

# Topological M Theory from Pure Spinor Formalism

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We construct multiloop superparticle amplitudes in 11d using the pure spinor formalism. We explain how this construction emerges in the superparticle limit of the multiloop pure spinor superstring amplitudes prescription. We then argue that this construction points to some evidence for the existence of a topological M theory based on a relation between the ghost number of the full-fledged supersymmetric critical models and the dimension of the spacetime for topological models. In particular, we show that the extensions at higher orders of the previous results for the tree and one-loop level expansion for the superparticle in 11 dimensions is related to a topological model in 7 dimensions.

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## 1. Introduction

We learned from [1,2] that stringy and membrane corrections to 11d supergravity can be captured by the superparticle limit of superstring or supermembrane. This was confirmed by the recent work [3] where the covariant quantized version of superparticle with the method of pure spinors [4] was employed. However, that work was limited to tree and one-loop analysis and the measure for such 11d superparticle amplitudes was discussed. Importantly it was remarked in [3] that the *full*<sup>1</sup> two derivatives effective action for the 11d supergravity can be obtained from the Chern-Simons action

$$S_{M-th} = \int \langle U^{(3)} Q U^{(3)} \rangle + \langle U^{(3)} [U^{(1)}, U^{(3)}] \rangle + \dots \quad (1.1)$$

where  $\langle \cdot \cdot \cdot \rangle$  is a bracket defined with the tree-level measure from the highest scalar element in the (restricted) zero momentum cohomology<sup>2</sup> group  $H^{(7)}(Q|p.s.)$  for Berkovits' Pure Spinor formalism [4,3] with the normalization

$$\langle \lambda^7 \theta^9 \rangle = 1, \quad (1.2)$$

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<sup>1</sup> We mean that not only the cubic and quartic couplings necessary for the linear supersymmetry are correctly described by 3-point and 4-point amplitudes in this theory, but as well the non linear terms needed for covariant answer. For instance from the 3 gravitons scattering one can complete the linearized equation of motion derived in [4]. Details will be given elsewhere [5].

<sup>2</sup> The abbreviation *p.s.* stands for “pure spinors” and reminds that the cohomology is computed in the restricted functional space (see below for the explicit form of the constraint).

where  $\lambda^A$  are the 11d pure spinors (see below for their definition) and  $\theta^A$  are 11d Majorana spinors of  $Spin(1, 10)$ . This formula states that the ghost number of the scalar measure for  $[\mathcal{D}\lambda]$  is +16 [3,6]. More generally the total ghost number for the measure of integration  $\mathcal{D}\lambda$  is the sum of the ghost number and the fermion number of the vacuum defined by  $\langle \lambda^p \theta^n \rangle = 1$ .

The pure spinor approach is based on a BRST operator  $Q = \lambda^A d_A$  such that  $Q^2 = P_M (\lambda \Gamma^M \lambda)$  and  $\lambda^A$  a commuting spinor.  $d_A$  is the fermionic constraint for the 11d Brink-Schwarz superparticle

$$S = \int d\tau \left( P_M \Pi^M - \frac{e}{2} P_M P^M \right), \quad (1.3)$$

where  $\Pi^M = \dot{x}^M + \frac{i}{2} \theta \Gamma^M \dot{\theta}$  is the supersymmetric line element and  $P^M$  is the conjugate momentum to the bosonic coordinate  $x^M$ . Together with  $\theta^A$ , they form the coordinates of 11d superspace. The Dirac matrices  $\Gamma_{AB}^M$  are symmetric and real and satisfy the Fierz identities  $(\Gamma_M)_{(AB} (\Gamma^{MN})_{CD)} = 0$ .

The BRST operator  $Q$  squares to zero modulo the reducible constraints

$$\lambda^A (\Gamma^M)_{AB} \lambda^B = 0, \quad \text{with} \begin{cases} m = 0, \dots, 9 & \text{for } d=10 \\ m = 0, \dots, 9, 11 & \text{for } d=11 \end{cases} \quad (1.4)$$

The quantized theory has the gauged fixed action [7]

$$S = \int d\tau \left( P_M \dot{x}^M - \frac{1}{2} P_M P^M + p_A \dot{\theta}^A + w_A \dot{\lambda}^A \right), \quad (1.5)$$

where  $p_A = d_A - \frac{i}{2} P_M (\Gamma^M \theta)_A$  and  $w_A$  is the conjugate momentum of  $\lambda^A$ . The action is invariant under the gauge transformation  $\delta w_A = \zeta_M (\Gamma^M) \lambda_A$  generated by the pure spinor constraints (1.4).

The physical states are identified by the BRST cohomology and for our purposes we are interested in two types of cohomologies: for  $P_M \neq 0$  (where the only non-vanishing cohomologies  $H^{(3)}(Q|p.s.) \simeq H^{(4)}(Q|p.s.) \neq 0$  and  $H^{(n)}(Q|p.s.) = 0$  for  $n \neq 3, 4$ ) and the zero momentum cohomology for  $P_M = 0$  (where  $H^{(n)}(Q|p.s.) \neq 0$  for  $0 \leq n \leq 7$ ). In the latter case,  $Q^2 = 0$ , since  $P_M = 0$ , but we still define the cohomology as the constrained cohomology. Actually the (restricted) zero momentum cohomology group  $H^{(*)}(Q|p.s.)$  contains all the fields of 11d supergravity, the ghosts, the ghost-for-ghosts and the antifields for their symmetries [4,8,9].

The pure spinor constraints (1.4) are reducible and the cohomology is best studied by introducing new ghosts at each level of reducibility and redefining the BRST operators

$Q$ . This amounts to relaxing the constraints and replaced them by new terms in the BRST charge. This approach has been pursued and developed in [10,11,12,13]; there a suitable treatment of the ghost-for-ghost system is obtained by introducing a new quantum number (the grading) and requiring that physical states are in a restricted functional space. Furthermore, in [9], it is shown that a straightforward application of the Homological Perturbation Theory techniques (see for example refs. [14], and [15,16] for the application to string theory) leads to an infinite set of ghost-for-ghosts and the cohomology  $H^{(*)}(Q|p.s.)$  is obtained as a relative cohomology  $H^{(*)}(Q, H^{(*)}(Q'))$  of a second BRST charge  $Q'$ . This charge implements the constraints at the quantum level.

Analogously to 11d superparticle, one can study 10d SYM theory or N=2 10d supergravities as the zero slope approximation of the open/closed superstrings. Denoting by  $Q_o, Q_{L/R}$  and  $Q$  the BRST operators for the open, the closed superstrings and for the supermembrane or their respective superparticle limit, one finds that the zero momentum cohomology for the case of open/closed superstring [8] and the supermembrane [4] reveals that the highest element is contained in the groups  $H^{(3)}(Q_o|p.s.), H_L^{(3)}(Q_L|p.s.) \otimes H^{(3)}(Q_R|p.s.)$  and  $H^{(7)}(Q|p.s.)$ , respectively.

A multiloop prescription for the closed string was constructed in [6], whose particle limit leads to a correct prescription for higher loop computations in quantum field theory using world-line formalism. Here we construct the measure of integration for all higher-loop amplitudes for the 11d superparticle.

One of the major difficulties for a prescription that works for higher loop expansion is that the model is supposed to describe a theory of supergravity. Indeed, by a simple dimensional reduction it reduces to 10d type IIA superparticle. The latter is a particle limit of superstring quantized with the pure spinors constraints. The construction of multiloop formalism can be done by analogy with superstring prescription of [6]. The measure that we are looking for has to fulfill the following requirements; 1) saturation of ghost zero modes for  $\lambda^\alpha$  (those zero modes are bosonic and therefore Dirac delta functions are needed to avoid divergences; those delta functions are indeed present into the picture changing operators); 2) saturation of bosonic ghost  $w_A$ , (again those zero modes are absorbed by the delta functions of picture changing operators); 3) the zero modes of the fermionic  $d_A$ 's which has to saturate the Berezin integration and finally, 4) the number of zero modes of  $\theta^A$  that have to select the correct term of the effective action matching the various non-renormalization theorems for higher-derivative terms.

These requirements imply that the number of insertions of “anti-ghost”  $b$ -field has to be equal to  $6(g - 1)$  for the 10d superparticle, and we will find that the correct number of insertions of  $b$ -field has to be  $7(g - 1)$  which seems to suggest a corresponding number of moduli. In addition, the number of picture changing operators are accordingly obtained. This counting of moduli and the number of insertions match with those of a topological model in 7 dimensions. This existence of this model has been conjectured earlier (see the talk by N. Nekrasov [17]) and a proposal was recently formulated by Gerasimov and Shatashvili [18] (see as well [19]).

We argued that all the topological theories can be derived from the pure spinor approach to open/closed Superstring Field Theory [20] and 11d action [3] by a consistent reduction (see the Table).

highest state	dimension	ghost anomaly	top model
$\lambda^3\theta^5$	$d = 3$	$-8$	open string
$\lambda^6\theta^{10}$	$d = 6$	$-2 \times 8$	closed B-model
$\lambda^7\theta^9$	$d = 7$	$-16$	Top. M-theory [18]

**Table:** This table lists the single state of highest ghost number for the pure spinor cohomology  $H^{(*)}(Q|p.s.)$  for “open” models and in  $H^{(*)}(Q_L|p.s.) \otimes H^{(*)}(Q_R|p.s.)$  for the “closed” models. This state is used for defining the measure of integration for the pure spinor tree-level amplitudes. The dimension is the one of the target space once boundary conditions on the fermionic variables are enforced. The last column lists the name of the theory: in  $d = 3$  and  $d = 7$  we have “open” models and in  $d = 6$  we have “closed” models. Finally the ghost anomaly of the model is the sum of the ghost number and fermionic number of the highest state for “open” models, or half of it for the left and right sector for “closed” models.

After this analysis have been performed a new interesting paper appeared [19] on the archive. There several forms of Form Theories of Gravity in 6 and lower dimensions are studied and their lifting to a 7 dimensional topological M theory. Our result is a rather strong piece of evidence for a topological M-theory at *the quantum level*. Indeed, we focused on the relation between the insertions needed to saturated the path integral measure for ghosts and Grassmann variables that yielded the dimension of the target space

theory which seems to point out that there is a relation between observables in topological M theory and the observables of physical M-theory.

We believe that the present framework gives a complementary view on twisted topological models, where the difficult part is to find the original  $N = (2, 2)$  superstring model from where they originate. For example, in [21], a worldsheet analysis is performed, but a derivation of this model by considering a topological version of the superstring theory on a  $G_2$  manifold is still missing.

The paper is organized as follows. In sec. 2, we present a prescription for higher loop contributions to 11d supergravity corrections by means of worldline methods. Then we discuss the different insertions needed to reabsorb the zero modes and we demonstrate that, at two loops, the zero mode saturation selects the term  $\nabla^4 R^4$  term of the effective action. In sec. 3, we show that, by choosing a suitable gauge fixing for the picture changing operators (the correspondent gauge parameters are parametrized by a spinor  $C_A$  and the 2-form  $B_{MN}$ ), the prescription given in sec. 2 can be reduced to the 10d superparticle prescription given in [6]. In sec. 4, the relation between the ghost number of the tree level measure and a corresponding topological model is exploited. We conclude in section 5 with a dictionary between the pure spinors supersparticle approach of this paper with topological string and M theory. The appendix contains some proof of identities of the main text.

## 2. Higher loop amplitudes for pure spinor superparticle formalism

We briefly recall some ingredients of the multiloop formalism for pure spinor superstrings constructed in [6] and extended to 11d superparticle in [3].

By the analysis pursued in there, we recall that at tree and one-loop the amplitude prescription has a suitable number of unintegrated vertex operators. However, for  $g \geq 2$  only integrated vertex operators, denoted by  $\int d\tau V(\tau)$  with  $\tau$  the world-line coordinate, are needed. The pure spinor formulation is based on the following conjugated pairs of variables  $(\theta^A, p_A)$  and  $(\lambda^A, \omega_A)$  where  $\lambda^A$  is constrained by (1.4).

The fermionic variables  $\theta^A$  have 32 components realized as  $\theta^A = (\vartheta^a, \tilde{\vartheta}^{\tilde{a}})$  with  $a, \tilde{a} = 1, \dots, 16$  for the ten dimensional case or as a 32-component Majorana spinor in eleven dimensions, and the pure spinor  $\lambda^A$ , satisfying (1.4), has 11 complex components in ten dimensions or 23 complex components in eleven dimensions. The conjugated variables  $p_A$

and  $w_A$  have zero modes at higher-loop  $g$  given by  $32 \times g$  for  $p_A$  (in the following we will work with the independent field  $d_A = p_A + \dots$ ) and  $11 \times g$  or  $23 \times g$  for  $w_A$  in 10 and 11 dimensions respectively.<sup>3</sup> For saturating the bosonic ghosts, we need in the path integral measure a corresponding number of Dirac delta function to soak up their zero modes. This can be done by respecting the BRST invariance (and the decoupling of BRST exact operators) by introducing the picture changing operators [22]

$$\begin{aligned} Z_B &= \{Q, \Theta(B_{MN}(\lambda\Gamma^{MN}w))\} = B_{MN}(\lambda\Gamma^{MN}d) \delta(B_{MN}(\lambda\Gamma^{MN}w)) , \\ Z_J &= \{Q, \Theta(\lambda w)\} = (\lambda^A d_A) \delta(\lambda^A w_A) , \\ Y_C &= C_A \theta^A \delta(C_A \lambda^A) \end{aligned} \quad (2.1)$$

where  $B_{MN}$  and  $C_A$  are gauge fixing parameters, and  $\Theta(x)$  is the Heavyside step function. There will be needed as much insertion of  $Z_{B,J}$  as the number of components for  $w_A$ . The parameter  $B_{MN}$  can be chosen in such a way that no normal ordering is needed in the expression for  $Z_B$ . Another ingredient needed is the picture changed anti-ghost  $b_B$ , which satisfies<sup>4</sup>

$$\{Q, b_B\} = Z_B T \quad (2.2)$$

where  $T$  is the stress energy tensor (for the superparticle  $T = P^M P_M$  and  $b_B = \Theta(B_{MN} \lambda \Gamma^{MN} w) T$  see [3] or [23] for more comments). The number of insertion of  $b_B$ -anti-ghost in the multiloop amplitude is the number of its zero modes given by

$$c(g) = 7(g - 1) . \quad (2.3)$$

The path integral measure for  $\lambda^A$  and for the conjugate  $w_A$  are symbolically given by  $[\mathcal{D}\lambda]_{+16}$  and  $[\mathcal{D}N]_{-16}$  where the superscript indicates the ghost charge (their complete expressions are given in [3]). We denote by  $\tau_i$  the Schwinger parameters.

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<sup>3</sup> For the 10d case this is justified by the fact that  $\theta^A$  is a periodic fermion of conformal weight 0 and  $p_A$ , its conjugated momentum, has conformal weight 1. Applying the Riemann-Roch theorem gives that  $\#(\theta)_0 - \#(p)_0 = 1 - g$ , where each component of  $\theta^A$  has one zero mode  $\#(\theta)_0 = 1$ , giving that each components of  $p_A$  has  $g$  zero modes  $\#(p)_0 = g$ .

<sup>4</sup> In the superparticle limit the  $b$ -field is a density. In string theory this quantity is the density formed by the inner product between the Beltrami differential such that  $\int b(t) = \int \mu_{\bar{z}}^z(t) b_{zz}$ .

Finally, the  $g$ -loop  $N$ -point correlation function is given by

$$\begin{aligned} \mathcal{A}_N^g &= \int \mathcal{D}\lambda d^{32}\theta \prod_{i=1}^g \mathcal{D}N_i d^{32}d_i \prod_{j=1}^{c(g)} \int dt_j b_B(t_j) \times \\ &\times \prod_{k=c(g)+1}^{22g} Z_{B_k} \prod_{l=1}^g Z_{J_l} \prod_{m=1}^{23} Y_{C_m} \prod_{n=1}^N \int d\tau_n V_n^{(0)}(\tau_n) \end{aligned} \quad (2.4)$$

As at one-loop (see [3]) we can count the zero modes.

- ▷ The  $\lambda$ -ghost number: The measure  $[\mathcal{D}\lambda]$  contributes to +16, each  $[\mathcal{D}N]$  factors to -16, the  $Z_{B,J}$  collectively contributes to  $23g - c(g)$  and  $Y_C$  to -23, for a total of

$$16 - 16g + 23g - c(g) - 23 = (23 - 16)(g - 1) - c(g) = 7(g - 1) - c(g) \quad (2.5)$$

which is zero for (2.3).

- ▷ We have to saturate the 32 zero modes for the  $\theta^A$ . We have 23 of them from the  $Y_C$ , so we should get 9 of them from the vertex operators.
- ▷ We have  $32g$  zero modes for the  $d_A$  which have to be soaked up by the  $23g - c(g)$  from the  $Z_{B,J}$ , we have  $2N - M$  zero modes from the vertex operators, if  $M$  counts the number of zero modes for the  $N^{MN} = (\lambda\Gamma^{MN}w)$ . For the  $d$ 's coming from the insertions  $b_B$ , we have to use the engineering dimension discussed in [6] and we found that  $c(g) b_B$  contribute to  $8c(g)/3 + 4M/3$ . For a non vanishing amplitude there should be enough  $d$  zero modes coming from the  $b_B$  insertion giving

$$\frac{5}{3}c(g) + \frac{M}{3} + 2N \geq 9g \quad (2.6)$$

Any multiloop prescription should agree with the non-renormalisation theorem in ten [24,25,26] and eleven dimensions [2] that states, for instance, that  $R^4$  is not renormalized above one-loop and that the four gravitons amplitude contributes to at least  $\nabla^4 R^4$  from two-loop and higher.<sup>5</sup> These theorems are consequences of supersymmetry therefore accessible by zero modes counting. We will show the number of zero modes (2.3) for the  $b_B$ -field is the *only* value compatible with the  $R^4$  non-renormalisation theorem.

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<sup>5</sup> The results of [2] point the fact that the  $D^4 R^4$  is as well not renormalized by higher-loop amplitudes, but this result is not a consequence of supersymmetry alone. For instance the absence of corrections from the 3 loops amplitude would be obtained after integration over the moduli and summing all the superparticle diagrams.



We consider four gravitons scattering ( $N = 4$  in (2.4)) at  $g \geq 2$  loop order. The  $R^4$  non-renormalisation theorems stipulate that from two-loop the four gravitons amplitude contributes to the eleven dimensions effective action to at least  $\int d^{11}x \nabla^4 R^4$  to where a suitably contraction of the covariant derivatives  $\nabla_M$  and the Riemann tensor  $R_{MNPQ}$  is understood. Recalling that the integrated vertex operators for the graviton have the structure [6,3]

$$V^{(0)} = \int dt (\dots + \mathcal{M}^{MN} \mathcal{M}^{PQ} \mathcal{R}_{MNPQ} + \dots) e^{ik \cdot X}$$

where we introduced the Lorentz generator  $\mathcal{M}^{MN} = (d\Gamma^{MN}\theta) + (\lambda\Gamma^{MN}w)$  and the superfield  $\mathcal{R}_{MNPQ}(x, \theta) = R_{MNPQ}(x) + \theta^2 \nabla R_{MNPQ}(x) + \dots$ , we only need the following structure from the vertex operators (see as well [6])

$$\prod_{i=1}^4 V_n^{(0)}(\tau_n) \sim (d\Gamma^{M_i N_i} \theta) \prod_{r=1}^7 (\lambda \Gamma^{P_i Q_i} w) \mathcal{R}_{M_i N_i P_i Q_i}(x, \theta) \quad (2.7)$$

The  $\theta^A$  coordinate zero modes counting showed that 9  $\theta$ 's should comes from the vertex operator part (2.7) which implies that 8  $\theta$ 's have to be extracted from the curvature superfields and the expression contributes to four derivatives. With  $N = 4$  and  $M = 7$  we can check that (2.6) is always satisfied for  $g \geq 2$ .

### 3. Reduction to 10d

Now, since this seems to give the correct counting for all loop amplitude, we would like to provide also an heuristic argument to support the number  $c(g) = 7(g - 1)$  as the correct number of insertions of  $b_B$  by comparing (2.4) with type IIA superstring amplitudes. We recall that the superstring amplitudes (for  $g > 1$ ) are computed by the prescription of [6]

$$\mathcal{A}_N^g = \int \mathcal{D}\lambda_L d^{32}\theta_L \mathcal{D}\lambda_R d^{32}\theta_R \prod_{i=1}^g \mathcal{D}N_{i,L} d^{32}d_{i,L} \prod_{i=1}^g \mathcal{D}N_{i,R} d^{32}d_{i,R} \quad (3.1)$$

$$\left| \prod_{j=1}^{3(g-1)} \int dz_j (\mu|b_{B,L})(z_j) \prod_{k=3(g-1)+1}^{11g} Z_{B_k} \prod_{m=1}^{22} Y_{C_m} \right|^2 \prod_{n=1}^N \int d^2 z_n V_n^{(0)}(z_n, \bar{z}_n)$$

where  $L/R$  refers to the left- and right-mover sectors of the superstrings. In the following we will focus on the superparticle limit of this amplitude.<sup>6</sup>

The measure, the picture changing operators and the insertions (except the vertices) can be factorized into left and right-parts. There the usual counting of moduli  $6(g-1)$  (the number of moduli for a punctured Riemann surface) leads to  $6(g-1)$  insertions of  $b_B$ 's. They are folded with the Beltrami differentials and each of  $b_B$  carries one picture changing operator  $Z_B$ . Notice also that the number of picture lowering operators  $Y_C$  soak up correctly the 22 zero modes for left and right-pure spinors  $\lambda_{L/R}$ . The following relation  $(2N^{mn} - \delta^{mn} J) (\gamma_m)_{ab} = 0$  valid at the classical level where  $N^{mn} = (\lambda\gamma^{mn}w)/2$  and  $J = \lambda w$  states that one can trade the ghost current  $J$  for one Lorentz generator. We make the same choice in the definition of the multiloop amplitude in 11d (2.4). This will make connection between the superstring prescription (3.1) and the 11d prescription (2.4) clearer.

Reducing the superstring to superparticle, it is easy to show that the above prescription is still valid and provide the correct results for radiative corrections to the four gravitons scattering at two loops. The difference between the 11 dimension superparticle and the 10 dimensions N=2 superparticle can be seen directly by counting the number of  $b_B$  insertions, since (3.1) has  $6(g-1)$  insertion when the 11d superparticle needs  $6(g-1)+1$  insertions.<sup>7</sup>

The supplementary zero modes arises when relaxing the constraints  $\lambda\Gamma^{11}\lambda = 0$  which is the eleven dimensions implementation of the condition  $b_0^- = 0$  [4,3]. Using the Fierz

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<sup>6</sup> In this limit the counting of moduli is the same as it can be understood from the plumbing fixture procedure. Namely adding a loop to a vacuum superparticle loop diagram requires 3 parameters: two for the position of each insertion point (the punctures) and one for the length of the line connecting the two punctures. The amplitudes are then constructed by distributing the vertex operators on the internal lines of the vacuum diagram.

<sup>7</sup> In the superparticle limit there is no Riemann-Roch theorem. There is no Riemann-Roch theorem as well for a 3d membrane world-volume theory without any boundaries. Thinking the superparticle prescription as a limit of superstring amplitudes, one can contemplate the possibility of higher spin ghosts. The Riemann-Roch theorem would require a non integer spin 9/4 ghost system, which does not seem realistic.

identity<sup>8</sup>  $(\Gamma^{MN})_{(AB}(\Gamma_N)_{CD)} \equiv 0$ , we can see that

$$(\lambda\Gamma^{MN}\lambda)(\lambda\Gamma_N\lambda) \equiv 0 \quad (3.2)$$

from which it follows

$$\begin{aligned} (\lambda\Gamma^{11n}\lambda)(\lambda\Gamma_n\lambda) &\equiv 0, \\ (\lambda\Gamma^{mn}\lambda)(\lambda\Gamma_n\lambda) + (\lambda\Gamma^{m11}\lambda)(\lambda\Gamma_{11}\lambda) &\equiv 0, \end{aligned} \quad (3.3)$$

where  $m, n = 0, \dots, 9$ . Imposing the pure spinor constraint  $\lambda\Gamma^m\lambda = 0$  for  $m = 0, \dots, 9$ , the first equation is automatically solved and the second implies either  $\lambda\Gamma^{11}\lambda = 0$  or  $\lambda\Gamma^{11}\Gamma^m\lambda = 0$ . The pure spinor condition in 11d requires that  $\lambda\Gamma^{11}\lambda = 0$ , but if we relax this condition we automatically get the second option  $\lambda\Gamma^{11}\Gamma^m\lambda = 0$ . Using the chiral decomposition of the pure spinor  $\lambda^A = (\lambda_L^\alpha, \lambda_{\tilde{\alpha}, R})$  these two equations are

$$\begin{aligned} \lambda_L^\alpha \gamma_{\alpha\beta}^m \lambda_L^\beta + \lambda_{\tilde{\alpha}, R} \gamma^{m, \tilde{\alpha}\tilde{\beta}} \lambda_{\tilde{\beta}, R} &= 0 \\ (\lambda_L^\alpha \lambda_{\alpha, R}) \lambda_L^\alpha \gamma_{\alpha\beta}^m \lambda_L^\beta &= 0 \end{aligned} \quad (3.4)$$

The choice  $\lambda_L^\alpha \gamma_{\alpha\beta}^m \lambda_L^\beta = 0$  corresponds to pure spinor conditions in 10d for Type IIA superstring (the left and right pure spinors have opposite chirality) found in [8].

Performing this reduction the 23 components of the 11d pure spinor decompose according  $\lambda = (\lambda_L, \lambda_R, \rho_\lambda)$  where  $\lambda_{L,R}$  are the 11 components of 10d pure spinors of [4] and  $\rho_\lambda$  is an extra scalar component arising from the rescaling  $(\lambda_L, \lambda_R) \rightarrow (\rho_\lambda \lambda_L, \rho_\lambda^{-1} \lambda_R)$  preserving the 11d pure spinor (1.4) constraints. Likewise for the conjugated ghost  $w = (w_L, w_R, \rho_w)$ . The measures constructed in [3] decomposes as

$$\begin{aligned} [\mathcal{D}^{23}\lambda]_{+16} &= [\mathcal{D}^{11}\lambda_L]_{+8} \wedge [\mathcal{D}^{11}\lambda_R]_{+8} \wedge [\mathcal{D}\rho_\lambda]_{+0} \\ [\mathcal{D}^{23}w]_{-16} &= [\mathcal{D}^{11}w_L]_{-8} \wedge [\mathcal{D}^{11}w_R]_{-8} \wedge [\mathcal{D}\rho_w]_{+0} \end{aligned} \quad (3.5)$$

The amplitude (2.4) has  $2 \times (11g - 3(g - 1)) + 1$  insertions of the picture raising operators  $Z_B$  which is one more than for the superstring amplitude (3.1), likewise the the number

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<sup>8</sup> We use the following notations:  $G \equiv 0$  for identities true independently of any constraints and  $G = 0$  for constraints. For instance

$$(\lambda\Gamma_{MN}\Gamma^P\lambda)(\lambda\Gamma_{MN}\lambda) \equiv 2(\lambda\Gamma_M\lambda)(\lambda\Gamma^{MN}\lambda) = 0$$

where the equality a consequence of (1.4).

of picture lowering operators  $Y_C$ . But the 11d multiloop amplitude has  $6(g-1) + (g-1)$  insertions of  $b$ -field. The extra  $g-1$   $b$ -fields and the extra  $Z_B$  are exactly the number needed for saturating the  $g$  zero modes for  $\rho_w$ .

▷ *The cohomology for the relaxed constraint*

When relaxing the constraint  $\lambda\Gamma^{11}\lambda = 0$ , the BRST operator for the 11d superparticle  $Q = \lambda^A d_A$  is no longer nilpotent since  $Q^2 = P_{11}\lambda\Gamma^{11}\lambda$ . For  $P_{11} \neq 0$ , we can anyway obtain a nilpotent BRST operator by adding a new pair of ghost fields  $(c, b)$  with the commutation relation  $\{b, c\} = 1$  such that

$$Q_M = Q + cP_{11} - \frac{1}{2}b\lambda\Gamma^{11}\lambda. \quad (3.6)$$

is now nilpotent since  $\{Q_M, P_{11}\} = 0$ . An operator/state in the cohomology of  $Q$  depends on the space-time coordinates  $x^M = (x^m, x^{11})$  and the pure spinor ghost  $\lambda^A$ , and an operator/state in the cohomology for  $Q_M$  depends as well on the  $c$  ghost. In order to prove the equivalence between the cohomology of the original BRST operator and the new one  $Q_M$ , we observe that given a vertex operator  $U^{(n)}(x^M, \lambda)$  of a given ghost number  $n$ , in the constrained cohomology  $\{Q, U^{(n)}\} = \lambda\Gamma^{11}\lambda W^{(n-1)}$  where  $W^{(n-1)}(x^M, \lambda)$  is an auxiliary vertex operator with ghost number  $n-1$ . Acting again with the BRST operator from the left, one gets  $\lambda\Gamma^{11}\lambda(\partial_{11}U^{(n)} - \{Q_M, W^{(n-1)}\}) = 0$ . And since  $\lambda\Gamma^{11}\lambda$  is non-vanishing we conclude that  $\{Q_M, W^{(n-1)}\} = \partial_{11}U^{(n)}$  (notice that we cannot add a second term proportional to  $\lambda\Gamma^{11}\Gamma^m\lambda$  since this quantity vanishes because we assumed that  $\lambda\Gamma^{11}\lambda \neq 0$  in (3.3)). Thus, we can construct the new vertex operator

$$U_M^{(n)}(x^M, \lambda, c) = U^{(n)}(x, \lambda) - cW^{(n-1)}(x, \lambda) \quad (3.7)$$

which satisfies  $\{Q_M, U_M^{(n)}\} = 0$ .

The amplitude is well defined as long as there is enough insertions of  $\delta(w_A)$ . General considerations [27] on picture changing operators ensure that the generic form of a picture raising operator is  $Z_B = \{Q, \Theta(B^A w_A)\}$  and of a picture lowering operator is  $Y_C = C_A \theta^A \delta(C_A \lambda^A)$  and that the amplitude is independent of the gauge fixing parameters  $B^A$  and  $C_A$ . In order to perform the reduction of the 11d superparticle multiloop prescription to the 10d prescription we have to choose appropriately the parameters  $B_{MN}$  and  $C_A$  in the picture lowering and raising operators. We choose the gauge parameters  $B_{MN}$  with the Lorentz indices along the ten dimensional directions  $B_{mn}$  with  $m, n = 0, \dots, 9$  such that

$B_{MN}(\lambda\Gamma^{MN}w) = B_{mn} [(\lambda_L\gamma^{mn}w_L) + (\gamma_R\gamma^{mn}w_R)]$ . And we make a different choice for the gauge fixing constants appearing in the ‘extra’ picture raising and lowering operators

$$\begin{aligned} Z_{11} &= \{Q, \Theta(w\Gamma^{11}w)\} = (w\Gamma^{11}d)\delta(w\Gamma^{11}w) \\ Y_{11} &= \lambda\Gamma^{11}\theta\delta(\lambda\Gamma^{11}\lambda) \\ \{Q, b_{11}\} &= Z_{11}T \iff b_{11} = \Theta(w\Gamma^{11}w)T. \end{aligned} \quad (3.8)$$

First of all we remark that  $Z_{11}$  and  $Y_{11}$  still have ghost number  $+1$  and  $-1$  respectively. These operators are in fact taking care of the zero modes for the scalar ghost component  $\rho_\lambda$  and  $\rho_w$  appearing in (3.5). The choice of the gauge parameter in  $Z_{11}$ , breaks the gauge symmetry of  $w_L$  and  $w_R$  generated by the 10 pure spinor constraints. However, the variation is cancelled by the delta function of the remaining PCO as explained below.

We have to notice the following properties: the combinations  $\hat{\gamma} = \lambda_L^\alpha\lambda_{\alpha,R}$  and  $\hat{b} = w_{\alpha L}w_R^\alpha$  have ghost number  $+2$  and  $-2$ , they are commuting and scalar combination of the pure spinor ghost fields and their conjugates. Moreover, the combinations  $\theta_L^\alpha\lambda_{\alpha R} + \lambda_L^\alpha\theta_{\alpha R}$  and  $w_{\alpha L}d_R^\alpha + d_{\alpha L}w_R^\alpha$  have ghost number  $+1$  and  $-1$ , they are anticommuting and they are also scalars. Let us denote the first two combinations as  $\hat{\gamma}$  and  $\hat{\beta}$ , and the second pair as  $\hat{c}$  and  $\hat{b}$ . Then we observe that the BRST variations of those fields are

$$Q\hat{c} = \hat{\gamma}, \quad Q\hat{\gamma} = 0, \quad Q\hat{\beta} = \hat{b}, \quad Q\hat{b} = P_m(\lambda_L\gamma^m w_R + w_L\gamma^m \lambda_R) \quad (3.9)$$

The last transformation implies that  $Q$  is not nilpotent on the field  $\hat{b}$ . However, if the field  $\hat{b}$  is inserted in the correlation functions, there are the picture changing operator  $Z_B$  containing the delta function  $\delta(B_{MN}\lambda\Gamma^{MN}w)$ . By choosing  $B_{m11} = P_m$ , the variation of  $\hat{b}$  vanishes (changing the gauge parameters  $B_{MN}$  is a BRST exact operation and the amplitude will not change under it). This allows us to view the quartet  $\hat{c}, \hat{b}$  and  $\hat{\gamma}$  and  $\hat{\beta}$  as a topological quartet with an effective BRST charge  $\hat{Q} = \hat{b}\hat{\gamma}$ . This system decouples from the rest of the theory when reducing the amplitude from 11d to 10d. As a further confirmation of this, we notice that for such simple topological model, one can construct the picture changing operators (know also the picture operator in [28,29,30])  $\hat{c}\delta(\hat{\gamma})$  which is BRST invariant (but not BRST exact) and the  $\hat{b}\delta(\hat{\beta}) = \{Q, \Theta(\hat{\beta})\}$  which is the picture raising operator. Those picture changing operator obtained by the gauge fixing in (3.8). The insertions of  $\hat{c}\delta(\hat{\gamma})$  and  $\prod_{k=1}^{g-1} \hat{b}\delta(\hat{\beta})$  in the amplitudes can be established by observing that this system corresponds to Liouville theory with a given background charge [30]. This

is a first step to have a derivation of the higher genus expansion of the amplitudes in [6] and in the present paper.

With these choices the multiloop amplitude (2.4) can be rewritten as the 10d prescription with the factorized expression for the

$$\int \mathcal{D}\rho_\lambda \prod_{i=1}^g \mathcal{D}(\rho_w)_i \int dX_{11} \prod_{i=1}^g d(P_{11})_i Z_{11} Y_{11} \prod_{i=1}^{g-1} b_{11}^i \times \left| \int [\mathcal{D}^{11}\lambda]_{+8} [\mathcal{D}^{11g}w]_{+8} \prod_{i=1}^{11g} Z_{B^i} \prod_{j=1}^{11} Y_C \right|^2 \int \mathcal{V} \dots \int \mathcal{V} \quad (3.10)$$

which is equivalent to the multiloop prescription given in [6] with the replacement of  $g$  of the  $Z_{B^i}$  by  $Z_{J^i}$ .

▷ *First case  $P_{11} = 0$  : the perturbative string amplitudes*

In this case the BRST charge  $Q$  is nilpotent and is the sum  $Q_L + Q_R$  of the BRST charge for the left and right movers for the superstring. All the states in the Hilbert space are independent of  $(X_{11}, P_{11})$  and the ghost  $(c, b)$ . Therefore the first line in (3.10) factorizes completely and we are left with the perturbative superstring multiloop amplitude given in [6].

▷ *Second case  $P_{11} \neq 0$ : Non perturbative contributions*

For constant  $P_{11} = M$ ,  $Q$  is the BRST charge for a D0-brane [31] where  $M$  is its mass. We showed earlier the equivalence between the cohomology of  $Q$  and  $Q_M$ . In the case of a compactification on a circle along the 11<sup>th</sup> dimension, one has  $\partial_{11} U_M^{(n)} = \frac{k}{R} U_M^{(n)}$  where  $k$  is an integer and  $R$  is the radius of the circle  $S^1$  of the compactification, so the loop amplitude prescription (2.4) gives perturbative and non-perturbative amplitudes (with D0-branes) for type IIA. Even for external states independent of  $X_{11}$  and the value of  $P_{11}$ , the intermediate states running the loops will carry a D0-brane charge giving rise to non-perturbative corrections as computed in [1].

#### 4. Relation between ghost number and dimension

In the present section, we propose some pieces of evidence pointing out some relations between the tree level measure for the supersymmetric models (quantized in the pure spinor formalism) and corresponding topological theories.

The pure spinors approach in 10 (respectively in 11 dimensions) gives rise to  $N = 1$  super Yang–Mills (respectively supergravity) equation of motions in 10d (respectively in eleven dimensions), but we show that by an appropriate choice of boundary condition on the fermionic variables  $\theta$ , open string topological model, as well as A/B (closed) string topological model and the 7d topological model of [18] and be derived.

#### 4.1. 10d, the tree level measure and open topological models

The relation seems to point out that to the  $N=1$  10d open superstring is characterized by a ghost number 3 measure, this number has led to the construction of a string field theory-like action [7] of the form (where we neglect for the moment the interactions and also all the complications of the BV formalism by restricting our attention to ghost number one, for a more general situation see for example [32])

$$S_{SYM} = \text{Tr} \left\langle \mathcal{U}^{(1)} Q_o \mathcal{U}^{(1)} \right\rangle + \dots \quad (4.1)$$

for 10d  $N = 1$  super Yang–Mills theory. To define the vertex operator and the fields, we started from superstring type IIB and we identify on a D9-brane the field as  $\theta_L = \theta_R \equiv \theta$ ,  $\lambda_L = \lambda_R \equiv \lambda$ ,  $d_{z\alpha} = d_{\bar{z}\alpha}$  and  $w_{\alpha z} = w_{\alpha \bar{z}}$ . This corresponds to a specific choice of boundary conditions and they implies that  $Q_L = Q_R \equiv Q_o$ . For a more generic situation we refer to [33]. The ghost number of the vertex operator  $\mathcal{U}^{(1)}$  is one and it contains the physical fields [34]

$$\mathcal{U}^{(1)} = \frac{1}{2} (\lambda \gamma^m \theta) a_m(x, \theta) + \frac{i}{12} (\theta \gamma^{mnp} \theta) (\lambda \gamma_{mnp} \chi) + \dots \quad (4.2)$$

The bracket  $\langle \cdot, \cdot \rangle$  is computed with the measure  $\int d\mu_5^{(3)} W_5^{(3)} = 1$  where

$$W_5^{(3)} = \lambda \gamma^{m_1} \theta \lambda \gamma^{m_2} \theta \lambda \gamma^{m_3} \theta \theta \gamma_{m_1 m_2 m_3} \theta. \quad (4.3)$$

This measure factor is defined uniquely by the fact that in ten dimensions for the pure spinor  $\lambda$  satisfying (1.4), and the Fierz identity

$$(\lambda \gamma_m)_\alpha (\lambda \gamma^{m n_1 \dots n_4} \lambda) = 0 \quad (4.4)$$

implies that (see Appendix A)

$$\begin{aligned} \epsilon_{m_1 \dots m_r}{}^{n_1 \dots n_{d-r}} (\lambda \gamma^{m_1} \theta) \dots (\lambda \gamma^{m_r} \theta) &= 0 \quad \text{for } r \geq 6 \\ (\lambda \gamma^{m_1} \theta) \dots (\lambda \gamma^{m_5} \theta) &= -\frac{1}{4} (\lambda \gamma^{m_1 \dots m_5} \lambda) W_5^{(3)} \end{aligned} \quad (4.5)$$

The first formula states that only five  $c^m = \lambda \gamma^m \theta$  are linearly independent, and the second states that on the constraint (1.4)  $W_5^{(3)}$  is the volume density. The  $c^m$  are anti-commuting variables<sup>9</sup> with ghost number +1. The open string BRST charge  $Q_o$  reads

$$Q_o = \lambda^\alpha d_\alpha = \lambda^\alpha p_\alpha - \frac{1}{2} c^m P_m + \frac{1}{4} c^m (\theta \gamma_m \partial \theta) \quad (4.6)$$

▷ *The A-model*

We now show how to obtain the A-model by projecting the action (4.1). For this we restrict ourselves to the space defined by

$$\delta_\theta^2 = \frac{1}{7!} (\theta \gamma_{m_1 \dots m_7} \theta) \delta^{(7)}(y) dy^{m_1} \wedge \dots \wedge dy^{m_7} \quad (4.7)$$

and define the new bracket with the insertion of this  $\delta$ -function<sup>10</sup>

$$\langle \dots \rangle_{CS} = \int d\mu_5^{(3)} \delta_\theta^2 \quad (4.8)$$

with this definition it is not difficult to see that (see Appendix A for details)

$$\begin{aligned} \text{Tr} \langle \mathcal{U}^{(1)} Q_o \mathcal{U}^{(1)} \rangle_{CS} &= \int d^{10}x \int d\mu_5^{(3)} \delta_\theta^2 \mathcal{U}^{(1)} Q_o \mathcal{U}^{(1)} \\ &= \int d^7y \delta^{(7)}(y) \int d^3x \text{Tr} (A_m \partial_n A_p + \chi \gamma_{mnp} \chi) \epsilon^{mnp} \end{aligned} \quad (4.9)$$

The interaction term can be computed along the same line giving

$$\langle \mathcal{U}^{(1)} \mathcal{U}^{(1)} \mathcal{U}^{(1)} \rangle_{CS} = \int d^3x \text{Tr} (A \wedge A \wedge A) . \quad (4.10)$$

Higher point interactions are defined as

$$\text{Tr} \left\langle \mathcal{U}^{(1)} \mathcal{U}^{(1)} \mathcal{U}^{(1)} \left( \int \mathcal{V}^{(0)} \right)^n \right\rangle_{CS} \quad (4.11)$$

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<sup>9</sup> These correspond to the twisted fermions for the topological sigma model on the world-sheet [35].

<sup>10</sup> This constraint amounts to put D-branes boundary conditions on which the superstring ends. The different types of D-branes in the pure spinor formalism are studied in [31] and they coincide with the one from the usual RNS formulation.



The zero picture vertex operator reads [34]

$$\begin{aligned}
\mathcal{V}^{(0)} &= \partial\theta^\alpha a_\alpha(x, \theta) + \Pi^m a_m(x, \theta) + d^\alpha W_\alpha + \frac{1}{2} N_{mn} F^{mn} \\
\Pi^m &= \partial x^m + \frac{1}{2} \theta \gamma^m \partial \theta \\
D_{(\alpha} a_{\beta)} &= (\gamma^m)_{\alpha\beta} a_m; \quad D_\alpha W_\beta = \frac{1}{4} (\gamma_{mn})_{\alpha\beta} F^{mn}
\end{aligned} \tag{4.12}$$

after integrating over the pure spinor  $\lambda$  and the fermions  $\theta$  one is left with the reduced amplitude

$$S_{higher} = \int d^3x \text{Tr} \left( A \wedge A \wedge A \ll (e^{ik \cdot x})^3 \left( \int \widehat{\mathcal{V}}^{(0)} \right)^n \gg \right) \tag{4.13}$$

where only the part  $\widehat{\mathcal{V}}^{(0)} = \partial x^m a_m(x) e^{ik \cdot x}$  of the zero ghost picture vertex operator can contribute. It is important to remark that the gaugino cannot contribute to this interaction term because of the restriction on the number of  $\theta$ s, and being non dynamical it can be integrated out completely. All the higher-point amplitude contains the inverse of the space-time metric  $g^{mn}$  and therefore can be scaled away in the limit  $g_{mn} \rightarrow t^2 g_{mn}$  with  $t \rightarrow \infty$ .

By projecting the 10d pure spinors approach of [8] on a 3 dimensional space using (4.8) and scaling out the metric, we reproduce Witten's 3 dimensional Chern-Simons theory [36] which is the Chern-Simons theory on  $T^*M$  is the string field theory description of the open topological A model (see for example the review [37]). On the restricted space defined by the constraint (4.7), the BRST operator  $Q_o$  reduces to the de Rham differential  $d = c^m \partial_m$ .

▷ *The B model*

We also have to take into account the existence of the topological B model. This is characterized by the fact that, unless we restrict to a Calabi-Yau manifold, the U(1) charge associated to the ghost number is anomalous. We can reproduce the topological B model, by starting from closed superstring of type IIB, and we observe the the ghost number (1, 0) vertex operators of the form

$$\mathcal{U}_L^{(1,0)} = \lambda_L^\alpha A_\alpha(x, \theta_L, \theta_R), \tag{4.14}$$

are BRST closed under  $Q_L$  if [34]  $(\gamma^{m_1 \dots m_5})^{\alpha\beta} D_{L,(\alpha} A_{\beta)} = 0$  where the superfields  $A_\alpha$  depends on both of the coordinates  $\theta_L$  and  $\theta_R$ . This implies that only for  $\theta_R = 0$ , the equations of motion describes the SYM theory on shell. However, the combination

$\mathcal{U}_L^{(1,0)} Q_L \mathcal{U}_L^{(1,0)} \Big|_{\theta_R=0}$  inserted into the tree-level path integral measure vanishes, because of the integration over the  $\theta_R$  variables. Therefore, the only way to get a non-trivial result we have to insert  $W_{5,R}^{(3)}$  the unique element of  $H^{(3)}(Q_R|p.s.)$  and the action is

$$S_h = \text{Tr} \left\langle W_{5,R}^{(3)} \left( \mathcal{U}_L^{(1,0)} Q_L \mathcal{U}_L^{(1,0)} \right) \right\rangle_{CS} + \text{Tr} \left\langle W_{5,R}^{(3)} \left( \mathcal{U}_L^{(1,0)} \right)^3 \right\rangle_{CS} + \dots \quad (4.15)$$

We have as well inserted a  $\delta_{\theta_L}^2$  in the measure for the left fermions as indicated by the subscript  $CS$  on the bracket. Notice that the presence of  $W_{5,R}^{(3)}$  has two purposes: *i*) it saturates the ghost charge of the vacuum and *ii*) by inserting the vertex  $W_{5,R}^{(3)}$ , the Grassmann variables  $\theta_R$  are totally soaked up, and projects  $\mathcal{U}_L^{(1,0)} Q_L \mathcal{U}_L^{(1,0)}$  on the space  $\theta_R = 0$ . As for the A model Chern-Simons action, all higher-point amplitudes are suppressed by scaling away the metric.

There is a close analogy with the holomorphic Chern-Simons theory for the topological B model (here  $\mathcal{M}_6$  is a Calabi-Yau 3-fold)

$$S_{hCS} = \int_{\mathcal{M}_6} \Omega \wedge \text{Tr} \left( A \bar{\partial} A + \frac{2}{3} A^3 \right) . \quad (4.16)$$

The globally defined holomorphic 3-form  $\Omega$  is replaced by the scalar (gauge singlet) measure  $W_{5,R}^{(3)}$  in (4.15). The latter is needed to compensate the ghost current anomaly, in the same way the presence of  $\Omega$  is needed in order to compensate the ghost anomaly of the topological model [38]. The vertex operator  $W_{5,R}^{(3)} = c_R^m c_R^n c_R^p (\theta_R \gamma_{mnp} \theta_R)$ , with  $c_R^m = \lambda_R \gamma^m \theta_R$  can be view as defining the holomorphic 3-form with the identification<sup>11</sup> the anti-commuting ghost  $c^m$  with one forms and  $\theta_R \gamma_{mnp} \theta_R$  with the 3-form  $\Omega_{mnp}$  of SU(3)-structure manifold. We recall that in the superparticle limit, the variable  $\theta$  reduces to its zero mode.

#### 4.2. 10d, $N = 2$ , the tree level measure and the A and B topological models

For the closed topological A/B models, the situation is very similar. Starting from pure spinor superstrings, the tree level measure is obtained by duplicating the  $W_5^{(3)}$  for the left- and right-movers that we denote by  $W_{5,L}^{(3)} W_{5,R}^{(3)}$ . This measure is BRST closed and not BRST exact, so it belongs to the cohomology  $H^{(3)}(Q_L|p.s.) \otimes H^{(3)}(Q_R|p.s.)$ . To construct

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<sup>11</sup> In the case of RNS, the vertex operator  $W_{5,R}^{(3)}$  is replaced by  $c \partial c \partial^2 c e^{2\phi}$  whose interpretation from the target physics is rather obscure. On the other side, in pure spinor formulation the explicit super-Poincaré invariance and the usage of superspace renders the interpretation rather transparent.

a string field theory model, the vertex operator must have ghost number 2,  $\mathcal{U}^{(1,1)}$  and therefore one has to insert an operator  $c_0^-$  to construct a kinetic term (see for example [32])

$$S_{\text{closed}} = \langle \mathcal{U}^{(1,1)} c_0^- (Q_L + Q_R) \mathcal{U}^{(1,1)} \rangle + \langle \mathcal{U}^{(1,1)} \mathcal{U}^{(1,1)} \mathcal{U}^{(1,1)} \rangle. \quad (4.17)$$

However, for pure spinor formulation in 10d there is no  $c_0^-$  to construct the kinetic term, we will show in the next subsection how this arises by reducing the 11d construction of [6,3].<sup>12</sup> By comparing with the topological model, we have to consider closed A/B models whose string field theory description is provided in [42] and in [43] (a string field theory for topological A model is also covered in [44]) and the action is written in terms of a (1,1) form  $A'$

$$S_{KS} = \frac{1}{2} \int A' \frac{1}{\partial} \bar{\partial} A' + \frac{1}{6} \int (A' \wedge A') \wedge A' \quad (4.18)$$

where the inverse differential operator (well-defined on the massive states of the theory) coincides with the ghost field  $c_0^-$ , and being  $b_0^- \equiv \partial$  and  $L_0 - \bar{L}_0 = \Delta - \bar{\Delta}$  with  $\Delta = \partial^\dagger \partial$ , one gets that  $\{c_0^-, b_0^-\} = 1$  on the massive states. Therefore,  $S_{KS} = (A', c_0^- Q A')$  where the differential  $\bar{\partial}$  is identified with the BRST operator. The same mapping is applied for the A-model of [36,44] with the action

$$S_{KG} = \frac{1}{2} \int K \frac{1}{d^{c\dagger}} dK + \frac{1}{6} \int K \wedge K \wedge K \quad (4.19)$$

where now  $b_0^- \equiv d^{c\dagger}$  and  $L_0 - \bar{L}_0 = \Delta - \bar{\Delta}$  with  $\Delta = d^{c\dagger} d$ .

The problem to construct a string field theory action for closed topological model is very similar to the construction above of string field theory for type IIA/B for the full-fledged superstring with pure spinors since there is no  $c_0^-$ .

Notice again the relation between the dimension of the spacetime for the topological model and the ghost number for the level measure and the counting of  $b_B$  insertion. As suggested in [4], the counting of degrees-of-freedom for N=2 type IIA/B superparticle models reveals that there are 8 bosonic degrees of freedom versus 20 fermions degrees-of-freedom. Four of the latter are interpreted as coming from  $c_{0,L}, c_{0,R}$  and  $b_{0,L}, b_{0,R}$  and therefore the level matching condition is not automatically implemented. On the other side, for 11d superparticle, this naive counting of degrees-of-freedom shows that there are 9 bosonic degrees-of-freedom, but only 18 fermion degrees-of-freedom. From the latter 2

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<sup>12</sup> In [39], the first author proposed an action with an infinite number of auxiliary fields (as suggested in [40] and [41]) and this points out that it can be replaced by a non-local action.

of them are read as the  $b_0^+$  and  $c_0^+$ , while  $b_0^-$  and  $c_0^-$  are automatically taken into account. This seems to suggest that in 11d a string field theory action can be indeed found.

In the next section, we explain the origin of the  $c_0^-$  from 11d and as well how the action (4.18) and (4.19) can be derived along the line of the previous sections.

#### 4.3. 11d, the tree level measure and Gerasimov-Shatashvili topological model

We briefly recall some ingredients of 11d pure spinor formalisms. We describe the tree level measure (while the all loop amplitude are described in the previous section) and we argue that from the string field theory action (for the massless fields, so a quantum field theory), which was established in [4] and extended beyond the kinetic term in [3] one can obtain a string field theory action for type IIA string theory. The relation with topological models is seen in the following way: from the tree level measure and from higher loop expansion we found that the dimension of the spacetime for the corresponding topological model should be 7. Recently in [18], it was pointed out that there is a description of the closed topological model type B (whose string field theory is identified with Kodaira-Spencer theory) in term of a local action in one higher dimension. We show that the form of the 7d Hamiltonian of [18] can be indeed guessed from the string field theory for the present 11d superparticle description.

First, we discuss the tree level measure for 11d, then we write the supergravity action in a Chern-Simons form, the relation with the functional by Gerasimov-Shatashvili, and finally we show that reducing from 11d to 10d we found that precisely the eleventh component of the pure spinor constraint leads to  $c_0^-$  discussed above.

$\lambda^A$  denotes a Majorana commuting spinor in 11d,  $A = 1, \dots, 32$ , and it satisfies the 11d pure spinor condition

$$\lambda^A \Gamma_{AB}^M \lambda^B = 0, \quad (4.20)$$

with  $M = 0, \dots, 10$  (notice that the  $\dim_{\mathbf{C}} Spin(10, 1) = 32$ , and the Majorana condition reduce it to  $\dim_{\mathbf{R}} Spin(10, 1) = 32$ . To solve the pure spinor constraints in 11d with signature  $(10, 1)$  we have to use Dirac complex spinors  $\lambda^A$ ).  $\Gamma_{AB}^M$  are  $32 \times 32$  symmetric Dirac matrices. Since the  $\lambda^A \lambda^B$  is a symmetric bi-spinor it can be decomposed into a basis of Dirac matrices as follows

$$32 \lambda^A \lambda^B = \Gamma_M^{AB} (\lambda \Gamma^M \lambda) + \frac{1}{2!} \Gamma_{[MN]}^{AB} (\lambda \Gamma^{[MN]} \lambda) + \frac{1}{5!} \Gamma_{[MNPQR]}^{AB} (\lambda \Gamma^{[MNPQR]} \lambda), \quad (4.21)$$

The first term vanishes thanks to the pure spinor constraint and the pure spinor satisfies the Fierz identity

$$(\lambda\Gamma^M)_A (\lambda\Gamma_{MN}\lambda) = 0 \quad (4.22)$$

This Fierz identity implies that zero momentum cohomology of the BRST operator  $Q$  with the pure spinor condition stop at  $\lambda$ -ghost number 7

$$W_9^{(7)} = \lambda\Gamma^{M_1}\theta \dots \lambda\Gamma^{M_7}\theta\theta\Gamma_{M_1\dots M_7}\theta, \quad (4.23)$$

and that the eleven dimensions supergravity fields and antifields belong to  $H^{(3)}(Q|p.s.) \oplus H^{(4)}(Q|p.s.)$ .

With the measure  $\int d\mu_9^{(7)} W_9^{(7)} = 1$ , one can construct the target space action  $S_{11d}$  by observing that the vertex operator  $\mathcal{U}^{(3)}$  contains the supergravity fields and the BRST charge has ghost number 1. As shown in [4] and extended at non-linear level in [3] we have<sup>13</sup>

$$S_{11d} = \langle \mathcal{U}^{(3)} Q \mathcal{U}^{(3)} \rangle + \langle \mathcal{U}^{(3)} [\mathcal{U}^{(1)}, \mathcal{U}^{(3)}] \rangle + \dots \quad (4.24)$$

As before we restrict the integration by specifying boundary conditions with the insertion of

$$\delta_\theta^2 = \frac{1}{4!} (\theta\gamma_{m_1\dots m_4}\theta) \delta^4(y) dy^{m_1} \wedge \dots \wedge dy^{m_4} \quad (4.25)$$

As before we consider the action

$$S_H = \langle \mathcal{U}^{(3)} Q \mathcal{U}^{(3)} \rangle_{CS} + \langle \mathcal{U}^{(3)} [\mathcal{U}^{(1)}, \mathcal{U}^{(3)}] \rangle_{CS} + \dots \quad (4.26)$$

The vertex operators  $\mathcal{U}^{(3)}$  contains the graviton and the 3-form at order  $\lambda^3\theta^3$  and the gravitino at order  $\lambda^3\theta^4$ , with the expression [4]

$$\begin{aligned} \mathcal{U}^{(3)} = & (\lambda\Gamma^{(M}\theta)(\lambda\Gamma^{N)K}\theta)(\lambda\Gamma_K\theta) g_{MN} \\ & + (\lambda\Gamma^M\theta) (\lambda\Gamma^N\theta) (\lambda\Gamma^P\theta) C_{MNP} \\ & + (\lambda\Gamma^M\theta) [(\lambda\Gamma^N\theta)(\lambda\Gamma^P\theta)(\theta\Gamma_{NP}\Psi_M) - (\lambda\Gamma^{NP}\theta)(\lambda\Gamma_N\theta)(\theta\Gamma_P\Psi_M)] . \end{aligned} \quad (4.27)$$

For the interaction term we just need to know that in  $\mathcal{U}^{(1)}$  all the physical fields appear at least at order  $\theta^2$  [3]. This forbids any contributions from the interactions and we are left with the *exact* seven dimensional action for the 3-form

$$S_H = \int d^7x (C \wedge dC + \Psi_{m_1} \Gamma^{m_1 m_2} \Psi_{m_2}) \quad (4.28)$$

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<sup>13</sup> The ellipsis stand for the quartic terms, *e.g.* the four fermions terms, computed in [5], and for higher-point interactions (4-point and higher).

where  $C$  is the three form and  $d$  is the de Rham differential. As before upon the restriction imposed by (4.25), the BRST operator  $Q$  reduced to  $c^M \partial_M$ . Gerasimov and Shatashvili showed that by Hamiltonian reduction how to obtain from (4.28) the Kodaira-Spencer theory of [42] by analyzing the a suitable wave function for the path integral. Again the fermion being non-dynamical they can be integrated out from (4.28).

▷ *The level matching condition*

We now show that from the 11d analysis, we can recover the insertion a candidate for  $c_0^-$  confirming the conjecture in [4].

The element  $W_9^{(7)}$  in (4.23) has ghost number 7 and is of order  $\theta^9$ . If we relax the constraint  $\lambda\Gamma^{11}\lambda = 0$ , then

$$Q_M W_9^{(7)} = (\lambda\Gamma^{11}\lambda) W_8^{(6)}, \quad (4.29)$$

where  $W_8^{(6)}$  is the vertex operator to be identified with the closed string zero momentum cohomology at highest ghost number. By a simple counting, one sees that the BRST differential  $Q_M$  reduces the number of  $\theta$ 's by an unity and therefore  $W_8^{(6)} \sim \lambda^6 \theta^8$  which does not match the states  $(\lambda^6 \theta^{10})$  in  $H^{(3)}(Q_L|p.s.) \otimes H^{(3)}(Q_R|p.s.)$ . But, we have also to recall that by eliminating the constraint  $\lambda\Gamma^{11}\lambda = 0$ , the number of possible invariants with ghost number seven increases. There is another term of the form  $W_{11}^{(7)} \sim \lambda^7 \theta^{11}$  such that

$$Q_M W_{11}^{(7)} = (\lambda\Gamma^{11}\lambda) W_{10}^{(6)}. \quad (4.30)$$

Solving this equation at zero momentum, with  $Q_M = Q_L + Q_R$  gives (see Appendix A for an alternative derivation)

$$W_{11}^{(7)} = (\lambda\Gamma^{11}\theta) W_{5,L}^{(3)} W_{5,R}^{(3)}. \quad (4.31)$$

where  $Q_{L/R} W_{5,L/R}^{(3)} = 0$ . This gives the relation between the tree level measure for 11d and that of the type IIA N=2 superparticle

$$\langle \mathcal{U}^{(3)} Q \mathcal{U}^{(3)} \rangle_{\lambda\Gamma^{11}\lambda \neq 0} = \langle \mathcal{U}^{(1,1)} c_0^- (Q_L + Q_R) \mathcal{U}^{(1,1)} \rangle \quad (4.32)$$

with

$$c_0^- = \lambda\Gamma^{11}\theta. \quad (4.33)$$

Notice that the factor  $\lambda\Gamma^{11}\theta$  impose the addition constraint  $c_0^-$  for the level matching.

We are finally able to confirm explicitly the conjectured relation between the 11d measure needed to write the type IIA string action in a covariant way. The Kähler two-form of the action (4.19) for the A model arises from the ghost number 2 element of the cohomology

$$K = \frac{1}{2} \lambda_L \gamma^m \theta_L \lambda_R \gamma^n \theta_R (\theta_L \gamma_m \gamma_n \theta_R) \quad (4.34)$$

which correspond to a vertex operator of IIA superstring  $\mathcal{U}^{(1,1)} = \lambda^\alpha \lambda^{\tilde{\alpha}} A_{\alpha\tilde{\alpha}}$  with constant RR field  $P_{\tilde{\beta}}^\alpha = \delta_{\tilde{\beta}}^\alpha f$  where  $f$  is a constant coefficient. (This is dual to  $\star F_{10}$ . The potential  $F_{10} = dC_9$  couples to the  $D_8$  branes.) Notice that it is peculiar that this non-propagating degrees-of-freedom of the superstring provides here the Kähler form. Inserted in (4.17) we reproduce the action (4.19).

And what about type IIB? As is well known, the problem of self-dual 5-form affects the construction of a kinetic term for string field theory in the usual way. However, there are several alternatives: one is to use an infinite number of field or non-polynomial expressions as we discussed above.

## 5. A dictionary

In the present section, we propose a dictionary between pure spinor formulation of superstrings, superparticle and supermembranes and topological theories on manifold with special holonomies.

Let us start from the case of open superstring. We found that the monomial  $W_5^{(3)}$ , dual to the path integral measure on the zero modes, yields the 3-form  $\theta \gamma_{mnp} \theta$ . This form resemble the usual calibration for compactification of string theory on a space with special holonomy. The spinor bilinear  $\theta \gamma_{mnp} \theta$ , built from the  $\theta^A$  zero modes, can be identified with the 3-form for a 3-fold (a Lagrangian submanifold) coinciding with its volume form. This provides a dictionary between the open superstring with topological A model. If we consider the supersymmetric sector of the heterotic string as the pure spinor string theory, we can identify the 3-form  $\theta_L \gamma_{mnp} \theta_L$  as the holomorphic 3-form. If the right moving sector is provided by a topological string on a 3-fold CY, we can construct a topological B model.

Let us now consider closed superstring model. In that case the volume form is provided by the product  $W_{5,L}^{(3)} W_{5,R}^{(3)}$ . In this case we identified the holomorphic sector of the CY with the left moving sector of superstring and vice-versa the anti-holomorphic sector with the right movers. In this way we can identify the 3-form  $\theta_L \gamma_{mnp} \theta_L$  with the holomorphic

3-form and  $\theta_R \gamma_{mnp} \theta_R$  with the antiholomorphic component. Notice that they identify the CY space with holonomy  $SU(3)$ .

In the case of 11d, we have found that the tree level measure for superparticle (and for supermembrane) is  $W_9^{(7)}$  giving the four form  $\theta \Gamma_{MNPQ} \theta$ . The dimension of the manifold is identified by the total ghost number of  $W_9^{(7)}$  (and from the number of  $b$ -field insertion in the higher-loop formula). The four form  $\theta \Gamma_{MNPQ} \theta$ , restricted to 7 dimensions is dual to the 3-form which is together with the four form provide the complete characterization of the  $G_2$ -holonomy space.

The construction of this paper exhibits special states in the pure spinor cohomology associated with invariant forms characterizing manifolds of special holonomy  $SU(3)$  and  $G_2$ . The vertex operators for these forms are part of the measure of integration of the effective Chern-Simons models, and are crucial for consistency of the model (with the boundary discussed in the main text). The forms are made from the zero modes of the fermionic coordinates due to the superparticle approximation, but a similar construction from a pure spinor formulation of the superstrings [34] and the supermembrane [4] would give non-constant invariant forms (the superparticle is the zero mode approximation of the superstring or the supermembrane). Finally, in order to verify the correctness of the present dictionary, it would be interesting to provide also a mapping between the amplitudes and try to see which sectors of the correlation functions can be indeed computed by the using topological models.

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## Appendix A. Technicalities and proofs of various identities

- ▷ **The gaugino part of (4.9):** For this it is the convenient to use the following representation of the integration measure (4.3)

$$\int d\mu_5^{(3)} (\lambda \gamma^{m_1} \theta) \cdots (\lambda \gamma^{m_3} \theta) (\theta \gamma^{m_4 \cdots m_{10}} \theta) = \frac{1}{7!} \epsilon^{m_1 \cdots m_{10}} \quad (\text{A.1})$$



and the following Fierz identity

$$(\theta\gamma^{mnp}\theta)(\lambda\gamma_{mnp}\chi)(\theta\gamma^{rst}\theta)(\lambda\gamma_{rst}\chi) \propto (\theta\gamma^{p_1\cdots p_3}\theta)(\theta\gamma^{p_4st}\theta)(\lambda\gamma_{p_1\cdots p_5}\lambda)(\chi\gamma^{stp_5}\chi) \quad (\text{A.2})$$

▷ **Proof of the identity (4.5).**

In 10d the pure spinors  $\lambda$  satisfies the Fierz identities

$$\begin{aligned} \lambda_\alpha\lambda_\beta &= \frac{1}{16 \cdot 5!} (\lambda\gamma_{p_1\cdots p_5}\lambda) (\gamma^{p_1\cdots p_5})_{\alpha\beta} \\ (\lambda\gamma_m)_\alpha (\lambda\gamma^{mn_1\cdots n_4}\lambda) &= 0 \end{aligned} \quad (\text{A.3})$$

Considering

$$\begin{aligned} 16 \cdot 5! (\lambda\gamma^m\theta)(\lambda\gamma^n\theta) &= (\lambda\gamma_{p_1\cdots p_5}\lambda)(\theta\gamma^m\gamma^{p_1\cdots p_5}\gamma^n\theta) \\ &= (\lambda\gamma_{p_1\cdots p_5}\lambda) \left[ (\theta\gamma^{mp_1\cdots p_5}n\theta) - 20\delta_{[p_1p_2}^{mn} (\theta\gamma_{p_3\cdots p_5]}\theta) \right] \end{aligned} \quad (\text{A.4})$$

we can show that

$$\begin{aligned} \epsilon_{m_1\cdots m_r}^{n_1\cdots n_{10-r}} (\lambda\gamma^{m_1}\theta) \cdots (\lambda\gamma^{m_r}\theta) &= -\frac{1}{16 \cdot 5!} (\lambda\gamma_{p_1\cdots p_5}\lambda) (\lambda\gamma^{m_1}\theta) \cdots (\lambda\gamma^{m_{r-2}}\theta) \\ &\times \left[ \frac{8!}{3!} \delta_{m_1\cdots m_{r-2} n_1\cdots n_{10-r}}^{p_1\cdots p_5 q_1\cdots q_3} (\theta\gamma_{q_1\cdots q_3}\theta) + 20 \epsilon_{m_1\cdots m_{r-2}}^{n_1\cdots n_{10-r} p_1 p_2} (\theta\gamma^{p_3\cdots p_5}\theta) \right] \end{aligned} \quad (\text{A.5})$$

The first term vanishes for  $r-2 > 3$  because of the second identity in (A.3). The second term vanished because

$$\begin{aligned} (\lambda\gamma_{p_1\cdots p_5}\lambda) \epsilon_{m_1\cdots m_{r-2}}^{n_1\cdots n_{10-r} p_1 p_2} &= 2! (\lambda\gamma^{m_1\cdots m_{r-2} n_1\cdots n_{10-r}} \gamma_{p_3\cdots p_5}\lambda) \\ &= \frac{2!8!}{5!} (\lambda\gamma^{[m_1\cdots m_{r-2} n_1\cdots n_{10-r}]} \lambda) \delta_{p_3\cdots p_5}^{n_8\cdots n_{10-r}} \end{aligned} \quad (\text{A.6})$$

which vanish for  $r-2 > 3$  when plugged back into the second term of (A.5).

▷ **Derivation of (4.31)**

We start from the ten-dimensional left and right measures

$$\begin{aligned} (\lambda_L\gamma^{m_1}\theta_L) \cdots (\lambda_L\gamma^{m_5}\theta_L) &= (\lambda_L\gamma^{m_1\cdots m_5}\lambda_L) W_{5,L}^{(3)} \\ (\lambda_R\gamma^{m_1}\theta_R) \cdots (\lambda_R\gamma^{m_5}\theta_R) &= (\lambda_R\gamma^{m_1\cdots m_5}\lambda_R) W_{5,R}^{(3)}. \end{aligned} \quad (\text{A.7})$$

Multiplying these two equations using that for  $m = 0, \dots, 9$   $\lambda\Gamma^m\theta = \lambda_L\gamma^m\theta_L + \lambda_R\gamma^m\theta_R$  and that  $\wedge^r (\lambda_{L/R}\gamma^m\theta_{L/R}) = 0$  for  $r > 5$  and  $(\lambda_{L/R}\gamma^{[m_1\cdots m_5}\lambda_{L/R})(\lambda_{L/R}\gamma^{m_6\cdots m_{10]}}\lambda_{L/R}) = 0$  for each of the ten-dimensional chiral pure spinors, we get that

$$(\lambda\Gamma^{m_1}\theta) \cdots (\lambda\Gamma^{m_{10}}\theta) = (\lambda\Gamma^{11}\Gamma^{m_1\cdots m_5}\lambda)(\lambda\Gamma^{11}\Gamma^{m_6\cdots m_{10}}\lambda) W_{5,L}^{(3)} W_{5,R}^{(3)} \quad (\text{A.8})$$

Multiplying this equation by  $\lambda\Gamma^{11}\theta$  and Fierzing the  $\lambda$ s on the right-hand-side we have

$$\epsilon_{M_1\cdots M_{11}} (\lambda\Gamma^{M_1}\theta) \cdots (\lambda\Gamma^{M_{11}}\theta) \propto (\lambda\Gamma^{11}\lambda)^2 (\lambda\Gamma^{11}\theta) W_{5,L}^{(3)} W_{5,R}^{(3)}. \quad (\text{A.9})$$

Which gives the vertex operator  $W_{11}^{(7)}$  of (4.31).

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