Toninelli, Biroli, and Fisher Reply: *On the universality of jamming percolation.*—In Ref. [1], we introduced a class of kinetically constrained models which display a dynamical glass transition: Above a critical density ρ_c, there appears an infinite cluster of forever *frozen* particles. At ρ_c, the density of frozen particles is discontinuous, while as $\rho \nearrow \rho_c$ there is an exponentially diverging crossover length. This *jamming percolation* behavior in two dimensions is a consequence of two perpendicular directed-percolation (DP)-like processes which together can form a frozen network of DP segments ending at T junctions with perpendicular DP segments.

In Ref. [1], we focused on a particular example: the "knights" model. As correctly pointed out by Jeng and Schwarz (JS) [2], we overlooked some frozen structures which are *not* simple DP paths: These "thicker" directed structures lower the critical density. Here we argue that, nevertheless, the full directed processes are in the DP universality class, and the T junctions between these give rise to a jamming percolation transition with the *same* universal properties [1]. Moreover, for a simpler model our results are rigorous. The "spiral" model is similar to the knights model except that blocking of a particle is by either its N, NE and S, SW or its W, NW and E, SE pairs of neighbors as in Fig. 1(b) [cf. Fig. 1(b) in Ref. [1]]. There are two directed processes, in the NNE-SSW and the ESE-WNW directions, which are congruent to DP processes on a square lattice. As infinite occupied DP paths are here necessary and sufficient for an infinite frozen structure, $\rho_c = \rho_c^{\mathrm{DP}}$ and our results are rigorous [3].

We claim that the knights and spiral models are in the same universality class. In each diagonal direction of the knights model, there are two infinite sequences of thicker and thicker DP processes, SDP and NDP, for which the existence of percolating occupied paths are, respectively, *sufficient* and *necessary* for an infinite blocked cluster. JS's structures are in the SDP sequence. We conjecture that the limits of *both* sequences belong to the DP universality class and the limit points of their critical densities coincide at some ρ_c^∞. Thus, there are two perpendicular sets of DP paths and effective T junctions between them which cause a jamming transition at $\rho_c^{\mathrm{knights}} = \rho_c^\infty$. To test this, we performed simulations on two types of long diagonal strips of length L and width $W \propto L^\zeta$, with $\zeta \simeq 0.63$ the DP anisotropy exponent. Boundary conditions empty on the sides and filled on top and bottom focus on SDP paths: $P_S(\rho, L)$ is the probability of such a frozen spanning cluster. Boundary conditions filled on one side but empty on the other side and on top and bottom focus on NDP paths (which prevent the arbitrary expansion of large holes [3]): $P_N(\rho, L)$ is the probability of some frozen particles in the *open half* of such strips. Both the P_S and the P_N data cross at the *same* critical density $\rho_c \simeq 0.6359$ and display good scaling with $(\rho - \rho_c)L^{1/\nu}$, where $\nu \simeq 1.73$ is the parallel correlation length exponent for DP: Figure 1(a) shows data for P_S. We find the *same* behavior for the spiral model (with a different ρ_c). This yields strong support for the conjectured universality of the DP-like processes on large length scales. Thus, the arguments in Ref. [1] for the glass transition can be applied to both models. As predicted, the density of *frozen* particles ϕ is nonzero at ρ_c: Figure 1(c) shows $\phi(\rho_c, L)$ for the knights model. The T-junction interactions between the two DP processes are thus crucial as the density of unidirectional DP clusters vanishes at the transition.

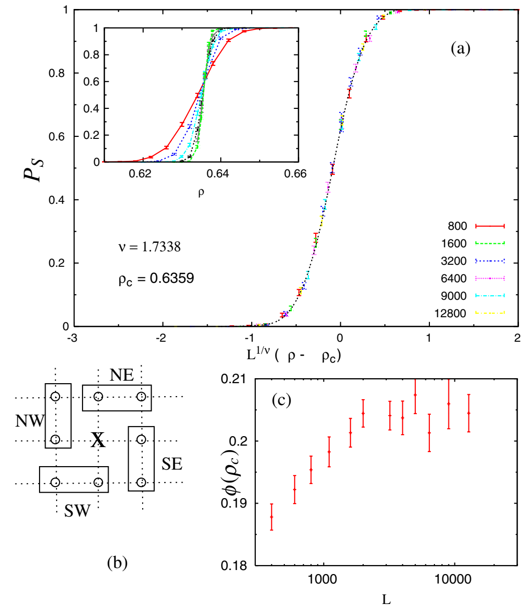

FIG. 1 (color online). (a) P_S as a function of ρ. (b) NE, NW, SE, and SW neighbors. (c) $\phi(\rho_c, L)$ as a function of L.

To summarize, our results in Ref. [1] are rigorous for the spiral model with $\rho_c = \rho_c^{\mathrm{DP}}$. For the knights models, $\rho_c \neq \rho_c^{\mathrm{DP}}$ as JS show. Nevertheless, our numerical results strongly suggest that the critical behavior is the same.

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