

v_4 in ideal and viscous hydrodynamics for RHIC and LHC

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We compute $v_4/(v_2)^2$ in ideal and viscous hydrodynamics. We investigate its sensitivity to details of the hydrodynamic model and compare the results to experimental data from RHIC. Whereas v_2 has a significant sensitivity only to initial eccentricity and viscosity while being insensitive to freeze out temperature, we find that $v_4/(v_2)^2$ is quite insensitive to initial eccentricity. On the other hand, it can still be sensitive to shear viscosity in addition to freeze out temperature, although viscous effects do not universally increase $v_4/(v_2)^2$ as originally predicted. Consistent with data, we find no dependence on particle species. We also make a prediction for $v_4/(v_2)^2$ in heavy ion collisions at the LHC.

I. INTRODUCTION

Much can be learned about the bulk properties of the medium created in relativistic heavy ion collisions by studying the azimuthal distribution of emitted particles. Typically, the dependence on the azimuthal angle ϕ with respect to the collision plane is written as a Fourier series

$$E \frac{d^3 N}{d^3 \mathbf{p}} = v_0 \left[1 + \sum_{n=1}^{\infty} 2v_n \cos(n\phi) \right]. \quad (1)$$

In a collision between identical nuclei at midrapidity, the sine terms and all odd terms are negligible when averaged over many events. There has been much study of the elliptic flow coefficient, v_2 [1], which has been an important observable for determining, e.g., the viscosity of the medium [2–6]. Less studied, however is the next harmonic v_4 [7, 8]. In this article we use viscous and ideal hydrodynamic simulations to determine what v_4 can tell us about the heavy ion collision fireball created at the Relativistic Heavy Ion Collider (RHIC), and to make predictions for planned heavy ion collisions at the Large Hadron Collider (LHC).

For details of the hydrodynamic model used, the reader is directed to Ref. [2]. As a brief summary, we solve 2+1 dimensional conformal viscous hydrodynamic equations (with zero bulk viscosity) using both Glauber- and CGC-type initial conditions and a Cooper-Frye freeze out routine followed by resonance feed down. All calculations use the same parameters as those that gave the best fit to RHIC data [2] (see Table I), except where specified.

II. REVIEW

First recall the simplest prediction for v_4 [9] that predicts $v_4 = 0.5(v_2)^2$ for a boosted thermal freeze out distribution. To summarize, in the low temperature limit, one can solve the zero-viscosity Cooper-Frye freeze out formula with Boltzmann statistics

$$E \frac{d^3 N}{d^3 \mathbf{p}} \propto \int p_\mu d\Sigma^\mu \exp\left(-\frac{p_\mu u^\mu}{T}\right) \quad (2)$$

Init. cond.	η/s	T_i [GeV]	T_f [GeV]	τ_0 [fm/c]
Glauber	10^{-4}	0.340	0.14	1
Glauber	0.08	0.333	0.14	1
Glauber	0.16	0.327	0.14	1
CGC	10^{-4}	0.310	0.14	1
CGC	0.16	0.299	0.14	1

TABLE I: Parameters used for the viscous hydrodynamics simulations except as indicated in each relevant section. The bold faced lines give the best fit to v_2 , $\langle p_t \rangle$, and multiplicity [2].

by performing a saddle point approximation. At large transverse momentum (i.e., where the minimum of $p \cdot u > m$), the dominant part of the integral comes from the part of the freeze out surface where the fluid velocity is parallel to the momentum of the emitted particle, and is at a maximum. We can decompose this maximum fluid 4-velocity magnitude u_{\max} at each angle ϕ in a Fourier series

$$u_{\max}(\phi) = U(1 + 2V_2 \cos(2\phi) + 2V_4 \cos(4\phi) + \dots). \quad (3)$$

Plugging this into Eq. (2) in a saddle point approximation and expanding to leading order in V_2 and V_4 , one obtains

$$v_2(p_t) = \frac{V_2 U}{T} (p_t - m_t v) \quad (4)$$

$$\begin{aligned} v_4(p_t) &= \frac{1}{2} \frac{(V_2 U)^2}{T^2} (p_t - m_t v)^2 + \frac{V_4 U}{T} (p_t - m_t v) \\ &= \frac{1}{2} v_2(p_t)^2 + \frac{V_4}{V_2} v_2(p_t), \end{aligned} \quad (5)$$

with $m_t = \sqrt{p_t^2 + m^2}$ and $v \equiv U/\sqrt{1+U^2}$. At a large enough p_t , the first term in Eq. (5) dominates, and it is in this sense that it can be said that v_4 is largely generated by an intrinsic elliptic flow (i.e., V_2) rather than a fourth order moment of the fluid flow (V_4). As one relaxes the large p_t limit, the ratio of v_4 to $(v_2)^2$ should have corrections from 1/2 that depend on the particular collision, but go like $1/p_t$ (or equivalently $1/v_2$).

This derivation is the motivation for studying the quantity $v_4/(v_2)^2$ rather than v_4 alone, and is the quantity to be studied here.

Equation (5) is quite general and does not depend on the details of the freeze out surface or the fluid flow that produces it. It should be noted, however, that several assumptions must be made in order to derive it: small temperature, large transverse momentum, small v_2 and v_4 , and zero viscosity. As these restrictions are relaxed, the behavior becomes more complicated and more sensitive to details. Therefore, in order to fully understand the final results of the most realistic simulations as well as to make contact with previous results from ideal hydrodynamics and transport simulations, it will be useful to gain insight by starting with the simple case of vanishing viscosity and low freeze out temperature, and then adding viscosity and a realistic freeze out temperature once the generic features of the simpler case are understood.

III. REVIEW OF RHIC DATA

v_4 has been measured at RHIC by both the STAR [11–16] and PHENIX [17–20] collaborations. They both find a significant deviation of $v_4/(v_2)^2$ from a constant 1/2. Rather, the reported value of the ratio is close to a constant value of 1 for a large range of transverse momentum ($p_t \gtrsim 500$ MeV) [20], centrality (~ 10 – 70%) [16, 20], and for all identified particles that have been measured (pions, kaons, and protons [18, 19]). It is larger for more central events, while there is no clear consensus for small p_t and very peripheral events. The result from PHENIX tends to be somewhat smaller than from STAR, likely indicating different sensitivities to non-flow effects, although all results are still well above 0.5.

Much of this discrepancy between prediction and experiment can be understood by noting that both v_4 and v_2 are averaged over many events before the ratio is taken. Even if the ratio $v_4/(v_2)^2$ were exactly 1/2 in each event, any fluctuation event-by-event (both at a given centrality and within a centrality bin) will tend to increase the measured $v_4/(v_2)^2$. In fact, using a Glauber Monte Carlo model of eccentricity fluctuations can explain most of the difference, as demonstrated in Ref. [10]. With this understanding, all calculations here—since they lack fluctuations—should be understood as relating to measured data by an overall (momentum independent) multiplicative factor that depends on the centrality bin chosen.

IV. RHIC RESULTS

We begin by noting that, since v_4 tends to be much smaller than v_2 , numerical errors can be of greater concern. We estimated this uncertainty in two ways. First, we looked at the results at $b = 0$, where v_4 should be zero theoretically. However, the use of a square lattice as well

as the presence of numerical/rounding errors results in a calculated v_4 that can be as much as $\sim 5\%$ of the value calculated at, e.g., $b = 6$ fm (or a much smaller fraction of the value for more peripheral collisions.) Second, we reran a number of simulations with half the usual grid spacing (0.2 fm instead of 0.4 fm). Similarly, we found that $v_4/(v_2)^2$ changed by less than 5%–10% as long as we only considered collisions for $b > 5$ – 6 fm. Therefore, we only report the results for $b = 6$ fm and larger. Reliable results outside this range can be obtained with more computer time or a better hydrodynamics algorithm (as claimed in Ref. [21]).

In addition to numerical uncertainties, there are several other concerns to keep in mind. Although the results are numerically stable above $p_t \sim 0.2$ – 0.3 GeV, they are moderately sensitive to details (e.g., small changes in parameters or the neglect of resonance feed down) up to ~ 0.5 – 1 GeV, although they seem quite robust at larger p_t . Of course, one must also note that in reality the hydrodynamic description should break down at some large value of p_t and therefore one has less and less confidence in the results as p_t increases. However, it is not clear *a priori* where exactly this break down should occur, and so it is still useful to calculate quantities even at large p_t . Only comparison to data can then give information about this break down of the hydrodynamic description.

None of these concerns should change the trends and conclusions reported here.

Figure 1 shows the results for pions with $\eta/s = 0.0001$ (corresponding to ideal hydrodynamics), and all parameters as specified in Table I except a small freeze out temperature of 100 MeV. As found previously in ideal hydrodynamic simulations [9, 16], $v_4/(v_2)^2$ approaches a constant value of roughly 1/2 at asymptotically large values of p_t , and increases as $1/p_t$ for decreasing p_t . In contrast to expectations and data, however, there is a definite dependence on impact parameter. Part—but not all—of this dependence as well as the deviation from 1/2 can be understood by the fact that here v_2 is unphysically large, as we have turned off viscous effects. For example, with CGC initial conditions at $b = 11$ fm and $p_t = 3.5$ GeV, v_2 reaches 70%. $v_4/(v_2)^2$ would then be expected to reach almost 0.62 instead of 0.50 at large p_t if one assumes V_4 of the fluid to be negligible (see Ref. [10]).

Figure 1 also makes it apparent that, unlike v_2 itself, the results for $v_4/(v_2)^2$ are quite insensitive to the initial eccentricity. Although there is a dependence on impact parameter, it is small, and there is little difference for Glauber and CGC initial conditions, despite having a very different initial eccentricity (and therefore very different v_2 for the same viscosity—see, e.g., Fig. 4 and 8 of Ref. [2]). Indeed, CGC initial conditions, with a larger initial eccentricity, lead to a smaller value than Glauber while more peripheral collisions have a larger value than more central collisions, despite also having a larger eccentricity.

The effect of raising the freeze out temperature to a more realistic value can be seen in Fig. 2. The choice

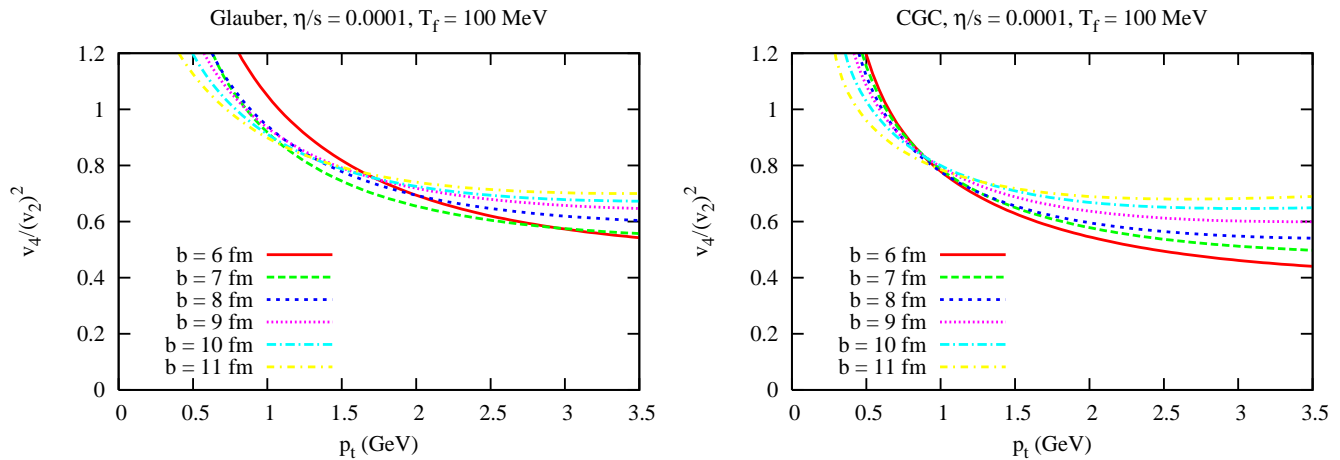


FIG. 1: (Color online) Pion $v_4/(v_2)^2$ from ideal hydrodynamics calculations with Glauber and CGC initial conditions and the freeze out temperature lowered to 100 MeV

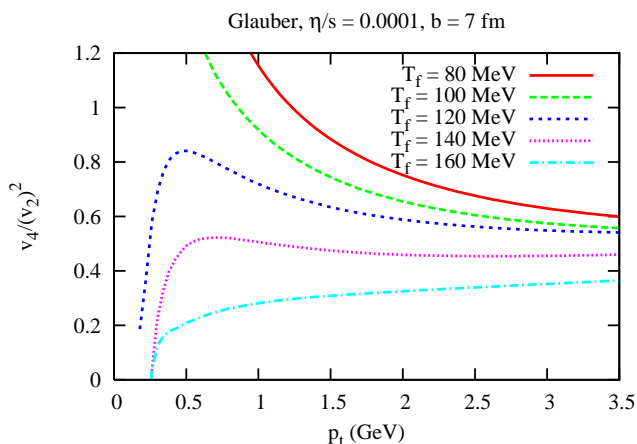


FIG. 2: (Color online) Pion $v_4/(v_2)^2$ for ideal hydrodynamics calculations with varying freeze out temperatures.

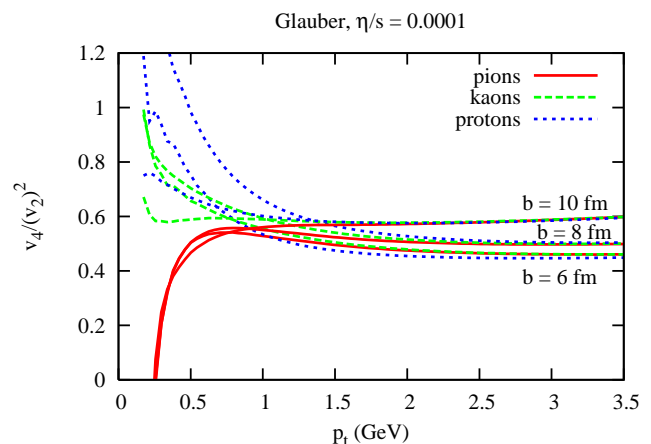


FIG. 3: (Color online) $v_4/(v_2)^2$ as a function of p_t for identified particles from ideal hydrodynamics with Glauber initial conditions and 140 MeV freeze out temperature.

of freeze out temperature makes a significant difference to the shape of the $v_4/(v_2)^2$ curve. Going to the best-fit freeze out temperature of 140 MeV results in a very flat curve as a function of p_t (reminiscent of the data), which is not the case when T_f is significantly larger or smaller. The sensitivity to T_f is perhaps not a complete surprise, since it is known from transport calculations that adjusting the treatment of freeze out does affect v_4 [23].

Figure 3 shows the results from ideal hydrodynamics for identified particles when the freeze out temperature is raised to 140 MeV. Data show approximately the same value for identified particles in the measured momentum range [18, 19]. This is confirmed in our calculation for $p_t \gtrsim 1-1.5$ GeV, below which it begins to depend on the parameters of the simulation as well as increasing numerical error (especially for the heavier particles for which $v_2 \sim 0$).

Finally, Fig. 4 shows the results of viscous hydrody-

amic simulations for several impact parameters, with ideal hydrodynamics results included for comparison. Also shown is experimental data from STAR [16] and PHENIX [20]. Since eccentricity fluctuations increase $v_4/(v_2)^2$ by a factor $K \equiv \langle \epsilon^4 \rangle / \langle \epsilon^2 \rangle^2$ [10], we have estimated K in each centrality bin using the PHOBOS Glauber Monte-Carlo [24], and scaled down the data by K . This scaling removes most of the (already small) centrality dependence for both STAR and PHENIX in this centrality range: the residual centrality dependence is much smaller than in our ideal hydrodynamics results. The larger values of $v_4/(v_2)^2$ for STAR as compared to PHENIX can most likely be attributed to larger nonflow effects [10].

It turns out that different choices for the form of the viscous correction δf to the distribution function at freeze out can significantly change the shape of these

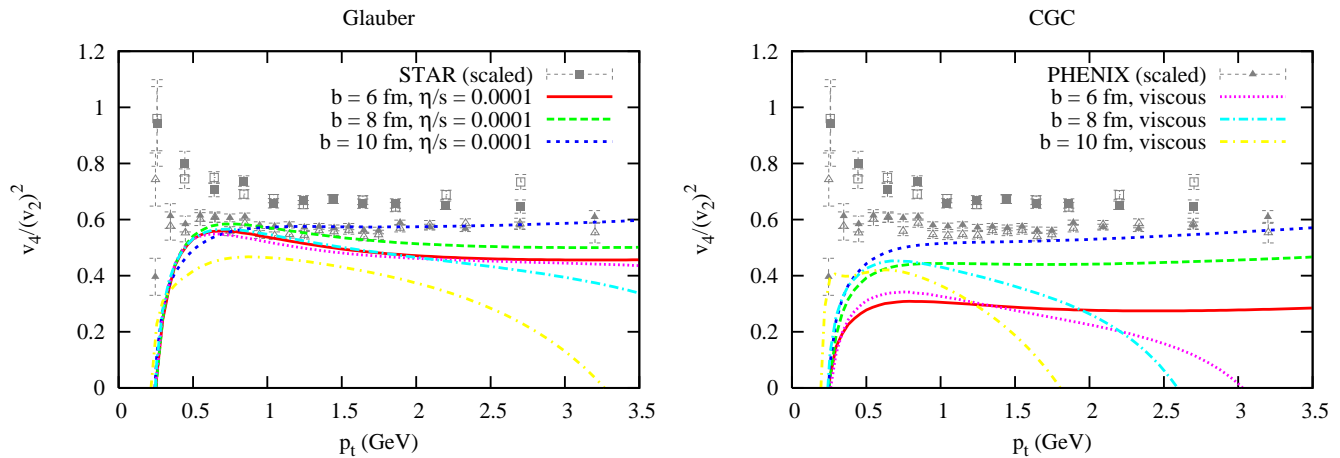


FIG. 4: (Color online) Charged hadron $v_4/(v_2)^2$ as a function of p_t from ideal and viscous hydrodynamics with Glauber and CGC initial conditions. Experimental results have been scaled down to account for the estimated effect of fluctuations [10]. Filled and open squares are for charged hadrons at 10-20% (with scale factor $K = 1.56$) and 40-50% ($K = 1.38$) centrality, respectively, from STAR [16]. Filled and open triangles are for charged hadrons at 15-20% ($K = 1.47$) and 45-50% ($K = 1.38$) centrality, respectively, from PHENIX [20]. Viscous results correspond to the bold lines in Table I.

curves (see Ref. [22]). Several aspects seem to be universal for reasonable choices of δf , however. In contrast to expectations [25] as well as the results at low freeze out temperature (not shown), viscosity does not universally increase the ratio of $v_4/(v_2)^2$. In fact, in most cases at this realistic freeze out temperature, the value was found to decrease compared to ideal hydrodynamics. Rather than making the agreement with data better, viscosity tends to make it worse. In particular, the correction induced by viscous effects depends strongly on centrality: viscous effects are larger for the smaller systems created in peripheral collisions. It should be noted that choosing a δf with a weaker momentum dependence than the standard quadratic ansatz (used in Fig. 4 as well as all previous viscous hydrodynamics results) can flatten these curves as well as reduce the strong impact parameter dependence (even potentially reducing it beyond that of ideal hydrodynamics), and this could in principle provide information about the dynamics of the hadron resonance gas at freeze out [22].

V. LHC PREDICTIONS

Using what we have learned from RHIC, it is then possible to predict what will be seen in heavy ion collisions at the LHC. Using the same values for τ_0 , T_f , and η/s that led to the best fit of RHIC data [2], T_i was adjusted to reproduce the expected multiplicity at top-energy Pb-Pb collisions at the LHC, as were the appropriate nuclear parameters (the same as for the LHC predictions in Ref. [26], to which we refer the reader for details. For reference T_i is 420 MeV and 390 MeV for Glauber and CGC initial conditions, respectively.)

The results are shown in Fig. 5. LHC results for ideal

hydrodynamics are similar to RHIC results with a lower freeze out temperature (see Fig. 2). Specifically, there is an increase at small p_t and viscous corrections can be positive as well as negative. However, here the viscous corrections are much smaller than for realistic RHIC simulations. This means that—unlike for RHIC—the choice for the viscous correction to the non-equilibrium distribution function (δf) is relatively unimportant, and a comparatively robust prediction can be made, with just a correction factor expected from fluctuations put in by hand. For example, a measurement of $v_4/(v_2)^2$ in a centrality bin of 20–60% is expected to be larger by a factor of ~ 1.34 . Although there is some dependence on initial conditions and impact parameter, the results are very similar to those obtained for RHIC, so it is reasonable to make the prediction that $v_4/(v_2)^2$ measured at the LHC will be similar to that measured at RHIC, except for a small increase with decreasing p_t .

VI. CONCLUSION

In summary, we calculated $v_4/(v_2)^2$ in ideal hydrodynamics as well as the first calculation using viscous hydrodynamic simulations of heavy ion collisions at RHIC as well as the LHC. We confirm previous results from low T_f ideal hydrodynamics and transport calculations. This observable is found to be insensitive to initial eccentricity, but to depend strongly on the freeze-out temperature T_f . In ideal hydrodynamics, the dependence on p_t is found to be flat—as in RHIC data—only if T_f is near 140 MeV, which is precisely the value providing the best fit to p_t spectra and v_2 . In addition, $v_4/(v_2)^2$ was found to be the same for pions, kaons, and protons—also in accord with data. There is an unexpected dependence on impact

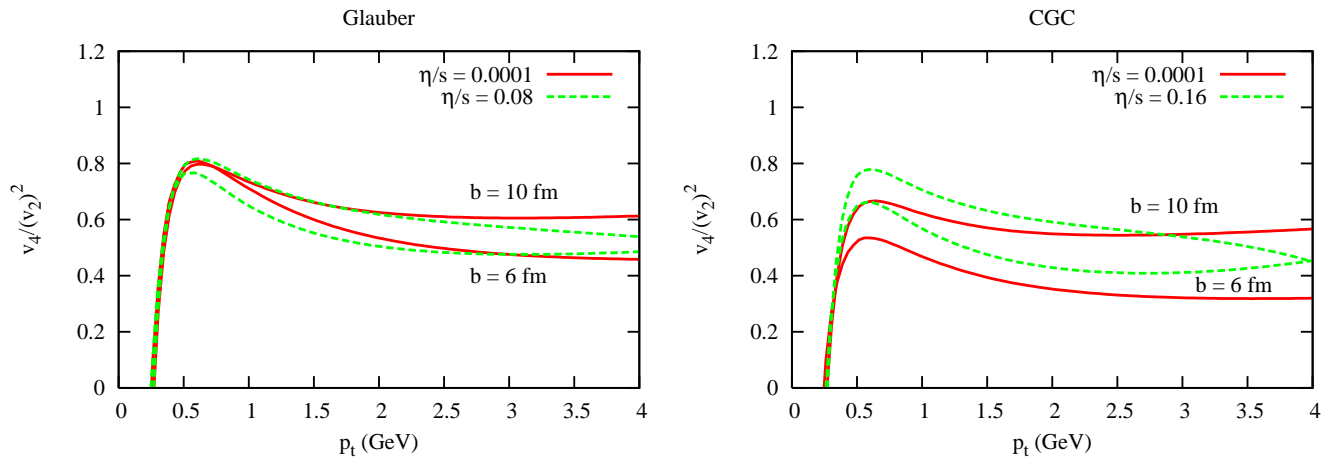


FIG. 5: (Color online) Charged hadron $v_4/(v_2)^2$ for 5.5 TeV Pb-Pb collisions at the LHC. The same parameters were used as for the predictions in Ref. [26]. The measured ratio of separately averaged quantities should be larger by a factor of ~ 1.25 – 1.4 , depending on the centrality bin ($K = 1.34$ for a bin at 20–60% centrality).

parameter, however, that is not seen in data. Adding viscosity using the standard freeze out scheme was seen to make worse both the shape of the curve as a function of p_t and the dependence on impact parameter (although both aspects could potentially be fixed by using a correct form for the viscous correction to the equilibrium distribution function at freeze out—see Ref. [22]).

For LHC, this uncertainty in the freeze out scheme is less relevant, and we find that $v_4/(v_2)^2$ should be similar to the value at RHIC, with a slight increase at small transverse momentum.

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