Localized gravity in non-compact superstring models *

Emmanuel Kohlprath [a] **and** Pierre Vanhove [b]

[a] *Centre de Physique Théorique*
École Polytechnique
91128 Palaiseau-Cedex, France
email: emmanuel.kohlprath@cpht.polytechnique.fr
[b] *Service de Physique Théorique, CEA/DSM/PhT*
Unité de recherche associée au CNRS
CEA/Saclay, 91191 Gif-sur-Yvette, France
email: pierre.vanhove@cea.fr

Abstract We discuss a string-theory-derived mechanism for localized gravity, which produces a deviation from Newton's law of gravitation at cosmological distances. This mechanism can be realized for general non-compact Calabi-Yau manifolds, orbifolds and orientifolds. After discussing the cross-over scale and the thickness in these models we show that the localized higher derivative terms can be safely neglected at observable distances. We conclude by some observations on the massless open string spectrum for the orientifold models.

1 Introduction

Extra dimensions are a natural concept in string theory which brings many new options on how to think about gravity couplings in our world. Interestingly, when the extra dimensions are non-compact the dilution of the gravitational interactions into the bulk affects the effective four-dimensional potential over cosmological scale. This was first pointed out in the so-called DGP model [1] where gravity is quasi-localized in four dimensions. Remarkably, this model has late time self-expanding cosmological solutions, which

*SPhT-T04/115, CPHT-PC-051.0904, hep-th/yymmnnn

do not need the introduction of a cosmological constant [2, 3, 4]. The main properties of the DGP model with one or more non-compact extra-dimensions have been reviewed in [5]. In the following we discuss a string theoretical setup of induced gravity. We first consider the issue of localizing gravitational interactions following the references [6, 7] (see [8] for a different realization and [9] for a review) and then address the question about constructing a consistent gauge theory sector within the model of localized gravity.

2 The induced gravity model

The DGP model and its generalization are specified by a bulk Einstein-Hilbert (EH) term and a four-dimensional term

$$M^{2+n}\int_{M_{4+n}} d^4x d^n y\,\sqrt{|G|}\,\mathcal{R}_{(4+n)} + M_{pl}^2\int_{M_4} d^4x\sqrt{|g|}\,\mathcal{R}_{(4)}\,, \tag{1}$$

with M and $M_{pl}(=: \sqrt{r_c^n M^{2+n}})$ the (possibly independent) respective Planck scales. The scale $M \geq 1$ TeV would be related to the short-distance scale below which UV quantum gravity or stringy effects are seen. $M_{pl} \sim 10^{19}$ GeV is our four-dimensional Planck mass. The four-dimensional metric is the restriction of the bulk metric $g_{\mu\nu} = G_{\mu\nu}|$ and we assume the WORLD[1] rigid, allowing the gauge $G_{i\mu}| = 0$ with $i \geq 5$. Finally no extrinsic curvature terms (as the Gibbons–Hawking term) are needed.

The effective potential between two test masses in four dimensions [12]

$$\begin{aligned}
\int d^3x\, e^{-ip\cdot x}\, V(x) &= \frac{D(p)}{1 + r_c^n\, p^2\, D(p)}\,\Big[\tilde{T}_{\mu\nu}T^{\mu\nu} - g(p)\,\tilde{T}^{\mu}_{\mu}\,T^{\nu}_{\nu}\Big] &(2)\\
D(p) &= \int d^n q\, \frac{f_w(q)}{p^2 + q^2} &(3)\\
g(p) &= \frac{1}{2}\Big[\frac{(2-2n)p^2 D(p) - 2/r_c^n}{(2-2n)p^2 D(p) - (2+n)/r_c^n}\Big] &(4)
\end{aligned}$$

is a function of the bulk graviton retarded Green's function $G(x,0;0,0) = \int d^4p\, e^{ip\cdot x}\, D(p)$ evaluated for two points localized on the WORLD ($y = y' = 0$). The integral (3) is UV-divergent for $n > 1$ unless there is a non-trivial brane thickness profile $f_w(q)$ of width w. If the four-dimensional WORLD has zero

[1] We avoid calling M_4 a brane, since gravity localizes on singularities of orbifold fixed points [6], orientifold planes [7], and intersection of branes [10]. In all these mechanisms, four-dimensional gravity is induced by loops of localized twisted fields coupled to the background metric. These mechanisms are string theory realizations of the field theory scenario of [1].

thickness, $f_w(q) \sim 1$, the bulk graviton does not have a normalizable wave function. It therefore cannot contribute to the induced potential, which always takes the form $V(p) \sim 1/p^2$ and Newton's law remains four-dimensional at all distances. For a non-zero thickness w, there is only one crossover length scale, R_c:

$$R_c = w\left(\frac{r_c}{w}\right)^{\frac{n}{2}}, \tag{5}$$

above which one obtains a higher-dimensional behaviour [8]. Therefore the effective potential presents two regimes: (i) at short distances ($w \ll r \ll R_c$) the gravitational interactions are mediated by the localized four-dimensional graviton and Newton's potential on the WORLD is given by $V(r) \sim 1/r$ and, (ii) at large distances ($r \gg R_c$) the modes of the bulk graviton dominate, changing the potential. For $n = 1$ the expressions (2) and (3) are finite and unambiguously give $V(r) \sim 1/r$ for $r \gg r_c$.[2] For a co-dimension bigger than 1, the precise behaviour for large-distance interactions depends *crucially* on the UV completion of the theory. Embedding this scenario in string theory will allow us to derive unambiguously all the physical parameters of the model.

At this point we stress a fundamental difference with the *finite extra dimensions* scenarios. In these cases Newton's law gets higher-dimensional at distances smaller than the characteristic size of the extra dimensions.

3 String Theory realization

We explain following [6] how to realize (1) with $n = 6$ as the low-energy effective action of string theory on a non-compact six-dimensional manifold $\mathcal{M}_6$. We work in the context of $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supergravities in four dimensions but the mechanism for localizing gravity is independent of the number of supersymmetries. Of course for $\mathcal{N} \geq 3$ supersymmetries, there is no localization.

In string perturbation, corrections to the four-dimensional Planck mass are in general very restrictive. In the heterotic string, they vanish to all orders in perturbation theory [13]; in type I theory, there are moduli-dependent corrections generated by open strings [14], but they vanish when the manifold $\mathcal{M}_6$ is decompactified; in type II theories, they are constant, independent of

[2] For $n = 1$ the propagator (3) is not UV-divergent, but (5) predicts a critical radius $R_c = \sqrt{w r_c} \ll r_c$ below which graviton's Kaluza–Klein excitations (induced by the cutoff) become massless, and the theory is five-dimensional. See [5] for a lucid discussion of the perturbation theory for the 5d model.

the moduli of the manifold $\mathcal{M}_6$, and receive contributions only from tree and one-loop levels (at least for supersymmetric backgrounds) [6, 15, 16].

The origin of the two EH terms in (1) can be traced back to the perturbative corrections to the eight-derivative effective action of type II strings in ten dimensions. These corrections include the tree-level and one-loop terms given by:[3]

$$
\begin{aligned}
&\frac{M_s^8}{(2\pi)^7}\int_{M_{10}} d^{10}x\sqrt{|G|}\,\frac{1}{g_s^2}\mathcal{R}_{(10)} + \frac{M_s^2}{3(4\pi)^7}\int_{M_{10}} d^{10}x\sqrt{|G|}\left(\frac{2\zeta(3)}{g_s^2}+4\zeta(2)\right)t_8t_8R^4\\
-&\frac{M_s^2}{3(4\pi)^7}\int_{M_{10}}\left(\frac{2\zeta(3)}{g_s^2}\mp 4\zeta(2)\right)R\wedge R\wedge R\wedge R\wedge e\wedge e+\cdots
\end{aligned}
\tag{6}
$$

where M_s is the string scale and ϕ is the dilaton field determining the string coupling $g_s = e^{\langle\phi\rangle}$.

On a direct product space-time $\mathcal{M}_6 \times \mathbb{R}^4$ the term $t_8t_8R^4$ contributes in four dimensions to R^2 and R^4 terms [16] (and to a cosmological constant which is zero due to $\mathcal{N} = 2$ supersymmetry [6]). At the level of zero modes the second R^4 term splits as $\frac{1}{3!(4\pi)^3}\int_{M_6} R\wedge R\wedge R \times \int_{M_4}\mathcal{R}_{(4)} = \chi\int_{M_4}\mathcal{R}_{(4)}$, and we have

$$
\frac{M_s^8}{(2\pi)^7}\int_{M_4\times M_6} d^{10}x\sqrt{|G|}\,\frac{1}{g_s^2}\,\mathcal{R}_{(10)} + \frac{\chi M_s^2}{(2\pi)^4}\int_{M_4} d^4x\sqrt{|g|}\left(-\frac{2\zeta(3)}{g_s^2}\pm 4\zeta(2)\right)\mathcal{R}_{(4)}\,,
\tag{7}
$$

which gives the expressions for the Planck masses M and M_{pl} for type II. A number of conclusions (confirmed by string calculations in [6, 7, 10]) can be reached by looking closely at (7):

$\triangleright$ $M_{pl} \gg M$ requires a large non-zero Euler characteristic for M_6 and/or a weak string coupling constant $g_s \to 0$ ($M_s g_s^{-1/4}$ gives the scale of the $\mathcal{R}_{(10)}$ term and $M_s g_s^{-1}$ the scale of the tree level $\mathcal{R}_{(4)}$ term).

$\triangleright$ Since χ is a topological invariant the localized $\mathcal{R}_{(4)}$ term coming from the closed string sector is universal, independent of the background geometry and dependent only on the internal topology.[4] It is a matter of simple

[3]The rank-eight tensor t_8 is defined as $t_8M^4 \equiv -6(\mathrm{tr}M^2)^2 + 24\mathrm{tr}M^4$, and the $\pm$ sign depends on the chirality (type IIA/B) of the theory. See [17] for more details.

[4]In type IIA/B, χ counts the difference between the numbers of $\mathcal{N} = 2$ vector multiplets and hypermultiplets: $\chi = \mp 2(n_V - n_H)$ (where the graviton multiplet counts as one vector). Field theory computations of [11] show that the Planck mass renormalization depends on the UV behaviour of the matter fields coupling to the external metric. But, even in the supersymmetric case, the corrections are not obviously given by an index.

inspection to see that if one wants to have a localized EH term in less than ten dimensions, namely something linear in curvature, with non-compact internal space in all directions, *the only possible dimension is four* (or five in the strong coupling limit but this setup does not have an interesting phenomenology).

▷ The width is given by the four-dimensional induced Planck mass [6]

$$\omega \simeq M_{pl}^{-1}. \tag{8}$$

4 Orbifold and Orientifold models

The supersymmetric $\mathbb{Z}_N$ orbifolds or orientifolds do not have a tree level induced EH term, but one-loop contributions to the induced Einstein term from the torus $\mathcal{T}$, the annulus $\mathcal{A}$, the Moebius strip $\mathcal{M}$ and the Klein bottle $\mathcal{K}$. We present the arguments of [6, 7] that show that the (quasi-)localization of gravity is purely a closed string sector phenomenon and that the open string sector contributions from $\mathcal{A}$, $\mathcal{M}$ and $\mathcal{K}$ are always subleading or vanishing. Therefore orbifold and orientifold models give the same estimates for the width and for the crossover scale.

▷ The closed string one-loop graviton amplitudes (from the torus) take the form of sums of quasi-localized contributions at the positions of the fixed points x_f [6]. Focusing on one particular fixed point $x_f = 0$ and sending the radii to infinity, we obtain the effective action for the quasi-localized EH term

$$\frac{\chi M_s^2}{24\pi^2} \int d^4x d^6y \sqrt{g} f_w(y)\, \mathcal{R}_{(4)} \tag{9}$$

where $\delta M_{pl}^2 = M_s^2 \times \mathcal{O}(N)$ as $N \to \infty$. For odd N orientifold models the torus contribution is given by one half of the orbifold result (9) and is $\mathcal{O}(N)$. For a more general non-compact background, the Euler number can be distributed over the various fixed points of the internal space, giving rise to different localized terms, with a different value for the induced Planck mass.[5]

▷ The open string sector given by the sum of the contributions $\mathcal{A} + \mathcal{M} + \mathcal{K}$ is always subleading as compared with the torus contribution since $\delta M_{pl}^2 \sim M_s^2 \times \mathcal{O}(1)$ for large-N (actually it even vanishes for orientifold models with odd N that have no $\mathcal{N} = 2$ sectors). Importantly the twisted tadpole cancellation conditions imply that the open string sector contribution to the

[5]For instance, keeping two fixed points we obtained the bi-gravity scenario discussed in [18], with (possibly) different Planck mass at each fixed point depending on the distribution of the twisted fields in the model.

induced Planck mass is ultraviolet finite, and no new scale can arise in these models.

$\triangleright$ As pointed out in [8] higher derivative R^2 terms both on the WORLD and in the bulk can drastically change the picture as they introduce new scales. The induced R^2 terms can be determined similarly to the induced Einstein term by considering the piece in forth order in momentum of graviton amplitudes. The crucial observation is that the torus contribution is again of $\mathcal{O}(N)$ and the sum of the contributions of annulus, Moebius strip and Klein bottle is $\mathcal{O}(1)$ and therefore subleading as compared to the torus contribution. The leading contribution to the terms in the effective action is then

$$\Delta\mathcal{L}^{(4)}_{\text{eff}} = NM_s^2 b\sqrt{|g|}\mathcal{R}_{(4)} + Nc_1\sqrt{|g|}R^2 + Nc_2\sqrt{|g|}R_{\mu\nu}R^{\mu\nu} + Nc_3\sqrt{|g|}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \tag{10}$$

with numbers $b, c_i,\ i = 1, 2, 3$. The induced R^2 terms are negligible for $p^2\, M_s^{-2} \ll b/c_i$. Gravity is only measured above 1 mm so even if M_s is as low as 1 TeV the induced R^2 terms can be neglegted if

$$\frac{b}{c_i} \gg 10^{-32}. \tag{11}$$

E.g. if one computes the forth order piece of the off-shell two gravitons amplitude similarly to [7] one can determine the non-ambiguous coefficient c_3 (the value of c_1 and c_2 are affected by field redefinitions). Though the authors did not evaluate the final world sheet integral explicitly the expression is similar to the one for the coefficient b and (11) is satisfied. It is generic that $\mathcal{O}(c_i) = \mathcal{O}(b)$ as long as the corresponding term is not protected by some symmetry, and the hierarchy is controlled by the value of M_s. The discussion generalizes for all induced higher derivative terms. The conclusion is that the induced higher derivative terms in the orbifold [6] and orientifold models [7] of induced gravity can be neglegted at observable distances. We expect this to be valid for general non-compact Calabi-Yau, too. In contrast to them higher derivative terms in the bulk need further study (see [8] for instance).

5 Phenomelogical implications

The crossover radius of eq. (5) is given by the string parameters (for $n = 6$)

$$R_c = \frac{r_c^3}{w^2} \simeq (2\pi)^{7/2} g_s \frac{M_{pl}^3}{M_s^4} \simeq g_s \times 10^{34}\ \text{cm}\,, \tag{12}$$

for $M^8 = M_s^8/((2\pi)^7 g_s^2)$ and $M_s \simeq 1$ TeV. Because R_c has to be of cosmological scale, the string coupling can be relatively small, and $|\chi| \simeq 10^3 g_s^2\, M_{pl}^2/M_s^2 \sim$

$g_s^2 \times 10^{35}$ must be very large. The hierarchy is obtained mainly thanks to the large value of χ, so that lowering the bound on R_c lowers the value of χ. Our actual knowledge of gravity at very large distances indicates [19] that R_c should be of the order of the Hubble radius $R_c \simeq 10^{28}$ cm, which implies $g_s \geq 10^{-6}$ and $|\chi| \geq 10^{23}$. A large Euler number implies only a large number of closed string massless particles. All these particles are localized at the fixed points and should have sufficiently suppressed gravitational-type couplings, so that their presence with such a huge multiplicity does not contradict observations. In orbifold models we can for instance introduce the observable gauge and matter sectors on D3-branes placed at the position where gravity localization occurs and they are otherwise unconstrained. In orientifold models we already have an open sector and we will determine the massless open string spectrum for some examples in the next section. Note that these results depend crucially on the scaling of the width w in terms of the Planck length: $w \sim M_{pl}^{-\nu}$, implies $R_c \sim M_{pl}^{2\nu+1}$ in string units. If there are models with $\nu > 1$, the required value of χ will be much lower, becoming $\mathcal{O}(1)$ for $\nu \geq 3/2$. In this case, the hierarchy will be determined by tuning the string coupling to infinitesimal values, $g_s \sim 10^{-16}$.

6 Localization of gauge interactions

After having discussed how gravity is localized in non-compact orbifold and orientifold models, we now discuss the localization of gauge interactions. We saw that the hierarchy between the bulk and induced Planck mass required a huge number of twisted fields (i.e. N has to be large). We analyze the open spectrum of the non-compact $\mathbb{Z}_N$ orientifolds constructed in [7] in order to determine if a consistent gauge theory sector can be induced together with gravity.

Let us consider the supersymmetric non-compact type IIB $\mathbb{Z}_N$ orientifold models constructed in [7] and defined by the combined action ΩJ of the worldsheet parity transformation Ω and

$$Z^i \to e^{2i\pi v_i}\, Z_i; \quad J Z^i = -Z^i; \quad \sum_{i=1}^{3} v_i = 0 \ , \tag{13}$$

where $Z^i := X^{2i+2} + i\, X^{2i+3}$ for $i = 1, 2, 3$. The last condition ensure that the model is supersymmetric. We assume N odd, therefore only D3-branes are needed. We assume that they are sitting on top of the orientifold $O3_+$-planes. Since we consider non-compact orientifolds the $\mathbb{Z}_N$ orbifold action need not act cristallographically and we do not need to impose the untwisted tadpole cancellation condition. The Chan-Paton implementation matrices for the n_3 D3-branes will be denoted by the $n_3 \times n_3$ matrices $\gamma_{k,3} = \gamma_{1,3}^k$, $k = 1, \ldots, N-1$.

The massless open string spectrum in the non-compact $\mathbb{Z}_N$ orientifold models of [7] can be determined using the method of [20, 21]. The twisted tadpole cancellation conditions on the Chan-Paton implementation matrices are

$$\begin{aligned} \mathrm{Tr}\ \gamma_{2k,3} &= \pm 4 \prod_{i=1}^{3} \frac{1}{2\cos(\pi k v_i)} && (14) \\ &= \pm 4 \prod_{i=1}^{3} \frac{1}{1+e^{2\pi k v_i}}, \quad k = 1, \ldots, N-1 && (15) \end{aligned}$$

with the positive sign for the SO projection and the negative sign for the Sp projection. Let us assume that N is a prime number and that $v = (1/N, 1/N, -2/N)$. We define $\alpha := e^{2\pi i/N}$ and use $-1 = \sum_{k=1}^{N-1} \alpha^{k\lambda}$, $\lambda = 1, \ldots, N-1$ to find

$$\mathrm{Tr}\ \gamma_{2k,3} = \pm 4 \left(\frac{1}{1+\alpha^k}\right)^2 \frac{1}{1+\alpha^{(N-2)k}} = \mp 4 \left(\sum_{j=1}^{\frac{N-1}{2}} \alpha^{(2j-1)k}\right)^2 \sum_{j=1}^{\frac{N-1}{2}} \alpha^{(2j-1)(N-2)k}. \tag{16}$$

As $\gamma_{1,3} = \gamma_{1,3}^{N+1} = \gamma_{2\frac{N+1}{2},3}$ we arrive at

$$\mathrm{Tr}\ \gamma_{1,3} = \mp \sum_{j_1,j_2,j_3=1}^{\frac{N-1}{2}} \alpha^{j_1+j_2+j_3(N-2)}. \tag{17}$$

There are two cases to distinguish: A) $\frac{N-1}{4} \in \mathbb{Z}$ and B) $\frac{N+1}{4} \in \mathbb{Z}$. We get

$$\begin{aligned} A)\ \mathrm{Tr}\ \gamma_{1,3} &= \mp 4 \Big(\frac{N-1}{4} + \frac{N-1}{4}(\alpha^{-2}+\alpha^2) \\ &\quad + \sum_{j=1}^{\frac{N-1}{4}-1} \Big(\frac{N-1}{4} - j\Big)(\alpha^{-4j} + \alpha^{4j} + \alpha^{-(4j+2)} + \alpha^{4j+2})\Big) \\ B)\ \mathrm{Tr}\ \gamma_{1,3} &= \mp 4 \left(\frac{N+1}{4} + \sum_{j=1}^{\frac{N+1}{4}-1} \Big(\frac{N+1}{4} - j\Big)(\alpha^{-(4j-2)} + \alpha^{4j-2} + \alpha^{-4j} + \alpha^{4j}) \right) \end{aligned} \tag{18}$$

so that $\gamma_{1,3}$ is a $n_3 \times n_3$-block diagonal matrix reading

$$
\begin{aligned}
A)\ \gamma_{1,3} = \mathrm{diag}(&I_{M\mp 4\frac{N-1}{4}}, \alpha^{-2} I_{M\mp 4\frac{N-1}{4}}, \alpha^{2} I_{M\mp 4\frac{N-1}{4}}, \ldots, \\
&\alpha^{-4j} I_{M\mp 4\left(\frac{N-1}{4}-j\right)}, \alpha^{4j} I_{M\mp 4\left(\frac{N-1}{4}-j\right)}, \alpha^{-(4j+2)} I_{M\mp 4\left(\frac{N-1}{4}-j\right)}, \alpha^{4j+2} I_{M\mp 4\left(\frac{N-1}{4}-j\right)}, \\
&\ldots, \alpha^{-(N-1)} I_M, \alpha^{N-1} I_M) \\
B)\ \gamma_{1,3} = \mathrm{diag}(&I_{M\mp 4\frac{N+1}{4}}, \ldots, \alpha^{-(4j-2)} I_{M\mp 4\left(\frac{N+1}{4}-j\right)}, \alpha^{4j-2} I_{M\mp 4\left(\frac{N+1}{4}-j\right)}, \\
&\alpha^{-4j} I_{M\mp 4\left(\frac{N+1}{4}-j\right)}, \alpha^{4j} I_{M\mp 4\left(\frac{N+1}{4}-j\right)}, \ldots, \alpha^{-(N-1)} I_M, \alpha^{N-1} I_M),
\end{aligned} \tag{19}
$$

where $M \in \mathbb{N}$ with $M \geq \pm(N-1)$ for A) and $M \geq \pm(N+1)$ for B). For both A) and B) this gives

$$
n_3 = NM \mp \frac{1}{2}(N^2 - 1) \ . \tag{20}
$$

The freedom of choosing M comes from the fact that we have a non-compact model and do not need to impose the untwisted tadpole cancellation condition.

Let us consider the SO projection (the Sp projection works the same). Similarly to [21] we find the gauge groups

$$
\begin{aligned}
A)\quad & SO\Big(M - 4\frac{N-1}{4}\Big) \times U\Big(M - 4\frac{N-1}{4}\Big) \times \\
& \times \prod_{j=1}^{\frac{N-5}{4}} \Big(U\Big(M - 4\Big(\frac{N-1}{4} - j\Big)\Big)\Big)^2 \times U(M) \qquad (21) \\
B)\quad & SO\Big(M - 4\frac{N+1}{4}\Big) \times \prod_{j=1}^{\frac{N-3}{4}} \Big(U\Big(M - 4\Big(\frac{N+1}{4} - j\Big)\Big)\Big)^2 \times U(M)
\end{aligned}
$$

which we can write as $SO(n_0) \times \prod_{j=1}^{\frac{N-1}{2}} U(n_j)$ and the chiral spectrum

$$
2\left(\sum_{j=1}^{\frac{N-1}{2}} (\Box_{j-1}, \bar{\Box}_j) + \begin{smallmatrix}\Box\\\Box\end{smallmatrix}_{\frac{N-1}{2}}\right) + \left((\bar{\Box}_{\frac{N-1}{2}-1}, \bar{\Box}_{\frac{N-1}{2}}) + \begin{smallmatrix}\Box\\\Box\end{smallmatrix}_1 + (\Box_2, \Box_0) + \sum_{j=1}^{\frac{N-1}{2}-3} (\Box_{j+2}, \bar{\Box}_j)\right) . \tag{22}
$$

One can check that the model is free of non-Abelian and $U(1)^3$ anomalies. Calling n_v the number of $\mathcal{N}=1$ vector multiplets and n_{ch} the number of $\mathcal{N}=1$ chiral multiplets we get

$$
n_v = c_v + \frac{4N}{3} + \frac{N^3}{6} + \frac{M^2 N}{2} - \frac{MN^2}{2} \tag{23}
$$

$$n_{ch} = c_{ch} - 5N + \frac{N^3}{2} + \frac{3M^2N}{2} - \frac{3MN^2}{2}, \tag{24}$$

where $c_v = -3/2$, $c_{ch} = 9/2$ for the case A) and $c_v = 3/2$, $c_{ch} = 3/2$ for the case B). Notice that for any choice M in (21) the number of $U(n_j)$ subgroups is $\mathcal{O}(N)$ and some of the n_j themself are $\mathcal{O}(N)$ as $N \to \infty$. It does therefore not seem natural to get the standard model gauge group in these models. Notice that this was derived for the choice that N is prime and $v = (1/N, 1/N, -2/N)$. For N prime and $N-1 \in 4\mathbb{Z}$ with $v_1 = v_2 = -v_3/2 = \frac{N-1}{4N}$ or $N+1 \in 4\mathbb{Z}$ with $v_1 = v_2 = -v_3/2 = \frac{N+1}{4N}$ one derives similar results and the conclusions are the same.

7 Conclusions

We have discussed a mechanism in string theory to realize induced gravity. This can be applied to non-compact Calabi-Yau manifolds, orbifolds and orientifolds. The hierarchy between the ten-dimensional and the four-dimensional Einstein term is due to a large Euler number and/or weak string coupling. The thickness is given by the induced Planck mass. The induced higher derivative terms are negligible at observable distances and we get four-dimensional Einstein gravity between 1 mm and the Hubble scale. The standard model can be realized on the WORLD where gravity is localized in a multitude of ways. However, the easiest orientifold realizations (though one can easily include the standard model) seem to give way to large gauge groups and too many chiral fields as to find these models natural and therefore of immediate phenomenological interest. One will have to consider more general models that may or may not be orientifold models. We leave this for future work.

Acknowledgements

We thank I. Antoniadis, C. Deffayet, G. Gabadadze and E. Kiritsis for useful discussions. The work reported here has been supported in part by the European Commission under the RTN contract HPRN-CT-2000-00148 and EC Excellence Grant MEXT-CT-2003-509661. E.K. has been supported by the Austrian FWF project No. J2259-N02. P.V. thanks the organizers of Quarks 2004 for the invitation to present this work.

References

[1] G.R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485** (2000) 208 [arXiv:hep-th/0005016].

[2] C. Deffayet, Phys. Lett. B **502** (2001) 199 [arXiv:hep-th/0010186].

[3] C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D **65** (2002) 044023 [arXiv:astro-ph/0105068].

[4] G. R. Dvali and G. Gabadadze, Phys. Rev. D **63** (2001) 065007 [arXiv:hep-th/0008054]; G. R. Dvali, G. Gabadadze, M. Kolanovic and F. Nitti, Phys. Rev. D **64** (2001) 084004 [arXiv:hep-ph/0102216]; G. Dvali, G. Gabadadze and M. Shifman, arXiv:hep-th/0208096; G. Dvali, G. Gabadadze and M. Shifman, Phys. Rev. D **67** (2003) 044020 [arXiv:hep-th/0202174].

[5] G. Gabadadze, arXiv:hep-th/0408118.

[6] I. Antoniadis, R. Minasian and P. Vanhove, Nucl. Phys. B **648** (2003) 69 [arXiv:hep-th/0209030].

[7] E. Kohlprath, Nucl. Phys. B **697** (2004) 243 [arXiv:hep-th/0311251].

[8] E. Kiritsis, N. Tetradis and T. N. Tomaras, JHEP **0108** (2001) 012 [arXiv:hep-th/0106050].

[9] E. Kiritsis, Fortsch. Phys. **52** (2004) 568, [arXiv:hep-th/0310001].

[10] F. Epple, arXiv:hep-th/0408105.

[11] S. L. Adler, Rev. Mod. Phys. **54** (1982) 729 [Erratum-ibid. **55** (1983) 837].

[12] G. Gabadadze and M. Shifman, Phys. Rev. D **69** (2004) 124032 [hep-th/0312289]

[13] I. Antoniadis, E. Gava and K. S. Narain, Phys. Lett. B **283** (1992) 209 [arXiv:hep-th/9203071].

[14] I. Antoniadis, C. Bachas, C. Fabre, H. Partouche and T. R. Taylor, Nucl. Phys. B **489** (1997) 160 [arXiv:hep-th/9608012]. I. Antoniadis, H. Partouche and T. R. Taylor, Nucl. Phys. B **499** (1997) 29 [arXiv:hep-th/9703076].

[15] E. Kiritsis and C. Kounnas, Nucl. Phys. B **442** (1995) 472 [arXiv:hep-th/9501020].

[16] I. Antoniadis, S. Ferrara, R. Minasian and K. S. Narain, Nucl. Phys. B **507** (1997) 571 [arXiv:hep-th/9707013].

[17] K. Peeters, P. Vanhove and A. Westerberg, Class. Quant. Grav. **18** (2001) 843 [arXiv:hep-th/0010167].

[18] A. Padilla, Class. Quant. Grav. **21** (2004) 2899 [arXiv:hep-th/0402079].

[19] A. Lue and G. Starkman, Phys. Rev. D **67** (2003) 064002 [arXiv:astro-ph/0212083].

[20] G. Aldazabal, A. Font, L.E. Ibanez und G. Violero, Nucl. Phys. B **536** (1998) 29 [arXiv:hep-th/9804026].

[21] L.E. Ibanez, R. Rabadan and A.M. Uranga, Nucl. Phys. B **542** (1999) 112 [arXiv:hep-th/9808139].