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# Spiral model, jamming percolation and glass-jamming transitions

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**Abstract.** The Spiral Model (SM) corresponds to a new class of kinetically constrained models introduced in joint works with Fisher [9,10] which provide the first example of finite dimensional models with an ideal *glass-jamming transition*. This is due to an underlying *jamming percolation* transition which has unconventional features: it is discontinuous (i.e. the percolating cluster is compact at the transition) and the typical size of the clusters diverges faster than any power law, leading to a Vogel-Fulcher-like divergence of the relaxation time. Here we present a detailed physical analysis of SM, see [6] for rigorous proofs.

**PACS.** 64.70.Pf Glass transitions – 05.20.-y Classical statistical mechanics – 05.50.+q Lattice theory and statistics – 61.43.Fs Glasses

## 1 Introduction

Theoretical progress in understanding the glass and jamming transition, and more generally glassy dynamics, is hampered by the shortage of finite dimensional models that display the basic phenomenological ingredients and that are simple enough to be fully analyzed. Kinetically Constrained Models (KCM) [1] are an exception. They have been introduced few decades ago [2–4] as models for glass-forming liquids. They are based on the assumption that a particle does not (or cannot) move if surrounded by too many others. This can be also interpreted in terms of dynamic facilitation [5]. All KCM share two basics properties: particles (or spins) can move (or flip) only if a certain constraint on the number of occupied neighbors is verified. Once the constraint is verified the dynamical rules are such that the resulting Boltzmann distribution is trivial, i.e. uncorrelated from site to site. As a consequence the glass transition, if any, is purely dynamical in these models. Furthermore, another advantage is that the study of the dynamical transition can be reduced to a highly correlated percolation problem. In fact, in these models a particle (or a spin) can be blocked if it has too many occupied (up) neighbors which can be blocked by their neighbors and so on and so forth. Using the fact that the Boltzmann equilibrium distribution is trivial one can prove [7,8] that the only dynamical transition that can take place in these systems corresponds to a *jamming percolation* where an infinite cluster of mutually blocked particles (spins) appears. In [9,10] we introduced a new class of KCM which displays such a transition on a finite dimensional lattice at a finite density of occupied sites (up spins), $p_c < 1$. We will thus refer to these models as Jamming Percolation (JP) models. Here we will review the easiest example of a JP model, namely the two-dimensional spin model which has been introduced in [6,10] and dubbed *Spiral Model* (SM). For SM the existence of a jamming transition has been rigorously proved [6] and the exact value of $p_c$ has been identified: $p_c$ coincides with the critical threshold of directed site percolation (DP) in two dimensions, $p_c^{DP} \simeq 0.705$. Contrary to recent claims [11] our proof for SM does not need any modification (we will pinpoint in Sect. 4.1 the incorrect assumption of [11]). This jamming transition has remarkable properties: the density of the frozen cluster, $\Phi(p)$, is discontinuous at the transition but the cross-over length over which the system is still ergodic (or liquid) diverges. Furthermore the time scale for relaxation (and also the cross-over length) diverges faster than any power law. These properties are quite unusual but are exactly what is often assumed the real glass or jamming transition should display (if they exist). They have been also rigorously proved in [6] modulo the standard conjecture on the existence of two different correlation lengths for DP [12].

In the following we will sketch in a physical and hopefully transparent way the arguments which lead to the above results providing the tools needed to analyze this transition and explaining the underlying mechanism: it is the consequence of two perpendicular directed percolation processes which together can form a compact network of

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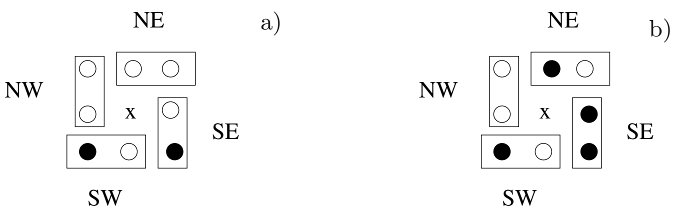


**Fig. 1.** Site $x$ and its NE, NW, SE and SW neighbours. Filled (empty) dots stand for up (down) spins. The constraint is (is not) verified at $x$ in case (a) (in case (b)).

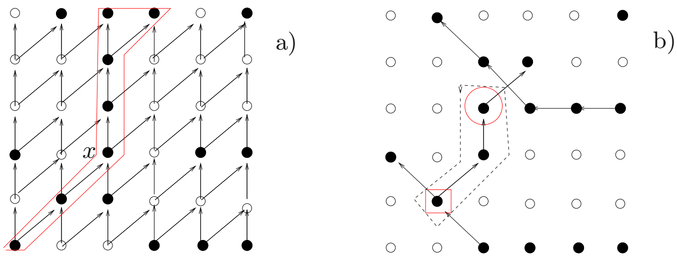


**Fig. 2.** a) The directed lattice obtained drawing arrows from each site towards its NE neighbours. Particles inside the continuous line belong to a NE-SW spanning cluster, thus they are blocked. b) A NE-SW non spanning cluster crossing two NW-SE clusters. We depict both types of T-junctions: the bottom (top) one occurs with (without) a site in common for the NE-SW and NW-SE cluster. In both cases the last point of NE-SW before the crossing is blocked by the presence of the NW-SE cluster, thus all sites inside the dashed line cannot be erased before erasing at least one site of the NE-SW cluster.

frozen directed paths at criticality. In the final section we shall discuss the generality of our approach and the universality of the jamming percolation transition of SM.

## 2 The spiral model and its related percolation problem

Consider a square lattice and, for each site $x$, define among its first and second neighbours the couples of its North-East (NE), South-West (SW), North-West (NW) and South-East(SE) neighbours as in Figure 1, namely $\mathrm{NE} = (x + e_2, x + e_1 + e_2)$, $\mathrm{SW} = (x - e_2, x - e_1 - e_2)$, $\mathrm{NW} = (x - e_1, x - e_1 + e_2)$ and $\mathrm{SE} = (x + e_1, x + e_1 - e_2)$, where $e_1$ and $e_2$ are the coordinate unit vectors. The spiral model is a stochastic spin lattice model where a spin can flip if and only if the following constraint is verified: both its NE *and/or* both its SW neighbours are down *and* both its SE *and/or* both its NW neighbours are down too (see Fig. 1). A complementary way to describe the kinetic rule consists in stating the conditions under which a spin *cannot* flip: in the case of the Spiral model this happens when there is at least an up spin in one of the neighboring couples (NE, SW, NW, SE) and in its diametrically opposite counterpart.

If the constraint is verified then the spin flip rate is $p$ from down to up and $1 - p$ from up to down. As a consequence, the invariant probability measure reached for $p < p_c$ is the Bernoulli product measure, i.e. independent from site to site. It is such that a spin is up with probability $p$ and down with probability $1 - p$. As explained in the introduction the dynamical transition takes place when an infinite cluster of mutually blocked spins appears with probability one with respect to the Bernoulli product measure. An easy way to unveil the existence of this cluster is to run the following cellular automaton. Down (up) spins are mapped to empty (occupied) sites and evolution obeys a deterministic discrete time dynamics: at each time step empty sites remain empty, while occupied sites get emptied provided the kinetic constraint of the Spiral Model is verified. The cluster of particles (if any) which remains in the final stationary configuration coincides with all the up spins that are mutually blocked under the stochastic SM dynamics. Note that the kinetic rules can be also rephrased by saying that at least one among the four sets $\mathrm{NE} \cup \mathrm{SE}$, $\mathrm{SE} \cup \mathrm{SW}$, $\mathrm{SW} \cup \mathrm{NW}$ and $\mathrm{NW} \cup \mathrm{NE}$ should be completely empty. From this perspective (and identifying an occupied site with high density regions in a liquid) SM encodes in a very simplified way the cage effect that emerges in liquids and granular media close to the glass and jamming transition.

## 3 Critical density

### 3.1 Occurrence of blocked clusters for $\mathbf{p > p_c^{DP}}$

In order to establish $p_c < 1$ we will identify a set of blocked clusters and show that they exist with probability one (with respect to the Bernoulli measure) for $p > p_c^{DP}$, where $p_c^{DP}$ is the critical threshold of directed site percolation in two dimensions. Therefore $p_c \leq p_c^{DP} < 1$.

Let us start by recalling the definition and a few basic results on DP (see e.g. [12]). Take a square lattice with randomly (independent) occupied sites and put two arrows going out from each site $x$ towards its neighbours in the positive coordinate directions, $x + e_1$ and $x + e_2$. On this directed lattice a continuous percolation transition occurs at a non trivial critical density $p_c^{DP} \simeq 0.705$ (a percolating cluster is here an occupied cluster which spans the lattice following the direction of the arrows). This transition is second order, as for site percolation, but belongs to a different universality class. In particular, due the anisotropy of the lattice, the typical sizes of the incipient percolating cluster in the parallel $(e_1 + e_2)$ and transverse $(e_1 - e_2)$ directions diverge with different exponents, $\xi_{\parallel} \simeq (p_c^{DP} - p)^{-\nu_{\parallel}}$ and $\xi_{\parallel} \simeq \xi_{\perp}^{z}$ with $\nu_{\parallel} \simeq 1.74$ and $z \simeq 1.58$.

Back to the Spiral Model, let us consider the directed lattice that is obtained from the square lattice putting two arrows from each site towards its NE neighbours, as in Fig. 2a). This lattice is equivalent to the one of DP, simply tilted and squeezed. Therefore, for $p > p_c^{DP}$, there exists a cluster of occupied sites which spans the lattice following the direction of the arrows (cluster inside the continuous line in Fig. 2a). We denote by NE-SW *clusters* the occupied sets which follow the arrows of such lattice and NW-SW *clusters* those that follow instead the arrows drawn starting from each site towards its NW neighbours. Consider now a site in the interior of a spanning NE-SW cluster, e.g. site $x$ in Figure 2a: by definition there is at least one occupied site in both its NE and SW neighbouring couples, therefore $x$ is occupied and blocked with respect to the updating rule of SM. Thus, the presence of the DP cluster implies a blocked cluster and $p_c \leq p_c^{DP}$ follows. Note that these results would remain true also for a different updating rule with the milder requirement that only at least one among the two couples of NE and SW sites is completely empty (and no requirement on the NW-SE direction). However, as we shall see, the coexistence of the constraint in the NE-SW and NW-SE directions is crucial to find a discontinuous transition for SM, otherwise we would have a standard DP-like continuous transition.

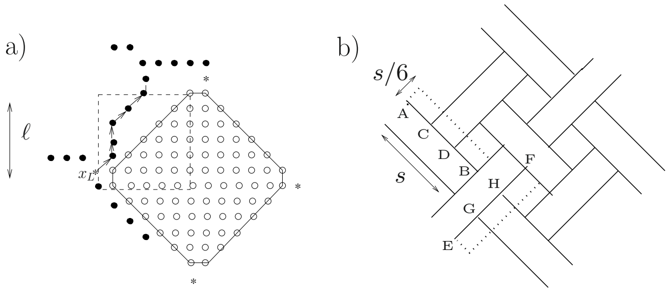


**Fig. 3.** a) $Q_\ell$ (continuous line), $S_\ell$ (dashed line) and the four sites $(*)$ which should be emptied to expand $Q_\ell$. We draw the necessary condition for $x$ to be frozen: it should belong to a NE-SW cluster spanning $S_\ell$ and supported by NW-SE cluster from the exterior. b) The frozen structure described in the text: continuous lines stand for occupied NE-SW or SW-NE clusters. Each of these clusters is blocked since it ends in a T-junction with a cluster along the transverse direction. The dotted rectangle adjacent to cluster AB (EF) are the regions in which this cluster can be displaced and yet a frozen backbone is preserved: the T-junctions in C and D (G and H) will be displaced but not disrupted.

### 3.2 Absence of blocked clusters for $p < p_c^{DP}$

Before showing that below $p_c^{DP}$ blocked clusters do not occur, a few remarks are in order. If instead of SM we were considering the milder rules described at the end of previous section, the result would follow immediately since the presence of a blocked cluster would imply the existence of a DP one. On the other hand for SM rules, since blocking can occur along either the NE-SW or the NW-SE direction (or both), a directed path implies a blocked cluster but the converse is not true. This is because a NE-SW non spanning cluster can be blocked if both its ends are blocked by a T-junction with NW-SE paths, as shown in Figure 2b (see Sect. 4.1 for a detailed definition of T-junction). By using such T-junctions it is also possible to construct frozen clusters which do not contain a percolating DP cluster neither in the NE-SW nor in the NW-SE direction: all NE-SW (NW-SE) clusters are finite and are blocked at both ends by T-junctions with finite NW-SE (NE-SW) ones (see Fig. 3b). As we will show in Section 4.2 these T-junctions are crucial to make the behavior of the transition for SM very different from DP, although they share the same critical density. This also means that the fact that spanning DP clusters do not occur for $p < p_c^{DP}$ is not sufficient to conclude that also blocked clusters do not occur. What strategy could one use? Recalling bootstrap percolation results [13], a possible idea is to search for proper unstable voids from which we can iteratively empty the whole lattice. Of course, since we already know that blocked clusters occur when $p \geq p_c^{DP}$, something should prevent this unstable voids to expand at high density.

Consider the region $Q_\ell$ inside the continuous line in Figure 3a, namely a "square" of linear side $\sqrt{2}\ell$ tilted of 45 degrees with respect to the coordinate axis and with each of the four vertexes composed by two sites. If $Q_\ell$ is empty and the four sites external and adjacent to each vertex denoted by $*$ in Figure 3a are also empty, then it is possible to enlarge the empty region $Q_\ell$ to $Q_{\ell+1}$. Indeed, as can be directly checked, all the sites external to the top right side can be subsequently emptied starting from the top one and going downwards. For the sites external to the other three sides of $Q_\ell$ we can proceed analogously, some care is only required in deciding whether to start from top sites and go downwards or bottom ones and go upwards. Therefore we can expand $Q_\ell$ of one step provided all the four $*$ sites are empty or can be emptied after some iterations of the cellular automaton. Let us focus on one of these $*$ sites, e.g. the left one, $x_L$ in Figure 3a. As it can be proved by an iterative procedure (see [6]), in order for $x_L$ not to be emptyable there should exist a NE-SW cluster which spans (from $x_L$ to the top edge) the square $S_\ell$ of size $\ell$ containing the top left part of $Q_\ell$ (region inside the dashed line in Fig. 3a). This is due to the fact that any directed path in the NW-SE direction can be unblocked starting from the empty part of $S_\ell$ below the diagonal in the $e_1 + e_2$ direction. Therefore the only way in which $x_L$ can be blocked is that it belongs to a NE-SW cluster that is either supported by NW-SE clusters running outside $S_\ell$ or that is infinite. In any case this NE-SW cluster has to be at least of length $\ell$. As a consequence, for large $\ell$, ($\ell \gg \xi_\parallel$) the cost for a one step expansion of $Q_\ell$ is proportional to the probability of not finding such a DP path for any of the four * sites, $1 - 4\exp(-c\ell/\xi_\parallel)$. Thanks to the positive correlation among events at different $\ell$'s, the probability that the emptying procedure can be continued up to infinity is bounded from below by the product of these single step probabilities which goes to a strictly positive value for $p < p_c^{DP}$ since $\xi_\parallel < \infty$. Note that, as we already knew from the results of Section 3.1, this is not true for $p > p_c^{DP}$: the presence of long DP paths prevents the expansion of voids. As a conclusion, the probability of emptying the whole lattice starting from an empty square $Q_\ell$ centered around a *given point* in the lattice and with $\ell \gg \xi_\parallel$ is finite (although very small). Since there is an infinite number of points in the lattice, there will be at least one (actually a finite fraction) of sites from which the whole lattice can be emptied[1] for $p < p_c^{DP}$. This, together with the result of Section 3.1, yields $p_c = p_c^{DP}$.

## 4 Critical behavior

### 4.1 T-junctions

One of the most important characteristic of the SM model, already alluded to in the previous sections, is that a directed path in the NE-SW direction can be supported by another path running in the NW-SE direction (and vice versa) via a T-junction. Let us discuss this point in detail since recently it has been incorrectly claimed that this is not true for the SM model. There are only two possible types of crossing of a NE-SW path with a NW-SE one: either they have one point in common or not. In the latter case they should cross as in the upper crossing of Figure 2b. In both cases we call the crossing a T-junction and the key observation is that if a NE-SW path ends in two T-junctions with NW-SE paths (or vice versa), it does not need to continue beyond the crossings in order to be blocked, as long as the NW-SE paths are blocked. If the T-junction occurs with a site in common (site inside the square of Fig. 3b) this is a trivial consequence of the fact that this point belongs to the NW-SE path. In the other case it can be easily checked that the last point belonging to the NE-SW path (site inside the circle of Fig. 2b) is blocked thanks to the one above it, which belongs to the NW-SE path. All other possible crossings are related to these two cases by symmetry. In [11] it is stated that in order for a NE-SW path to stabilize a NW-SE path (or the converse) they shouldn't only cross but also have a point in common and since this may not happen our proof for the SM needs a modification. As explained above this conclusion is incorrect and our proof for the SM model does not need any modification (see also [6] for further details).

### 4.2 SM: discontinuity of the transition

In the previous sections, we have shown that the percolation transition due to the occurrence of a frozen backbone for the Spiral Model occurs at $p_c^{DP}$. We will now explain why the density of the frozen cluster is discontinuous, $\Phi(p_c^{DP}) > 0$ (the frozen structures are compact rather than fractal at criticality).

By translation invariance $\Phi(p_c^{DP})$ is equal the probability that a given point, e.g. the origin, is blocked (i.e. it belongs to an infinite blocked structure). In order to show that $\Phi(p_c^{DP}) > 0$ we will then construct a set of configurations, $\mathcal{B}$, for which the origin is blocked and such that $P_{\mathcal{B}}(p_c^{DP}) > 0$. Since $\Phi(p_c^{DP}) \geq P_{\mathcal{B}}$ our result implies $\Phi(p_c^{DP}) > 0$. In order to define $\mathcal{B}$, consider a configuration in which the origin belongs to a NE-SW path of finite length $\ell_0/2$: this occurs with some finite probability $q_0 > 0$. Now focus on the infinite sequence of pairs of rectangles of increasing size $\ell_i \times \ell_i/12$ with $\ell_1 = \ell_0$, $\ell_i = 2\ell_{i-2}$ and intersecting as in Figure 4a. A configuration belongs to $\mathcal{B}$ if *each* of these rectangles with long side along the NE-SW (NW-SE) diagonal contains a NE-SW (NW-SE) percolating path (dotted lines in Fig. 4b). If this is the case then the infinite backbone of particles containing the origin (cluster inside the continuous line in Fig. 4c) survives thanks to the T-junctions among paths in intersecting rectangles. Therefore $\Phi(p) > q_o \prod_{i=1,\infty} P(\ell_i)^2$, where $P(\ell_i)$ is the probability that a rectangle of size $\ell_i \times 1/12\ell_i$ with short side in the transverse direction is spanned by a DP cluster. Recall that there is a parallel and a transverse length for DP with different exponents, i.e. a cluster of parallel length $\ell$ has typically transverse length $\ell^{1/z}$ [12]. Let us divide the $\ell_i \times 1/12\ell_i$ rectangle into $\ell_i^{1-1/z}$ slices of size $\ell_i \times 1/12\ell_i^{1/z}$. For each slice the probability of having a DP cluster along the parallel direction at $p_c^{DP}$ is order unity. Thus, the probability of *not* having a DP cluster in each of the slice is $1 - P(\ell_i) = O[\exp(-c\ell_i^{1-1/z})]$. From this result and the above inequality we get $\Phi(p_c^{DP}) > 0$. Therefore the infinite cluster of jamming percolation is "compact" with dimension $d = 2$ at the transition.

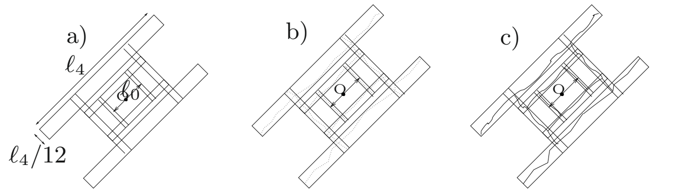


**Fig. 4.** a) The sequence of intersecting rectangles. b) Dotted non straight line stand for NE-SW (NW-SE) clusters spanning the rectangles. c) Frozen structure containing the origin.

### 4.3 Crossover length

Let us now focus on the divergence of the incipient blocked cluster approaching the transition. This can be studied analyzing the typical size, $L_c$, below which frozen clusters occur on finite lattices.

We first obtain a lower bound on $L_c$ constructing explicitly blocked structures that exist with finite probability as long as $L < L_c^{lb}$. Consider NE-SW and NW-SE paths of length $s$ intersecting as in Figure 3b. This type of structure can be emptied completely only starting from its border since each finite directed path terminates on both ends into T-junctions with a path in the transverse direction. Therefore it is frozen if we continue the construction up to the border of the lattice and we take periodic boundary conditions. Thus the probability that there exists a frozen cluster, $1-R(L,p)$, is bounded from below by the probability that *each* of the $O(L/s)^2$ dotted rectangles in Figure 3b contains at least one path connecting its short sides. This

[1] Mathematically, one would say that the ergodic theorem implies that in the thermodynamic limit with probability one the final configuration is completely empty.

leads to $R(L,p) \le (L/s)^2 \exp(-cs^{1-1/z})$ provided $s \le \xi_\parallel$ (for $s > \xi_\parallel$ the probability of having a DP cluster in a rectangle starts to decrease and cannot be bounded anymore by $1 - O[\exp(-c\ell_i^{1-1/z})]$). Thus taking $s \propto \xi_\parallel$, we get $\log L_c \ge k_l |p - p_c^{DP}|^{-\mu}$, where $\mu = \nu_\parallel(1-1/z) \simeq 0.64$.

In order to establish an upper bound on $L_c$, we determine the size $L_c^{up}$ above which unstable voids, that can be expanded until emptying the whole lattice, occur typically. The results in Section 3.1 imply that the probability of expanding an empty nucleus to infinity is dominated by the probability of expanding it up to $\ell = \xi_\parallel$. Indeed, above this size the probability of an event which prevents expansion is exponentially suppressed. Therefore, considering the $O(L/\xi_\parallel)^2$ possible positions for the region that it is guaranteed to be emptyable up to size $\xi_\parallel$, we can bound the probability that a lattice of linear size $L$ is emptyable as $R(L,p) \ge L^2\delta$, where $\delta$ is the probability that a small empty nucleus can be expanded until size $\xi_\parallel$.In the emptying procedure described in Section 3.1 we evaluated the cost for expanding of one step the empty region $Q_\ell$. Analogously, the cost of expanding directly from $Q_\ell$ to $Q_{2\ell}$ can be bounded from below by $C^{\ell^{1-1/z}}$, with $C$ a positive constant independent from $\ell$. This can be done by dividing the region contained in $Q_{2\ell}$ and not in $Q_\ell$ into $\ell^{1-1/z}$ strips with parallel and transverse length of order $\ell$ and $\ell^z$, requiring that none of them contains a DP path which percolates in the transverse direction and using for this event the scaling hypothesis of directed percolation when $p \nearrow p_c^{DP}$. Thus for the expansion up to size $\xi_\parallel$ we get $\delta \ge \prod_{i=1}^{\log_2 \xi_\parallel} C^{2^{i(1-1/z)}} \simeq \exp(-C'\xi_\parallel^{1-1/z})$, with $C' > 0$ . This, together with above inequality, yields $\log L_c \le k_u |p - p_c^{DP}|^{-\mu}$.

As a consequence upper and lower bound leads to the same scaling at leading order and imply that the crossover length diverges with an essential singularity, i.e. faster than any power law for $p \nearrow p_c^{DP}$.

## 5 Discussion

Let us first discuss the dynamical behavior of the SM model. The results of the previous sections have important consequence on the dynamics of SM. First, incipient blocked clusters can be unblocked only from the boundary. As a consequence the relaxation timescale is expected to scale at least as (but likely larger than) their typical size: $\propto \exp(k/|p-p_c^{DP}|^\mu)$. Indeed this can be proved rigorously [8]. Furthermore, since the fraction of blocked sites is finite at the transition, two point dynamical correlation functions, e.g. spin-spin correlations, will show a plateau like super-cooled liquids approaching the glass transition. The plateau, also called Edwards-Anderson parameter in the context of spin-glasses, corresponds to the frozen spin fluctuations. These two dynamical characteristics are remarkable since they lead to a dynamics qualitatively similar to the one of glass-forming liquids. It would be very interesting to perform more detailed investigations and comparisons, in particular by numerical simulations.

The extension and universality of the jamming percolation transition of SM remain fundamental questions to be investigated. As it has been discussed in [10] (see also [11,14]), it is possible to identify a class of rules which give rise to a jamming transition and belong to the same universality class of SM: as $p \nearrow p_c$ the divergence of the incipient frozen cluster follows the same scaling and the transition remains discontinuous. For all these models the jamming transition is a consequence of the existence of (at least) two transverse directed percolation (DP)-like processes which can form a network of finite DP-clusters blocked by T-junctions with clusters in the transverse direction. A model that belongs to this class is for example the Knight model defined in [9]. Note that in general, at variance with SM, it will not be possible to determine analytically the exact value of $p_c$ (this was possible for SM thanks to the fact that in each of the two transverse directions the blocked clusters can be exactly mapped into those of conventional 2 dimensional DP). Neither it is possible to generalize the rigorous proofs obtained for the SM. However, it is nevertheless possible to obtain numerically a reliable estimate of $p_c$ and a confirmation that the transition has the same properties of SM. This is done analyzing finite size effects with proper choices of the geometry and boundary conditions which allow to focus separately on each of the two transverse directions. In this way one can verify that on long length scales the two independent directional processes are in the DP universality class and obtain a good numerical estimate of $p_c$. Using suitable boundary conditions and geometries is particularly important for jamming percolation since, as for bootstrap percolation, convergence to the asymptotic results can be extremely slow. For an extended discussion on this we refer to [10], where the value of the critical density for the Knight model has been derived. The result is $p_c^{Knight} \simeq 0.635$ and differs from our original conjecture $p_c^{Knight} = p_c^{DP}$ [9] which was due to the overlooking of some blocked structures [14].

Finally, we also expect that suitable generalizations of the spiral model in three dimensions will display similar transitions although it may not be possible to find a model in which all arguments can be made rigorous and the transition value established analytically (at variance with the case of SM in two dimensions). This is certainly an issue worth investigating further. Also because dimensions higher than two provide more possible choices for the blocking rules. Thus, other type of jamming transition may be unveiled.

We thank D.S. Fisher for very important collaborations on this subject.

## References

1. F. Ritort, P. Sollich, Adv. in Phys. **52**, 219 (2003)
2. G.H. Fredericksson, H.C. Andersen, Phys. Rev. Lett. **53**, 1244 (1984)
3. W. Kob, H.C. Andersen, Phys. Rev. E **48**, 4364 (1993)

4. J. Jackle, S. Eisinger, Z. Phys. B **84**, 115 (1991)
5. J.P. Garrahan, D. Chandler, PNAS **100**, 9710 (2003)
6. C. Toninelli, G. Biroli, *Spiral Model: a new cellular automaton with a discontinuous glass transition* e-print `arXiv:0709.0378`
7. C. Toninelli, G. Biroli, J. Stat. Phys. **117**, 27 (2004)
8. N. Cancrini, F. Martinelli, C. Roberto, C. Toninelli, *Kinetically constrained spin models*, to appear in Probab. Th. and Rel. Fields, preprint `math.PR/0610106`
9. C. Toninelli, G. Biroli, D.S. Fisher, Phys. Rev. Lett. **96**, 035702 (2006)
10. C. Toninelli, G. Biroli, D.S. Fisher, Phys. Rev. Lett. **98**, 129602 (2007)
11. M. Jeng, J.M. Schwarz, *On the study of jamming percolation*, `arXiv:0708.0582` (v1)
12. H. Hinrichsen, Adv. in Phys. **49**, 815 (2000); R. Durrett, Ann. Prob. **12**, 999
13. R.H. Schonmann, Ann. of Probab. **20**, 174 (1992)
14. M. Jeng, J.M. Schwarz, Phys. Rev. Lett. **98**, 129601 (2007)