Centrality dependence of elliptic flow, the hydrodynamic limit, and the viscosity of hot QCD

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We show that the centrality and system-size dependence of elliptic flow measured at the BNL Relativistic Heavy Ion Collider (RHIC) are fully described by a simple model based on eccentricity scaling and incomplete thermalization. We argue that the elliptic flow is at least 25% below the (ideal) "hydrodynamic limit," even for the most central Au-Au collisions. This lack of perfect equilibration allows for estimates of the effective parton cross section in the quark-gluon plasma and of its viscosity to entropy density ratio. We also show how the initial conditions affect the transport coefficients and thermodynamic quantities extracted from the data, in particular, the viscosity and the speed of sound.

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When two ultrarelativistic nuclei collide at a nonzero impact parameter, their overlap area in the transverse plane has a short axis, parallel to the impact parameter, and a long axis perpendicular to it. This almond shape of the initial profile is converted by the pressure gradient into a momentum asymmetry, so that more particles are emitted along the short axis [1]. The magnitude of this effect is characterized by elliptic flow, defined as

$$v_2 \equiv \langle \cos 2(\varphi - \Phi_R) \rangle, \tag{1}$$

where φ is the azimuthal angle of an outgoing particle, Φ_R is the azimuthal angle of the impact parameter, and angular brackets denote an average over many particles and many events. The unexpected large magnitude of elliptic flow at the BNL Relativistic Heavy Ion Collider (RHIC) [2] has generated a lot of activity in recent years.

Elliptic flow results from the interactions between the produced particles and can be used to probe local thermodynamic equilibrium. If the produced matter equilibrates, it behaves as an ideal fluid. Hydrodynamics predicts that at a given energy, v_2 scales like the eccentricity ε of the almond [1,3]. It is independent of its transverse size R, as a consequence of the scale invariance of ideal-fluid dynamics. If, on the other hand, equilibration is incomplete, then eccentricity scaling is broken and v_2/ε also depends on the Knudsen number $K = \lambda/R$, where λ is the length scale over which a parton is deflected by a large angle.

Here, we show that the centrality dependence of v_2/ε, for both Au – Au and Cu – Cu collisions, can be described by the following simple formula [4]:

$$\frac{v_2}{\varepsilon} = \frac{v_2^{\text{hydro}}}{\varepsilon} \frac{1}{1 + K/K_0}. \tag{2}$$

v_2/ε is largest in the hydrodynamic limit $K \to 0$. The first-order corrections to this limit, corresponding to viscous effects, are linear in K. For a large mean-free path, far from the hydrodynamic limit, $v_2/\varepsilon \sim 1/K$ vanishes like the number of collisions per particle. One expects the transition between these two regimes to occur when $\lambda \simeq R$ and, hence, that $K_0 \simeq 1$. A recent transport calculation [5] in two spatial dimensions indeed obtained $K_0 \simeq 0.7$.

Elliptic flow develops gradually during the early stages of the collision. Because of the strong longitudinal expansion, the thermodynamic properties of the medium depend on the time τ, of course. The average particle density, for instance, decreases like $1/\tau$ (if their number is approximately conserved, see recent discussion in [6]):

$$\rho(\tau) = \frac{1}{\tau S} \frac{dN}{dy}, \tag{3}$$

where dN/dy denotes the total (charged + neutral) multiplicity per unit rapidity, and S is the transverse overlap area between the two nuclei. The quantities that we shall extract from v_2 should be interpreted as averages over the transverse area S and over some time interval around R/c_s, which is the typical timescale for the buildup of v_2 in hydrodynamics [4]. c_s denotes the velocity of sound.

The Knudsen number K is defined by evaluating the mean-free path $\lambda = 1/\sigma\rho$ (σ is a partonic cross section) at $\tau = R/c_s$. Thus,

$$\frac{1}{K} = \frac{\sigma}{S} \frac{dN}{dy} c_s. \tag{4}$$

The purpose of this article is to show that the centrality and system-size dependence of the data for v_2 at RHIC is described very well by Eqs. (2) and (4). This provides three important pieces of information. First, such a fit allows us to "measure" the Knudsen number corresponding to a given centrality, which quantifies how close the dense matter produced in heavy-ion collisions at RHIC is to perfect fluidity. Second, the extrapolation to $K = 0$ allows us to read off the limiting value for $v_2^{\text{hydro}}/\varepsilon$ extracted from the *data*; this is useful for constraining the equation of state (EoS) of QCD via hydrodynamic simulations, and we shall also see that it exhibits a rather surprising dependence on the initial conditions. Finally, using Eq. (4), we can convert the Knudsen number into the typical partonic cross section σ (and viscosity) in the quark-gluon plasma (QGP). Because only the combination $K_0\sigma c_s$ actually appears in Eq. (2), uncertainties in K_0 or c_s then translate into corresponding uncertainties of σ. Unless mentioned otherwise, our standard choice is $c_s = 1/\sqrt{3} \simeq 0.58$ (ideal QGP) and $K_0 = 0.7$. Letting $K_0 = 1$ and $c_s^2 = 2/3$[1] instead reduces the estimated σ by a factor of two; on the other hand, taking $K_0 = 0.5$ and $c_s^2 = 1/6$ increases σ by the same factor.

For the elliptic flow, v_2, we use PHOBOS data for Au-Au [7] and Cu-Cu [8] collisions. The same analysis could be carried out using data from PHENIX [9] or STAR [10]. The initial eccentricity ε and the transverse density $(1/S)(dN/dy)$ are evaluated using a model of the collision. Two such models are compared. The remaining parameters v_2^{hydro} and σ are fit to the data. The first step is to plot v_2/ε versus $(1/S)(dN/dy)$ [11]. Such plots have already been obtained at the CERN Super Proton Synchrotron (SPS) and RHIC [12], and they are puzzling: while v_2/ε increases with centrality, it shows no hint of the *saturation* predicted by Eq. (2) for $K/K_0 \lesssim 1$, suggesting that the system is far from equilibrium [4]. On the other hand, the value of v_2 for central Au-Au collisions at RHIC is about as high as predicted by hydrodynamics, which is widely considered as key evidence that a "perfect liquid" has been created at RHIC [13].

It has been understood only recently that the eccentricity of the overlap zone has so far been underestimated as the result of two effects. The first effect is fluctuations in initial conditions [14]: the timescale of the nucleus-nucleus collision at RHIC is so short that each nucleus remains in a frozen configuration, with its nucleons distributed according to the nuclear wave function. Fluctuations in the nucleon positions result in fluctuations of the overlap area. Their effect on elliptic flow was first pointed out in Ref. [15]. It was later realized by the PHOBOS Collaboration [8,16] that the orientation of the almond may also fluctuate, so that Φ_R in Eq. (1) is no longer the direction of impact parameter, but the minor axis of the ellipse defined by the positions of the nucleons. These fluctuations explain both the large magnitude of v_2 for small systems, such as Cu-Cu collisions, as well as the nonzero magnitude of v_2 in central collisions, where the eccentricity would otherwise vanish. They have to be taken into account to observe the expected saturation of v_2/ε at high density mentioned above.

The eccentricity is usually estimated from the distribution of participant nucleons in the transverse plane (Glauber model). More precisely, we assume here that the density distribution of the produced particles is given by a fixed 80:20% superposition of participant and binary-collision scaling, respectively [17]. For Au-Au collisions, this simple model reproduces the centrality dependence of the multiplicity reasonably well (we assume that charged particles are 2/3 of the total multiplicity and that $dN/d\eta \simeq 0.8 dN/dy$ at midrapidity), while it underestimates it for central Cu-Cu collisions by about 10%.

At high energies a second effect that increases the eccentricity is perturbative gluon saturation, which determines the $p_\perp$-integrated multiplicity from weak-coupling QCD without additional models for soft particle production. High-density QCD [the "color-glass condensate" (CGC)] predicts a different distribution of produced gluons, $dN/d^2\mathbf{r}_\perp dy$, which gives a similar centrality dependence of the multiplicity [17] but a larger eccentricity [18,19]. When particle production is dominated by transverse momenta below the saturation scale of the denser nucleus, then $dN/d^2\mathbf{r}_\perp dy \sim \min(n_{\text{part}}^A(\mathbf{r}_\perp), n_{\text{part}}^B(\mathbf{r}_\perp))$ traces the participant density of the more dilute collision partner, rather than the average as in the Glauber model [19]. Precise figures depend on how the saturation scale is defined [20]. Naively, the larger initial eccentricity predicted by the gluon saturation approach is expected to require more dissipation to reproduce the same experimentally measured v_2. Somewhat surprisingly, we find that this expectation is incorrect, which underscores the nontrivial role played by the initial conditions.

Both effects, fluctuations and gluon saturation, were recently combined by Drescher and Nara [21]. In their approach, the saturation momenta and the unintegrated gluon distribution functions of the colliding nuclei are determined for each configuration individually. The finite interaction range of the nucleons is also taken into account. Upon convolution of the projectile and target unintegrated gluon distribution functions and averaging over configurations, the model leads to a very good description of the multiplicity for both Au-Au and Cu-Cu collisions over the entire available range of centralities.

Having determined the density distributions of produced particles from either model as described above, we obtain the eccentricity via [15,22]

$$\varepsilon = \sqrt{\langle \varepsilon_{\text{part}}^2 \rangle}, \quad \varepsilon_{\text{part}} = \frac{\sqrt{\left(\sigma_y^2 - \sigma_x^2\right)^2 + 4\sigma_{xy}^2}}{\sigma_x^2 + \sigma_y^2}. \tag{5}$$

σ_x, σ_y are the respective root-mean-square widths of the density distributions, and $\sigma_{xy} = \overline{xy} - \bar{x}\bar{y}$ (a bar denotes a convolution with the density distribution for a given configuration while brackets stand for averages over configurations). The overlap area S is defined by $S \equiv 4\pi\sigma_x\sigma_y$ [5]. We find it more appropriate to define these moments via the number density distribution $dN/d^2\mathbf{r}_\perp dy$ rather than via the energy density distribution $dE_\perp/d^2\mathbf{r}_\perp dy$. The reason is twofold: first, v_2 is extracted experimentally from the azimuthal distribution of the particle number, not the transverse energy; second, our CGC approach describes the centrality dependence of the *measured* final-state multiplicity very well, which indicates that the ratio of final-state particles to initial-state gluons (including possible gluon multiplication processes [23]) is essentially constant.

Figures 1 and 2 display v_2/ε as a function of $(1/S)(dN/dy)$ for Au-Au and Cu-Cu collisions at various centralities, within the Glauber and CGC approaches, respectively. For both types of initial conditions, Cu-Cu and Au-Au collisions at the same $(1/S)(dN/dy)$ give the same v_2/ε within error bars. Eccentricity fluctuations are crucial for this agreement [8]. The figures also show that Eqs. (2) and (4) provide a good fit to the data, for both sets of initial conditions. On the other hand, the values of the fit parameters clearly depend on the initial conditions, which has important consequences for the physics.

[1]Such a "hard" EoS can arise from repulsive long-range interactions among the partons such as classical fields. We thank V. Koch for pointing this out to us.

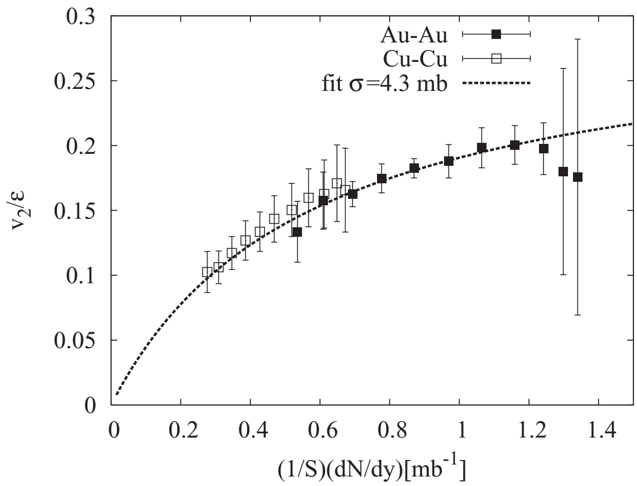

FIG. 1. Variation of the scaled elliptic flow with the density, assuming initial conditions from the Glauber model. The line is a two-parameter fit using Eqs. (2) and (4).

The first physical quantity extracted from the fit is the hydrodynamic limit, $v_2^{\text{hydro}}/\varepsilon$, obtained by extrapolating to $(1/S)(dN/dy) \to \infty$. The values are $v_2^{\text{hydro}}/\varepsilon = 0.30 \pm 0.02$ with the Glauber parameterization and $v_2^{\text{hydro}}/\varepsilon = 0.22 \pm 0.01$ with the CGC initial conditions. Comparing these numbers to the experimental data points one observes that deviations from ideal hydrodynamics are as large as 30%, even for central Au-Au collisions. This is our first important result.

So far, a quantitative extraction of the QCD EoS from the RHIC data via hydrodynamic analysis was hampered by the fact that v_2/ε had not been factorized into the perfect-fluid part $v_2^{\text{hydro}}/\varepsilon$ and the dissipative correction $1/(1 + K/K_0)$. For example, Huovinen found [24] that an EoS with a rapid crossover rather than a strong first-order phase transition, as favored by lattice QCD [25], overpredicted the flow data. This finding was rather puzzling, too, as it was widely believed that the RHIC data fully saturated the hydrodynamic limit.

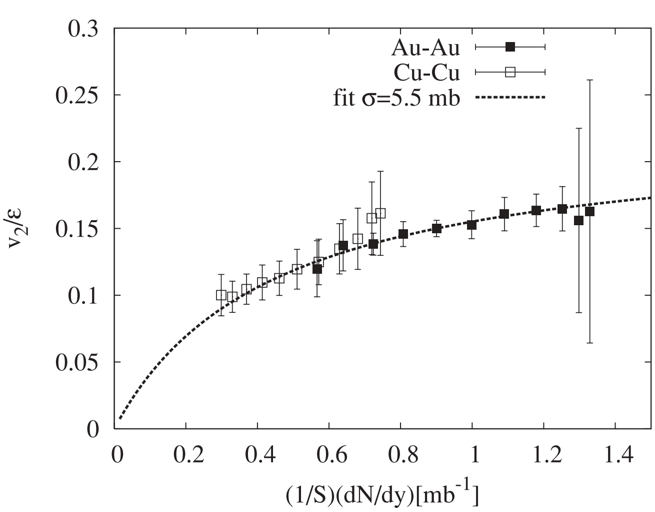

FIG. 2. Same as Fig. 1, but using CGC initial conditions.

Our results suggest that ideal hydrodynamics *should,* in fact, overpredict the measured flow; that is, one should not choose an EoS in perfect-fluid simulations that fits the data. Rather, the EoS could be extracted by comparing ideal hydrodynamics to $v_2^{\text{hydro}}/\varepsilon$.

The next result is that CGC initial conditions, which predict a higher initial eccentricity, ε, naturally lead to a lower hydrodynamic limit, $v_2^{\text{hydro}}/\varepsilon$. Now, close to the ideal-gas limit $(c_s = 1/\sqrt{3})$, $v_2^{\text{hydro}}/\varepsilon$ scales approximately like the sound velocity c_s [4]. This means that CGC initial conditions imply an average speed of sound lower (softer equation of state) than that implied by Glauber initial conditions, by a factor of $0.22/0.3 \simeq 0.73$.

The second fit parameter is the partonic cross section σ. The larger σ, the faster the saturation of v_2/ε as a function of $(1/S)(dN/dy)$. For our standard values of K_0 and c_s we obtain $\sigma = 4.3 \pm 0.6$ mb for Glauber initial conditions and $\sigma = 5.5 \pm 0.5$ mb for CGC initial conditions. These values are significantly smaller than those found in previous transport calculations [26], but match the findings of Ref. [27].

CGC initial conditions imply a value of σ larger than that implied by Glauber initial conditions, that is, a *lower* viscosity. This can be easily understood. As already mentioned above, the CGC model predicts a larger eccentricity ε than the Glauber model for semi-central collisions of large nuclei (when there is a large asymmetry in the local saturation scales of the collision partners, along a path in impact-parameter direction away from the origin [19]). However, for very peripheral collisions or small nuclei, there is of course very little asymmetry in the saturation scales, and the eccentricity approaches the same value as that in the Glauber model. This has been checked numerically in Fig. 7 of Ref. [21] and can also be clearly seen by comparing our figures: while in Fig. 2 v_2/ε for semi-central Au − Au collisions is lower than v_2/ε in Fig. 1, there is no visible difference for peripheral Cu − Cu collisions. In all, with CGC initial conditions the scaled flow grows less rapidly with the transverse density, which is the reason for the larger elementary cross section.

The dependence of σ on the initial conditions is probably even stronger than the numerical values above suggest, for the following reason. As alluded to above, our fit to the data really determines the product $K_0\sigma c_s$, rather than σ alone. It appears reasonable to assume that K_0 does not depend on the initial conditions. However, for consistency, the speed of sound c_s entering the Knudsen number should match the one underlying the hydrodynamic limit $v_2^{\text{hydro}}/\varepsilon$; hence, if CGC initial conditions require a smaller c_s by a factor 0.73, the elementary cross section obtained above should be rescaled accordingly. This leads to our final estimate $\sigma_{\text{CGC}} \simeq 7.6 \pm 0.7$ mb.

Our numerical results for σ should be taken as rough estimates rather than precise figures, because of the uncertainties related to the precise values of K_0 and c_s. It is, however, tempting to convert them into estimates of the shear viscosity η, which has been of great interest lately. A universal lower bound, $\eta/s \geqslant 1/4\pi$ (where s is the entropy density), has been conjectured using a correspondence with black-hole physics [28], and it has been argued that the viscosity of QCD might be close to the lower bound. Extrapolations of perturbative estimates to temperatures $T \simeq 200$ MeV, on the other hand, suggest that the viscosity of QCD could be much larger [29]. On the microscopic side, η is related to the scattering cross section σ. Following Teaney [30], the relation for a classical gas of massless particles with isotropic differential cross sections (which applies, for example, to a Boltzmann-transport model) is $\eta = 1.264T/\sigma$ [31]. On the other hand, the entropy density of a classical ultrarelativistic gas is $s = 4n$, with n the particle density, so that

$$\frac{\eta}{s} = 0.316\frac{T}{c\sigma n} = 0.316\frac{\lambda T}{c}. \qquad (6)$$

The relevant particle density in Au-Au collisions at RHIC, which is estimated at the time when v_2 develops [4], is 3.9 fm^{-3} for both Glauber and CGC initial conditions, and $T \simeq 200$ MeV. Our two estimates $\sigma = 4.3$ mb (Glauber initial conditions) and $\sigma = 7.6$ mb (CGC initial conditions) thus translate into $\lambda = 0.60$ fm, $\eta/s = 0.19$ and $\lambda = 0.34$ fm, $\eta/s = 0.11$, respectively. These values for η/s agree with those from Ref. [32] if the mean-free path is scaled to our result and also with estimates of η/s based on the observed energy loss and elliptic flow of heavy quarks [33], on transverse momentum correlations [34], or on bounds on entropy production [6]. Hence, for our best fit(s) η/s is slightly larger than the conjectured lower bound, but significantly smaller than extrapolations from perturbative estimates. On the other hand, our lower value is close to a recent lattice estimate [35] for SU(3) gluodynamics, which gives $\eta/s = 0.134 \pm 0.033$ at $T = 1.65T_c$.

A complementary approach to incorporate corrections from the ideal-fluid limit is viscous relativistic hydrodynamics. A formulation that is suitable for applications to high-energy heavy-ion collisions has been developed in recent years [36]. A first calculation of elliptic flow [37] shows that for Glauber initial conditions and $\eta/s = 0.16$, v_2 reaches about 70% of the ideal-fluid value for semi-central Au-Au collisions. It is interesting to note that our simple estimates are in good agreement with this finding. Using Eq. (6), $\eta/s = 0.16$ corresponds to $\sigma = 5.1$ mb, for which Eqs. (2) and (4) give $v_2/v_2^{\rm hydro} = 0.68$. The comparison to experimental data in Ref. [37], however, appears to favor lower values of η/s because the EoS used there underpredicts the $v_2^{\rm hydro}/\varepsilon \simeq 0.3$ required for Glauber initial conditions. Alternatively, simulations could be performed with CGC initial conditions that require only $v_2^{\rm hydro}/\varepsilon \simeq 0.22$.

In summary, we have shown that the centrality and system-size dependence of the *measured* v_2 can be understood as follows: v_2 scales like the initial eccentricity ε (as predicted by hydrodynamics), multiplied by a correction factor due to off-equilibrium (i.e., viscous) effects. This correction involves the multiplicity density in the overlap area, $(1/S)(dN/dy)$. Two types of initial conditions have been compared: a Glauber-type model and a CGC approach. PHOBOS data can be described with both. In particular, there is good agreement between Cu-Cu and Au-Au data. The resulting estimates for thermodynamic quantities and transport coefficients, on the other hand, depend significantly on the initial conditions.

Color glass condensate-type initial conditions require *lower* viscosity and a *softer* equation of state (smaller speed of sound). The scaled flow extrapolated to vanishing mean-free path is lower than that for Glauber initial conditions by a factor of $\simeq 0.22/0.3 = 0.73$; the effective speed of sound should also be lower by about the same factor. Our estimates for the viscosity are $\eta/s \simeq 0.19$ for Glauber initial conditions and $\eta/s \simeq 0.11$ for CGC initial conditions, but these numbers should be taken only as rough estimates.

We have also shown that the data for the scaled flow indeed *saturate* at high densities to a hydrodynamic limit. In central Au-Au collisions at RHIC, v_2 reaches 70% (resp. 75%) of the hydrodynamic limit for Glauber (CGC) initial conditions. The corrections to ideal hydrodynamics are therefore significant, but reasonably small compared to unity, implying that (viscous) hydrodynamics should be a valid approach for understanding flow at RHIC. Also, the asymptotic limit of v_2/ε has been isolated and could now be used to test realistic equations of state from lattice QCD with hydrodynamic simulations of heavy-ion collisions.

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