.

We run 3000 hydro events in each 10% centrality bin. Anisotropic flow is calculated accurately in every event [26]. The ratio r_n is displayed in Fig. 1 for $n = 2$ and $n = 3$. Deviations from the factorization limit $r = 1$ are already seen at low momentum but become larger as the difference between p_T^a and p_T^b increases, as expected from the general arguments above. Surprisingly, the breaking of factorization appears *larger* in hydrodynamics than in experiment.

The ALICE collaboration has studied factorization by performing a global fit of the measured correlation $V_{n\Delta}(p_T^a, p_T^b)$ by the right-hand side of Eq. (1), where $v_n(p_T)$ is a fit parameter [9]. The ratio of the measured correlation to the best fit differs from unity if factorization is broken. We can apply the same procedure to our hydrodynamic results. The result is shown in Fig. 2. Again, hydrodynamic calculations and experimental data show similar trends, with the noticeable difference that

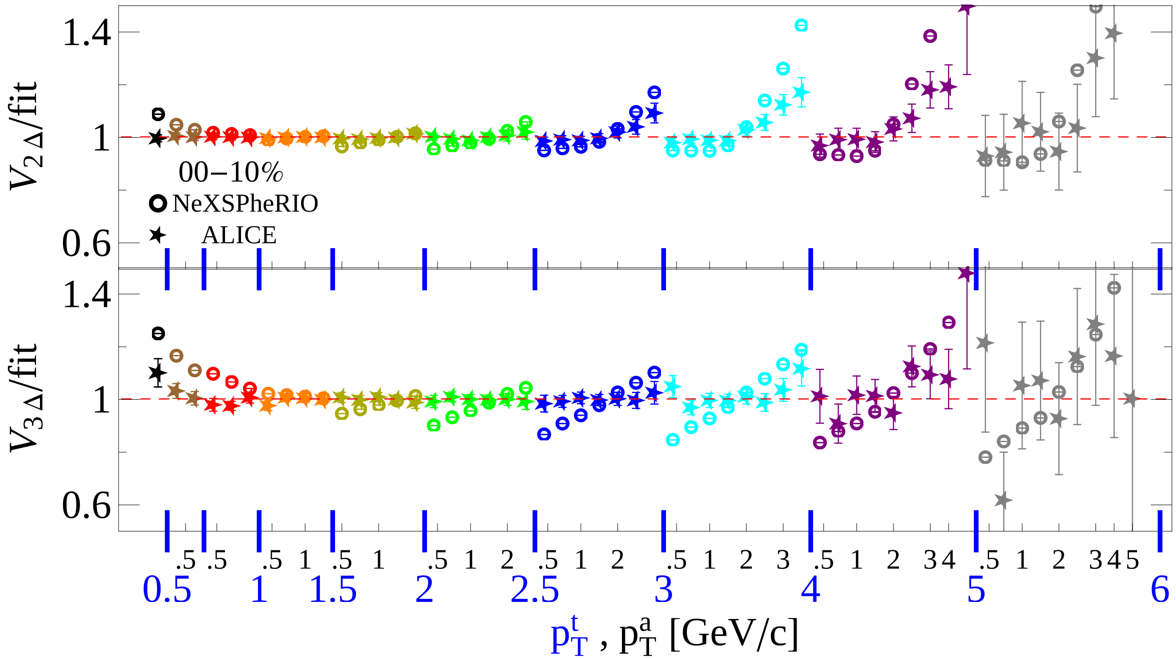

FIG. 2. (Color online) Ratio of the left-hand side to the right-hand side of Eq. (1). Filled stars: ALICE data for 0-10% central Pb-Pb collisions at 2.76 TeV [9]. Open cicles: ideal hydrodynamic calculations for 0-10% central Au-Au collisions at 200 GeV.

the breaking of factorization is significantly *stronger* in ideal hydrodynamics than in data.

Several effects can explain this discrepancy. First, the average p_T is significantly larger at LHC than at RHIC [27], so that it might be more natural to compare, e.g., 4 GeV at RHIC to 5 GeV at LHC, rather than doing the comparison at the same p_T. The second effect is viscosity, which is neglected in our calculation. Shear viscosity, in particular, tends to damp the effect of initial fluctuations [28]. It is therefore natural that it will also decrease the breaking of factorization induced by initial fluctuations. A similar observation is that the linear correlation between the initial eccentricity and the final anisotropic flow is stronger in viscous hydrodynamics [29] than in ideal hydrodynamics [26].

IV. CONCLUSIONS

We have demonstrated that the detailed structure of two-particle angular correlations contains much more information than traditional analyses of anisotropic flow, where the correlation is averaged over one of the particles [30]. Even though such two-dimensional analyses are much more demanding in terms of statistics than traditional analyses, they bring new, independent insight into the underlying physics.

In particular, we have shown that quantum fluctuations in the wavefunction of incoming nuclei result in a decoherence in the angular correlations produced by collective flow, which becomes increasingly important as the difference between particle momenta increases. Due to this effect, factorization of angular correlations is broken even if collective flow is the only source of correlations. Our numerical calculations show that factorization breaking is at least as strong in ideal hydrodynamics as in experimental data, thereby suggesting that all correlations below $p_T \sim 5$ GeV (for central Pb-Pb collisions at LHC near midrapidity) are actually dominated by flow. The sensitivity of this decoherence phenomenon to viscosity has not yet been investigated, but we anticipate that factorization should be restored as viscosity increases, thus potentially offering a new means of constraining the viscosity from data.

Decoherence also provides a natural explanation for the important observation that event-by-event fluctuations reduce elliptic flow at high p_T [31], thus improving agreement between hydrodynamics and experimental data. Indeed, v_2 at high p_T is inferred from azimuthal correlations between a high p_T particle and all other particles — mostly low p_T particles, and these azimuthal correlations are reduced due to the decoherence phenomenon. Note that the other main explanation for the reduction of v_2 at high p_T, viscosity, typically relies on the assumption of a quadratic momentum dependence of the viscous correction to the distribution function at freeze-out δ_f, which may not be correct [32].

In this paper, we have focused on the transverse momentum dependence of the correlations. The rapidity dependence of the correlation is also worth investigating. In particular, it was recently observed that azimuthal correlations decrease as a function of the relative pseudorapidity [33], at variance with common lore that correlations due to flow are essentially independent of rapidity. While standard models of initial conditions do predict a mild rapidity dependence of azimuthal correlations [34], longitudinal fluctuations [35] could also produce a decoherence effect similar to the one studied here. The detailed structure of two-particle correlations as a function of both particle momenta thus opens a new window on the study of flow fluctuations.

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