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First analysis of anisotropic flow with Lee–Yang zeros

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We report on the first analysis of directed and elliptic flow with the new method of Lee–Yang zeros. Experimental data are presented for Ru+Ru reactions at 1.69*A* GeV measured with the FOPI detector at SIS/GSI. The results obtained with several methods, based on the event-plane reconstruction, on Lee–Yang zeros, and on multiparticle cumulants (up to fifth order) applied for the first time at SIS energies, are compared. They show conclusive evidence that azimuthal correlations between nucleons and composite particles at this energy are largely dominated by anisotropic flow.

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The study of collective flow in relativistic heavy ion reactions is of great interest since it is expected to shed light on our knowledge about the properties of hot and dense nuclear matter and the underlying equation of state (EoS) [1]. As pointed out early on, nuclear collective flow is also influenced by the momentum-dependent interactions and the in-medium nucleon-nucleon cross section [2,3]. Both effects play a crucial role in the determination of the EoS and cannot be neglected at intermediate energies. In this regard both directed and elliptic flow are a field of intense experimental and theoretical researches (see [4] and references therein).

Most flow analyses, based either on the reaction plane reconstruction (the so-called event-plane method) [5] or on two-particle azimuthal correlations [6], rely on the assumption that the only correlations are those stemming from the existence of the reaction plane. Other correlations (usually called nonflow), such as small-angle correlations due to final state interactions and quantum statistical effects [7], correlations due to resonance decays [8] and mini-jet production [9] are neglected. In recent years, several alternative techniques were introduced, in which nonflow correlations can be unraveled. The cumulant method is based on a cumulant expansion of multiparticle (typically four-particle) correlations [10], which eliminates most nonflow correlations. It has been applied at ultrarelativistic energies, at RHIC and SPS, for directed and elliptic flow studies and also for higher harmonic measurements [11–15]. More recently, a new method based on an analogy with the Lee–Yang theory of phase transitions [16], where flow is extracted directly from the genuine correlation between a large number of particles, has been proposed [17–19]. This method is expected to provide the cleanest separation between flow and nonflow effects.

We present the first analysis of collective flow using the new method of Lee–Yang zeros. The cumulant method is also applied, for the first time at SIS energies. A comparison with results obtained with the event-plane method is performed. We are thus able to check for the first time the validity of standard methods at SIS energies, by investigating possible contributions of correlations unrelated to the reaction plane

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which could introduce distortions on directed and elliptic flow results.

The data set presented in this work concerns Ru+Ru reactions at 1.69*A* GeV measured with the FOPI detector installed at the SIS accelerator facility of GSI-Darmstadt. FOPI is an azimuthally symmetric apparatus made of several subdetectors which provide charge and mass determination over nearly the full 4π solid angle. The central part ($33^\circ < \theta_{\rm lab} < 150^\circ$) is placed in a superconducting solenoid and consists of a drift chamber (CDC) surrounded by a barrel of plastic scintillators. Particles measured in the CDC are identified by their mass using magnetic rigidity and energy loss. The forward part is composed of a wall of plastic scintillators ($1.2^\circ < \theta_{\rm lab} < 30^\circ$) and an other drift chamber (Helitron) mounted inside the superconducting solenoid. The plastic wall provides charge identification of the reaction products, combining time of flight and specific energy loss informations. For the present analysis, the forward wall and the CDC were used. More details on the configuration and performances of the different components of the FOPI apparatus can be found in Ref. [20].

The events are sorted out according to their degree of centrality by imposing conditions on the multiplicity of charged particles measured in the outer plastic wall ($7^\circ < \theta_{\rm lab} < 30^\circ$) [21], named PMUL. The flow analysis presented here was carried out for about 2.9×10^6 events belonging to the centrality class labeled PMUL4, which corresponds to a mean geometrical impact parameter of 2.9 fm and to a geometrical impact parameter range from 1.6 to 3.9 fm, obtained assuming a sharp-cutoff approximation [22].

We recall that directed flow (v_1) and elliptic flow (v_2) are quantified by Fourier coefficients of the azimuthal distributions [23], $v_n = \langle \cos n(\varphi - \varphi_R)\rangle$, where φ is the particle azimuthal angle and φ_R is the azimuth of the reaction plane.

In the conventional method, the reaction plane is estimated event by event according to the standard transverse momentum procedure devised in [5], which allows to construct the event-plane vector

$$\mathbf{Q} = \sum_\nu \omega_\nu \mathbf{u}_\nu. \tag{1}$$

The sum runs over all charged particles in the event, except pions identified in the CDC. $\mathbf{u}_\nu$ is the unit vector parallel to the particle transverse momentum (i.e., $\mathbf{u}_\nu = (\cos\varphi_\nu, \sin\varphi_\nu)$, where φ_ν is the particle azimuth), and ω_ν is a weight to improve the resolution, depending on the scaled center-of-mass (c.m.) rapidity $y^{(0)} = (y/y_p)_{\rm c.m.}$ (the subscript p refers to the projectile): $\omega_\nu = -1$ for $y^{(0)} < -0.3, \omega_\nu = +1$ for $y^{(0)} > 0.3$, and $\omega_\nu = 0$ otherwise. The azimuth of $\mathbf{Q}$, denoted by Ψ_R, is an estimate of φ_R.

The Fourier coefficients v_n are calculated using the formula

$$v_n\{EP\} \equiv \frac{\langle \cos n(\varphi - \Psi_R)\rangle}{\langle \cos n\Delta\varphi_R\rangle}, \tag{2}$$

where $\{EP\}$ stands for "event-plane." In the numerator, the particle of interest is excluded from the sum in Eq. (1) to avoid autocorrelation effects. The resolution factor $1/\langle\cos n\Delta\varphi_R\rangle$ is an estimate of the error $\Delta\varphi_R = \Psi_R - \varphi_R$ on the determination of the reaction plane. This factor is calculated according to the procedure proposed in [24], and involves the correlation between randomly chosen subevents [5]. The numerical values for the PMUL4 centrality class are $1/\langle\cos\Delta\varphi_R\rangle = 1.17$ for directed flow and $1/\langle\cos 2\Delta\varphi_R\rangle = 1.69$ for elliptic flow, corresponding to a resolution parameter $\chi \approx 1.47$ [24]. Several procedures have been developed in order to take into account correlations due to overall transverse momentum conservation, by using standard methods [25,26]. Here, to subtract these correlations the event-plane method has been improved by introducing a recoil correction, as proposed in Ref. [25].

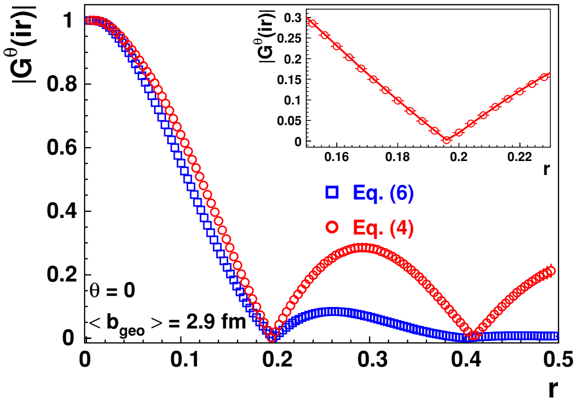

FIG. 1. (Color online) $|G^\theta(ir)|$ versus r for $\theta = 0$. A zoom of $|G^\theta(ir)|$ [Eq. (4)] around its first minimum is shown in the insert and the solid line is just to guide the eye. See text for details.

Let us now recall the principle of the Lee–Yang zeros procedure to analyze flow. A more complete description of the method can be found in Refs. [17–19]. The method is based on the location of the zeros, in the complex plane, of a generating function of azimuthal correlations, in close analogy with the theory of phase transitions of Lee and Yang [16]. The first step of the procedure is to determine the "integrated" directed flow, defined as the average projection of $\mathbf{Q}$ on the (true) reaction plane

$$V_1 \equiv \langle Q_x \cos\varphi_R + Q_y \sin\varphi_R\rangle_{\rm events}, \tag{3}$$

where Q_x, Q_y are the components of $\mathbf{Q}$, and the average is taken over events in a centrality class. For this purpose, one introduces the following complex-valued generating function [19]:

$$G^\theta(ir) = \left\langle \prod_\nu [1 + ir\omega_\nu \cos(\varphi_\nu - \theta)] \right\rangle_{\rm events}, \tag{4}$$

where r is a positive real variable, θ is an arbitrary reference angle, ω_ν is the same weight as in Eq. (1), and φ_ν is the particle azimuthal angle.

Figure 1 displays the amplitude of the generating function $|G^\theta(ir)|$ versus r for $\theta = 0$ (circles). It starts at a value of 1 for $r = 0$ and quickly decreases as r increases, which means that particles are strongly correlated: for uncorrelated particles, indeed, $|G^\theta(ir)|$ is identically equal to unity within statistical fluctuations. A sharp minimum of $|G^\theta(ir)|$ then occurs, which is in fact compatible, within statistical fluctuations, with a zero of $G^\theta(ir)$ (see insert in Fig. 1). Following the general arguments presented in Refs. [17–19], this is a clear indication that correlations are due to collective flow. The position r_0^θ of the first minimum yields an estimate V_1^θ of the integrated flow V_1:

$$V_1^\theta = \frac{j_{01}}{r_0^\theta}, \tag{5}$$

where $j_{01} = 2.40483$ is the first root of the Bessel function $J_0(x)$. Strictly speaking, like other flow analysis methods, the present one is only able to determine the absolute value of V_1. The sign is assumed to be positive at these energies.

A potential limitation of the method comes from statistical errors, which can be much larger than with the event-plane method. The reason why statistical errors depend on the method used is that the reaction plane is unknown, and that the v_n are obtained through the indirect observation of a correlated emission [5]. The statistical errors depend on the observables used to characterize this correlation. The important quantity here is the resolution parameter χ, related to the well-known event-plane resolution [24]. If $\chi > 1$, which means that the reaction plane can be reconstructed with reasonable accuracy, all methods yield statistical errors of the same order of magnitude as if the reaction plane was exactly known while systematic errors from nonflow effects are expected to be much smaller with Lee–Yang zeros. If $\chi < 0.5$, on the other hand, statistical errors prevent the use of Lee–Yang zeros. For the present analysis, we find $\chi \approx 1.45$, which definitely indicates that statistical errors are not a problem here (see Figs. 2 and 3).

An alternative form of the generating function, which can be used instead of Eq. (4), is [17]

$$G^\theta(ir) = \left\langle \exp\left(ir \sum_\nu \omega_\nu \cos(\varphi_\nu - \theta)\right)\right\rangle_{\text{events}}. \tag{6}$$

This gives the squares in Fig. 1. There is no obvious relation between Eqs. (4) and (6). In particular, the latter differs from unity for $r > 0$ even if particles are uncorrelated because of “autocorrelation” terms. But quite remarkably, the first minimum occurs at the same place with either form. This is a further indication that it is due to flow, as anticipated in Ref. [17] (see in particular Appendix A). The analysis of differential directed and elliptic flow presented below was carried out using Eq. (4). The results obtained with Eq. (6) are very similar, thereby confirming that the method is insensitive to autocorrelations.

Once the first minimum r_0^θ has been determined, the Fourier coefficients are estimated from the following equation:

$$v_n^\theta \propto \mathrm{Re}\left\langle \cos n(\varphi - \theta) \prod_\nu{}' \left[1 + ir_0^\theta \omega_\nu \cos(\varphi_\nu - \theta)\right]\right\rangle, \tag{7}$$

where φ is the azimuthal angle of the analyzed particle, and the notation $\prod'$ means that the particle of interest is excluded from the product in order to avoid autocorrelations. The average is over a particle type in a given phase-space region, in all events. A proportionality constant ensures that the result is consistent with the estimate of the integrated flow V_1^θ. Its expression can be found in Ref. [19].

The procedure is repeated for several values of θ (typically, five equally spaced values from 0 to $4\pi/5$), and the results are found to be independent of θ except for statistical fluctuations. This demonstrates that the results are not affected by detector azimuthal asymmetries. The final estimates shown below are averaged over θ, which reduces the statistical errors by about a factor of 2.

Before we come to the results, let us say a few words about the cumulant method [10], which has been already applied by several experiments [11–15]. This method makes use of multiparticle correlations to estimate directed and elliptic flow. One can construct several independent estimates of v_1 and v_2, depending on how many particles are correlated: 2 or 4 for v_1, 3 or 5 for v_2. The number of particles involved is referred to as the order of the cumulant. Lowest-order estimates of v_1 and v_2 are not corrected for nonflow effects, and therefore are expected to be similar to estimates from the event-plane method without recoil correction. The higher the order, the smaller the bias from nonflow correlations. Lee–Yang zeros are essentially the limit of cumulants when the order goes to infinity, and therefore minimize the bias from nonflow effects.

The features of differential directed and elliptic flow at SIS energies have been discussed in several publications [27, 28]. Here, we focus on the comparison between the different procedures investigated in this work.

In the following figures only statistical errors are shown. Possible sources of systematic uncertainties have been studied using IQMD events [29] passed through a complete GEANT simulation of the detector. We found that the full simulation underestimates v_1 of protons (deuterons) by about 6% (4%), in relative value, in the phase space region under consideration and for data integrated over transverse momentum (p_t). These distortions are mainly due to a track-density effect which leads to a loss of particles in the directed flow direction. They are independent of the procedure.

The differential directed flow calculated with the method of Lee–Yang zeros (circles) is shown in Fig. 2 for protons (upper panel) and deuterons (lower panel) in a rapidity window in the backward hemisphere. The values are compared to those obtained from the event-plane analysis (squares). Also shown are the second (stars) and fourth (crosses) cumulant values.

A first look at Fig. 2 shows that all methods give similar results. This proves that azimuthal correlations between nucleons and composite particles at SIS energies are dominated by anisotropic flow and that nonflow correlations, if any, are of smaller magnitude. Figure 2 also shows that cumulants and Lee–Yang zeros can be successfully used to analyze anisotropic flow at SIS energies.

A detailed examination of the results in Fig. 2 reveals however that there are small differences between the methods, beyond statistical errors at high p_t. First, there is a small difference between the event-plane method and the second-order cumulant. This is due to the recoil correction for overall momentum conservation, which is applied in the event-plane method, but not in the cumulant method. We have checked that the event-plane method without recoil correction and the second-order cumulant give compatible results. It is important to emphasize that four-particle cumulants and event-plane results differ. The difference between second-order cumulant and fourth-order cumulant is also observed in analyses of elliptic flow at RHIC [11], where discrepancies are larger in relative value. There, it was suggested that the difference may be due to fluctuations of the flow within the sample of

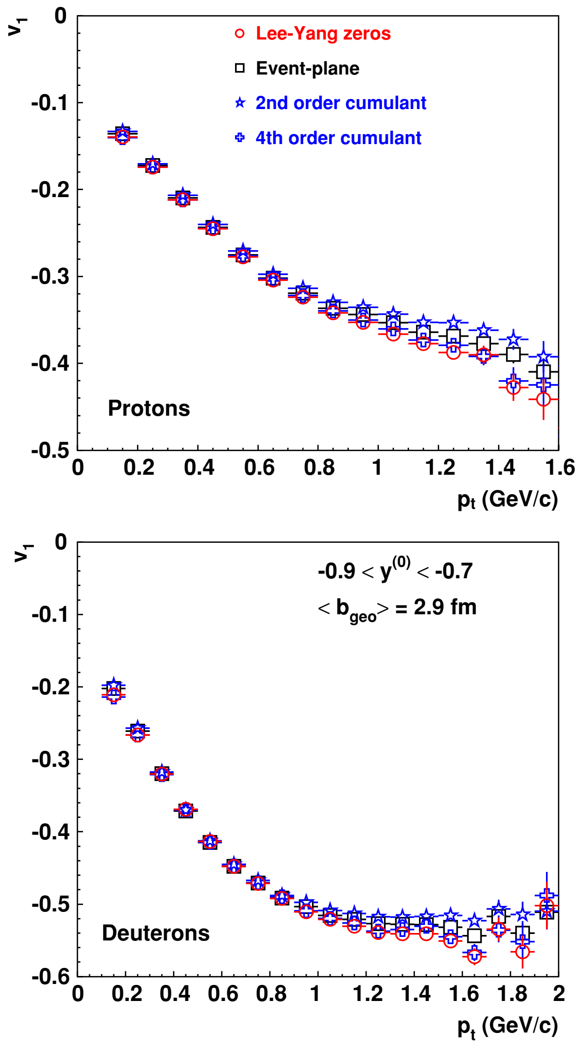

FIG. 2. (Color online) v_1 versus transverse momentum for protons (upper panel) and deuterons (lower panel) measured in semicentral events and in a rapidity window in backward hemisphere. See text for details.

events, corresponding to variations in the impact parameter or in the initial conditions [11,30]. This effect leads to smaller flow estimates with the four-particle cumulant than with the second-order cumulant, in absolute value, independently of p_t. The opposite behavior is evidenced in Fig. 2, from which one concludes that such fluctuations are not responsible for the observed differences. It seems therefore more likely that they are due to nonflow correlations. The fact that they increase with p_t suggests that they are mostly due to overall transverse momentum conservation [31]. The recoil correction which has been used in the event-plane method to correct for this effect relies on a nonrelativistic formalism [25] and that may explain the difference relative to the fourth-order cumulant. Moreover, it is worth stressing the fact that overall momentum conservation, which is a long-range effect involving all particles, effectively behaves as a short-range correlation [17,31]. As a consequence it is eliminated by using fourth-order cumulant. On the other hand, results from four-particle cumulants and Lee–Yang zeros are perfectly compatible. This lends support to the idea that both methods are able to extract reliably the genuine collective flow at SIS energies.

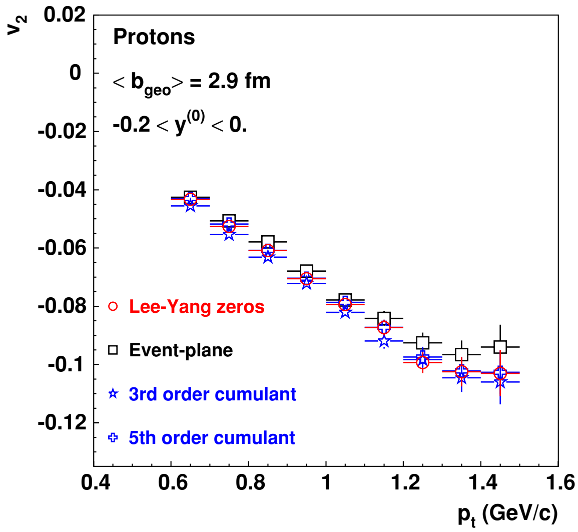

FIG. 3. (Color online) v_2 versus transverse momentum for protons measured in semicentral events and around midrapidity. See text for details.

Figure 3 displays the proton differential elliptic flow, estimated from the same methods. The results now concern the midrapidity region ($-0.2 < y^{(0)} < 0.$) and the corresponding p_t range of the CDC acceptance. Effects of nonflow correlations such as correlations due to momentum conservation are expected to be less pronounced on elliptic flow than on directed flow. Indeed, the differences between the methods are smaller, in absolute value, than for directed flow and can be considered as almost negligible within statistical error bars. However the general trend seems to be that the lowest-order cumulant (stars) gives a slightly larger signal than Lee–Yang zeros (circles) and fifth-order cumulant (crosses), at all p_t. This small difference could be due to impact parameter fluctuations [11] within the PMUL4 centrality bin which increase the estimates of v_2 from cumulants, in absolute value, independently of p_t. This bias is expected to be more pronounced for the third-order cumulant than for high-order cumulants. v_2 values from the event-plane method (squares) are not distorted by such fluctuations because of the high accuracy on the reaction plane determination. Similar trends have been also obtained for deuterons.

In summary, we have presented the first analysis of directed and elliptic flow in heavy-ion collisions using the method of Lee–Yang zeros. Results were obtained from the FOPI experiment at GSI. Such method is expected to provide the best possible separation between correlations due to flow and other correlations. We were thus able to check explicitly that most azimuthal correlations between protons and composite particles at SIS energies are due to their correlation with the reaction plane of the collision. There is no evidence for event-by-event fluctuations of directed flow. Nonflow effects are small; they are clearly seen only on directed flow at high p_t, and may be entirely ascribed to global transverse momentum conservation. They are eliminated using four-particle cumulants or the Lee–Yang zeros procedure. Results were presented only for semicentral events, for sake of brevity. The analysis was also carried out for other centrality classes, covering an impact parameter range up to 7 fm, and led to similar conclusions.

Such analysis is promising for studying pion flow. Since most pions originate from Δ decays, the resulting nonflow correlations with protons may contaminate the flow analysis if standard procedures are applied. The Lee–Yang zeros method, which is insensitive to nonflow effects, should be able to provide reliable results.

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