Determining initial-state fluctuations from flow measurements in heavy-ion collisions

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We present a number of independent flow observables that can be measured using multiparticle azimuthal correlations in heavy-ion collisions. Some of these observables are already well known, such as $v_2\{2\}$ and $v_2\{4\}$, but most are new—in particular, joint correlations between v_1, v_2 and v_3. Taken together, these measurements will allow for a more precise determination of the medium properties than is currently possible. In particular, by taking ratios of these observables, we construct quantities which are insensitive to the hydrodynamic response of the medium, and thus directly probe the early-time, non-equilibrium QCD dynamics. We present predictions for these ratios using two Monte-Carlo models, and compare to available data.

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INTRODUCTION

Thermalization of the matter produced in ultrarelativistic nucleus-nucleus collisions results in strong collective motion. The clearest experimental signature of collective motion is obtained from azimuthal correlations between outgoing particles. It has been recently realized [1] that fluctuations [2] due to the internal structure of colliding nuclei, followed by collective flow, naturally generate specific patterns which are observed in these azimuthal correlations. In this Letter, we propose a number of independent flow measurements and study the possibility to constrain models of initial-state fluctuations directly from these experimental data.

FLOW OBSERVABLES

Correlations between particles emitted in relativistic heavy-ion collisions at large relative pseudorapidity $\Delta\eta$ are now understood as coming from collective flow [3]. According to this picture, particles in a given event are emitted independently according to some azimuthal distribution. The most general distribution can be written as a sum of Fourier components,

$$\frac{dN}{d\varphi} = \frac{N}{2\pi}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos(n\varphi - n\Psi_n)\right), \qquad (1)$$

where v_n is the n^{th} flow harmonic [4] and Ψ_n the corresponding reference angle, all of which fluctuate event-by-event.

The largest flow harmonic is elliptic flow, v_2 [5], which has been extensively studied at SPS [6], RHIC [7–9], and LHC [10]. Next is triangular flow, v_3 [1], which together with v_2 is responsible for the ridge and shoulder structures observed in two-particle correlations [11, 12]. In addition, quadrangular flow, v_4, has been measured in correlation with elliptic flow [13, 14]. Finally, directed flow, v_1, can be uniquely separated [15] into a rapidity-odd part, which is the traditional directed flow [6, 13, 16], and a rapidity-even part created by initial fluctuations [17]. In this work, we are concerned with experimental observables that can be constructed from phase-space-integrated, rapidity-even parts of v_1, v_2 and v_3, which will allow for a study of the global event shape and early-time dynamics. The study of v_4 is more complicated due to the large interference with v_2 [18], and is left for future work.

In practice, one cannot exactly reconstruct the underlying probability distribution from the finite sample of particles emitted in a given event. All known information about v_n is inferred from azimuthal correlations. Generally, a k-particle correlation is of the type

$$v\{n_1, n_2, \ldots, n_k\} = \langle\cos(n_1\varphi_1 + \ldots + n_k\varphi_k)\rangle, \qquad (2)$$

where $n_1, \ldots, n_k$ are integers, $\varphi_1, \ldots, \varphi_k$ are azimuthal angles of particles belonging to the same event, and angular brackets denote average over multiplets of particles and events in a centrality class. Since the impact parameter orientation is uncontrolled, the only measurable correlations have azimuthal symmetry: $n_1 + \ldots + n_k = 0$.

Inserting Eq. (1) into Eq. (2) gives

$$v\{n_1, \ldots, n_k\} = \langle v_{n_1} \ldots v_{n_k} \cos(n_1\Psi_{n_1} + \ldots + n_k\Psi_{n_k})\rangle, \qquad (3)$$

where the average is now only over events. To the extent that correlations are induced by collective flow, azimuthal correlations measure moments of the flow distribution.

The simplest v_n measurement is the pair correlation [19], which corresponds to the event-averaged root-mean-square v_n

$$v_n\{2\} \equiv \sqrt{v\{n, -n\}} \simeq \sqrt{\langle {v_n}^2\rangle}. \qquad (4)$$

Higher-order correlations yield higher moments of the v_n distribution:

$$v\{n, n, -n, -n\} \equiv 2v_n\{2\}^4 - v_n\{4\}^4 \simeq \langle {v_n}^4\rangle, \qquad (5)$$

where we have used the standard notation $v_n\{4\}$ for the 4-particle cumulant [20]. Only the n=2 harmonics $v_2\{2\}$ and $v_2\{4\}$ have been previously analyzed [6, 10, 21].

Finally, one can construct correlations involving mixed harmonics, as in previous analyses of v_4 [13] and v_1 [22]. The first non-trivial correlations between v_1, v_2 and v_3 are

$$\begin{aligned} v_{12} &\equiv v\{1,1,-2\}, & v_{13} &\equiv v\{1,1,1,-3\}, \\ v_{23} &\equiv v\{2,2,2,-3,-3\}, & v_{123} &\equiv v\{1,2,-3\}. \end{aligned} \tag{6}$$

One generally expects $v_1 < v_3 < v_2$. Thus correlations involving high powers of v_1 are more difficult to measure. However, we will see that all of these correlations are likely to be measurable at the LHC.

PREDICTIONS

The anisotropy in the distribution (1) has its origin in the anisotropy of the transverse density distribution at early times. Following Teaney and Yan [17], we define

$$\begin{aligned} \varepsilon_1 e^{i\Phi_1} &\equiv -\frac{\{r^3 e^{i\varphi}\}}{\{r^3\}} \\ \varepsilon_2 e^{2i\Phi_2} &\equiv -\frac{\{r^2 e^{2i\varphi}\}}{\{r^2\}} \\ \varepsilon_3 e^{3i\Phi_3} &\equiv -\frac{\{r^3 e^{3i\varphi}\}}{\{r^3\}}, \end{aligned} \tag{7}$$

where curly brackets denote an average over the transverse plane in a single event [23], weighted by the density at midrapidity, and the distribution is centered in each event, $\{re^{i\varphi}\} = 0$. In this equation, Φ_n is the minor orientation angle (corresponding, e.g., to the minor axis of the ellipse for n=2), and ε_n the magnitude of the respective anisotropy. Anisotropic flow scales like the initial anisotropy ε_n and develops along Φ_n [5]. It is therefore natural to assume $v_n = K_n \varepsilon_n$ and $\Psi_n = \Phi_n$, where K_n is the hydrodynamic response to the initial anisotropy in harmonic n. These relations are not exact, but event-by-event hydrodynamic calculations have shown that they hold approximately for v_1 [15], v_2 [24] and v_3 [25]. On the other hand, they are not valid for higher harmonics such as v_4 and v_5 [26]. Inserting these proportionality relations into Eq. (3), we obtain

$$v\{n_1, \ldots, n_k\} = K_{n_1} \ldots K_{n_k} \varepsilon\{n_1, \ldots, n_k\}, \tag{8}$$

where we have introduced the notation

$$\varepsilon\{n_1, \ldots, n_k\} \equiv \langle \varepsilon_{n_1} \ldots \varepsilon_{n_k} \cos(n_1\Phi_{n_1} + \ldots + n_k\Phi_{n_k}) \rangle \, . \tag{9}$$

Thus the measured correlations are sensitive to details of the hydrodynamic evolution mostly through the coefficients K_n, and to the initial-state dynamics through $\varepsilon\{n_1, \ldots, n_k\}$.

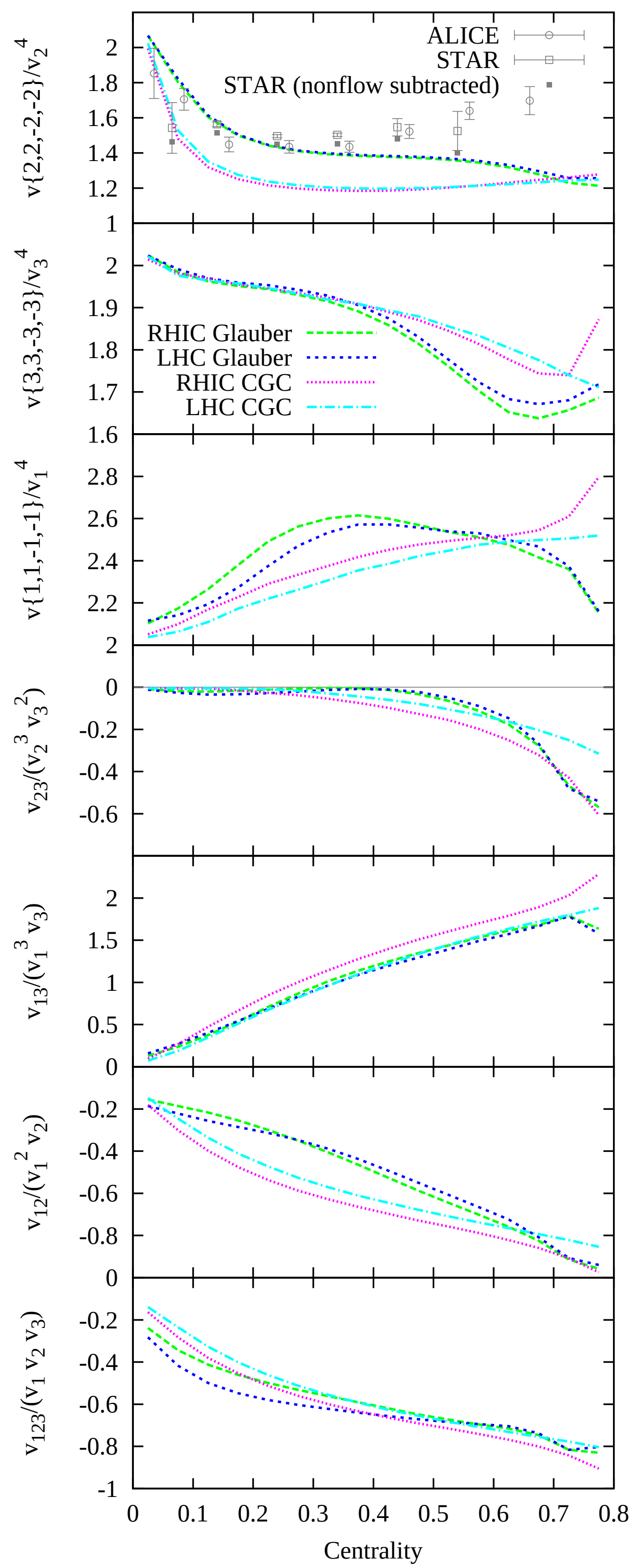

FIG. 1. (Color online) Predictions for ratios of various proposed measurements as a function of centrality (fraction of the total cross section) in Au-Au collisions at RHIC and Pb-Pb collisions at LHC, using a Glauber- and a CGC-type model with 100 million and 20 million events, respectively. The factors in the denominator are shorthand, $v_n \equiv v_n\{2\}$. See text for details.

One can eliminate the dependence on the unknown proportionality coefficients K_n and isolate initial-state effects by taking correlations (2), integrated over phase space, and scaling appropriately:

$$\frac{v\{n_1, n_2, \ldots, n_k\}}{v_{n_1}\{2\} \ldots v_{n_k}\{2\}} = \frac{\varepsilon\{n_1, n_2, \ldots, n_k\}}{\varepsilon_{n_1}\{2\} \ldots \varepsilon_{n_k}\{2\}}, \tag{10}$$

where $\varepsilon_n\{2\} \equiv \sqrt{\langle {\varepsilon_n}^2 \rangle}$. The left-hand side of Eq. (10) can be measured experimentally, while the right-hand side depends only on early-time dynamics, and can be calculated using a model of initial-state fluctuations.

In order to probe the sensitivity of these ratios to the model of initial conditions, we compute them using two models. First is the PHOBOS Glauber Monte-Carlo [27], with binary collision fraction $x = 0.145$ for RHIC collisions and $x = 0.18$ for LHC. The second uses the gluon density from a color-glass-condensate (CGC) inspired model — the MC-KLN [28], with rcBK unintegrated gluon densities [29]. The main difference between the two models is that the eccentricity is larger in the CGC model [30, 31]. The only source of fluctuations in both is the nucleonic structure of nuclei, with differences in the technical implementation. In reality there could be more sources of fluctuations.

Fig. 1 displays predictions for all of the scaled correlations in Au-Au collisions at 200 GeV per nucleon pair and Pb-Pb collisions at 2.76 TeV per nucleon pair.

The top three panels show $v\{n, n, -n, -n\}/v_n\{2\}^4 = 2 - v_n\{4\}^4/v_n\{2\}^4 = \langle {v_n}^4 \rangle / \langle {v_n}^2 \rangle^2$. For Gaussian fluctuations [32], the n=1 and 3 ratios are equal to 2 (i.e., $v_n\{4\}$=0), and likewise for n=2 in central collisions. However, this is expected only in the limit of a large system. A more detailed analysis [33] shows that, e.g., $v_3\{4\}$ should be smaller than $v_3\{2\}$ only by a factor $\sim$2 in mid-central collisions, in agreement with these results. Note that wherever the ratio is greater than 2, $v_n\{4\}$ is undefined. The top panel also shows existing data from STAR [34] and ALICE [10]. Neither measurement includes a rapidity gap, and thus may contain nonflow correlations (see discussion below). For STAR $v_2\{2\}$, we use both the raw data, and the value with an estimated correction for nonflow effects [35]. The data seem to favor the larger relative fluctuations of the Glauber model.

The bottom four panels display scaled mixed correlations, indicating non-trivial correlations between Ψ_1, Ψ_2 and Ψ_3. The scaled correlation v_{23} indicates a negligible correlation between Ψ_2 and Ψ_3 for most centralities in the Glauber model [1, 36], while the CGC model predicts a small anticorrelation. In contrast, Ψ_1 has both a strong correlation with Ψ_3 [37] (positive v_{13}) and a (weaker) anticorrelation with Ψ_2 [17] (negative v_{12}), though this decreases for central collisions. The dependence on impact parameter can be attributed to the intrinsic eccentricity of the nuclear overlap region [33]. The strong positive correlation between Ψ_1 and Ψ_3 explains why v_{12} and v_{123} [17] have the same sign and behave similarly.

ANALYSIS METHOD: WEIGHTS, RAPIDITY GAPS, AND STATISTICS

In practice, the average over particles in Eq. (2) is a weighted average, where each particle is given a weight: in a given harmonic n, one gives more weight to particles which have larger v_n in order to increase the resolution. Our goal here is to characterize initial-state fluctuations, which are approximately independent of rapidity [38]. Weights should therefore be chosen independent of (pseudo)rapidity (a nonstandard choice for odd harmonics [39]). The dependence on transverse momentum p_t should ideally mimic that of the flow coefficients themselves [40]. Simple choices are $w = p_t$ in harmonics 2 and 3, and $w = p_t - \langle p_t^2 \rangle / \langle p_t \rangle$ in harmonic 1 [41].

One must analyze the various correlations in such a way as to isolate the correlation induced by collective flow from other "nonflow" effects. Nonflow correlations are negligible at large relative pseudorapidity $\Delta\eta$ [3]; when analyzing $v\{n, -n\}$, they are easily suppressed by putting a rapidity gap between each pair, while $v\{n, n, -n, -n\}$ can then be obtained from the cumulant $v_n\{4\}$, which is insensitive to nonflow effects by construction. The correlations we have introduced in Eq. (6) involve between 3 and 5 particles, and it is not realistic to enforce a rapidity gap between all the particles. We therefore proceed to evaluate the order of magnitude of nonflow effects, in order to determine where rapidity gaps are important.

Nonflow effects are correlations between a small number of particles—typically pairs of particles [42]. One can estimate their order of magnitude by assuming that particles are emitted in collinear pairs. If there are $M \gg 1$ particles in each event, the probability that two random particles belong to the same pair is $1/(M-1) \simeq 1/M$.

Consider the first correlation in Eq. (6), v_{12}, which involves three particles. There are different nonflow contributions to this correlation corresponding to the different pairings. Pairing the first two particles (with harmonic 1) gives a nonflow correlation of order $(v_2)^2/M$, while pairing 1 or 2 with 3 gives a correlation of order $(v_1)^2/M$. Since $v_2 \gg v_1$, it is important to put a rapidity gap between the first two particles. On the other hand, there is no restriction for the third particle. For v_{13}, a similar discussion shows that there must be rapidity gaps between the first three particles (again those with harmonic 1).

For v_{23}, the largest nonflow correlation is between the two last particles (with harmonic 3), which gives a correction on the ratio (10) of order $v_6/(M v_3^2)$, which is small. Rapidity gaps are *not* needed for v_{23}.

Finally, for v_{123}, the largest nonflow correlation is between the first and the third particle (harmonics 1 and 3) and is of order v_2^2/M. The next-to-largest is between the first two particles, of order v_3^2/M, which is much smaller, except for central collisions.

The limiting factor in the ability to measure these high-order correlations is statistics. Assuming that multiplets are weakly correlated, the statistical error on $v\{n_1,\ldots,n_k\}$ in Eq. (2) is $1/\sqrt{2N}$, where N is the total number of multiplets. For $N_{\rm evts}$ events and M analyzed particles per event, $N\sim M^k N_{\rm evts}$. Thus the statistical error on the ratio in Eq. (10) is of order $N_{\rm evts}^{-1/2}/(\chi_{n_1}\ldots\chi_{n_k})$, where $\chi_n\equiv v_n\sqrt{M}$ is the resolution parameter [43] in harmonic n, and v_n here denotes a root-mean-square value over all particles. Since $v_1<v_3<v_2$, the limiting factor is the number of particles in harmonic 1, and to a lesser extent, the number of particles in harmonic 3. Any experiment able to analyze $v_3\{2\}$ should also be able to measure v_{23}. Similarly, any experiment able to analyze $v_1\{2\}$ can analyze v_{123} and v_{12}. $v_3\{4\}$ should be roughly as demanding. The most demanding measurements are v_{13} and $v_1\{4\}$, which will likely require a large pseudorapidity coverage, and may not be possible for RHIC detectors.

CONCLUSION

We have proposed a new set of independent flow observables in heavy-ion collisions which can be combined to tightly constrain theoretical models. In particular, certain ratios are constructed which are largely determined only by the initial state, and thus directly measure properties of the early-time system. We have presented predictions for these ratios using two common Monte-Carlo models, and compared to existing data.

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