

String loop corrections to the universal hypermultiplet

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Abstract

We study loop corrections to the universal dilaton supermultiplet for type IIA strings compactified on Calabi-Yau threefolds. We show that the corresponding quaternionic kinetic terms receive non-trivial one-loop contributions proportional to the Euler number of the Calabi-Yau manifold, while the higher-loop corrections can be absorbed by field redefinitions. The corrected metric is no longer Kähler. Our analysis implies in particular that the Calabi-Yau volume is renormalized by loop effects which are present even in higher orders, while there are also one-loop corrections to the Bianchi identities for the NS and RR field strengths.

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1. Introduction and discussion

Type II string compactifications on Calabi-Yau threefolds (CY_3) provide a theoretical framework for addressing several physically interesting problems. Away from possible brane insertions, the four-dimensional (4d) low energy massless spectrum is $\mathcal{N} = 2$ supersymmetric and has two separate and decoupled matter sectors involving, respectively, the vectors and hypermultiplets. At a generic point of the moduli space, the vectors are abelian and the hypermultiplets are neutral, while their corresponding multiplicities are given by the Betti numbers of the $(1,1)$ and $(1,2)$ forms of the CY_3 : $h_{(1,1)}$ ($h_{(1,2)}$) and $h_{(1,2)} + 1$ ($h_{(1,1)} + 1$) in type IIA (IIB) theory. The $+1$ stands for the so-called universal hypermultiplet, formed by the 4d dilaton, the axion dual to the NS-NS (Neveu-Schwarz–Neveu-Schwarz) 2-form and a complex RR (Ramond-Ramond) scalar C , obtained for instance in type IIA, by the 3-form gauge potential $C^{(3)} \equiv C\omega^{(3)}$ with $\omega^{(3)}$ the CY_3 holomorphic volume 3-form.

The kinetic terms of vector multiplets form a special Kähler manifold characterized by an analytic prepotential of the $\mathcal{N} = 2$ special geometry. Since the dilaton belongs to a hypermultiplet, the prepotential is determined exactly at the string tree-level. On the other hand, the kinetic terms of hypermultiplets form a quaternionic manifold, where radiative corrections are highly restricted by the structure of the universal hypermultiplet that contains the string coupling. Type IIB is invariant under S-duality, while in the strong coupling limit of type IIA, the hypermultiplet space is lifted to 5 dimensions, describing the complex structure moduli of M-theory compactified on the same Calabi-Yau, with the dilaton replaced by the CY_3 volume.

The one-loop corrections to the hypermultiplet metric were computed in [1] for directions orthogonal to the dilaton and were shown to be topological and proportional to the Euler number of the CY_3 manifold. Moreover, they can be easily understood as descending from the R^4 terms in ten dimensions.

In this work, we focus on just the universal hypermultiplet and study the perturbative string corrections. For this purpose, we can think of type IIA compactified on a CY_3 with no complex structure moduli ($h_{(1,2)} = 0$), so that the quaternionic manifold contains only the dilaton multiplet. The corresponding metric is then reduced to a 4d self-dual Einstein space of non-zero scalar curvature, while at tree-level it is further reduced to a symmetric coset space $SU(1,2)/U(2)$, which is also Kähler. At a generic order of perturbation theory, on the other hand, there are only three isometries corresponding to the three independent shifts of the NS-NS axion and the complex RR scalar, and they generate a Heisenberg algebra. Imposing just these isometries, one finds that there is one possible perturbative correction at the one-loop level, which destroys the Kähler structure of the manifold.

Comparing the above supergravity result with the general form of the effective action in the string frame, including a possible one-loop correction, one finds an apparent inconsistency with string perturbation theory. Moreover, the inconsistency persists even in the

absence of one-loop corrections to the scalar kinetic terms, due to the one-loop modification of the effective Planck mass that has been established by a string computation [1]. To resolve this puzzle, we need to introduce a renormalization of the CY_3 volume which may, in principle, receive contributions even from higher loops. This phenomenon is analogous to the renormalization of the 4d dilaton in the presence of higher order σ -model corrections to the prepotential.

Allowing for such one-loop redefinition of the volume, we find that the one-loop string effective action can be made compatible with the quaternionic structure of the universal hypermultiplet for two possible values of the one-loop correction to the hypermultiplet metric in the dilaton direction: either zero, or a precise non-zero value proportional to the Euler number. In the former case, the one-loop correction to the gravity kinetic terms is absorbed by a shift of the (inverse) string coupling. However, we show that an explicit one-loop string computation of the 3-point amplitude involving two RR scalars and one graviton or one NS-NS antisymmetric tensor selects the second non-vanishing value allowed by the field theory analysis. This result makes therefore the completion of the one-loop correction to the hypermultiplet metric along the dilaton direction. Moreover, our analysis suggests that the absence of higher loop perturbative corrections to the hypermultiplet metric should persist in the presence of additional hypermultiplets parameterizing the complex structure of the CY_3 .

Our paper consists of two parts. In Sections 2 to 7, the physical implications of the string loop corrections to the universal hypermultiplet metric are discussed, while the string computation is contained in Section 5. The second part, composed by three Appendices, contains the details of the computations needed in the main text. More precisely, in Section 2, we summarize the generic form of the four-dimensional type IIA action involving the vectors and hypermultiplets orthogonal to the volume and dilaton, respectively, as determined by the analysis of [1]. In Section 3, we describe the results of Calderbank and Pedersen for the general metric of one quaternion with two commuting isometries and reduce its form for the case of three isometries. In Section 4, we compare this metric with the general form of the one-loop corrected string effective action and show their compatibility upon introducing a redefinition of the CY_3 volume. In Section 5, we perform a one-loop string computation and determine the coefficients of the effective action. In Section 6, we discuss the strong coupling limit which lifts the hypermultiplet space to that of M-theory compactified on CY_3 . In Section 7, we show how our results can be obtained by reduction of the supersymmetric completion of the R^4 couplings in ten dimensions. Appendix A contains our conventions and useful properties of the Riemann tensor. We also present the details of the reduction of the R^4 terms from ten to four dimensions. Appendix B contains the information about the quaternionic geometry of the universal hypermultiplet sigma-model and the implementation of string loop corrections. Finally, in Appendix C, we present technical details for the one-loop string computation of the 3-point physical amplitude from which the one-loop correction to the universal hypermultiplet metric is extracted.

2. One-loop corrections to the non-universal directions

In this section, we briefly review the analysis of [1]. We consider type IIA compactified on a CY 3-fold with Betti numbers $h_{(1,1)}$ and $h_{(1,2)}$, leading to $h_{(1,1)} \mathcal{N} = 2$ vector multiplets and $h_{(1,2)} + 1$ hypermultiplets in four dimensions. The one-loop corrected string effective action in the string frame contains the terms

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{g^\sigma} \left[\left(\left(1 + \frac{\chi_T}{v_6}\right) e^{-2\phi_4} - \chi_1 \right) \mathcal{R}_{(4)} + \left(\left(1 - \frac{\chi_T}{v_6}\right) e^{-2\phi_4} - \chi_1 \right) G_{vv} (\partial v)^2 + \left(\left(1 + \frac{\chi_T}{v_6}\right) e^{-2\phi_4} + \chi_1 \right) G_{hh} (\partial h)^2 \right]. \quad (2.1)$$

G_{vv} is the metric of the $h_{(1,1)} - 1$ vector-multiplets orthogonal to the volume modulus of the internal manifold and G_{hh} is the metric of the $h_{(1,2)}$ non-universal hypermultiplets; $v_6 = V_6 (2\pi l_s)^{-6}$ is the normalized volume of the internal Calabi-Yau manifold with Euler number χ (in string units of length $l_s = \sqrt{\alpha'}$). $\chi_T = 2\zeta(3)\chi/(2\pi)^3$ and $\chi_1 = 4\zeta(2)\chi/(2\pi)^3$ are the tree-level and one-loop corrections, respectively¹. They descend from the $\alpha'^3 R^4$ -terms in ten dimensions [1,2,3]. Here we have dropped world-sheet instanton corrections that are exponentially suppressed in the large volume limit. The four-dimensional dilaton ϕ_4 is related to the ten-dimensional dilaton ϕ_{10} via $e^{-2\phi_4} = v_6 e^{-2\phi_{10}}$. From now on, we set $2\kappa_4^2 = 1$.

For type IIA reduction on a Calabi-Yau manifold, the universal hypermultiplet contains the dilaton $\tilde{\phi}_4$, while the universal vector-multiplet contains the volume \tilde{v}_6 . Because there is one conformal compensator associated with each of these multiplets [4], \tilde{v}_6 is the (σ -model) “loop” counting parameter for the corrections to the metric of vector multiplets, and $e^{\tilde{\phi}_4}$ the “loop” counting parameter for the corrections to the metric of hypermultiplets. In $\mathcal{N} = 2$ supergravity, the moduli space factorizes into a product of a special Kähler manifold (vector) and a quaternionic manifold (hyper). This imposes that $\tilde{\phi}_4$ and \tilde{v}_6 are a mixture of the dilaton ϕ_4 and the volume v_6 . From general arguments, it can be easily seen that the tree-level corrections force to redefine the dilaton, while the loop corrections lead to a redefinition of the volume

$$e^{-2\tilde{\phi}_4} \simeq e^{-2\phi_4} \left(1 + \mu_T \frac{\chi_T}{v_6} + \dots \right) ; \quad \tilde{v}_6 \simeq v_6 \left(1 - \frac{3\mu_1}{2} \chi_1 e^{2\phi_4} + \mathcal{O}(e^{4\phi_4}) \right) \quad (2.2)$$

No information regarding the mixing can be deduced from [1] since the analysis there was restricted to the non-universal directions. The value of μ_1 is determined in Section 4. An extension of this analysis to tree-level $\zeta(3)\alpha'^3 R^4$ -induced corrections to the universal vector-multiplet has been attempted in [5], where the tree-level mixing between the dilaton and the volume was discussed. Unfortunately the analysis of ref.[5] does not allow to derive the precise value of μ_T . In the following, we study the metric of the universal hypermultiplet obtained by compactification of type IIA/M-theory on CY_3 , taking into account one-loop corrections. This question was partly addressed by Strominger in [6], but with results different from ours.

¹ The normalizations of the R^4 -action follow those of [2], where $2\kappa_{10}^2 = (2\pi)^7 l_s^8$, $2\kappa_4^2 = 2\pi l_s^2$. We refer to the Appendix A for more detailed discussion of the R^4 terms.

3. The universal hypermultiplet metric

3.1. Calderbank-Pedersen metric

We now consider the simplest case of just the dilaton hypermultiplet. Calderbank and Pedersen [7] have shown that any self-dual Einstein metric of non-zero scalar curvature with two commuting isometries can be derived from a real potential $F(\eta, \rho)$ of two variables, and has the local form

$$ds_{CP}^2 = \frac{1}{F^2(\eta, \rho)} \left[\det Q \frac{d\rho^2 + d\eta^2}{4\rho^2} + \frac{1}{\det Q} \begin{pmatrix} d\phi & d\psi \end{pmatrix} N^t Q^2 N \begin{pmatrix} d\phi \\ d\psi \end{pmatrix} \right] \quad (3.1)$$

with

$$Q = \begin{pmatrix} \frac{1}{2}F - \rho\partial_\rho F & -\rho\partial_\eta F \\ -\rho\partial_\eta F & \frac{1}{2}F + \rho\partial_\rho F \end{pmatrix} ; \quad N = \begin{pmatrix} \sqrt{\rho} & 0 \\ \frac{\eta}{\sqrt{\rho}} & \frac{1}{\sqrt{\rho}} \end{pmatrix}. \quad (3.2)$$

The metric is Einstein $R_{mn} = 3g_{mn}$ and has a self-dual Weyl tensor $W_{rstu}^- = 0$ only if the potential $F(\eta, \rho)$ satisfies the Laplace equation,

$$\rho^2 (\partial_\rho^2 + \partial_\eta^2) F(\eta, \rho) = \frac{3}{4} F(\eta, \rho). \quad (3.3)$$

This metric is quaternionic Kähler, and thus invariant under local $\mathcal{N} = 2$ supersymmetry. For $\det(Q) < 0$, the metric g_{mn}^{CP} is negative definite with positive scalar curvature, therefore $-ds_{CP}^2$ is positive definite with a negative curvature scalar $\mathcal{R}(-g^{CP}) = -12$. The coupling to the supergravity multiplet is [8,9,10,11] $S = \int d^4x \sqrt{g^E} [R_E - ds_{CP}^2 + \text{fermions}]$.

The potential $F(\eta, \rho)$ completely specifies the metric. Its explicit form will reflect the loop and D2/M2 instanton corrections, which break the perturbative shift symmetry of η . As long as these corrections are compatible with the constraint (3.3) the metric remains quaternionic Kähler.

Coupling the quaternionic Kähler metric (3.1) to gravity and rescaling the space-time metric as $g_{mn} = \sqrt{\rho} F(\eta, \rho) \bar{g}_{mn}$ gives the effective action

$$\int d^4x \sqrt{\bar{g}} \sqrt{\rho} F(\eta, \rho) \left[\bar{\mathcal{R}}_{(4)} - ds_{CP}^2 + \frac{3}{2} (\partial_\mu \ln(\sqrt{\rho} F(\eta, \rho)))^2 \right]. \quad (3.4)$$

The tree-level universal hypermultiplet metric ($\chi_1 = 0$ in (2.1)) matches with the quaternionic Kähler metric (3.1) for $\sqrt{\rho} F(\eta, \rho) = \rho^2$ under the field identifications [12]

$$\rho^2 = e^{-2\phi_4}, \quad C = C_1 + iC_2 = \eta + \frac{i}{2}\phi, \quad \psi = D - C_1 C_2. \quad (3.5)$$

D is the scalar obtained after dualization of the NS B -field in four dimensions or the 3-form gauge potential of M-theory in five dimensions. The classical metric is Kähler and can be derived from the Kähler potential $\mathcal{K} = \ln(S + \bar{S} - 2C\bar{C})$ with $S = \exp(-2\phi_4) + 2iD + C\bar{C}$.

Moreover, it describes a symmetric coset space with eight isometries $SU(1,2)/U(2)$. For any function $F(\eta, \rho)$ independent of η , the metric has three $U(1)$ isometries ($\alpha_i \in \mathbb{R}$)

$$\eta \rightarrow \eta + \alpha_1 ; \quad \phi \rightarrow \phi + \alpha_2 ; \quad \psi \rightarrow \psi + \alpha_3 - \alpha_1 \phi . \quad (3.6)$$

These isometries can be identified with the perturbative Peccei-Quinn shift symmetries on the Ramond fields $C_{1,2}$ and the NS B -field:

$$C \rightarrow C + \alpha_1 + \frac{i}{2}\alpha_2 ; \quad D \rightarrow D + \left(\frac{1}{2}\alpha_2 C_1 - \alpha_1 C_2\right) + \left(\frac{1}{2}\alpha_1 \alpha_2 + \alpha_3\right) . \quad (3.7)$$

It is easy to verify that the three isometries satisfy the Heisenberg algebra, $[T_{1,2}, T_3] = 0$ and $[T_1, T_2] = T_3$, with T_i the generators associated to the transformation parameters α_i for $i = 1, 2, 3$.

As already mentioned, the quantum corrections to the metric are encoded in the solutions to (3.3). Here we will be interested only in the perturbative corrections. They must be such that they preserve all three PQ symmetries. The only possible deformation of the potential $F(\eta, \rho)$ compatible with the three $U(1)$ isometries and the constraint (3.3) is $\sqrt{\rho} F(\eta, \rho) = \rho^2 - \hat{\chi}$. The string frame expression (3.4) suggests the identification $\sqrt{\rho} F(\eta, \rho) = e^{-2\tilde{\phi}_4} - \chi_1$, with the Planck mass one-loop correction in (2.1) expressed in terms of the modified dilaton (2.2). In [6], Strominger has proposed that all loop corrections may be captured by modifying the map $\rho^2 = f(\exp(-2\tilde{\phi}_4))$, while keeping the potential $\sqrt{\rho} F(\eta, \rho) = \rho^2$. We will see below that this is not the case and $\hat{\chi}$ does not vanish, implying also the absence of a Kähler potential. Thus, the one-loop correction cannot be absorbed by field redefinitions and plays an important role.

Note that for a potential depending on η , only two $U(1)$ isometries (α_2, α_3) are left, and a particular solution to the constraint (3.3) is the D-instanton function [13,2,14,15,16,17] $F(\eta, \rho) = E_{3/2}$. Additional wrapped brane instantons (D4 and NS5) are expected to break all isometries, thus breaking the classical $SU(2,1)/U(2)$ [12,18,19,20,21] to a discrete set and should transform this potential into a quaternionic function $F(Q)$ [19,22,23].

3.2. Perturbations

Since we are interested in one-loop corrections to the hypermultiplet metric, we will ignore the tree-level corrections proportional to $\chi_T = 2\zeta(3)\chi/(2\pi)^3$. They only modify the $\mathcal{N} = 2$ prepotential. Therefore the dilaton is not redefined and from now on we can take $\mu_T = 0$ and $\tilde{\phi}_4 = \phi_4$.

The Calderbank-Pedersen metric for the one parameter family of potentials preserving the three $U(1)$ isometries,

$$F(\eta, \rho) = \rho^{3/2} - \hat{\chi} \rho^{-1/2} , \quad (3.8)$$

has the following form in quaternionic notation: $ds_{CP}^2 = -2(u\bar{u} + v\bar{v})$, with²

$$\begin{aligned} u &= \sqrt{\frac{\rho^2 + \hat{\chi}}{(\rho^2 - \hat{\chi})^2}} dC ; & v &= \sqrt{\frac{\rho^2}{4(\rho^2 + \hat{\chi})(\rho^2 - \hat{\chi})^2}} (dS + 2C d\bar{C}) \\ C &= i\eta + \frac{1}{2}\phi ; & S &= \rho^2 + 2\hat{\chi} \ln(\rho) + i(2\psi + \eta\phi) - C\bar{C} . \end{aligned} \quad (3.9)$$

To compare with Strominger's analysis (Section 6 of [6]), we study perturbations of this metric. We expand the ρ coordinate in a power series of $\exp(2\phi_4)$ as $\rho^2 = \exp(-2\phi_4) - 2\hat{\alpha}\hat{\chi} + \dots$. At the first order, the metric (3.1) is modified as (using $v + \bar{v} = 2d\ln\rho$ at tree level)

$$\delta_1 u = \frac{1}{2}(2\hat{\alpha} + 3)\hat{\chi} e^{2\phi_4} u ; \quad \delta_1 v = (2\hat{\alpha} + 1)\hat{\chi} e^{2\phi_4} v + \frac{1}{2}\hat{\chi} e^{2\phi_4} \bar{v} . \quad (3.10)$$

The \bar{v} contribution in $\delta_1 v$ was ruled out in [6] using the argument that such contributions introduce parity violating terms (under $D \rightarrow -D$, $C \leftrightarrow \bar{C}$). This is, however, not the case as long as vv and $\bar{v}\bar{v}$ appear in the metric with equal coefficients. Strominger's solution corresponds to the potential $F(\eta, \rho) = \rho^{3/2}$. In Appendix B, we show that (3.10) is a particular case of the most general physically acceptable one-loop deformation of the metric compatible with the quaternionic geometry. We also show there that the physically acceptable two-loop deformations of the metric are captured by the same potential (3.8), but the map between ρ and the dilaton has to be modified as

$$\rho^2 = e^{-2\phi_4} - 2\hat{\alpha}\hat{\chi} + \left(\frac{1}{2} - 2\hat{\gamma} + 4\hat{\alpha}\right)\hat{\chi}^2 e^{2\phi_4} , \quad (3.11)$$

with $\hat{\gamma}$ the only new parameter appearing at two loops (see Appendix B).

4. Type IIA string on CY_3

Having established the most general form of the perturbative quantum metric compatible with $\mathcal{N} = 2$ supersymmetry, we now find the precise identification of the coordinates

² This is a particular case of the general expression, valid for $\det(Q) < 0$,

$$\begin{aligned} \sqrt{2}u &= \frac{Q_{12}d\rho + A d\eta}{2\rho F} + i\frac{Q_{11}\alpha + Q_{12}\beta}{F\sqrt{-\det Q}} ; & \sqrt{2}v &= \frac{A d\rho - Q_{12}d\eta}{2\rho F} + i\frac{Q_{12}\alpha + Q_{22}\beta}{F\sqrt{-\det Q}} \\ A &= \sqrt{(\rho\partial_\rho F)^2 - F^2/4} ; & \alpha &= \sqrt{\rho}d\phi ; & \beta &= (d\psi + \eta d\phi)/\sqrt{\rho} . \end{aligned}$$

It is interesting to notice that once instantons are switched on, $Q_{12} \neq 0$ and the u and v one-forms will depend both on the RR and NS fields.

on the universal hypermultiplet target space with the string variables. To do so, we first have to recast (3.4) in a form that allows a direct comparison with a string action (in Einstein frame). We start from the metric associated with the one-parameter family of potentials (3.8):

$$-\frac{1}{2} ds_{CP}^2 = \frac{\rho^2 + \hat{\chi}}{(\rho^2 - \hat{\chi})^2} \left((d\rho)^2 + (d\eta)^2 + \frac{(d\phi)^2}{4} \right) + \frac{\rho^2}{(\rho^2 - \hat{\chi})^2 (\rho^2 + \hat{\chi})} (d\psi + \eta d\phi)^2. \quad (4.1)$$

As at the tree-level, we introduce the fields $C = \frac{1}{2}\phi + i\eta$ and keeping the definition for $D = \psi + \eta\phi/2$, we dualize the dD field-strength into a three-form H by adding the Lagrange multiplier $-\frac{1}{3} \int \epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho} \partial_\sigma D$. We thus obtain the following action³

$$S = -2 \int d^4x \sqrt{g^E} \left[\frac{\rho^2 + \hat{\chi}}{(\rho^2 - \hat{\chi})^2} (d\rho)^2 + \frac{\rho^2 + \hat{\chi}}{(\rho^2 - \hat{\chi})^2} |dC|^2 - \frac{(\rho^4 - \hat{\chi}^2)(\rho^2 - \hat{\chi})}{4\rho^2} |H|^2 + \frac{i}{2} \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} \partial_\rho C \partial_\sigma \bar{C} \right] \quad (4.2)$$

We now rescale the RR-scalars⁴ $C_{1,2}$ as $C = f^{-1/2}(\rho) C'$ and define $F' = f^{1/2}(\rho) dC$, where $f(\rho)$ is for the moment an arbitrary function. F' satisfies the modified Bianchi identity $d(f^{-1/2}(\rho) F') = 0$ solved by $F' = dC' + d \ln(f^{-1/2}) C'$. Similarly, we redefine the B -field as $B = f(\rho) B'$ and introduce $H' = dB' + d \ln f(\rho) \wedge B'$, so that the last term in (4.2) remains invariant. This leads to the action

$$S = -2 \int d^4x \sqrt{g^E} \left[\frac{\rho^2 + \hat{\chi}}{(\rho^2 - \hat{\chi})^2} (d\rho)^2 + f^{-1}(\rho) \frac{\rho^2 + \hat{\chi}}{(\rho^2 - \hat{\chi})^2} |F'|^2 - f^2(\rho) \frac{(\rho^4 - \hat{\chi}^2)(\rho^2 - \hat{\chi})}{4\rho^2} |H'|^2 + \frac{i}{2} \epsilon^{\mu\nu\rho\sigma} B'_{\mu\nu} F'_\rho \bar{F}'_\sigma \right], \quad (4.3)$$

which is to be compared with the string effective field theory.

On the other hand, we consider in the string frame the one-loop corrected action:

$$S_1 = \int d^4x \sqrt{g^\sigma} \left\{ (e^{-2\phi_4} - \chi_1) R_\sigma + 4 (e^{-2\phi_4} + \alpha) (d\phi_4)^2 + \frac{1}{2} (e^{-2\phi_4} + \beta) |\hat{H}|^2 - \frac{1}{2} (e^{-2\phi_4} + \gamma) |\hat{F}|^2 + \frac{i}{4} (e^{-2\phi_4} + \delta) \epsilon^{\mu\nu\rho\sigma} \hat{B}_{\mu\nu} \hat{F}_\rho \hat{\bar{F}}_\sigma \right\} \quad (4.4)$$

³ We use the convention for p -forms that $\sqrt{g} |F_{(p)}|^2 = F_{(p)} \wedge *F_{(p)} = \frac{\sqrt{g}}{p!} F_{\mu_1 \dots \mu_p} F^{\mu_1 \dots \mu_p}$.

⁴ The complexification of RR scalars is a consequence of string perturbation. The $\mathcal{N} = (2, 2)$ $U(1) \times U(1)$ -charge conservation forbids $dC dC$ or $d\bar{C} d\bar{C}$ terms at one-loop so that the only additional freedom is to rescale the fields $C_{1,2} \rightarrow \kappa_{1,2} C_{1,2}$ with $\kappa_1 \kappa_2 = 1$.

where all the coefficients α, β, γ and δ are proportional to the one-loop constant $\chi_1 = 4\zeta(2)\chi/(2\pi)^3$ and should be fixed by comparison with (4.3). Here we have introduced hatted variables, \hat{B} and \hat{C} , which are string variables that have to be related to their supergravity primed counterparts B' and C' in (4.3).

Because of the mixing (2.2) between the volume and the four-dimensional dilaton, the volume dependent terms are also needed:

$$S_2 = \int d^4x \sqrt{g^\sigma} \left\{ -\frac{1}{6} (e^{-2\phi_4} - \chi_1) (d \ln v_6)^2 + \mu_1 \chi_1 d \ln v_6 d\phi_4 \right\} \quad (4.5)$$

This mixing is necessary for the factorization of the vector and hypermultiplet moduli spaces, as required by $\mathcal{N} = 2$ supergravity. The factorization also requires the coefficient in front of $(d \ln v_6)^2$ to be independent of ϕ_4 in the Einstein frame⁵. By redefining the volume vector-modulus as

$$\tilde{v}_6 = v_6 (1 - \chi_1 e^{2\phi_4})^{\frac{3\mu_1}{2}} \quad (4.6)$$

we find that in the string frame the kinetic terms for the volume and the dilaton take the form:

$$\int d^4x \sqrt{g^\sigma} \left[4 \left(e^{-2\phi_4} + \alpha + \frac{3\mu_1^2}{8} \frac{\chi_1^2}{e^{-2\phi_4} - \chi_1} \right) (d\phi_4)^2 - \frac{1}{6} (e^{-2\phi_4} - \chi_1) (d \ln \tilde{v}_6)^2 \right] \quad (4.7)$$

Finally, in the Einstein frame, the action reads

$$\begin{aligned} S_1 + S_2 = \int d^4x \sqrt{g^E} \left\{ R_E - \frac{1}{6} (d \ln \tilde{v}_6)^2 \right. \\ - 2 \frac{e^{-4\phi_4} + 2(\chi_1 - \alpha) e^{-2\phi_4} + 2\alpha\chi_1 - 3\mu_1^2\chi_1^2/4}{(e^{-2\phi_4} - \chi_1)^2} (d\phi_4)^2 + \frac{1}{2} (e^{-2\phi_4} + \beta)(e^{-2\phi_4} - \chi_1) |\hat{H}|^2 \\ \left. - \frac{e^{-2\phi_4} + \gamma}{2(e^{-2\phi_4} - \chi_1)} |\hat{F}|^2 + \frac{i}{4} (e^{-2\phi_4} + \delta) \epsilon^{\mu\nu\rho\sigma} \hat{B}_{\mu\nu} \hat{F}_\rho \hat{F}_\sigma \right\}. \end{aligned} \quad (4.8)$$

Comparison of (4.3) with (4.8) leads to the following identifications of hatted and primed variables: $(\exp(-2\phi_4) + \delta)^{1/2} \hat{F} = 2F'$, $\hat{B} = B'$ (and $\hat{H} = H'$). Moreover, the matching between the two metrics appearing in the supergravity quaternionic sigma-model and in the Einstein frame 1-loop string effective action gives only two possible solutions, depending on whether $\hat{\chi} = 0$ or $\hat{\chi} \neq 0$.

When $\hat{\chi} = 0$, we recover the solution of [6]. The field identifications are the following:

$$\begin{aligned} \rho^2 = e^{-2\phi_4} - \chi_1, \quad C = \frac{1}{2}\phi + i\eta, \quad \psi = D - C_1 C_2, \\ f(\rho) = 1, \quad \hat{\chi} = 0, \quad \gamma = \delta, \quad \beta = -\chi_1, \quad \alpha = \chi_1, \quad \mu_1^2 = \frac{8}{3} \end{aligned} \quad (4.9)$$

⁵ In Appendix A the relation between the coefficient μ_1 and the R^4 -terms in ten dimensions is discussed.

and the kinetic terms for the modified volume and the dilaton in the string frame are:

$$\int d^4x \sqrt{g^\sigma} \left[4 \frac{e^{-4\phi_4}}{e^{-2\phi_4} - \chi_1} (d\phi_4)^2 - \frac{1}{6} (e^{-2\phi_4} - \chi_1) (d \ln \tilde{v}_6)^2 \right] . \quad (4.10)$$

The first term was already found in [24], but the mixing of the dilaton with the volume, necessary for the correct perturbative string interpretation, was not discussed there.

The solution with $\hat{\chi} \neq 0$ is new, and the field identifications are

$$\begin{aligned} \rho^2 &= e^{-2\phi_4} - \chi_1, \quad C = \frac{1}{2} \phi + i\eta, \quad \psi = D - C_1 C_2, \\ f(\rho) &= 1 - \chi_1 e^{2\phi_4}, \quad \hat{\chi} = -\chi_1, \quad \gamma = \beta = -2\chi_1, \quad 2\alpha = 5\chi_1, \quad \delta = 0, \quad \mu_1^2 = 4 \end{aligned} \quad (4.11)$$

Both solutions are consistent with $\mathcal{N} = 2$ supergravity, although only one solution corresponds to the low energy effective action of string theory. The one-loop corrections to the kinetic term of the RR fields and to the Chern-Simons term, depend on the parameters of the effective action (4.4). In the context of quantum field theory γ and β are wave-function renormalization, and δ the vertex correction. Field-theoretically the wave-function renormalization is fixed by the two-point function, but the closed string two-point amplitude vanishes on-shell and does not have a reliable off-shell extension. Therefore, we have to consider the three-point amplitude which computes the S -matrix elements corresponding to the renormalized couplings: $\langle GF\bar{F} \rangle = \chi_1/2$ and $\langle BF\bar{F} \rangle = \delta - \gamma - \beta/2$. For the solution (4.9) the tree-level and one-loop S -matrix elements are the same. We show by a direct one-loop string computation presented in Section 5 that this is not the case. This selects the solution (4.11). Thus, one concludes that the one-loop corrections to the universal hypermultiplet metric are physical. We will discuss the higher-loop corrections in Section 6. Note that the solution has a sign ambiguity $\mu_1 = \pm 2$, which we cannot determine by our analysis.

Remarks about the solution (4.11)

- ▷ The identification of the supergravity and string metrics required field redefinitions for both the NS-NS and RR fields:

$$\hat{H} = d\hat{B} - 2 \frac{\chi_1}{e^{-2\phi_4} - \chi_1} d\phi_4 \wedge \hat{B} \quad ; \quad \hat{F} = 2e^{\phi_4} (1 - \chi_1 e^{2\phi_4})^{1/2} dC . \quad (4.12)$$

These identifications in turn imply modifications of the Bianchi identities by terms proportional to χ_1 . In the background of the Calabi-Yau manifold, there is a non-trivial dilaton, $\phi_{10} = \phi_{10}^o - e^{2\phi_{10}^o} 2\zeta(2) E_6 / (2\pi)^3$, which leads to

$$e^{-2\phi_4} = e^{-2\phi_4^o} \left(1 + \chi_1 e^{2\phi_4^o} \right) . \quad (4.13)$$

Therefore the field redefinition of the RR field strengths

$$\hat{F} = 2 \left(e^{2\phi_4} (1 - \chi_1 e^{2\phi_4}) \right)^{1/2} dC = e^{\phi_4^o} dC \quad (4.14)$$

is the standard redefinition for this particular dilaton background. As for the modified Bianchi identity for the NS field, it simply leads to new interaction terms between the B -field and the dilaton, needed for supersymmetry of the one-loop action.

- ▷ There is one more crucial distinction between (4.9) and (4.11). By an interesting coincidence, the classical metric on $SU(2,1)/U(2)$ happened to be Kähler. It can be easily checked that the metric (4.1) admits a closed Kähler two-form only if $\hat{\chi} = 0$. Therefore the Kähler character of the tree-level metric (defined with $F(\eta, \rho) = \rho^{3/2}$) is lost once quantum corrections are turned on.

5. Reconstructing the four-dimensional effective action

5.1. Three-point S -matrix elements

In this Section, we perform a string one-loop computation which allows us to distinguish between the two solutions (4.9) and (4.11) which determine the effective action (4.4). One possibility is to study the kinetic terms of the NS 2-form field B . The one-loop correction to the H^2 -metric arises from the one-loop term $\alpha'^3 R^3 H^2$ obtained in [25] (given in the string frame):

$$\begin{aligned} \int d^{10}x \sqrt{G} \delta_{s_1 \dots s_9}^{r_1 \dots r_9} R_{s_1 s_2}^{r_1 r_2} R_{s_3 s_4}^{r_3 r_4} R_{s_5 s_6}^{r_5 r_6} \left(H_{r_7 r_8 s_9} H_{s_7 s_8 r_9} - \frac{1}{9} H_{r_7 r_8 r_9} H_{s_7 s_8 s_9} \right) \\ \rightarrow \chi \int d^4x \sqrt{g^\sigma} H_{r_1 r_2 r_3} H^{r_1 r_2 r_3} . \end{aligned} \quad (5.1)$$

This shows that $\beta \propto \int_{M_6} R \wedge R \wedge R \sim \chi$ is proportional to the Euler characteristic of the Calabi-Yau manifold. Unfortunately, because S -matrix elements evaluate the renormalized couplings, the three-point amplitude $\langle BBh \rangle = \beta - 2 \times \beta/2 + \chi_1/2$ (see below) does not contain any information about the value of β . The precise relation can be fixed by the analysis of a four point amplitude in four dimensions, but we will not do this here. We can nevertheless find some other appropriate three-point amplitudes which will make possible to discriminate between (4.9) and (4.11).

The field theory limit of the three-point one-loop amplitude decomposes into the three diagrams given in fig. 1, where the blob corresponds to the one-loop correction coefficients⁶

⁶ See [3,25,26,27] for a rational about extracting the low energy string effective action from S -matrix elements.

in the low-energy string effective action (4.4). If the external lines are RR-scalars and the wiggly line the graviton, the amplitude is

$$\langle GF\bar{F} \rangle = \frac{1}{2} F_\mu h^{\mu\nu} \bar{F}_\nu \left(\gamma - 2 \times \frac{\gamma}{2} + \frac{\chi_1}{2} \right) . \quad (5.2)$$

On the other hand, the correction to the Chern-Simons coupling is

$$\langle BF\bar{F} \rangle = \frac{i}{4} \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} F_\rho \bar{F}_\sigma \left(\delta - 2 \times \frac{\gamma}{2} - \frac{\beta}{2} \right) . \quad (5.3)$$

Thus, computing these string amplitudes will allow us to decide which of the two solutions for the hypermultiplet metric is chosen by string theory.

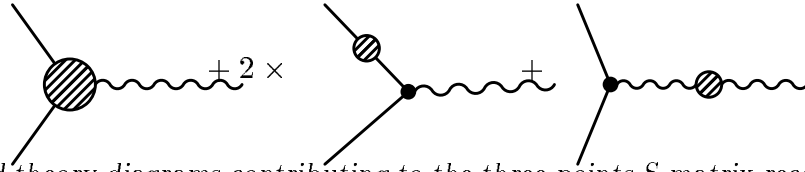


fig. 1 The field theory diagrams contributing to the three points S-matrix reconstructed from the effective action (4.4). The first diagram represents the one-loop correction to the vertex function, the second and the third the wave-function renormalization.

5.2. Definition of the vertex operators

We now compute the same scattering amplitudes in type IIA string theory at one loop. This requires the computation of the three-point torus correlation function of the vertex operators for the RR field strengths F , \bar{F} and the NS field (either the graviton or the anti-symmetric tensor). For the comparison with the field theory result we need the amplitude to $\mathcal{O}(k^2)$. Useful references for the one-loop calculations are [28,29,30]. We use the conventions of [28] which means e.g. that $\alpha' = 2$.

The NS-NS vertex operator is⁷

$$V_{\text{NS}}^{(-1,-1)} = \zeta_{\mu\nu} : \psi^\mu \tilde{\psi}^\nu e^{ik \cdot X} e^{-(\varphi+\bar{\varphi})} : \quad (5.4)$$

where $\zeta_{\mu\nu}$ is traceless and symmetric for the graviton h and antisymmetric for the antisymmetric tensor B . The vertex operators of the RR-scalars in the type IIA universal hypermultiplet in the $(-\frac{1}{2}, -\frac{1}{2})$ -ghost picture are composed of left- and right-moving $SO(4)_{\text{space-time}}$ spin fields S and \tilde{S} and internal $\mathcal{N} = (2, 2)$ superconformal fields Σ and its conjugate $\bar{\Sigma}$

$$\begin{aligned} V_F^{(-\frac{1}{2}, -\frac{1}{2})} &= \hat{F}_\mu : S^\alpha (\sigma^\mu)_{\alpha\dot{\beta}} \tilde{S}^{\dot{\beta}} \Sigma e^{-\frac{1}{2}(\varphi+\bar{\varphi})} e^{ik \cdot X} : \\ V_{\bar{F}}^{(-\frac{1}{2}, -\frac{1}{2})} &= \hat{\bar{F}}_\mu : S_{\dot{\alpha}} (\bar{\sigma}^\mu)^{\dot{\alpha}\beta} \tilde{S}_\beta \bar{\Sigma} e^{-\frac{1}{2}(\varphi+\bar{\varphi})} e^{ik \cdot X} : . \end{aligned} \quad (5.5)$$

⁷ Tilded fields are right movers.

To compute the correlation function, we rotate to Euclidean signature and bosonize, as usual, the complex world-sheet fermions and spin-fields according to $\psi^I \sim e^{i\phi^I}$, $\psi^{\bar{I}} \sim e^{-i\phi^I}$ ($I = 1, 2$) and $S_\alpha \sim e^{\pm \frac{i}{2}(\phi^1 + \phi^2)}$, $S_{\dot{\alpha}} \sim e^{\pm \frac{i}{2}(\phi^1 - \phi^2)}$. For simplicity, in these bosonization formulas we omit cocycle factors which are needed to obtain $SO(4)$ covariant correlation functions. Finally, the spinor indices are raised and lowered with the epsilon-tensor $\epsilon^{\alpha\beta}$ and $\epsilon^{\dot{\alpha}\dot{\beta}}$, with the conventions:

$$\begin{aligned} S_{(--)} &= e^{-\frac{i}{2}(\phi^1 + \phi^2)} = S^{(++)}, & \dot{S}_{(+-)} &= e^{\frac{i}{2}(\phi^1 - \phi^2)} = \dot{S}^{(-+)} \\ (\sigma^1)_{--, -+} &= (\bar{\sigma}^1)^{+-, ++}, & (\sigma^{\bar{1}})_{++, +-} &= (\bar{\sigma}^{\bar{1}})^{-+, --} \\ (\sigma^2)_{--, +-} &= (\bar{\sigma}^2)^{-+, ++}, & (\sigma^{\bar{2}})_{++, -+} &= (\bar{\sigma}^{\bar{2}})^{+-, --}. \end{aligned} \quad (5.6)$$

5.3. The tree-level amplitude

The tree-level amplitude is given by⁸

$$\begin{aligned} \langle V_{NS}^{(-1, -1)}(x) V_F^{(-\frac{1}{2}, -\frac{1}{2})}(u) V_{\bar{F}}^{(-\frac{1}{2}, -\frac{1}{2})}(v) \rangle &\sim \text{tr}(\bar{\sigma}^\kappa \sigma^\nu \bar{\sigma}^\lambda \sigma^\mu) \zeta_{\mu\nu} F_\lambda \bar{F}_\kappa \\ &= 2[2F \cdot h \cdot \bar{F} - i\epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} F_\rho \bar{F}_\sigma] \end{aligned} \quad (5.7)$$

where we have used

$$\psi^\mu S_\alpha S_{\dot{\beta}} \sim \frac{1}{\sqrt{2}}(\sigma^\mu)_{\alpha\dot{\beta}}, \quad \psi^\mu S^{\dot{\alpha}} S^\beta \sim \frac{1}{\sqrt{2}}(\bar{\sigma}^\mu)^{\dot{\alpha}\beta} \quad (5.8)$$

The one-loop amplitude will be again a linear combination of the same tensorial structures. The task is to decide whether they come with the same relative coefficients as the tree-level or not. This will be the criterion to decide between the two solutions we found in Section 4.

5.4. The one-loop amplitude

In Appendix C, we give some technical details and useful formulae for the one-loop computation, and we discuss the independence of the physical amplitude from the supercurrent insertion points. Here, we start from the representation with all the vertex operators in the canonical ghost picture.

On a toroidal world-sheet, the left- and right-moving superconformal ghost charges have to add up to zero separately. If all vertices are chosen in the canonical ghost picture — $(-1, -1)$ for NS-NS fields and $(-\frac{1}{2}, -\frac{1}{2})$ for RR-fields — we have also to insert two

⁸ We use Wess and Bagger [31] conventions $\sigma^a \bar{\sigma}^b + \sigma^b \bar{\sigma}^a = -2\eta^{ab}$ and the identities $\text{tr}(\sigma^a \bar{\sigma}^b \sigma^c \bar{\sigma}^d) = 2(\eta^{ab} \eta^{cd} + \eta^{bc} \eta^{ad} - \eta^{ac} \eta^{bd} - i\epsilon^{abcd})$ and $\sigma^a \bar{\sigma}^b \sigma^c = \eta^{ac} \sigma^b - \eta^{bc} \sigma^a - \eta^{ab} \sigma^c + i\epsilon^{abcd} \sigma^d$.

left-moving and two right-moving picture changing operators, $Y(z)$ and $\tilde{Y}(\bar{w})$. We thus have to compute⁹

$$\mathcal{A} = \sum_{s, \tilde{s}=1}^4 \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} Z(\tau, \bar{\tau}) \int d^2x \int d^2u \int d^2v \langle Y(z_1) Y(z_2) \tilde{Y}(\bar{w}_1) \tilde{Y}(\bar{w}_2) \times V_{\text{NS}}^{(-1, -1)}(x) V_F^{(-\frac{1}{2}, -\frac{1}{2})}(u) V_{\bar{F}}^{(-\frac{1}{2}, -\frac{1}{2})}(v) \rangle_{s, \tilde{s}} \quad (5.9)$$

where $Z(\tau, \bar{\tau}) = (64\pi^4 \tau_2^2 |\eta(\tau)|^4)^{-1}$ is the bosonic partition function, and the summation is over all possible 4×4 spin-structures s, \tilde{s} . We have normalized the short-distance singularity of bosonic correlators to one, while those for fermions and super-conformal ghosts to their respective partition functions.

The relevant part of the picture changing operators is $Y = e^\varphi T_F$ where $T_F = \partial X^\mu \psi_\mu + T_F^{\text{int}}$ is the matter part of the world-sheet supercurrent and φ and $\bar{\varphi}$ belong to the ‘bosonized’ superconformal ghost system. For compactifications on orbifolds $T_F^{\text{int}} = \sum_{I=3,4,5} \partial X^I \psi^{\bar{I}}$. We have to compute the following correlation function

$$\langle e^\varphi T_F(z_1) e^\varphi T_F(z_2) e^{\bar{\varphi}} \tilde{T}_F(\bar{w}_1) e^{\bar{\varphi}} \tilde{T}_F(\bar{w}_2) V_{\text{NS}}^{(-1, -1)}(x) V_F^{(-\frac{1}{2}, -\frac{1}{2})}(u) V_{\bar{F}}^{(-\frac{1}{2}, -\frac{1}{2})}(v) \rangle_{s, \tilde{s}} \quad (5.10)$$

We choose the polarizations of the RR and NS-NS states such that the tree-level amplitude is zero. At the level of conformal field theory this is achieved by non-conservation of $SO(4)$ charge. None of the two Lorentz structures appearing on the r.h.s. of (5.7) vanishes separately but they cancel¹⁰. The criterion which decides between the two solutions (4.9) and (4.11) is whether the one-loop amplitude also vanishes for this choice of polarization or not. Before embarking on the one-loop computation we remark that for this choice of polarizations there is no contribution from the second diagram of fig. 1 since it is proportional to the tree-level vertex (5.7). An appropriate choice which leads to a vanishing tree-level amplitude is:

$$\zeta_{12} F_{\bar{1}} \bar{F}_{\bar{2}} \langle \psi^1(x) S_{(- -)}^L(u) \dot{S}_{(+ -)}^L(v) \rangle_s \langle \tilde{\psi}^2(\bar{x}) \dot{S}_{(- +)}^R(\bar{u}) S_{(- -)}^R(\bar{v}) \rangle_{\tilde{s}} \quad (5.11)$$

The one-loop amplitude then becomes

$$\begin{aligned} \mathcal{A}_1 &= \zeta_{12} F_{\bar{1}} \bar{F}_{\bar{2}} \sum_{s, \tilde{s}} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} Z(\tau, \bar{\tau}) \int d^2x d^2u d^2v \langle \Sigma(u) \bar{\Sigma}(v) \rangle_s \langle \tilde{\Sigma}(\bar{u}) \tilde{\Sigma}(\bar{v}) \rangle_{\tilde{s}} \\ &\times \langle \partial X^{\mu_1}(z_1) \partial X^{\mu_2}(z_2) \bar{\partial} X^{\nu_1}(w_1) \bar{\partial} X^{\nu_2}(w_2) \prod_{i=1}^3 e^{ik^{(i)} \cdot X} \rangle \\ &\times \langle \psi^{\mu_1}(z_1) \psi^{\mu_2}(z_2) \psi^1(x) S_{(- -)}^L(u) \dot{S}_{(+ -)}^L(v) \rangle_s \langle e^{\varphi(z_1)} e^{\varphi(z_2)} e^{-\varphi(x)} e^{-\frac{\varphi(u)}{2}} e^{-\frac{\varphi(v)}{2}} \rangle_s \\ &\times \langle \tilde{\psi}^{\nu_1}(\bar{w}_1) \tilde{\psi}^{\nu_2}(\bar{w}_2) \tilde{\psi}^2(\bar{x}) \dot{S}_{(- +)}^R(\bar{u}) S_{(- -)}^R(\bar{v}) \rangle_{\tilde{s}} \langle e^{\bar{\varphi}(\bar{w}_1)} e^{\bar{\varphi}(\bar{w}_2)} e^{-\bar{\varphi}(\bar{x})} e^{-\frac{\bar{\varphi}(\bar{u})}{2}} e^{-\frac{\bar{\varphi}(\bar{v})}{2}} \rangle_{\tilde{s}} \end{aligned} \quad (5.12)$$

⁹ The existence of one conformal Killing vector on the torus allows to fix the position of one of the three vertex operators. We can instead use translational invariance to integrate over all three positions and compensate by dividing with τ_2 , the volume of the torus.

¹⁰ We have written (5.7) in Minkowski signature while the calculation below is performed in Euclidean signature.

with arbitrary positions of the four picture changers. The only choices of indices for the fermions which lead to non-vanishing fermionic correlation functions are, in a complex basis, $(\mu_1, \mu_2) = (\bar{1}, 2)$ or $(2, \bar{1})$ and $(\nu_1, \nu_2) = (\bar{2}, 1)$ or $(1, \bar{2})$ ¹¹. We consider the choice $(\mu_1, \mu_2, \nu_1, \nu_2) = (\bar{1}, 2, \bar{2}, 1)$. The others are obtained (up to a sign) by exchanging $z_1 \leftrightarrow z_2$ and/or $w_1 \leftrightarrow w_2$.

The fermionic contractions give

$$\begin{aligned} C_s^L &\equiv \frac{K}{\sqrt{8}} \delta_s \frac{\theta_1(z_1 - u) \theta_1(z_2 - x)}{\theta_1(x - u) \theta_1(z_1 - z_2)} \frac{\theta_s(x - z_1 - \frac{u-v}{2}) \theta_s(z_2 - \frac{u+v}{2})}{\theta_s(z_1 + z_2 - x - \frac{u+v}{2}) \theta_1(u - v)} \prod_{I=3}^5 \theta_{s, \mathbf{h}_I}(\frac{u-v}{2}) \\ C_{\bar{s}}^R &\equiv \frac{\bar{K}}{\sqrt{8}} \bar{\delta}_s \frac{\theta_1^*(\bar{w}_1 - \bar{v}) \theta_1^*(\bar{w}_2 - \bar{x})}{\theta_1^*(\bar{x} - \bar{v}) \theta_1^*(\bar{w}_1 - \bar{w}_2)} \frac{\theta_s^*(\bar{x} - \bar{w}_1 - \frac{\bar{u}-\bar{v}}{2}) \theta_s^*(\bar{w}_2 - \frac{\bar{u}+\bar{v}}{2})}{\theta_s^*(\bar{w}_1 + \bar{w}_2 - \bar{x} - \frac{\bar{u}+\bar{v}}{2}) \theta_1^*(\bar{u} - \bar{v})} \prod_{I=3}^5 \theta_{s, \mathbf{h}_I}^*(\frac{\bar{u}-\bar{v}}{2}) . \end{aligned} \quad (5.13)$$

The relative phases δ_s and $\bar{\delta}_s$ are determined by requiring periodicity in all position variables on the world-sheet torus. The normalization constants are obtained from the short distance behavior of the amplitudes and are found to be

$$K = \langle \mathbf{1}_{\psi+\varphi} \rangle_s \frac{\theta_1^{\frac{3}{2}}(0)}{\theta_s(0)} = \frac{\theta_1'(0)}{\eta(\tau)} = -2\pi \eta^2(\tau) = \bar{K}^* . \quad (5.14)$$

Where $\mathbf{1}_{\psi+\varphi}$ is the unit operator the space-time fermions and the superconformal ghosts. Sending $z_1 \rightarrow x$, and $w_1 \rightarrow x$ and fixing $\delta_1 = 1$, we get $\delta_s = (-1)^s$ for $s = 2, 3, 4$ (likewise for the right-movers). The sum over spin structures can then be done using the Riemann identity (see Appendix C) with the result

$$\sum_{s, \bar{s}} C_s^L C_{\bar{s}}^R = 2\pi^2 |\eta|^4 \prod_{I=3}^5 |\theta_{1, -\mathbf{h}_I}(0)|^2 . \quad (5.15)$$

This is related to the Euler characteristic of the compactification manifold by remarking that $\prod_{I=3}^5 \theta_{1, -\mathbf{h}_I}(0) = \text{Tr}_R(-)^{F_L}$ and that $\text{Tr}_{RR}(-)^{F_L + F_R} = \chi$

In the same limit at least one of the fermionic correlators for any of the other three possible index structures vanishes. In the choice (5.12), the only possible bosonic contractions are

$$\begin{aligned} &\langle \partial X^1(x) \bar{\partial} X^{\bar{1}}(w_2) \partial X^{\bar{2}}(z_2) \bar{\partial} X^2(x) \prod_{i=1}^3 e^{ik^{(i)} \cdot X} \rangle \\ &= \langle \partial X^1(x) \bar{\partial} X^{\bar{1}}(w_2) \rangle \langle \partial X^{\bar{2}}(z_2) \bar{\partial} X^2(x) \rangle \langle \prod_{i=1}^3 e^{ik^{(i)} \cdot X} \rangle \\ &\quad - \langle \bar{\partial} X^{\bar{1}}(w_2) \partial X^{\bar{2}}(z_2) \prod_{i=1}^3 e^{ik^{(i)} \cdot X} \rangle \left(k_{\bar{1}}^{(2)} \partial_x G_B(x - u) + k_{\bar{1}}^{(3)} \partial_x G_B(x - v) \right) \\ &\quad \times \left(k_{\bar{2}}^{(2)} \bar{\partial}_x G_B(x - u) + k_{\bar{2}}^{(3)} \bar{\partial}_x G_B(x - v) \right) \end{aligned} \quad (5.16)$$

¹¹ In our choice of polarizations for the external states, the internal part of the supercurrent cannot contribute.

The first term on the r.h.s. (second line) contributes only through zero modes, while the remaining terms are of order $\mathcal{O}(k^4)$. Indeed, for generic positions z_2 and w_2 of the supercurrent insertions, the integration over the positions of the vertex operators cannot lead to cancellation of these momentum factors. We are thus left with the first term which leads, upon integration over the positions x , u and v of the vertices, to

$$\begin{aligned}\mathcal{A}_1 &= 2\pi^4 \chi \zeta_{12} F_{\bar{1}} \bar{F}_{\bar{2}} \left[\int_{\mathcal{F}} d^2\tau Z(\tau, \bar{\tau}) |\eta|^4 + \mathcal{O}(k^2) \right] \\ &= \frac{\pi^2}{3 \cdot 2^5} \chi \zeta_{12} F_{\bar{1}} \bar{F}_{\bar{2}} + |F|^2 \mathcal{O}(k^2) .\end{aligned}\tag{5.17}$$

This shows that the one-loop amplitude is non-zero for a configuration that makes the tree-level amplitude to vanish. We conclude that the string one-loop correction to the universal hypermultiplet is given by the solution (4.11).

6. M-theory on CY_3

In this Section we discuss the M-theory lifting, or the strong coupling limit of the previous construction. Of course, the expression (3.1) should now describe the metric for the universal hypermultiplet obtained by compactification of $l_P^6 R^4$ -corrected M-theory to five-dimensions on a Calabi-Yau three-fold. However, the identification of coordinates in (3.1) with M-theory variables is subtle.

In five dimensions, the universal hypermultiplet is composed of [32] the (normalized) volume of the Calabi-Yau manifold $\hat{v}_6 = (\int \sqrt{g} d^6x) l_P^{-6}$, the three-form $C_{\mu\nu\rho}$, which in 5d is dual to a scalar, and the complex scalar \hat{C} obtained from the RR 3-form along the volume form of the CY_3 , $C^{(3)} = \hat{C} \omega^{(3)}$. The four-dimensional universal vector multiplet (which contained \tilde{v}_6) upon lifting becomes part of the gravity multiplet. Reduction of the $l_P^6 R^4$ -corrected action on a Calabi-Yau three-fold to five dimensions gives then the following universal hypermultiplet metric

$$\begin{aligned}S &= \int d^5x \sqrt{G} \left[(\hat{v}_6 - \chi_1) \mathcal{R}_{(5)} + \frac{5}{6} (\hat{v}_6 + \hat{\alpha}) (d \ln \hat{v}_6)^2 - \frac{1}{2} (\hat{v}_6 + \hat{\beta}) (F_4)^2 \right. \\ &\quad \left. - \frac{1}{2} (\hat{v}_6 + \hat{\gamma}) |\hat{F}|^2 + i (\hat{v}_6 + \hat{\delta}) C_3 \wedge \hat{F} \wedge \hat{\bar{F}} \right] ,\end{aligned}\tag{6.1}$$

where F_4 is the field-strength of the 5d RR 3-form C_3 . Because the relative normalization between $\mathcal{R}_{(11)}$ and R^4 terms in eleven dimension is the same as between the $\mathcal{R}_{(10)}$ and the one-loop R^4 for type IIA in ten dimensions [2], the Planck mass correction is again given by χ_1 .

In order to compare this action with the strong coupling limit of the low energy string effective action determined in the previous section, we should first stress their differences.

Since in the string frame supersymmetry remains exact loop by loop, the string action (4.4) + (4.5) is a one-loop exact supersymmetric effective action. Such a statement cannot be made for M-theory compactifications; rather, the effective action (6.1) should be thought as a large- \hat{v}_6 approximation of an exact supersymmetric action. We saw that in four dimensions a mixing of the dilaton with the volume, or in other words of the universal hyper- and vector multiplets, was necessary for obtaining a supersymmetric string effective action. As already mentioned, M-theory does not have a universal vector multiplet and thus the matching between the action (6.1) and the Calberbank-Pedersen metric (3.1) can only be done up to the order $\mathcal{O}(\hat{v}_6^{-1})$.

For the identification of (6.1) with the strong coupling limit of (4.4) + (4.5) we need the standard relation [33] between the five-dimensional (M-theory) and the four-dimensional (string) metrics:

$$R_5^{-1} l_s^{-2} ds_{(4)}^2 = l_P^{-2} [ds_{(5)}^2 - R_5^2 (dx^5 - C_\mu dx^\mu)^2]. \quad (6.2)$$

We thus identify the dilaton with the volume of the Calabi-Yau measured in l_P units

$$\hat{v}_6 = e^{-2\phi_4}, \quad (6.3)$$

and the members of the universal hypermultiplets are identified by $(\hat{v}_6, C_{\mu\nu\rho}, \hat{C}_1, \hat{C}_2) \rightarrow (\phi_4, R_5^{-1} B_{\mu\nu}, R_5^{-3/2} C_1, R_5^{-3/2} C_2)$. The reduction to four dimensions of the volume dependent part of (6.1) then gives

$$S_v = \int d^4x \sqrt{g} \left\{ (\hat{v}_6 - \chi_1) \left[\mathcal{R}_{(4)} - \frac{1}{6} (d \ln [R_5^3 (\hat{v}_6 - \chi_1)])^2 \right] + (\hat{v}_6 + \frac{5\hat{\alpha} + 2\chi_1}{6} + \frac{1}{6} \frac{\chi_1^2}{\hat{v}_6 - \chi_1}) (d \ln \hat{v}_6)^2 \right\} \quad (6.4)$$

This matches with the four-dimensional action determined in the previous section upon the following identification

$$R_5^3 (\hat{v}_6 - \chi_1) = \tilde{v}_6, \quad (6.5)$$

which implies that the strong coupling radius R_5 is modified by a volume dependent term (in M-theory units) as

$$R_5^3 \simeq e^{2\phi_{10}} \left(1 - \left(\frac{3\mu_1}{2} - 1 \right) \frac{\chi_1}{\hat{v}_6} + \mathcal{O}(\hat{v}_6^{-2}) \right) \quad (6.6)$$

This identification, up to terms of order $\mathcal{O}(\hat{v}_6^{-1})$, allows us to fix all the constants in (6.1) as $5\hat{\alpha} + 2\chi_1 = 6\alpha$, $\hat{\beta} = \beta$, $\hat{\gamma} = \gamma$ and $\hat{\delta} = \delta$, but does not provide any information about μ_1 .

7. Supersymmetry and higher-loop terms

As we have seen, the only perturbative corrections to the universal hypermultiplet are at one loop, and all other contributions from higher loop/derivative terms can be absorbed in field redefinitions. Similarly, non-universal hypermultiplets and vector multiplets get corrected only at one-loop and tree-level respectively [1]. All these corrections can be seen as descending from ten-dimensional R^4 terms [1,5,34] (see Appendix A for such a derivation for the universal hypermultiplet). Clearly string theory has many more terms which are higher-order in curvature, may contain higher numbers of derivatives, and appear at higher loops. The conclusion of our analysis in Appendix B is that, when reduced to four dimensions, the contributions from all these extra terms should be absorbable in field redefinitions of the dilaton. Here, we would like to examine what constraints are imposed by this property on certain higher loop/derivative terms in ten dimensions. Naturally, such indirect analysis can apply only to a very limited set of couplings.

The only terms that can possibly be constrained by such analysis are those that upon reduction to four dimensions survive the large volume limit. Using $e^{-2\phi_4} = v_6 e^{-2\phi_{10}}$, it is not hard to see that such terms are of the type R^{3m+1} (and $H^2 R^{3m}$ or $F_{RR}^2 R^{3m}$) where m counts the loops. In other words, these are exactly the same terms that lift to eleven dimensions. It is not very hard to see that at two-derivative level the only contribution comes from strictly factorized terms of the form $\mathcal{R} \mathcal{S}^{(m)}$, where \mathcal{R} is the Ricci scalar and $\mathcal{S}^{(m)} \sim R^{3m}$ is a fully-contracted combination of Riemann tensors. Similarly, corrections to the B -field kinetic term come from fully-factorized terms in ten dimensions. A detailed discussion of the $m = 1$ case can be found in Appendix A, and $\mathcal{S}^{(1)}$ coincides with the six-dimensional Euler density. These contributions in the ten-dimensional action have a priori ambiguities, due to possible field redefinitions, but are constrained by the $\mathcal{N} = 2$ supersymmetry in four dimensions. In order for the higher-order corrections to the universal hypermultiplet metric to be absorbable from two loops and higher, the corrections to the four-dimensional Einstein term and the kinetic term for the B -field must be exactly the same and of the form:

$$S = \int d^{10}x \sqrt{g^\sigma} (\mathcal{R}_{(10)} + \frac{1}{2} H \wedge *H) \sum_{m \geq 2} (e^{-2\phi_{10}})^{1-m} \mathcal{S}^{(m)} \quad (7.1)$$

The special geometry of $\mathcal{N} = 2$ supersymmetry requires that the integrals over the internal manifolds M_6 , $\int_{M_6} \mathcal{S}^{(m)} \equiv \delta_m^R$, are independent of Kähler moduli.

We can further specify the precise form of $\mathcal{S}^{(m)}$ in (7.1). To this end, we turn to the non-universal sector and consider the higher-order corrections to (2.1):

$$\begin{aligned} S = \int d^4x \sqrt{g^\sigma} & \left[\left(\left(1 + \frac{\chi_T}{v_6}\right) e^{-2\phi_4} - \chi_1 + \sum_{m \geq 2} (e^{-2\phi_4})^{1-m} \delta_m^R \right) \mathcal{R}_{(4)} \right. \\ & + \left(\left(1 - \frac{\chi_T}{v_6}\right) e^{-2\phi_4} - \chi_1 + \sum_{m \geq 2} (e^{-2\phi_4})^{1-m} \delta_m^V \right) G_{vv} (\partial v)^2 \\ & \left. + \left(\left(1 + \frac{\chi_T}{v_6}\right) e^{-2\phi_4} + \chi_1 + \sum_{m \geq 2} (e^{-2\phi_4})^{1-m} \delta_m^H \right) G_{hh} (\partial h)^2 \right], \end{aligned} \quad (7.2)$$

where as before, G_{vv} is the metric of the vector-multiplets orthogonal to the volume modulus of the internal manifold and G_{hh} is the metric of the hypermultiplets orthogonal to the universal direction. In order to be able to absorb all higher-loop corrections by redefining the dilaton, clearly $\delta_m^R = \delta_m^V = \delta_m^H$ should hold for all $m \geq 2$. Moreover, in order not to spoil the metric on the hypermultiplet moduli space, $\delta_m^R = \int_{M_6} \mathcal{S}^{(m)}$ should be independent of complex structure moduli as well. Fully contracted combinations of Riemann tensors which do not depend on the complex structure are known [35], and are the generalizations of the Euler density $\mathcal{S}^{(1)}$

$$R_{r_1 s_1}{}^{r_2 s_2} R_{r_2 s_2}{}^{r_3 s_3} \dots R_{r_{3m} s_{3m}}{}^{r_1 s_1} - 2^{3m-1} R_{r_1}{}^{r_2}{}_{s_1}{}^{s_2} R_{r_2}{}^{r_3}{}_{s_2}{}^{s_3} \dots R_{r_{3m}}{}^{r_1}{}_{s_{3m}}{}^{s_1} + \text{Ricci}$$

One thus reaches the conclusion that $\mathcal{S}^{(m)}$ in (7.1) are given precisely by these combinations.

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Appendix A. R^4 terms and corrections to the universal hypermultiplet

This Appendix has two parts. First, we collect some formulae needed for reducing the effective action from ten or eleven dimensions to four or five dimensions, respectively, on a Calabi-Yau manifold. We then discuss the reduction of R^4 terms and their relation to the corrections in the universal hypermultiplet.

A.1. Useful formulae

▷ Definitions:

The connection is

$$\Gamma_{NP}^M = \frac{1}{2} G^{MK} (\partial_N G_{PK} + \partial_P G_{NK} - \partial_K G_{NP}) . \quad (\text{A.1})$$

The Riemann tensor, defined as $[\nabla_M, \nabla_N]V_P = R_{MNP}{}^Q V_Q$, is

$$\begin{aligned} R_{MNPQ} = & \frac{1}{2} (\partial_{MQ}^2 G_{NP} + \partial_{NP}^2 G_{MQ} - \partial_{MP}^2 G_{NQ} - \partial_{NQ}^2 G_{MP}) \\ & + G_{KL} (\Gamma_{MQ}^K \Gamma_{NP}^L - \Gamma_{MP}^K \Gamma_{NQ}^L) \end{aligned} \quad (\text{A.2})$$

▷ **Weyl Rescaling:**

Under a Weyl rescaling of the metric $\bar{G}_{MN} = \Omega^2 G_{MN}$, the Ricci scalar transforms as

$$\bar{\mathcal{R}} = \Omega^{-2} (\mathcal{R} - 2(D-1)\nabla^2 \ln \Omega - (D-2)(D-1)(\nabla \ln \Omega)^2) \quad (\text{A.3})$$

$$\int d^D x \sqrt{\bar{g}} \bar{\mathcal{R}} = \int d^D x \sqrt{g} \Omega^{D-2} [\mathcal{R} + (D-2)(D-1)(\nabla \ln \Omega)^2] \quad (\text{A.4})$$

▷ **Compactification:**

We consider a background metric associated with the warped product space-time $\mathbb{R}^D \times \mathcal{M}_6$

$$G_{MN}(x, y) = \begin{pmatrix} g_{\mu\nu}(x) & 0 \\ 0 & v_6^{\frac{1}{3}}(x) \gamma_{ij}(y) \end{pmatrix}, \quad \int_{CY_3} \sqrt{\det(\gamma_{ij})} = 1, \quad (\text{A.5})$$

where $l_s^6 v_6$ is the volume of \mathcal{M}_6 . Since we are only interested in the universal multiplet part of the reduction of string/M-theory on a Calabi-Yau threefold, we assume that the internal metric is independent of the coordinate $x \in \mathbb{R}^D$. The non-vanishing components of the connection are

$$\Gamma_{ij}^\mu = -\frac{1}{6} \gamma_{ij} v_6^{\frac{1}{3}} \partial^\mu \ln v_6, \quad \Gamma_{\mu j}^i = \frac{1}{6} \delta_j^i \partial_\mu \ln v_6 \quad (\text{A.6})$$

and $\Gamma_{\mu\nu}^\rho, \Gamma_{ij}^k$. The latter, being the connections constructed from $g_{\mu\nu}$ and γ_{ij} , respectively, are independent of v_6 . The components of the Riemann tensor involving derivatives of the volume are

$$\begin{aligned} R_{ijkl}(G) &= v_6^{\frac{1}{3}} \hat{R}_{ijkl}(\gamma) - \frac{1}{36} v_6^{\frac{2}{3}} (\partial_\mu \ln v_6)^2 (\gamma_{ik} \gamma_{jl} - \gamma_{il} \gamma_{jk}) \\ R_{\mu i \nu j}(G) &= -\frac{1}{36} \gamma_{ij} v_6^{\frac{1}{3}} [6 \nabla_\mu \nabla_\nu \ln v_6 + \partial_\mu \ln v_6 \partial_\nu \ln v_6] \end{aligned} \quad (\text{A.7})$$

All other components are volume independent ($R_{\mu\nu\rho\sigma}$), or zero ($R_{\mu ijk}$, $R_{i\mu\nu\rho}$ and $R_{ij\mu\nu}$). Other useful results (needed for $d=6$) are

$$\begin{aligned} R_{ij}(G) &= -\frac{d}{36} v_6^{\frac{1}{3}} (\partial_\mu \ln v_6) (\partial^\mu \ln v_6) \gamma_{ij} - \frac{1}{6} v_6^{\frac{1}{3}} \nabla^\mu \nabla_\mu \ln v_6 \gamma_{ij} + \hat{R}_{ij}(\gamma) \\ R_{\mu\nu}(G) &= R_{\mu\nu}(g) - \frac{d}{6} \left(\nabla_\mu \nabla_\nu \ln v_6 + \frac{1}{6} (\partial_\mu \ln v_6) (\partial_\nu \ln v_6) \right) \end{aligned} \quad (\text{A.8})$$

from which

$$\mathcal{R}(G) = -\frac{1}{36} d(d+1) (\partial_\mu \ln v_6) (\partial^\mu \ln v_6) - \frac{d}{3} \nabla^\mu \nabla_\mu \ln v_6 + \mathcal{R}(g) + v_6^{-\frac{1}{3}} \hat{\mathcal{R}}(\gamma). \quad (\text{A.9})$$

Of course, in our case $\hat{R}_{ij}(\gamma) = 0$.

A.2. Reduction of the R^4 terms

We study the reduction of the one-loop R^4 couplings in type IIA or M-theory in the background geometry specified by (A.5). This section is similar to the Appendix A of [5] but differs in details; e.g. we do not assume that the dilaton dependence is captured by the replacement $R_{MN} \rightarrow R_{MN} + 2\nabla_M \nabla_N \phi_{10}$.

▷ Correction to the volume kinetic term in four dimensions.

We want to confirm the action (4.5) by considering the reduction of the ten-dimensional R^4 term in the Calabi-Yau geometry specified by (A.5). For this we rewrite the kinetic terms for $\mathcal{R}_{(4)}$, v_6 and ϕ_4 in (4.5) and (4.4) as

$$S_2 = \int d^4x \sqrt{g^\sigma} \left\{ (v_6 e^{-2\phi_{10}} - \chi_1) \mathcal{R}_{(4)} + \frac{1}{6} [5 v_6 e^{-2\phi_{10}} + (1 - 3\mu_1) \chi_1 + 6\alpha] (d \ln v_6)^2 \right. \\ \left. - [4 v_6 e^{-2\phi_{10}} + (4\alpha - \mu_1 \chi_1)] d \ln v_6 d\phi_{10} + 4 [v_6 e^{-2\phi_{10}} + \alpha] (d\phi_{10})^2 \right\} \quad (\text{A.10})$$

The tree-level mixing between ϕ_{10} and v_6 arises from the reduction $\square_{10} \phi_{10} = \square_4 \phi_{10} + (\partial^\mu \ln v_6)(\partial_\mu \phi_{10})$. The dilaton dependence is difficult to test, given the lack of knowledge concerning its couplings in ten dimensions¹². Using the ten-dimensional $\mathcal{O}(\alpha')$ on-shell condition $R_{MN} + 2\nabla_M \nabla_N \phi_{10} = 0$ the one-loop mixing may arise from the couplings $R_{MN} S^{MN}$ which we discuss below.

We nevertheless can test the coefficients for the volume kinetic term. In type IIA, the one-loop R^4 terms are $t_8 t_8 R^4 + \frac{1}{4} E_8$ where $E_8 = 8! \times \delta_{N_1 \dots N_8}^{M_1 \dots M_8} R_{M_1 M_2}^{N_1 N_2} \times \dots \times R_{M_7 M_8}^{N_7 N_8}$. This can be expressed in terms of a basis R_{4i} , $i = 1, \dots, 6$ and A_7 (using the notation of the Appendix B.2 of [3]) of seven scalars built from four Riemann tensors (since we are compactifying on a Calabi-Yau manifold, we do not need terms involving the Ricci tensor or Ricci scalar of the internal manifold)

$$\begin{aligned} R_{41} &= \text{tr}(R_{MN} R_{NP} R_{PQ} R_{QM}) \rightarrow -(S_2 - \frac{1}{4} S_1) \mathcal{A} ; \\ R_{42} &= \text{tr}(R_{MN} R_{NP} R_{MQ} R_{QP}) \rightarrow \frac{S_1}{4} \mathcal{A} ; \\ R_{43} &= \text{tr}(R_{MN} R_{PQ}) \text{tr}(R_{MN} R_{PQ}) \rightarrow 2 S_1 \mathcal{A} ; \\ R_{44} &= \text{tr}(R_{MN} R_{MN}) \text{tr}(R_{PQ} R_{PQ}) \rightarrow 0 ; \\ R_{45} &= \text{tr}(R_{MN} R_{NP}) \text{tr}(R_{PQ} R_{QM}) \rightarrow 0 ; \\ R_{46} &= \text{tr}(R_{MN} R_{PQ}) \text{tr}(R_{MP} R_{NQ}) \rightarrow S_1 \mathcal{A} ; \\ A_7 &= R^{PQRS} R_P{}^M{}_R{}^U R_M{}^V{}_Q{}^W R_{UVSW} \rightarrow (S_2 - \frac{1}{4} S_1) \mathcal{A} , \end{aligned} \quad (\text{A.11})$$

¹² E.g. the kinetic term for the dilaton receives corrections from 5-point contributions $R^3(\nabla \phi_{10})^2$ which are not all captured [25] by the modified connection scheme of Gross and Sloan [26].

where $\mathcal{A} = \frac{1}{9v_6}(\partial \ln v_6)^2$. We have also given the contributions of each scalar to the kinetic energy of the universal volume modulus v_6 . $S_1 = \hat{R}_{ijkl}\hat{R}^{klmn}\hat{R}_{mn}{}^{ij}$ and $S_2 = \hat{R}_{ikjl}\hat{R}^{imjn}\hat{R}_m{}^k{}_n{}^l$ form a basis of scalars built from three Riemann tensors. The indices are raised with the metric γ^{ij} . One can now show that [3]¹³

$$\begin{aligned} t_8 t_8 R^4 &= 192R_{41} + 384R_{42} + 24R_{43} + 12R_{44} - 192R_{45} - 96R_{46} \\ \frac{1}{4}E_8 &= t_8 t_8 R^4 + 192R_{46} - 768A_7 + 64\mathcal{R}\mathcal{S} - 768R_{MN}S^{MN} + \text{higher Ricci} \end{aligned} \quad (\text{A.12})$$

where $\mathcal{S} = S_{ij}\gamma^{ij} = S_1 - 2S_2 + \text{Ricci}$ is the six-dimensional Euler density (defined analogously to E_8). $\mathcal{R}\mathcal{S}$ contributes to the correction to $\mathcal{R}_{(4)}$, while $t_8 t_8 R^4$, R_{46} , A_7 , $\mathcal{R}\mathcal{S}$ and $R_{ij}S^{ij}$ contribute to the kinetic term of the volume. Because the Ricci terms in (A.12) are affected by field redefinitions, we introduce arbitrary coefficients $c_{1,2}$ in front of $64\mathcal{R}\mathcal{S}$ and $-768R_{MN}S^{MN}$. Using the components (A.7) before integration over the zero modes, we get

$$64(S_1 - 2S_2) \left[c_1 \mathcal{R}_{(4)} - \frac{6 + 12c_1 - 7c_2}{6} (d \ln v_6)^2 \right]. \quad (\text{A.13})$$

Comparison with (A.10) shows that neither of the values $\mu_1 = \pm 2$ is matched by the naive choice $c_1 = c_2 = 1$. Of course, there is no reason that this naive choice should work. Having fixed the sign ambiguity of μ_1 (which in principle can be done by a string computation), one could use (A.13) to determine $c_{1,2}$ in the given background.

▷ Reduction of $C_3 \wedge t_8 R^4$

The $\int C_3 \wedge t_8 R^4$ coupling [36,37] does not contribute to the two-derivative effective action in four dimensions. The indices should split into five external μ -type indices and six internal i -type indices. Plugging the expressions for the components of the Riemann tensor (A.7), we find that $C_{\mu ij}$ and C_{ijk} only appear with higher derivative terms $\mathcal{O}(\partial_\mu^4)$. Only $C_{\mu_1 \mu_2 \mu_3}$ could be associated with the two-derivate interaction originating from $\int \epsilon^{\mu_1 \dots \mu_5} C_{\mu_1 \dots \mu_3} t_8 (R_{\mu_4 i_1} R_{\mu_5 i_2} R_{i_3 i_4} R_{i_5 i_6}) \epsilon^{i_1 \dots i_6}$, but the antisymmetrisation on the $\mu_{4,5}$ -indices makes the term vanishing.

¹³ For a comparison with the expressions J_o of [5,35], one can use the identity $R^r{}_{[mnp]} = 0$ to find that $t_8 t_8 R^4 - \frac{1}{4}E_8 = 64J_o$. The six-dimensional Euler density is

$$E_6 = \frac{1}{3!(8\pi)^3} \epsilon_6 \epsilon_6 R^3 = \frac{1}{12(2\pi)^3} (S_1 - 2S_2 + \text{Ricci})$$

Appendix B. The Quaternionic Kähler geometry

Following [21,6], we study here loop corrections to the quaternionic geometry. Our notation is the same with that used in these papers.

Introduce the real vierbein

$$V = \begin{pmatrix} v_1 \\ v_2 \\ \bar{v}_2 \\ -\bar{v}_1 \end{pmatrix},$$

such that $ds^2 = {}^T V(\sigma_2 \otimes \sigma_2)V = v_1 \bar{v}_1 + v_2 \bar{v}_2$. We split the holonomy group $O(4) = Sp(1) \otimes Sp(1)$, as well as the connection $\Omega := P + Q$ with

$$P := -\frac{i}{2} p \cdot \sigma \otimes \mathbb{I}_2; \quad Q := -\frac{i}{2} \mathbb{I}_2 \otimes q \cdot \sigma,$$

where $\sigma^{i=1,2,3}$ are the Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The connections P and Q satisfy the tetrad postulate

$$dV + (P + Q) \wedge V = 0. \quad (\text{B.1})$$

For further reference, it is useful to remark that

$$P + Q = -\frac{i}{2} \begin{pmatrix} p_3 + q_3 & p^- & q^- & 0 \\ p^+ & -p_3 + q_3 & 0 & q^- \\ q^+ & 0 & p_3 - q_3 & p^- \\ 0 & q^+ & p^+ & -(p_3 + q_3) \end{pmatrix} \quad (\text{B.2})$$

where we have introduced $p^\pm = p_1 \pm ip_2$ and $q^\pm = q_1 \pm iq_2$. The reality of the vierbein implies that p_i and q_i are real.

We define the $Sp(1)$ curvatures as $\Omega^i = \frac{i}{2} V^\dagger \wedge (\sigma^i \otimes \mathbb{I}_2) V$, and the connection P satisfies the constraint [21,9]

$$dp^i + \frac{1}{2} \varepsilon^{ijk} p^j \wedge p^k = \Omega^i \iff \begin{cases} dp^+ - ip^+ \wedge p^3 = \Omega^1 + i\Omega^2 \\ dp^- + ip^- \wedge p^3 = \Omega^1 - i\Omega^2 \\ dp^3 + \frac{i}{2} p^+ \wedge p^- = \Omega^3 \end{cases} \quad (\text{B.3})$$

We study the perturbative deformations of the quaternionic geometry by solving (B.1) and (B.3) order by order in $\exp(2\phi_4)$: $V = \sum_n \exp(2n\phi_4) V_n$. The map between the sigma-model metric (3.1) and the dilaton can receive perturbative corrections as $\rho^2 = \exp(-2\phi_4) - 2\hat{\alpha} \hat{\chi} + \dots$.

B.1. Tree-level solution

The zero-th order solution is given by making the choice of the vierbein

$$V_o = \begin{pmatrix} u \\ v \\ \bar{v} \\ -\bar{u} \end{pmatrix} \quad (\text{B.4})$$

The $\text{Sp}(1)$ curvatures are

$$\begin{aligned} \Omega_o^1 &= i(\bar{u} \wedge v + \bar{v} \wedge u) \\ \Omega_o^2 &= \bar{u} \wedge v + u \wedge \bar{v} \\ \Omega_o^3 &= i(\bar{u} \wedge u + v \wedge \bar{v}) \end{aligned} \quad (\text{B.5})$$

The u and v are the isometry invariant coordinates defined as in the main text (see equation (3.9) for $\hat{\chi} = 0$), and satisfy the relations

$$du = \frac{1}{2} u \wedge (v + \bar{v}); \quad dv = v \wedge \bar{v} + u \wedge \bar{u} \quad (\text{B.6})$$

Equation (B.1) is solved by

$$\begin{aligned} p_o^+ &= 2i\bar{u}; \quad p_o^- = -2iu; \quad p_o^3 = \frac{i}{2}(v - \bar{v}) \\ q_o^+ &= q_o^- = 0; \quad q_o^3 = \frac{3i}{2}(\bar{v} - v) \end{aligned} \quad (\text{B.7})$$

B.2. One-loop correction

We consider now the one-loop corrections to the previous solution. The loop counting parameter is $\exp(-\phi_4) = \rho$. The most general expression for the one-loop correction to the vierbein which is invariant under the three isometries and which leads to a real metric is

$$V_1 = e^{2\phi_4} \begin{pmatrix} \alpha u + \bar{\beta} \bar{u} + \gamma v + \bar{\delta} \bar{v} \\ \alpha' u + \bar{\beta}' \bar{u} + \gamma' v + \bar{\delta}' \bar{v} \\ \bar{\alpha}' \bar{u} + \beta' u + \bar{\gamma}' \bar{v} + \delta' v \\ -(\bar{\alpha} \bar{u} + \beta u + \bar{\gamma} \bar{v} + \delta v) \end{pmatrix} \quad (\text{B.8})$$

At the first order, the metric is given by $ds^2 = {}^T(V_o + 2V_1)(\sigma_2 \otimes \sigma_2)V_o$.

Because the perturbative u -coordinate contains the field-strength of the RR-fields and the imaginary part of v -coordinate is the NS-axion, some physical constraints have to be imposed on possible deformations of the metric. Following arguments of [6], we find the following restrictions on the parameters appearing in (B.8):

- i) In order not to violate parity invariance, vv and $\bar{v}\bar{v}$ terms must appear in the combination $vv + \bar{v}\bar{v}$. This implies that δ' is a real parameter.
- ii) The real and imaginary parts of u are the RR-field strengths. Since string perturbation forbids any mixing between these fields (cf. footnote 4), uu and $\bar{u}\bar{u}$ terms can only appear in the combination $uu + \bar{u}\bar{u}$. This forces β to be a real parameter. The same conclusion can also be reached using parity invariance, since a parity transformation acts as $u \leftrightarrow \bar{u}$.
- iii) Amplitudes of odd powers of RR-fields vanish in string perturbation theory. This implies that $\beta' + \delta = 0$ and $\gamma + \bar{\alpha}' = 0$.

At the first order, the metric then reads

$$\begin{aligned} ds^2 &= {}^T(V_o + 2V_1)(\sigma_2 \otimes \sigma_2)V_o \\ &= 2(u\bar{u} + v\bar{v}) + \beta(u^2 + \bar{u}^2) + \delta'(v^2 + \bar{v}^2) + (\alpha + \bar{\alpha})u\bar{u} + (\gamma' + \bar{\gamma}')v\bar{v} . \end{aligned} \quad (\text{B.9})$$

Since the coefficients γ and δ and the imaginary parts of α and γ' do not appear in (B.9), we can set them to zero. The vierbein then takes the form (we drop primes)

$$V_1 = e^{2\phi_4} \begin{pmatrix} \alpha u + \beta \bar{u} \\ \gamma v + \delta \bar{v} \\ \gamma \bar{v} + \delta v \\ -(\alpha \bar{u} + \beta u) \end{pmatrix} \quad (\text{B.10})$$

where all coefficients are real. It is not difficult to prove order by order that this is the most general parameterization of the perturbative corrections to the vierbein (affecting the metric) for any V_n which satisfies the constraints i), ii) and iii).

The first-order corrections to the $\text{Sp}(1)$ connections are $\Omega_1^i = i V_o^\dagger (\sigma^i \otimes \mathbb{I}_2) V_1$:

$$\begin{aligned} e^{-2\phi_4} \Omega_1^1 &= (\alpha + \gamma) \Omega_o^1 + i(\beta - \delta) (u \wedge v - \bar{u} \wedge \bar{v}) \\ e^{-2\phi_4} \Omega_1^2 &= (\alpha + \gamma) \Omega_o^2 + (\beta + \delta) (u \wedge v + \bar{u} \wedge \bar{v}) \\ e^{-2\phi_4} \Omega_1^3 &= 2\alpha \Omega_o^3 + 2i(\alpha - \gamma) \bar{v} \wedge v \end{aligned} \quad (\text{B.11})$$

The first-order variation of (B.3) is

$$dp_1^i + \frac{1}{2} \varepsilon^{ijk} (p_1^j \wedge p_o^k + p_o^j \wedge p_1^k) = \Omega_1^i \quad (\text{B.12})$$

Using that $d \exp(2\phi_4) = -\exp(2\phi_4) (v + \bar{v})$, these equations imply

$$p_1^+ = 2i(\alpha - \delta) \bar{u}; \quad p_1^- = 2i(\delta - \alpha) u; \quad p_1^3 = i(\alpha - 2\delta) (v - \bar{v}) \quad (\text{B.13})$$

and the one-loop correction to the vierbein becomes

$$V_1 = e^{2\phi_4} \begin{pmatrix} \alpha u \\ 2(\alpha - 2\delta) v + \delta \bar{v} \\ 2(\alpha - 2\delta) \bar{v} + \delta v \\ -\alpha \bar{u} \end{pmatrix}. \quad (\text{B.14})$$

Solving the first-order linearization of (B.1) does not constrain α and δ .

We redefine $(\alpha, \delta) \rightarrow (\hat{\alpha} + 3\hat{\beta}, \hat{\beta}) \times \hat{\chi}$, and rewrite the one-loop correction to the vierbein (B.14) as:

$$V_1 = e^{2\phi_4} \hat{\chi} \begin{pmatrix} (\hat{\alpha} + 3\hat{\beta}) u \\ 2(\hat{\alpha} + \hat{\beta}) v + \hat{\beta} \bar{v} \\ 2(\hat{\alpha} + \hat{\beta}) \bar{v} + \hat{\beta} v \\ -(\hat{\alpha} + 3\hat{\beta}) \bar{u} \end{pmatrix} \quad (\text{B.15})$$

This two parameter deformation is obtained by linearization of the Calderbank-Pedersen metric (3.9), taking into account the one-loop correction to the dilaton $\rho^2 = \exp(-2\phi_4) - 2\hat{\alpha} \hat{\chi}$ and to the potential $F(\eta, \rho) = \rho^{3/2} - 2\hat{\beta} \hat{\chi} \rho^{-1/2}$. The analysis in [6] only considers the special case $\hat{\alpha} \neq 0$ and $\hat{\beta} = 0$. Note that $\hat{\beta} \neq 0$ is in fact a redundant parameter and we have set $\hat{\beta} = \frac{1}{2}$ in (3.8) and (3.11).

B.3. Two-loop correction

At two-loops, the most general parametrization of the modification of the vierbein compatible with the requirements i), ii) and iii) and affecting the metric is

$$V_2 = e^{4\phi_4} \begin{pmatrix} \tilde{\alpha} u + \tilde{\beta} \bar{u} \\ \tilde{\gamma} v + \tilde{\delta} \bar{v} \\ \tilde{\gamma} \bar{v} + \tilde{\delta} v \\ -(\tilde{\alpha} \bar{u} + \tilde{\beta} u) \end{pmatrix} \quad (\text{B.16})$$

where all parameters are real. Solving equation (B.3) at two-loop order shows, after tedious but straightforward calculation, that the vierbein is given by

$$V_2 = e^{4\phi_4} \begin{pmatrix} \left(\frac{3}{2}\alpha^2 - 4\alpha\delta + 3\delta^2 + \tilde{\delta}\right) u \\ \left(4\alpha^2 - 28\alpha\delta + \frac{99}{2}\delta^2 + 3\tilde{\delta}\right) v + \tilde{\delta} \bar{v} \\ \left(4\alpha^2 - 28\alpha\delta + \frac{99}{2}\delta^2 + 3\tilde{\delta}\right) \bar{v} + \tilde{\delta} v \\ - \left(\frac{3}{2}\alpha^2 - 4\alpha\delta + 3\delta^2 + \tilde{\delta}\right) \bar{u} \end{pmatrix}, \quad (\text{B.17})$$

where α and δ already appeared in V_1 and $\tilde{\delta}$ is unrestricted. The two-loop order variation of (B.2) does not restrict $\tilde{\delta}$ either. Switching to the variables $(\alpha, \delta, \tilde{\delta}) \rightarrow (\hat{\alpha} + 3\hat{\beta}, \hat{\beta}, \hat{\gamma}) \times \hat{\chi}$

the second-order modification of the vierbein becomes

$$V_2 = e^{4\phi_4} \hat{\chi} \begin{pmatrix} \left(\frac{3}{2}\hat{\alpha}^2 + 5\hat{\alpha}\hat{\beta} + \frac{9}{2}\hat{\beta}^2 + \hat{\gamma} \right) u \\ \left(4\hat{\alpha}^2 - 4\hat{\alpha}\hat{\beta} + \frac{3}{2}\hat{\beta}^2 + 3\hat{\gamma} \right) v + \hat{\gamma}\bar{v} \\ \left(4\hat{\alpha}^2 - 4\hat{\alpha}\hat{\beta} + \frac{3}{2}\hat{\beta}^2 + 3\hat{\gamma} \right) \bar{v} + \hat{\gamma}v \\ - \left(\frac{3}{2}\hat{\alpha} + 5\hat{\alpha}\hat{\beta} + \frac{9}{2}\hat{\beta}^2 + \hat{\gamma} \right) \bar{u} \end{pmatrix} \quad (\text{B.18})$$

This can be absorbed to a modification of the map between the dilaton and ρ at two-loops as

$$\rho^2 = e^{-2\phi_4} - 2\hat{\alpha}\hat{\chi} - 2(\hat{\gamma} - \hat{\beta}(\hat{\beta} + 4\hat{\alpha}))\hat{\chi}^2 e^{2\phi_4}, \quad (\text{B.19})$$

while keeping the potential $F(\eta, \rho) = \rho^{3/2} - 2\hat{\beta}\hat{\chi}\rho^{-1/2}$ unchanged.

Appendix C. Some details of the one-loop string computation

C.1. Dependence of the amplitude on the supercurrent insertions

We consider (5.9) where all vertex operators are in their superconformal ghost charge we had to insert two floating holomorphic picture changers [38] $Y(z_{1,2}) = \{Q_{BRST}, 2\xi(z_{1,2})\}$ and two floating anti-holomorphic picture changers $\tilde{Y}(\bar{w}_{1,2})$. Using the BRST invariance of the supercurrent and the vertex operators, we derive that the only contribution arises from the action of the BRST-charge on the measure of integration, which varies into a total derivative of the moduli [38],¹⁴

$$\begin{aligned} \partial_{z_1}(5.9) &= \sum_{s, \bar{s}=1}^4 \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \int d^2x \int d^2u \int d^2v \sum_{m=\tau, x, u, v} \frac{\partial}{\partial m} Z(\tau, \bar{\tau}) \\ &\quad \langle \partial\xi(z_1)Y(z_2)\tilde{Y}(w_1)\tilde{Y}(w_2) V_{\text{NS}}^{(-1, -1)}(x) V_F^{(-\frac{1}{2}, -\frac{1}{2})}(u) V_{\bar{F}}^{(-\frac{1}{2}, -\frac{1}{2})}(v) \rangle_{s, \bar{s}}. \end{aligned} \quad (\text{C.1})$$

By ghost charge conservation only the ghost-charge $+2$ part of the picture changer $Y(z_2)|_{+2} = \{\oint be^{2\phi-2\chi}, 2\xi(z_2)\}$ contributes to the amplitude. This is important in the computation of section 5, where we choose specific polarizations of the external states such that the tree level amplitude vanishes. For this choice (C.1) is also zero and we are thus free to place the picture changing operators at any convenient position and we do not have to worry about any contributions from the boundary of moduli space which might need regularization. Identical arguments and conclusions apply to the variation w.r.t. z_2 of the positions of the right-moving picture changers.

¹⁴ See as well [39] for an application to an amplitude closely related to the one studied in section 5.

C.2. Useful formulae

Here, we collect some results needed for the calculation of the one-loop amplitude we describe in section 5.

The bosonic correlation functions are reduced, via Wick's theorem, to two-point functions (and their derivatives):

$$\langle X^\mu(z)X^\nu(w) \rangle = -\eta^{\mu\nu} \ln |\chi(z-w)|^2 \equiv -\eta^{\mu\nu} G_B(z-w) \quad (\text{C.2})$$

with

$$G_B(z-w) = \ln \left| \frac{\theta_1(z-w)}{\theta_1'(0)} \right|^2 - \frac{2\pi}{\tau_2} (\text{Im}(z-w))^2 \quad (\text{C.3})$$

To be able to compute the internal $\mathcal{N} = (2,2)$ part, we consider compactifications on symmetric orbifolds. Our final conclusions should not depend on this choice. The SCFT is then realized via twisted complex free bosons X^I and fermions ψ^I , $I = 3, 4, 5$ with twists $\mathbf{h}_I = (h_I, g_I)$, where e.g. $X^I(z+1) = e^{2\pi i h_I} X^I(z)$ and $X^I(z+\tau) = e^{2\pi i g_I} X^I(z)$. Space-time supersymmetry requires $\sum_I h_I = \sum_I g_I = 0$. We bosonize the fermions $\psi^I = \exp(i\phi^I)$. The internal spin fields in (5.5) are expressed as $\Sigma(z, \bar{z}) = \Sigma(z)\tilde{\Sigma}(\bar{z})$, $\Sigma = \exp(i\frac{\sqrt{3}}{2}H)$ with $\sqrt{3}H = \sum_I \phi^I$ the left U(1)-current of the $\mathcal{N} = (2,2)$ superconformal theory with similar expressions for the right-movers $\tilde{\psi}^I$ and $\tilde{Z}(\bar{z})$. Then, for spin structure s

$$A^{\text{int}} = \langle \Sigma_L(u) \tilde{\Sigma}_L(v) \rangle_s = \frac{\theta_1'^{\frac{3}{4}}(0)}{\theta_1^{\frac{3}{4}}(u-v)} \prod_{I=3}^5 \theta_{s, \mathbf{h}_I} \left(\frac{u-v}{2} \right) \quad (\text{C.4})$$

The normalization is fixed by matching the singularity as $u \rightarrow v$ and using $\langle \mathbf{1}_{\text{int}} \rangle = \prod_{I=3}^5 \theta_{1, h_I}(0)$. $\mathbf{1}_{\text{int}}$ is the unit operator in the internal sector. To sum over the spin structures we will need to use the following Riemann identity which is valid for $h_1 = g_1 = 0$, $h_2 + h_3 + h_4 = g_2 + g_3 + g_4 = 0$:

$$\sum_{s=1}^4 (-1)^s \prod_{i=1}^4 \theta_{s, \mathbf{h}_i}(z_i) = -2 \prod_{i=1}^4 \theta_{1, -\mathbf{h}_i}(v_i) \quad (\text{C.5})$$

where

$$v_i = A_{ij} z_j \quad \text{with} \quad A_{ij} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}.$$

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