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Chiral splitting and world-sheet gravitinos in higher-derivative string amplitudes

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Abstract

We report on progress made in the construction of higher-derivative superinvariants for type-II theories in ten dimensions. The string amplitude calculations required for this analysis exhibit interesting features which have received little attention in the literature so far. We discuss two examples from a forthcoming publication: the construction of the $(H_{NS})^2R^3$ terms and the fermionic completion of the $\epsilon\epsilon R^4$ terms. We show that a correct answer requires very careful treatment of the chiral splitting theorem, implies unexpected new relations between fermionic correlators, and most interestingly, necessitates the use of worldsheet gravitino zero modes in the string vertex operators. In addition, we discuss the relation of our results to the predictions of the linear scalar superfield of the type-IIB theory and find (and explain) an interesting discrepancy.

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1 Introduction

In a recent paper [1] we have initiated an extensive project to determine the supersymmetric completion of the higher-derivative expansion of string effective actions in ten dimensions. Although several terms in these derivative expansions are known, they concern almost exclusively the pure graviton sector, except for a few isolated terms related to anomalies and a number of terms conjectured from duality symmetries. Apart from the obvious interest in extending our systematic results to include also the Neveu-Schwarz and Ramond gauge fields, we have argued in [1] that there are several reasons why the fermion bilinears are interesting as well. We have shown how, using a combination of string scattering amplitudes and component supersymmetry, one is able to determine these terms. The reader is referred to the introduction of [1] for further historical details and motivation.[1]

In the process of computing the string amplitudes which are required for the full construction of the type-II invariants at order $(\alpha')^3$, we realised that there are several interesting subtleties which deserve additional attention. The aim of the present letter is therefore to highlight these issues, separated from the (necessarily rather technical) bulk of a future publication in which the full invariants will be constructed. As we will show, our calculations (and the various consistency checks they have to satisfy) are an excellent way to probe some ill-appreciated features of the formalism for (one-loop) string amplitudes. In particular, we will highlight the role of left/right mixing contractions both in the amplitudes and in the picture-changing procedure (particularly relevant in the odd spin-structure sector), discuss in some detail the machinery necessary for the evaluation of fermionic correlators, and exhibit the role of world-sheet gravitino zero modes appearing in vertex operators.

[1] Our previous paper also dealt with an extension of the ten-dimensional results to eleven dimensions, but we will not discuss this issue in the present letter.

Let us start with a short reminder of the present knowledge of higher-derivative invariants in ten dimensions. The eight-derivative terms of the superstring effective actions in ten dimensions are constructed from two separate bosonic building blocks, which at linear order in the Neveu-Schwarz gauge field are given by

$$\begin{aligned} I_X &= t_8 t_8 W^4 + \tfrac{1}{2}\varepsilon_{10} t_8 B W^4\,, \\ I_Z &= \varepsilon_{10}\varepsilon_{10} W^4 + 4\,\varepsilon_{10} t_8 B W^4\,. \end{aligned} \tag{1.1}$$

Here, W denotes the Weyl tensor and B the Neveu-Schwarz two-form gauge field. The tensors t_8 and ε_{10} soak up the free indices. These two building blocks are separately invariant under a subset of the supersymmetry transformation rules, namely those that lead to variations which are completely independent of any of the gauge fields of the theory. This limited supersymmetry analysis does not predict the relative coefficient between the two building blocks; whether or not full supersymmetry fixes this coefficient remains an open question. This coefficient has, however, been determined in other ways, and it turns out that its value in the various string theories is such that

$$\begin{aligned} \mathcal{L}_{\text{heterotic}}\Big|_{(\alpha')^3} &= e^{-2\phi^H}\big(I_X - \tfrac{1}{8}I_Z\big) + I_X\,, \\ \mathcal{L}_{\text{IIA}}\Big|_{(\alpha')^3} &= e^{-2\phi^A}\big(I_X - \tfrac{1}{8}I_Z\big) + \big(I_X + \tfrac{1}{8}I_Z\big)\,, \\ \mathcal{L}_{\text{IIB}}\Big|_{(\alpha')^3} &= f(\Omega,\bar{\Omega})\big(I_X - \tfrac{1}{8}I_Z\big)\,, \end{aligned} \tag{1.2}$$

(Ω is the complexified coupling constant, and we have ignored the Yang-Mills part of the heterotic effective action for simplicity). As a result, the coefficient of the $B \wedge t_8 W^4$ term relative to the $t_8 t_8 W^4$ term is zero for the type-IIB theory, while in the type-IIA theory it is twice the coefficient of the heterotic theory. We will refer to this term as the "anomaly term" in the following.

In our previous paper, we have analysed in detail the fermionic completion of the I_X invariant and its associated supersymmetry algebra. In the present letter we discuss two specific calculations that are needed to extend this result to other terms in the effective actions. The first one, discussed in section 3.1 below, deals with the fermionic completion of the I_Z invariant. Whereas the sign flip of the $B\wedge W^4$ term between the two invariants in (1.1) is simple to understand in terms of a GSO projection sign, the related sign flip for the associated fermion bilinears is very much hidden. This example is particularly relevant in order to understand modifications to the supersymmetry algebra, which was our original motivation for [1].

The second example concerns bosonic terms, namely the $W^3 H^2_{(NS)}$ terms of the type-II theories. These are addressed in section 3.2. They have recently attracted more attention due to their relevance for the computation of $1/N$ corrections in conifold versions of the AdS/CFT conjecture (for more on this we refer to Frolov et al. [2] and references therein). The existing results on these terms are rather incomplete, mainly due to the fact that they were done in the light-cone gauge. Moreover, a comparison of these terms with the prediction of the linearised superfield construction has not appeared so far. We exhibit a covariant RNS calculation of these terms and show how the linearised superfield fails to reproduce these correctly.

In both of these calculations, a precise understanding of the string subtleties mentioned at the beginning of this introduction turns out to be crucial in order to arrive at a correct result. The examples discussed in the present letter are typical of the problems that arise in the full construction of the type-II invariants, to appear elsewhere.

2 Picture changing, zero modes and contact terms

Chiral splitting, or the separation of string amplitudes in a "left-moving, left-handed" and a "right-moving, right-handed" factor, is often implicitly used in string calculations. As explained by D'Hoker and Phong [3], the separation into two completely independent sectors is, however, a subtle issue. On-shell conditions on the fields, which constrain the fields to be either holomorphic or anti-holomorphic functions of the complex world-sheet coordinate, can obviously not be used inside the path integral, but this is not the only source of problems. For instance, the Green function of the bosons on a torus leads to

$$\big\langle \partial X(z)\bar\partial X(w)\big\rangle = \alpha'\pi\Big(\delta^{(2)}(z-w) - \frac{1}{\tau_2}\Big)\,, \tag{2.1}$$

a left/right-mixing correlator which receives contributions both from a contact interaction and from zero modes. In addition the action contains a four-fermi term which does not respect the separation of Weyl components.

In practise, however, few of these things seem to matter. The reason for this is reflected in the chiral splitting theorem of D'Hoker and Phong [3]. This theorem implies that there exist *effective* rules that summarise the consequences of the coupling of both sectors. In many cases the implication is simply that amplitudes can be obtained as the product of two chirally split factors. However, this is definitely *not* what happens in the most general case. In the present section we would like to recall these subtleties of the chiral splitting theorem and formulate them in such a way that they can be readily applied to string amplitude calculations.

We start with the world-sheet action for the superstring. While the graviton can always be gauge fixed to the flat metric, one has to be careful with the gravitino. We therefore keep the gauge-fixed gravitino present. The action for the $(1,1)$ supersymmetric string in a flat background is then given by

$$\begin{aligned} S_{X,\Psi,F} = \int \mathrm{d}^2 z\,\Big(&\partial X^\mu\bar\partial X_\mu - \Psi^\mu\bar\partial\Psi_\mu - \tilde\Psi^\mu\partial\tilde\Psi_\mu + F^\mu F_\mu \\ &+ \chi_-\bar\partial X^\mu\tilde\Psi_\mu + \tilde\chi_+\partial X^\mu\Psi_\mu + \tfrac{1}{2}\Psi\tilde\Psi\,\chi_-\tilde\chi_+\Big)\,. \end{aligned} \tag{2.2}$$

where Ψ and χ denote the left-handed component of the world-sheet fermion and gravitino respectively, and tildes denote the opposite chirality components. In addition there are of course the ghost terms. We keep the auxiliary field because a correct BRST treatment requires a symmetry algebra which closes off-shell.

Because the BRST charge contains the derivative of the scalar fields, any verification of BRST invariance of vertex operators which also contain these factors is made more difficult because of the contraction (2.1). Therefore, it is useful to start the analysis of the vertex operators in the ghost picture for which they contain (apart from plane wave exponentials) only world-sheet spinors. These preferred graviton, two-form gauge field and dilaton operators are given by

$$V_\zeta^{(-1,-1)} = \int \mathrm{d}^2 z\,\Big(\zeta_{\mu\nu}\,\Psi^\mu\tilde\Psi^\nu\,e^{-\phi-\tilde\phi}e^{ik\cdot X}\Big)\,. \tag{2.3}$$

For target-space fermions, one has to take operators in the $(-1/2,-1/2)$ picture to achieve a similar goal; more on those operators later. Using this class of vertex operators, all contractions of the type (2.1) come from the insertion of picture-changing operators. They come in two different types: those from the requirement of a fixed total ghost charge, and those from the integration over odd supermoduli. Although the effective rules for these different contractions are the same, they arise in completely different ways, as we will now show.

Let us first consider the latter class. In this case the gravitino is gauge-fixed to a non-zero function, and a Berezin integral has to be performed over the odd supermodulus. Contractions of the type (2.1) can only occur when both sectors are in the odd spin-structure sector. A generic correlator takes the form

$$\begin{aligned}\Big\langle V_1(z_1)\cdots V_n(z_n)\Big\rangle &= \int\mathcal{D}\chi\mathcal{D}\tilde{\chi}\mathcal{D}X\mathcal{D}\Psi\,\Big[V_1(z_1)\cdots V_n(z_n)\Big]\,\exp\left(-S[X,\Psi]\right)\\ &\times e^{\phi+\tilde{\phi}}\Bigg(1-\frac{1}{2\pi\alpha'}\int \mathrm{d}^2z\,\tilde{\chi}_+\Psi\partial X(z)-\frac{1}{2\pi\alpha'}\int\mathrm{d}^2z\,\chi_-\tilde{\Psi}\bar{\partial}X(z)+\frac{1}{4\pi\alpha'}\int\mathrm{d}^2z\,\tilde{\chi}_+\chi_-\Psi\tilde{\Psi}(z)\\ &\qquad+\frac{1}{(2\pi\alpha')^2}\int\mathrm{d}^2w\!\int\mathrm{d}^2z\,\tilde{\chi}_+\Psi\partial X(w)\chi_-\tilde{\Psi}\bar{\partial}X(z)\Bigg)\,, \end{aligned} \tag{2.4}$$

where we have expanded in powers of the world-sheet gravitino. The supersymmetry ghost e^{ϕ} arises from the super-Beltrami differentials. The integral over the gravitino zero modes picks out those terms which contain both of the gravitino chiralities. These contributions are usually interpreted as insertions of picture changing operators, as the factors multiplying the gravitino terms indeed form the normal picture-changing operators (see below). One now observes immediately that the $\delta^{(2)}(z-w)$ part of the contraction (2.1) in the fifth term cancels the fourth term. As a consequence, one finds the effective rule that *left/right contractions between bosons of odd spin-structure picture changers only involve the zero mode part.*

Picture changers of the former class, inserted by hand to balance the ghost charge, behave differently. One does not usually keep them explicit inside the path integral, but instead works out the vertex operators in a new ghost picture. This is, however, also plagued with subtleties arising from the contraction (2.1) and the presence of gravitino modes.[2] We will first re-examine this picture-changing procedure, after which we will derive an effective rule for the contractions between the bosons that arise from these picture changers and those that arise from the odd-supermoduli integration.

The picture-changing procedure is defined as the BRST commutator with an insertion of the ghost zero-mode ξ,

$$V^{(q+1,\tilde{q})}=\frac{2}{\alpha'}\big[Q^L_{\text{BRST}},\xi\,V^{(q,\tilde{q})}\big]\,, \tag{2.5}$$

and similar for the picture-changing on the right. In the presence of world-sheet gravitino zero modes, the formulation of an appropriate Q_{BRST} is troublesome, so we prefer to use directly the BRST transformation rules. These can be found in Ohta [6] and the γ-ghost dependent parts read in our conventions

$$\begin{aligned} &\delta(\partial X^\mu)=\gamma\partial\Psi^\mu+\tilde{\gamma}\partial\tilde{\Psi}^\mu\,, && \delta(\bar{\partial}X^\mu)=\tilde{\gamma}\bar{\partial}\tilde{\Psi}^\mu+\gamma\bar{\partial}\Psi^\mu\,,\\ &\delta(\Psi^\mu)=\gamma\Big(\partial X^\mu-\tfrac{1}{2}\tilde{\Psi}^\mu\chi_-\Big)+\tilde{\gamma}F^\mu\,, && \delta(\tilde{\Psi}^\mu)=\tilde{\gamma}\Big(\bar{\partial}X^\mu-\tfrac{1}{2}\Psi^\mu\tilde{\chi}_+\Big)-\gamma F^\mu\,.\\ &\delta(F^\mu)=\gamma\Big(\bar{\partial}\Psi^\mu+\partial X^\mu\tilde{\chi}_+-\tfrac{1}{2}\tilde{\Psi}^\mu\tilde{\chi}_+\chi_-\Big)+\tilde{\gamma}\Big(\partial\Psi^\mu+\bar{\partial}X^\mu\chi_--\tfrac{1}{2}\Psi^\mu\chi_-\tilde{\chi}_+\Big)\,, \\ &\delta(e^{ikX})=-i\,\gamma\,k\cdot\Psi^\mu-i\,\tilde{\gamma}\,k\cdot\tilde{\Psi}^\mu\,. \end{aligned} \tag{2.6}$$

[2]The importance of careful treatment of gravitino modes was recently stressed again in the context of two-loop calculations by D'Hoker and Phong [4, 5].

Note the terms proportional to the equations of motion in the first line; these are absolutely crucial to arrive at gauge-invariant results later. In the standard approach, when these rules are obtained from an OPE with the BRST charge, these terms arise from left/right-mixing contractions (2.1).

Starting from the vertex operators (2.3) in their canonical picture, we can now apply (2.6), with the symbolic rule that $\gamma \to e^{\phi}$. This corresponds to the usual picture changing prescription in the operator formalism, as given in (2.5). The first increase of ghost number is only made slightly non-standard by the presence of the gravitino gauge-fixing functions. Continuing, to for instance $V^{(0,0)}$ or beyond, the contraction (2.1) is also going to play a role. For the type-II operators, we find after the first step

$$V^{(-1,0)} = \int \mathrm{d}^2 z\, \zeta_{\mu\nu} \Big(\Psi^\mu \big(\bar\partial X^\nu - ik\cdot\tilde\Psi\tilde\Psi^\nu + \tfrac{1}{2}\Psi^\nu \tilde\chi_+ \big) + F^\mu \tilde\Psi^\nu \Big) e^{-\phi} e^{ik\cdot X} \,, \tag{2.7}$$

$$V^{(0,-1)} = \int \mathrm{d}^2 z\, \zeta_{\mu\nu} \Big(\big(\partial X^\mu - ik\cdot\Psi\Psi^\mu - \tfrac{1}{2}\tilde\Psi^\mu \chi_- \big)\tilde\Psi^\nu + \Psi^\mu F^\nu \Big) e^{-\tilde\phi} e^{ik\cdot X} \,. \tag{2.8}$$

Some subtleties arise when the polarisation is a pure trace, but we will not need the dilaton operator here, so we refrain from discussing these issues (see also Terao and Uehara [7]). The presence of the third term, involving the gravitino, will be crucial to derive the correct effective rules for the bosonic contractions. Applying the picture-changing operator once more, we now also encounter contractions of the type (2.1). When the polarisation tensor is taken to be that of the NS two-form, the various terms quadratic in fermions can be grouped by performing one partial integration. One then obtains the operator using $H_{\mu\nu\rho} = 3\, ik_{[\mu}B_{\nu\rho]}$

$$\begin{aligned} V_B^{(0,0)} = \int \mathrm{d}^2 z\, B_{\mu\nu} &\Big(\partial X^\mu \bar\partial X^\nu - k\cdot\Psi\Psi^\mu\, k\cdot\tilde\Psi\tilde\Psi^\nu + \tfrac{1}{2}\Psi^\mu\tilde\Psi^\nu\, \chi_-\tilde\chi_+ \Big)\, e^{ik_\rho X^\rho} \\ &- \frac{1}{6}\int \mathrm{d}^2 z\, H_{\mu\nu\rho} \Big(\Psi^\mu\Psi^\nu\Psi^\rho \tilde\chi_+ - \tilde\Psi^\mu\tilde\Psi^\nu\tilde\Psi^\rho \chi_- \Big)\, e^{ik_\rho X^\rho} \\ &- \frac{1}{2}\int \mathrm{d}^2 z\, H_{\mu\nu\rho} \Big(\Psi^\mu\Psi^\nu\bar\partial X^\rho - \tilde\Psi^\mu\tilde\Psi^\nu\partial X^\rho - 2\Psi^\mu\tilde\Psi^\nu F^\rho \Big)\, e^{ik_\rho X^\rho} \,. \end{aligned} \tag{2.9}$$

The last line corresponds to the additional terms that were used by Green and Seiberg [8] in order to generate contact terms (required to satisfy world-sheet supersymmetric Ward identities). Here we see them arise naturally from the operator $V^{(-1,0)}$ after picture changing.

Only with this full vertex operator will string amplitudes be gauge-invariant; we will discuss an explicit example in section 3.2. Note that after insertion of a delta-function supported world-sheet gravitino, the second line of the above expression will no longer contain a z-integral, but instead depend on the location of the gravitino. The last line of (2.9) was also used in this gauge-invariant form by Gutperle [9], although there the term was added by hand, not derived from picture changing. Similar on-shell vanishing terms have also been exhibited, though again not through picture changing, for the heterotic string gauge field vertex operators by Green and Seiberg [8].

We should mention that the vertex operators in the various pictures, including the gravitino terms and the contact terms, can also be obtained directly from superspace, avoiding the picture-changing operation altogether. With the condition $\Gamma^m \chi_m = 0$ on the gravitino The necessary ingredients are the supervielbein determinant, $E = e$, as well as the supercovariant derivatives of the superfield $\Phi = X^\mu + \theta\Psi^\mu + \bar\theta\tilde\Psi^\mu + \bar\theta\theta F^\mu$,

$$\begin{aligned} D_-\Phi^\mu &= \tilde\Psi^\mu + \theta F^\mu + \bar\theta(\bar\partial X^\mu + \tfrac{1}{2}\tilde\chi_+\Psi^\mu) + \theta\bar\theta(-\bar\partial\Psi^\mu - \tfrac{1}{2}\tilde\chi_+\partial X^\mu - \tfrac{1}{4}\tilde\chi_+\chi_-\tilde\Psi^\mu)\,,\\ D_+\Phi^\mu &= \Psi^\mu + \theta(\partial X^\mu + \tfrac{1}{2}\chi_-\tilde\Psi^\mu) - \bar\theta F^\mu + \theta\bar\theta(-\partial\tilde\Psi^\mu - \tfrac{1}{2}\chi_-\bar\partial X^\mu - \tfrac{1}{4}\chi_-\tilde\chi_+\Psi^\mu)\,. \end{aligned} \tag{2.10}$$

The vertex operators in the various pictures now follow from $\tilde V = E\, D_-\Phi D_+\Phi \exp(k\Phi)$ and

$$V^{(-1,-1)} = \tilde V\Big|_{\substack{\theta=0\\ \bar\theta=0}}\,,\quad V^{(0,-1)} = \int {\rm d}\theta\,\tilde V\Big|_{\substack{\theta=0\\ \bar\theta=0}}\,,\quad V^{(0,0)} = \int {\rm d}\theta{\rm d}\bar\theta\,\tilde V\Big|_{\substack{\theta=0\\ \bar\theta=0}}\,. \tag{2.11}$$

This procedure was followed in D'Hoker and Phong [10], but the new terms have to our knowledge not yet been used in actual amplitude calculations. It is important to understand that the superfield approach only applies in the case of the (NS,NS) vertex operators, since only for these operators is a superspace expression available. The spacetime gravitino vertex operator, for instance, is expected to receive world-sheet gravitino terms at higher ghost picture as well, but this can only be derived using the picture-changing procedure based on the transformation rules (2.6). The same thing holds true for (R,R) gauge field vertex operators.

Let us now return to the derivation of an effective rule for the boson contractions between normal picture changers and those arising from the integration over odd supermoduli. This proceeds simply by inserting $(-1,0)$ and $(0,0)$ operators into the path integral (2.4). For $(-1,0)$ operators, the contraction of the boson in (2.7) with the second term in brackets in (2.4) leads to $\zeta_{\mu\nu}\Psi^\mu\Psi^\nu\chi_-$, which is precisely cancelled by presence of the third term in the vertex operator. Again, the zero mode part of the bosonic contraction is left. Similarly, for $(0,0)$ operators, such cancellations occur. We have therefore deduced the second effective rule, namely that *left/right contractions between bosons of odd spin-structure picture-changing operators and bosons of vertex operators again only involve the zero mode part.*

Finally, the amplitudes exhibit contractions between bosons of two vertex operators. These contractions are not related to any special cancellation mechanism, and therefore *left/right contractions between bosons of vertex operators involve both the contact term and the zero mode part.* However, the effective rule is different, due to the fact that the δ-function part of the contraction is cancelled by poles arising from the contraction of plane wave exponentials.

3 R^4 invariants in IIA and IIB supergravity

3.1 Gravitino bilinears in the $\epsilon\epsilon R^4$ invariant

Our first example illustrating the above mentioned string-amplitude subtleties involves the computation of terms which are needed for the fermionic completion of the I_Z invariant discussed in the introduction. One of the most intriguing aspects of the superinvariants is how terms with very different tensorial structures talk to each other under supersymmetry variations. For instance, an ϵ_{10} tensor can in this way, via an intermediate Hodge dualisation, be related to a t_8 structure. In the present section we will see a similar phenomenon, namely the appearance of a t_8 tensor from a correlator with ten world-sheet fermions in the odd spin-structure sector.

We will here focus on a particular fermionic bilinear in the action, namely the one that is needed to cancel terms that arise from variation under supersymmetry of the B-field in the anomaly term. This variation produces (among other terms) a fermionic contribution

$$\delta_\epsilon(B \wedge t_8 W^4) \to t_8^{(s)}\,(\bar\psi_m \Gamma^{m r_1\cdots r_8}\epsilon) W_{r_1 r_2 s_1 s_2} W_{r_3 r_4 s_3 s_4} W_{r_5 r_6 s_5 s_6} W_{r_7 r_8 s_7 s_8}\,. \tag{3.1}$$

This contribution can only be cancelled by adding a fermion bilinear to the action. For the heterotic string the required term is

$$S^{\text{heterotic}}_{\bar\psi\Gamma^{[7]}\psi_2 W^3} = \int \mathrm{d}^{10}x\, e\, t_8^{(s)}\,(\bar\psi_m \Gamma^{m r_1\cdots r_6}\psi_{s_7 s_8}) W_{r_1 r_2 s_1 s_2} W_{r_3 r_4 s_3 s_4} W_{r_5 r_6 s_5 s_6}\,, \tag{3.2}$$

with the overall coefficient determined by the supersymmetry transformation rules. In [1] we have discussed the origin of this fermion bilinear for the heterotic theory. It arises from an amplitude in the odd spin-structure sector, just like the bosonic anomaly term.

In the type-II theories the situation is more complicated. There we have both an odd/even and an even/odd spin-structure sector, with a relative minus sign between the two in the type-IIB case. Therefore, the bosonic $B \wedge t_8 W^4$ term receives two contributions, which add up for the type-IIA theory while they cancel for the type-IIB theory (see Vafa and Witten [11]). This is again reflected in the sign difference for the one-loop invariants appearing in (1.2). Because of the supersymmetry argument sketched above, there should be a similar addition/cancellation mechanism for the associated fermion bilinears, which are now given by

$$S^{\text{type-II}}_{\bar\psi\Gamma^{[7]}\psi_2 W^3} = \int \mathrm{d}^{10}x\, e\, t_8^{(s)}\,(\bar\psi^I_m \Gamma^{m r_1\cdots r_6}\psi^I_{s_7 s_8}) W_{r_1 r_2 s_1 s_2} W_{r_3 r_4 s_3 s_4} W_{r_5 r_6 s_5 s_6}\,. \tag{3.3}$$

Here $I = 1, 2$ labels the two Majorana-Weyl gravitinos. However, the cancellation cannot simply be between the odd/even and even/odd sectors. The reason is that these two sectors produce terms with fermions of opposite spacetime chirality, which can never add or subtract. A new mechanism is required; we will show below that it depends crucially on the left/right mixing discussed in the previous section, as well as on some intricate identities between world-sheet fermion correlators.

The only way in which terms of the form (3.3) can be made to add or cancel for *both* fermion types in the same way, is to have them arise from the odd/odd spin-structure sector. Amplitudes computed there differ by a minus sign (due to the GSO projection) between the type-IIA and type-IIB theories. The surprising element is that the odd/odd spin-structure sector in fact produces the t_8 tensor of (3.3), something that is not at all obvious from the RNS fermionic correlator. This new odd/odd term which we expect to contribute to the coefficient of (3.3) is given by the following five-point amplitude:

$$\begin{aligned}\mathcal{A}_5^{\text{odd/odd}} = \Big\langle Y(w_1)\tilde Y(\tilde w_1) \prod_{i=2}^{4} \oint \frac{\mathrm{d}w_i}{2\pi i}\, Y(w_i) \prod_{j=2}^{4} \oint \frac{\mathrm{d}\tilde w_i}{2\pi i}\, \tilde Y(\tilde w_i) \\ \times \prod_{m=1}^{2} \int \mathrm{d}^2 y_m\, V_{\psi_m}^{(-1/2,-1)}(y_m) \prod_{n=1}^{3} \int \mathrm{d}^2 z_n\, V_{g_n}^{(-1,-1)}(z_n) \Big\rangle_{\text{odd/odd}}\,.\end{aligned} \tag{3.4}$$

We have written it in the most general form, with all the vertex operators in their canonical pictures and the two odd spin-structure picture-changing operators inserted at arbitrary points.

Part of this amplitude has been computed previously by Lin [12], namely the terms which do not involve a bosonic zero-mode contraction of two odd spin-structure picture-changing operators. However, there is no $\Gamma^{[7]}$ contribution from these terms. Indeed, there is in general no way in which the pure left/right separation can lead to a result which has a gravitino index contracted with the gamma matrix, as in (3.2). We will therefore immediately turn our attention to the case in which the two picture-changing operators are contracted through their bosonic zero modes.

In order to evaluate this part of the amplitude, it is convenient to first eliminate some of the picture-changing contour integrals. Having chosen the two picture changers which are to be contracted through their bosonic zero modes, all the bosons of the remaining picture changers are to be contracted with plane-wave exponentials (to produce the $(0,0)$ ghost-picture operators and the associated effective contraction rules which we discussed in section 2). The odd/odd spin-structure contribution to the left/right mixing part of the amplitude then reduces to the much simpler expression

$$\begin{aligned}\mathcal{A}_5^{\text{odd/odd}}\Big|_{\text{left/right}} &= (\bar\psi^{(1)}_m)_a(\psi^{(2)}_{np})_b\, W^{(3)}_{r_1r_2s_1s_2}\, W^{(4)}_{r_3r_4s_3s_4}\, W^{(5)}_{r_5r_6s_5s_6}\\ &\times\Big\langle \Psi^l(w_1)\ S^a(y_1)\quad S^b(y_2)\quad :\Psi^{r_1}\Psi^{r_2}:(z_1)\ :\Psi^{r_3}\Psi^{r_4}:(z_2)\ :\Psi^{r_5}\Psi^{r_6}:(z_3)\\ &\qquad \tilde\Psi^l(\tilde w_1)\ \tilde\Psi^m(y_1)\ :\tilde\Psi^n\,\tilde\Psi^p:(y_2)\ :\tilde\Psi^{s_1}\tilde\Psi^{s_2}:(z_1)\ :\tilde\Psi^{s_3}\tilde\Psi^{s_4}:(z_2)\ :\tilde\Psi^{s_5}\tilde\Psi^{s_6}:(z_3)\Big\rangle_{\text{odd}}\\ &\times\Big\langle e^{\phi(w_1)}\,e^{-\phi(y_1)/2}\,e^{-\phi(y_2)/2}\Big\rangle_{\text{odd}}\Big\langle e^{\tilde\phi(\tilde w_1)}\,e^{-\tilde\phi(\tilde y_1)}\Big\rangle_{\text{odd}}.\end{aligned} \tag{3.5}$$

Integration over the insertion points as in (3.4) is implicitly understood from now on. We have also replaced the linearised expressions involving the graviton polarisations with the full Weyl tensors. Bracketed superscripts on the polarisation tensors are used to identify the five external states.

The above amplitude is to be added to

$$\begin{aligned}\mathcal{A}_5^{\text{odd/even}}\Big|_{\text{left/right}} &= (\bar\psi^{(1)}_m)_a(\psi^{(2)}_{np})_b\, W^{(3)}_{r_1r_2s_1s_2}\, W^{(4)}_{r_3r_4s_3s_4}\, W^{(5)}_{r_5r_6s_5s_6}\\ &\times\Big\langle \Psi^l(w_1)\,S^a(y_1)\quad S^b(y_2)\quad :\Psi^{r_1}\Psi^{r_2}:(z_1)\ :\Psi^{r_3}\Psi^{r_4}:(z_2)\ :\Psi^{r_5}\Psi^{r_6}:(z_3)\Big\rangle_{\text{odd}}\\ &\times\Big\langle \frac{\eta^{lm}}{\bar w_1-\bar y_1}\quad :\tilde\Psi^n\,\tilde\Psi^p:(y_2)\ :\tilde\Psi^{s_1}\tilde\Psi^{s_2}:(z_1)\ :\tilde\Psi^{s_3}\tilde\Psi^{s_4}:(z_2)\ :\tilde\Psi^{s_5}\tilde\Psi^{s_6}:(z_3)\Big\rangle_{\text{even}}\\ &\times\Big\langle e^{\phi(w_1)}\,e^{-\phi(y_1)/2}\,e^{-\phi(y_2)/2}\Big\rangle_{\text{odd}}\Big\langle e^{\tilde\phi(\tilde w_1)}\,e^{-\tilde\phi(\tilde y_1)}\Big\rangle_{\text{even}},\end{aligned} \tag{3.6}$$

which arises by considering the even spin-structure sector for the right-moving fermions. Note that the difference with the odd/odd amplitude (3.4) is minimal: the un-integrated picture-changing operator on the right was dropped in exchange for an integrated one. As we have already observed that the integrand of (3.5) is independent of the insertion point $\tilde w_1$, it is clear that we need a different type of contraction in order to obtain the odd/even amplitude. Instead of keeping the zero modes on the right, we have now contracted one picture-changing operator fermion with a fermion from the gravitino. Explicitly, this means that we consider the

l-m contraction. This produces a pole and therefore a non-vanishing result after integration over the picture-changer insertion point. An epsilon tensor arises from the left-handed sector. Having contracted these two fermions, the tensor structure for the right-handed fermion sector is then fixed by symmetry arguments which we will not repeat.

The total amplitude can thus be written in a compact way as

$$\mathcal{A}_5\Big|_{\text{left/right}} = (\bar\psi^{(1)}_m\, T^{r_1\cdots r_6}{}_l \psi^{(2)}_{s_7 s_8})\, W^{(3)}_{r_1 r_2 s_1 s_2}\, W^{(4)}_{r_3 r_4 s_3 s_4}\, W^{(5)}_{r_5 r_6 s_5 s_6} \times \begin{cases} \varepsilon_{(10)}^{l m s_1\cdots s_8} & \text{odd/odd}\,, \\ \eta^{lm} t_8^{s_1\cdots s_8} & \text{odd/even}\,, \end{cases} \tag{3.7}$$

where—taking into account the symmetries imposed by the contracting tensors—T has the gamma-matrix expansion[3]

$$\begin{aligned} T^{r_1\cdots r_6\, l} &= A(y_i, z_i, w_i)\, \Gamma^{r_1\cdots r_6}\Gamma^l \\ &\quad + \tfrac12\, C_1(y_i,z_i,w_i)\,\big(\eta^{r_1 r_3}\eta^{r_2 r_4}\Gamma^{r_5 r_6} + 5 \text{ permutations}\big)\Gamma^l \\ &\quad + \tfrac12\, C_2(y_i,z_i,w_i)\,\big(\eta^{r_1 r_3}\eta^{r_2 r_5}\Gamma^{r_6 r_4} + 23 \text{ permutations}\big)\Gamma^l\,. \end{aligned} \tag{3.8}$$

For the odd/even amplitude we have to focus on the $\Gamma^{[7]}$ terms that arise from these expressions, while for the odd/odd amplitude it is the $\Gamma^{[3]}$ terms that are relevant to obtain a contribution to the effective action of the form (3.3). In the former case we simply have to compute the coefficient function A. In the latter case we first have to eliminate the epsilon tensor in (3.7) by dualising the gamma matrix onto which it is contracted; this leads to

$$\varepsilon_{(10)}^{s_1\cdots s_8 lm}\,\Gamma_l\,\psi_{s_7 s_8} = \Gamma_{[9]}^{s_1\cdots s_8 m}\,\psi_{s_7 s_8} = \widetilde{\mathcal{E}}(\psi) - 42\,\Gamma_{[5]}^{[s_1\cdots s_5}\,\psi^{s_6 m]}\,, \tag{3.9}$$

where $\widetilde{\mathcal{E}}(\psi)$ denotes the gravitino equation of motion. Multiplying the right-hand side with the other gamma matrices that appear in the decomposition of the T tensor, we find that the restriction of the five-point amplitude to $\Gamma^{[7]}$ terms in the odd/odd sector is independent of A,

$$\begin{aligned} &\mathcal{A}_5^{\text{odd/odd}}\Big|_{\text{left/right},\Gamma^{[7]}} \\ &\quad = -6\,\bar\psi^{(1)}_m \Gamma^{m r_1\cdots r_6}\Big(C_1\ \mathrm{tr}\big(W^{(3)}_{r_1 r_2} W^{(4)}_{r_3 r_4}\big)\,\mathrm{tr}(W^{(5)}_{r_5 r_6}\psi^{(2)}) - 4\,C_2\ \mathrm{tr}\big(W^{(3)}_{r_1 r_2} W^{(4)}_{r_3 r_4} W^{(5)}_{r_5 r_6}\psi^{(2)}\big)\Big)\,, \end{aligned} \tag{3.10}$$

where permutation of the three graviton polarisation tensors is understood. Provided we can show that the coefficient functions satisfy $C_1 = C_2 = C$, the resulting amplitude will thus take the form

$$\mathcal{A}_5^{\text{odd/odd}}\Big|_{\text{left/right},\Gamma^{[7]}} = C\, t_8^{s_1\cdots s_8} (\bar\psi^{(1)}_m \Gamma^{m r_1\cdots r_6} \psi^{(2)}_{s_7 s_8})\, W^{(3)}_{r_1 r_2 s_1 s_2} W^{(4)}_{r_3 r_4 s_3 s_4} W^{(5)}_{r_5 r_6 s_5 s_6}\,, \tag{3.11}$$

and can potentially cancel the odd/even part of the amplitude for the type-IIB string.

In order to evaluate the precise expressions for A, C_1 and C_2, we evaluate the amplitude for particular values of the r, a and b-indices (for a combination which yields a non-zero element of the gamma matrix under consideration, where we employ the helicity basis of appendix A.2). These are listed, together with the fermion combinations that appear in the resulting correlators, in table 1. The fermionic part of the amplitude can now be deduced

[3] In addition to the complete symmetry in the exchange of r-index pairs and the pair-wise antisymmetries, we also use the observation that any contraction between l and an r index leads to a Weyl tensor antisymmetrised in three indices and hence vanishes.

$r_1 = 1$	$r_2 = 3$	$r_3 = 1,2$	$r_4 = 3,4$	$r_5 = 5$	$r_6 = 8$	$l = 7$	a	b
$\Psi(z_1)$	0	$\overline{\Psi}(z_2)$	0	0	0	0	$S_+(y_1)$	$S_-(y_2)$
0	$\Psi(z_1)$	0	$\overline{\Psi}(z_2)$	0	0	0	$S_+(y_1)$	$S_-(y_2)$
0	0	0	0	$\Psi(z_3)$	0	0	$S_-(y_1)$	$S_-(y_2)$
0	0	0	0	0	$\overline{\Psi}(z_3)$	$\Psi(w_1)$	$S_+(y_1)$	$S_-(y_2)$
0	0	0	0	0	0	0	$S_+(y_1)$	$S_-(y_2)$

$r_1 = 1$	$r_2 = 3$	$r_3 = 1$	$r_4 = 8$	$r_5 = 5$	$r_6 = 3$	$l = 7$	a	b
$\Psi(z_1)$	0	$\overline{\Psi}(z_2)$	0	0	0	0	$S_+(y_1)$	$S_-(y_2)$
0	$\Psi(z_1)$	0	0	0	$\overline{\Psi}(z_3)$	0	$S_+(y_1)$	$S_-(y_2)$
0	0	0	0	$\Psi(z_3)$	0	0	$S_-(y_1)$	$S_-(y_2)$
0	0	0	$\overline{\Psi}(z_2)$	0	0	$\Psi(w_1)$	$S_+(y_1)$	$S_-(y_2)$
0	0	0	0	0	0	0	$S_+(y_1)$	$S_-(y_2)$

$$(\Gamma^{57}\Gamma^{8})_{ab} = \sigma^2 \otimes \sigma^1 \otimes \mathbb{1} \otimes \sigma^1 \otimes \sigma^1 \,,$$
$$(\Gamma^{1324578})_{ab} = \sigma^1 \otimes \sigma^2 \otimes \mathbb{1} \otimes \sigma^1 \otimes \sigma^1 \,.$$

Table 1: The fermion distributions used for the computation of $\mathcal{A}_5^{\text{odd/odd}}$ and $\mathcal{A}_5^{\text{odd/even}}$, together with the gamma-matrix products that determine them. The fermions are defined in (A.17).

using the approach of Atick and Sen [13] or an equivalent construction based on bosonisation. Since the result is by construction independent of the picture-changer insertion point, we can evaluate it for $w_1 = z_1$ and find indeed that $C_1 = C_2$.

Unfortunately, it is not so easy to see whether the coefficient function A is related to C in such a way as required by supersymmetry. Although the distribution for the fermions is similar, in the sense that $(\Gamma^{1324578})_{ab}$ leads to the same type of correlators as $(\Gamma^{57}\Gamma^{8})_{ab}$, they are not identical. It therefore seems that cancellation of the terms in (3.3) appears only after the insertion-point integrals have been performed.

The fact that both odd/even and odd/odd amplitudes contribute to the same term in the effective action will also be relevant for other fermion bilinears. It is indeed a crucial observation in order to construct the I_Z invariant as an effective action arising from string amplitudes. More details about cancellation mechanisms analogous to the one described here will appear in a forthcoming publication.

3.2 Interactions involving the NS two-form

3.2.1 String amplitudes

While the higher-derivative terms involving only gravitons have been known now for quite some time, substantially less is known about terms which involve Neveu-Schwarz or Ramond gauge fields.[4] In the present section we will discuss the terms in the effective action which contain two powers of the Neveu-Schwarz three-form field strength and three powers of the Weyl tensor. We will see that the subtleties discussed in section 2 are *crucial* in order to arrive at the correct answer.

The terms under consideration have been used in a recent paper by Frolov et al. [2] in the context of the AdS/CFT correspondence. We would like to comment here in some detail on the covariant calculation of several terms in the required amplitudes, since they all originate from the picture-changing subtleties discussed in section 2. As in the previous section we restrict ourselves to a one-loop analysis.

In the odd/odd spin structure we find the result

$$\begin{aligned}\mathcal{A}_{\text{odd/odd}} = {}& k^{(1)}_{r_7} B^{(1)}_{r_8 s_9} k^{(2)}_{s_7} B^{(2)}_{s_8 r_9}\, W^{(3)}_{r_1 r_2 s_1 s_2} W^{(4)}_{r_3 r_4 s_3 s_4} W^{(5)}_{r_5 r_6 s_5 s_6}\\ &\times\Big\langle \partial X\cdot\Psi(w_1)\ :\Psi^{r_7}\Psi^{r_8}:(z_1)\quad \Psi^{r_9}(z_2)\qquad :\Psi^{r_1}\Psi^{r_2}:(z_3)\ :\Psi^{r_3}\Psi^{r_4}:(z_4)\ :\Psi^{r_5}\Psi^{r_6}:(z_5)\\ &\quad \bar\partial X\cdot\tilde\Psi(w_2)\quad \tilde\Psi^{s_9}(z_1)\qquad :\tilde\Psi^{s_7}\tilde\Psi^{s_8}:(z_2)\ :\tilde\Psi^{s_1}\tilde\Psi^{s_2}:(z_3)\ :\tilde\Psi^{s_3}\tilde\Psi^{s_4}:(z_4)\ :\tilde\Psi^{s_5}\tilde\Psi^{s_6}:(z_5)\Big\rangle\,,\end{aligned}\tag{3.12}$$

where picture changers from the integration over the odd supermoduli were inserted. The zero-mode term in (2.1) leads to an amplitude that can be reproduced by a term in the effective action of the form[5]

$$\mathcal{L}_{\text{odd/odd}} = \epsilon^{r_1\cdots r_9 m}\epsilon^{s_1\cdots s_9}{}_m W_{r_1 r_2 s_1 s_2} W_{r_3 r_4 s_3 s_4} W_{r_5 r_6 s_5 s_6}\Big(H_{r_7 r_8 s_9} H_{s_7 s_8 r_9} - \tfrac{1}{9} H_{r_7 r_8 r_9} H_{s_7 s_8 s_9}\Big)\,.\tag{3.13}$$

Despite the many antisymmetrisations this term is not a total derivative.

In the odd/even sector (and similarly in the even/odd sector) there are three different terms that contribute to the amplitude. Two of them are standard in the sense that they only involve the left/right-mixing contraction (2.1). The third one is unusual because it relies on the presence of world-sheet gravitino terms in the $V_B^{(0,0)}$ vertex operator. Since the odd/even contribution is parity odd, one expects it to be absent from the effective action. The fact that the sum of the three terms indeed vanishes (for both the type-IIA as well as the type-IIB theory) shows that the world-sheet gravitino terms in vertex operators cannot be ignored.

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[4]These terms are closely related to the eleven-dimensional terms which involve the four-form field strength. For all applications so far (see for instance Becker and Becker [14]) the knowledge about only the anomaly term $C\wedge t_8 W^4$ was sufficient. A large-volume limit was taken, and in this limit all other terms involving the gauge field scale differently.

[5]As explained in e.g. Gross and Sloan [15], one needs to subtract five-point functions obtained as tree-level graphs formed from lower-point vertices before a link with the effective action can be made. Fortunately, in our case, the fact that these lower-order vertices come with two derivatives implies that one needs at least four three-point vertices in order to produce an eight-derivative amplitude. Such graphs have, however, at least six external lines. The subtraction problem is therefore absent for the terms under consideration and we can deduce the effective action immediately from the five-point string amplitude.

Let us first discuss the two standard terms, one given by

$$\mathcal{A}^1_{\text{odd/even}} = \tfrac{3}{2}k^{(1)}_{[r_7}B^{(1)}_{r_8 m]}k^{(2)}_{s_7}B^{(2)}_{r_9 s_8}\, W^{(3)}_{r_1r_2s_1s_2}W^{(4)}_{r_3r_4s_3s_4}W^{(5)}_{r_5r_6s_5s_6}$$
$$\times\Big\langle \begin{matrix} \partial X\cdot\Psi(w_1) & :\Psi^{r_7}\Psi^{r_8}:(z_1) & \Psi^{r_9}(z_2) & :\Psi^{r_1}\Psi^{r_2}:(z_3) & :\Psi^{r_3}\Psi^{r_4}:(z_4) & :\Psi^{r_5}\Psi^{r_6}:(z_5) \\ & \bar\partial X^m(z_1) & :\tilde\Psi^{s_7}\tilde\Psi^{s_8}:(z_2) & :\tilde\Psi^{s_1}\tilde\Psi^{s_2}:(z_3) & :\tilde\Psi^{s_3}\tilde\Psi^{s_4}:(z_4) & :\tilde\Psi^{s_5}\tilde\Psi^{s_6}:(z_5) \end{matrix}\Big\rangle\,, \tag{3.14}$$

and the other one obtained by using a different part of the gauge field vertex operator:

$$\mathcal{A}^2_{\text{odd/even}} = k^{(1)}_{r_7}k^{(1)}_{s_7}B^{(1)}_{r_8 s_8}B^{(2)}_{r_9 m}\, W^{(3)}_{r_1r_2s_1s_2}W^{(4)}_{r_3r_4s_3s_4}W^{(5)}_{r_5r_6s_5s_6}$$
$$\times\Big\langle \begin{matrix} \partial X\cdot\Psi(w_1) & :\Psi^{r_7}\Psi^{r_8}:(z_1) & \Psi^{r_9}(z_2) & :\Psi^{r_1}\Psi^{r_2}:(z_3) & :\Psi^{r_3}\Psi^{r_4}:(z_4) & :\Psi^{r_5}\Psi^{r_6}:(z_5) \\ & :\tilde\Psi^{s_7}\tilde\Psi^{s_8}:(z_2) & \bar\partial X^m(z_1) & :\tilde\Psi^{s_1}\tilde\Psi^{s_2}:(z_3) & :\tilde\Psi^{s_3}\tilde\Psi^{s_4}:(z_4) & :\tilde\Psi^{s_5}\tilde\Psi^{s_6}:(z_5) \end{matrix}\Big\rangle\,. \tag{3.15}$$

Their sum gives a non-vanishing parity-odd contribution to the effective action:

$$\mathcal{L}^{1+2}_{\text{odd/even}} = (-\tfrac{3}{2}+1)\,\epsilon^{mr_1\cdots r_9}t_8^{s_1\cdots s_8}W_{r_1r_2s_1s_2}W_{r_3r_4s_3s_4}W_{r_5r_6s_5s_6}\,H_{r_7r_8m}H_{s_7s_8r_9}\,. \tag{3.16}$$

Fortunately, there is a third odd/even amplitude,

$$\mathcal{A}^3_{\text{odd/even}} = \tfrac{1}{2}k^{(1)}_{[r_7}B^{(1)}_{r_8 m]}k^{(2)}_{s_7}B^{(2)}_{r_9 s_8}\, W^{(3)}_{r_1r_2s_1s_2}W^{(4)}_{r_3r_4s_3s_4}W^{(5)}_{r_5r_6s_5s_6}$$
$$\times\Big\langle \begin{matrix} :\Psi^{r_7}\Psi^{r_8}\Psi^m:(y) & \Psi^{r_9}(z_2) & :\Psi^{r_1}\Psi^{r_2}:(z_3) & :\Psi^{r_3}\Psi^{r_4}:(z_4) & :\Psi^{r_5}\Psi^{r_6}:(z_5) \\ & :\tilde\Psi^{s_7}\tilde\Psi^{s_8}:(z_2) & :\tilde\Psi^{s_1}\tilde\Psi^{s_2}:(z_3) & :\tilde\Psi^{s_3}\tilde\Psi^{s_4}:(z_4) & :\tilde\Psi^{s_5}\tilde\Psi^{s_6}:(z_5) \end{matrix}\Big\rangle\,, \tag{3.17}$$

obtained by taking the first term in brackets in (2.4) and compensating for the missing world-sheet gravitino by using a gravitino term in (2.9) (y is the arbitrary and un-integrated location of the gravitino support, crucial for the $1/\tau_2$ factors to conspire into a modular invariant result; see also the comment below (2.9)). This term leads to an effective-action term which precisely cancels (3.16), thus removing the parity-odd term altogether.[6]

This mechanism could in fact have been observed much earlier, namely in the computation of the $B\wedge t_8W^4$ term in the effective action. If, in this calculation, one takes the graviton vertex operator in the $(-1,0)$ picture (instead of the B operator, as is usually done), then the amplitude comes out with a factor $3/2$ with respect to the usual result. Inclusion of the world-sheet gravitino terms in the B vertex operator gives another contribution which corrects this mismatch.

Finally, in the even/even spin-structure sector one encounters the term

$$\mathcal{A}_{\text{even/even}} = k^{(1)}_{[r_7}B^{(1)}_{r_8 n]}k^{(2)}_{[s_7}B^{(2)}_{m s_8]}\, W^{(3)}_{r_1r_2s_1s_2}W^{(4)}_{r_3r_4s_3s_4}W^{(5)}_{r_5r_6s_5s_6}$$

[6] We should perhaps mention that this cancellation mechanism is completely different from the one that cancels the $B\wedge t_8W^4$ term in the type-IIB action. The latter arose because of a sign flip between the odd/even and the even/odd sector. As a result of the GSO projection, which adds another sign between these two terms for the type-IIA theory, this type of cancellation mechanism can only work for one of the two type-II theories, not for both [11]. Note in addition that the world-sheet gravitino term is absent from $V_g^{(0,0)}$, implying that world-sheet gravitinos do not play a role in the determination of the $B\wedge t_8W^4$ term in the effective action.

$$\times\Big\langle :\Psi^{r_7}\Psi^{r_8}:(z_1) \quad \partial X^m(z_2) \qquad :\Psi^{r_1}\Psi^{r_2}:(z_3) \;\; :\Psi^{r_3}\Psi^{r_4}:(z_4) \;\; :\Psi^{r_5}\Psi^{r_6}:(z_5)$$
$$\bar\partial X^n(z_1) \quad :\tilde\Psi^{s_7}\tilde\Psi^{s_8}:(z_2) \;\; :\tilde\Psi^{s_1}\tilde\Psi^{s_2}:(z_3) \;\; :\tilde\Psi^{s_3}\tilde\Psi^{s_4}:(z_4) \;\; :\tilde\Psi^{s_5}\tilde\Psi^{s_6}:(z_5)\Big\rangle . \tag{3.18}$$

Taking into account the contractions of the plane-wave factors (which are not displayed above), we find that only the zero-mode term of (2.1) contributes. In the effective action this amplitude therefore leads to

$$\mathcal{L}_{\text{even/even}} = t_8^{r_1\cdots r_8} t_8^{s_1\cdots s_8}\, W_{r_1r_2s_1s_2}W_{r_3r_4s_3s_4}W_{r_5r_6s_5s_6}\, H_{r_7r_8m}H_{s_7s_8}{}^{m} . \tag{3.19}$$

Collecting the results (3.13) and (3.19), we finally arrive at the effective action

$$\begin{aligned}\mathcal{L}_{W^3H^2} = {} & t_8^{r_1\cdots r_8} t_8^{s_1\cdots s_8}\, W_{r_1r_2s_1s_2}W_{r_3r_4s_3s_4}W_{r_5r_6s_5s_6}\, H_{r_7r_8m}H_{s_7s_8}{}^{m} \\ & \pm \epsilon^{r_1\cdots r_9 m}\epsilon^{s_1\cdots s_9}{}_{m} W_{r_1r_2s_1s_2}W_{r_3r_4s_3s_4}W_{r_5r_6s_5s_6}\Big(H_{r_7r_8s_9}H_{s_7s_8r_9} - \tfrac{1}{9}H_{r_7r_8r_9}H_{s_7s_8s_9}\Big) ,\end{aligned} \tag{3.20}$$

the relative sign differing between the two type-II theories (we have suppressed an unknown relative normalisation factor between the even/even and odd/odd parts of the effective action).

We would like to stress that our results provide further illustration of the fact that the H_{NS} dependence of the effective action *cannot* be fully absorbed in a modified spin connection (see Metsaev and Tseytlin [16] for another example). A part of the invariant can be grouped in this way, but there are remaining terms which do not have the correct tensorial structure to allow for such rewriting. Another interesting observation is that the above result does *not* factorise as

$$\Big(t_8^{r_1\cdots r_8} t_8^{s_1\cdots s_8} \pm \epsilon^{r_1\cdots r_8 np}\epsilon^{s_1\cdots s_8}{}_{np}\Big) W_{r_1r_2s_1s_2}W_{r_3r_4s_3s_4}W_{r_5r_6s_5s_6}\, H_{r_7r_8m}H_{s_7s_8}{}^{m} , \tag{3.21}$$

in contrast to statements that can be found in the literature. This conclusion is based on the fact that our result (3.20) can be shown to contain, after expansion of the double epsilon-tensor contraction, terms with no contractions of indices between two gauge field strengths. Such terms are manifestly absent from (3.21).

3.2.2 Comparison with the linearised superfield

An elegant method for deriving higher-derivative terms in the $N = 1$ effective action is to make use of the fact that the lowest order on-shell supergravity theory can be expressed entirely in terms of a single scalar superfield, as shown by Nilsson [17]. Taking powers of this superfield and integrating over sixteen fermionic coordinates produces W^4 terms familiar from string-amplitude calculations (see Nilsson and Tollstén [18] and Kallosh [19]). A similar procedure can be used in the type-IIB theory, where Howe and West [20] have shown that a formulation based solely on a single scalar superfield exists, albeit now only at the linearised level.

In the present section we would like to compare the predictions from a scalar superfield approach with the results of the string-amplitude calculations discussed above. As we shall see, the scalar superfield does *not* reproduce the W^3H^2 terms obtained from string theory. We will conclude the section with a discussion of possible reasons for this discrepancy.

The expansion of the superfield in components can be found in e.g. Nilsson [17]. For our purposes the terms of interest are

$$\Delta = \cdots + (\theta\mathcal{C}\Gamma^{\mu\nu\rho}\theta)\, H_{\mu\nu\rho} + \cdots + (\theta\mathcal{C}\Gamma^{\mu\nu\kappa}\theta)(\theta\mathcal{C}\Gamma_{\kappa}{}^{\rho\sigma}\theta)\, W_{\mu\nu\rho\sigma} + \cdots . \tag{3.22}$$

(We assume the remaining gauge fields having been set to zero and leave out the dilaton dependence as these fields are irrelevant for our argument.) Higher-order θ terms are formed either from auxiliary fields, from equation-of-motion terms or from terms non-linear in the component fields; we will return to the latter below. The type-IIB action has been conjectured to be given by a θ^{16} integral of some function of the superfield (see e.g. Green and Sethi [21] for more information),

$$S_{\rm IIB} = \int \mathrm{d}^{10}x\, \mathrm{d}^{16}\theta\ \det e\, F[\tau_0 + \Delta]\,, \tag{3.23}$$

where $\tau_0 = C_0 + ie^{-\phi_0}$ is a constant background. The integral over sixteen thetas is, in general, very hard to evaluate. However, already the knowledge of the superfield expansion can be used to draw conclusions about the structure of the terms arising from the integration; the W^3H^2 terms under consideration is a case in point.

We are interested in the W^3H^2 terms arising from the fifth power of the superfield:

$$\mathcal{L}_{W^3H^2}\Big|_{\Delta^5} = \int \mathrm{d}^{16}\theta \left((\theta\mathcal{C}\Gamma^{m_1m_2k}\theta)\, (\theta\mathcal{C}\Gamma_k{}^{n_1n_2}\theta) W_{m_1m_2n_1n_2}\right)^3 \left((\theta\mathcal{C}\Gamma^{r_1r_2r_3}\theta) H_{r_1r_2r_3}\right)^2. \tag{3.24}$$

This is to be compared with the expression that yields the W^4 terms,

$$\mathcal{L}_{W^4} = \int \mathrm{d}^{16}\theta\, \Delta^4 = \int \mathrm{d}^{16}\theta \left((\theta\mathcal{C}\Gamma^{m_1m_2k}\theta)\, (\theta\mathcal{C}\Gamma_k{}^{n_1n_2}\theta)\, W_{m_1m_2n_1n_2}\right)^4. \tag{3.25}$$

In order to determine the possible contractions of the two gauge field strengths, we employ a group-theoretical argument. The θ^4 expression in the second factor of (3.24) decomposes as

$$(\mathbf{16}\otimes\mathbf{16}\otimes\mathbf{16}\otimes\mathbf{16})_a = \mathbf{770}\oplus\mathbf{1050}^+\,, \tag{3.26}$$

on which the product of the two gauge field strengths hence are projected (the subscript a denotes the restriction to the fully antisymmetrised part). In general, the symmetric tensor product of two third-rank antisymmetric tensors contains the irreducible representations

$$\Big([1^3]\otimes[1^3]\Big)_s = \underset{\mathbf{1}}{\cdot} \oplus \underset{\mathbf{54}}{\widetilde{[2]}} \oplus \underset{\mathbf{210}}{[1^4]} \oplus \underset{\mathbf{770}}{\widetilde{[2,2]}} \oplus \underset{\mathbf{1050^+}}{\widetilde{[2,1^4]}^{+}} \oplus \underset{\mathbf{1050^-}}{\widetilde{[2,1^4]}^{-}} \oplus \underset{\mathbf{4125}}{\widetilde{[2,2,2]}}\,. \tag{3.27}$$

Explicitly, this corresponds to

$$\begin{aligned}
\mathbf{210}:&\quad \tfrac{1}{3}\big(H^m{}_{r_1r_2}H_{s_1s_2m} - 2H^m{}_{r_1s_2}H_{s_1r_2m}\big)\,,\\
\mathbf{770}:&\quad \tfrac{2}{3}\big(H^m{}_{r_1r_2}H_{s_1s_2m} + H^m{}_{r_1s_2}H_{s_1r_2m}\big) - \text{trace terms}\,,\\
\mathbf{1050}^\pm:&\quad \tfrac{1}{2}(\Pi_\pm)_{r_1r_2r_3s_1s_2}{}^{n_1n_2n_3n_4n_5}\big(H_{n_1n_2n_3}H_{n_4n_5s_3} - 3\,H_{n_1n_2s_3}H_{n_3n_4n_5}\big)\,,\\
\mathbf{4125}:&\quad \tfrac{1}{2}\big(H_{r_1r_2r_3}H_{s_1s_2s_3} + 3\,H_{r_1r_2s_3}H_{s_1s_2r_3}\big) - \text{trace terms}\,.
\end{aligned} \tag{3.28}$$

where $(\Pi_\pm)_{n_1\ldots n_5}{}^{p_1\cdots p_5} = \frac{1}{2}(\delta^{p_1\cdots p_5}_{n_1\ldots n_5} \mp \frac{1}{5!}\epsilon_{n_1\ldots n_5}{}^{p_1\cdots p_5})$ are projection operators onto self-dual and anti-self-dual five-forms, respectively. In (3.28) we implicitly assume full antisymmetrisations (separately) in the r- and s-indices, as well as symmetry under the exchange $r \leftrightarrow s$ for $\mathbf{1050}^\pm$.

Let us now compare the above result to the action (3.20) obtained from string amplitudes. The **1** and **54** representations are proportional to the equations of motion and can therefore be absorbed by field redefinitions, so we will focus attention on the remaining terms. For a comparison of the other representations we have worked out the double epsilon contraction arising from the odd/odd sector. Rather surprisingly, we find that

$$\epsilon^{r_1\cdots r_9 m}\epsilon^{s_1\cdots s_9}{}_m\, W_{r_1r_2s_1s_2}W_{r_3r_4s_3s_4}W_{r_5r_6s_5s_6}\Big(H_{r_7r_8r_9}H_{s_7s_8s_9} - 3\,H_{r_7r_8s_9}H_{s_7s_8r_9}\Big) = 0\,. \tag{3.29}$$

An analysis of the H^2 representation content shows that this identity combined with the more readily derived result

$$\epsilon^{r_1\cdots r_9 m}\epsilon^{s_1\cdots s_9}{}_m\, W_{r_1r_2s_1s_2}W_{r_3r_4s_3s_4}W_{r_5r_6s_5s_6}\,\epsilon_{r_7r_8r_9s_7s_8}{}^{n_1\ldots n_5}H_{n_1n_2n_3}H_{n_4n_5s_9} = 0\,, \tag{3.30}$$

together imply that the $(\epsilon^2W^3)^{r_7r_8r_9s_7s_8s_9}$ tensor in the string-theory effective-action term (3.13) projects to zero the representations **210**, $\mathbf{1050^-}$ and $\mathbf{1050^+}$. The **4125** part of (3.13) is however non-zero, in contradiction with the prediction from the linear scalar superfield integral.

Two possible sources for this discrepancy can readily be identified. First, in a more complete superspace treatment the vielbein determinant in the integral (3.23) would be replaced by its superfield extension, whose θ expansion might produce additional gauge-field terms of the kind under consideration. Another reason why one should not have expected a match is that the current knowledge of the theta expansion of the scalar superfield is incomplete. While the only R- and H-dependent terms of the superfield that have been given in the literature are those displayed in (3.22), they by no means constitute the complete story; the complete scalar superfield contains non-linear terms in the component fields.

We expect, e.g., a term $WH\theta^6$, with some internal contractions between the Weyl tensor and the gauge field strength, which will contribute to the W^3H^2 terms that arise from the Δ^4 integral. The possible terms can be classified. The product of six fermionic coordinates leads to

$$(\otimes^6\mathbf{16})_a = \underset{\mathbf{3696}}{\tilde{\ydiagram{2,2,1,1,1}}^{\,+}} \oplus \underset{\mathbf{4312}}{\tilde{\ydiagram{3,1,1}}}\,. \tag{3.31}$$

The contractions of one Weyl tensor and one (NS,NS) three-form gauge field strength that correspond to these two representations are

$$RH_{\tilde{\ydiagram{2,2,1,1,1}}^{+}} = W^{mn}{}_{[r_1r_2}H_{r_3r_4r_5]}\,,\quad RH_{\tilde{\ydiagram{3,1,1}}} = \tfrac{1}{6}W^{p(m}{}_{[r_1r_2}H^{n)}{}_{r_3]p}\,. \tag{3.32}$$

Since a θ^4H^2 expression thus cannot be factored out, the simple representation-theory argument given at the beginning of this section no longer applies. It is extremely tedious to find the precise way in which these higher-order θ terms arise (either by solving the Bianchi identities or by using a gauge-completion procedure), and we therefore refrain from following that route. It should be clear that any conclusions drawn from the scalar superfield which involve other gauge-field terms will similarly suffer from our incomplete understanding of its component expansion.

4 Discussion and conclusions

In this paper we have analysed in detail a number of string-amplitude computations which exhibit interesting and often overlooked features. While motivated by an ongoing programme that aims at the full construction (at order $(\alpha')^3$) of higher-derivative superinvariants for the type-II theories, the calculations are relevant in a very general setting. It is important to understand these issues in detail because there are very few practical checks that one can make on the not yet determined terms in the effective action. In particular, while we were previously [1] able to use supersymmetry constraints to massage our string calculations, this is a much more computationally intensive check once gauge fields are included.

We have focused on three main issues. The first one concerns the application of the chiral splitting theorem. We have shown its role in the picture-changing operation and exhibited the importance of left/right mixing contractions. Our second point of emphasis is the role of world-sheet gravitino zero modes in physical vertex operators. These lead to new terms which have so far not received much attention, yet are extremely relevant for obtaining consistent amplitudes (and are expected to lead to even more serious problems when omitted at higher genus). The third feature is that of the tensor structure arising from fermionic correlators. Here we have shown that the effective action receives contributions from various terms which at first sight look completely different, yet combine in a subtle way once the fermionic correlators are worked out and the signs of the GSO projection are taken into account.

Finally, we have discussed the relation of our string-based approach to the construction of higher-derivative superinvariants using (linearised) scalar superfields. We have shown limitations of the applicability of the latter approach by exhibiting a mismatch between its predictions and the string-amplitude calculations. We have traced this discrepancy to the omission of terms in the superfield which are non-linear in the component fields.

The observations made in the present letter are relevant for the full construction of the various other terms in the superstring effective action, including those with Ramond-Ramond gauge fields. This project is in progress and will be reported on in a forthcoming publication.

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A Appendix

A.1 Operator product expansions

Our conventions for the normalisations of the fields are summarised by the following operator products and the short distance behaviour of the correlators on the torus $\mathcal{T}$ with modular parameter τ. For the bosons we use

$$\langle X(z)X(w)\rangle_{\mathcal{T}} \simeq -\frac{\alpha'}{2}\ln|z-w|^2 - \frac{\pi\alpha'}{2\,\tau_2}(z-w-\bar{z}+\bar{w})^2\,, \tag{A.1}$$

$$\langle \partial X(z)\partial X(w)\rangle_{\mathcal{T}} \simeq -\frac{\alpha'}{2}\frac{1}{(z-w)^2} + \frac{\pi\alpha'}{\tau_2}\,, \tag{A.2}$$

$$\left\langle \partial X(z)\, e^{ikX(w)}\right\rangle_{\mathcal{T}} \simeq -\frac{\alpha'}{2}\frac{ik}{z-w}e^{ikX(w)} - \frac{2\pi ik\alpha'}{\tau_2}(z-w-\bar{z}+\bar{w})e^{ikX(w)}\,, \tag{A.3}$$

$$\left\langle e^{ipX(z)}\, e^{iqX(w)}\right\rangle_{\mathcal{T}} \simeq |z-w|^{\alpha' pq}\, e^{i(p+q)X(z)}\,, \tag{A.4}$$

$$\left\langle \partial X(z)\bar{\partial}X(w)\right\rangle_{\mathcal{T}} = \alpha'\pi\,\delta^{(2)}(z-w) - \frac{\pi\alpha'}{\tau_2}\,, \tag{A.5}$$

$$\langle F^{\mu}(z)F^{\nu}(w)\rangle_{\mathcal{T}} = \alpha'\delta^{(2)}(z-w)\,. \tag{A.6}$$

The fermion normalisations are fixed by

$$\Psi^{\mu}(z)\Psi^{\mu}(w) = \frac{\alpha'}{2}\frac{1}{z-w} + \cdots\,, \qquad \tilde{\Psi}^{\mu}(z)\tilde{\Psi}^{\mu}(w) = \frac{\alpha'}{2}\frac{1}{\bar{z}-\bar{w}} + \cdots\,, \tag{A.7}$$

and for the ghosts one has

$$e^{-\chi(w)}e^{\chi(z)} = \frac{1}{z-w} + \mathcal{O}(1)\,, \tag{A.8}$$

$$e^{-\phi(w)}e^{\phi(z)} = z-w + \mathcal{O}((z-w)^2)\,. \tag{A.9}$$

The delta function can be represented as

$$2\pi\,\delta^{(2)}(z-w) = \bar{\partial}_z\frac{1}{z-w}\,. \tag{A.10}$$

Finally, observe that the fermions anti-commute, $\{\Psi^1,\Psi^2\}=0$, $\{\Psi^1,\tilde{\Psi}^2\}=0$, $\{e^{\phi},e^{\tilde{\phi}}\}=0$ and so on. In order to check correctness of our expressions without deriving them from the covariant formalism, one can use the following U(1) charges inherited from the two-dimensional word-sheet complex structure,

$$\begin{aligned}
\Psi &= (-\tfrac{1}{2},0)\,, & \tilde{\Psi} &= (0,-\tfrac{1}{2})\,,\\
\theta &= (\tfrac{1}{2},0)\,, & \bar{\theta} &= (0,\tfrac{1}{2})\,,\\
\partial &= (-1,0)\,, & \bar{\partial} &= (0,-1)\,,\\
\chi_- &= (-1,\tfrac{1}{2})\,, & \tilde{\chi}_+ &= (\tfrac{1}{2},-1)\,,\\
\gamma &= (\tfrac{1}{2},0)\,, & \tilde{\gamma} &= (0,\tfrac{1}{2})\,,\\
F &= (-\tfrac{1}{2},-\tfrac{1}{2})\,.
\end{aligned} \tag{A.11}$$

A.2 Gamma matrices in the helicity basis

In the computation of the string amplitudes we make use of gamma matrices in the so-called helicity basis, also found in appendix A of Atick and Sen [13]. The Euclidean gamma matrices with mixed indices $(\gamma^r)_a{}^b$ in this basis are given by

$$\begin{aligned}
(\gamma^1)_a{}^b &= \frac{\sigma^1 - i\sigma^2}{2} \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1}\,, & (\gamma^{\bar{3}})_a{}^b &= \sigma^3 \otimes \sigma^3 \otimes \frac{\sigma^1 + i\sigma^2}{2} \otimes \mathbb{1} \otimes \mathbb{1}\,,\\
(\gamma^{\bar{1}})_a{}^b &= \frac{\sigma^1 + i\sigma^2}{2} \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1}\,, & (\gamma^4)_a{}^b &= \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \frac{\sigma^1 - i\sigma^2}{2} \otimes \mathbb{1}\,,\\
(\gamma^2)_a{}^b &= \sigma^3 \otimes \frac{\sigma^1 - i\sigma^2}{2} \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1}\,, & (\gamma^{\bar{4}})_a{}^b &= \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \frac{\sigma^1 + i\sigma^2}{2} \otimes \mathbb{1}\,,\\
(\gamma^{\bar{2}})_a{}^b &= \sigma^3 \otimes \frac{\sigma^1 + i\sigma^2}{2} \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1}\,, & (\gamma^5)_a{}^b &= \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \frac{\sigma^1 - i\sigma^2}{2}\,,\\
(\gamma^3)_a{}^b &= \sigma^3 \otimes \sigma^3 \otimes \frac{\sigma^1 - i\sigma^2}{2} \otimes \mathbb{1} \otimes \mathbb{1}\,, & (\gamma^{\bar{5}})_a{}^b &= \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \sigma^3 \otimes \frac{\sigma^1 + i\sigma^2}{2}\,,
\end{aligned} \tag{A.12}$$

and from here it is easy to compute products of them. The algebra satisfied by these matrices is

$$\{\gamma^r, \gamma^{\bar{s}}\} = \delta^{rs}\,, \quad \{\gamma^r, \gamma^s\} = 0\,. \tag{A.13}$$

The relation to the basis used in the main text is given by

$$\Gamma^1 = (\gamma^1 + \gamma^{\bar{1}})\,, \quad \Gamma^2 = -i(\gamma^1 - \gamma^{\bar{1}})\,, \tag{A.14}$$

and so on, which implies that

$$\{\Gamma^r, \Gamma^s\}_a{}^b = 2\,\delta^{rs}(\mathbb{1})_a{}^b\,. \tag{A.15}$$

Index raising is done by multiplying with the ϵ^{ab} symbol on the right; this matrix reads

$$\epsilon^{ab} = -\sigma^1 \otimes \sigma^2 \otimes \sigma^1 \otimes \sigma^2 \otimes \sigma^1\,. \tag{A.16}$$

and the inverse satisfies $\epsilon_{ab} = \epsilon^{ab}$. In the notation of Atick and Sen [13] we have

$$\Psi^1_{AS} = \tfrac{1}{2}(\Psi^1 + i\Psi^2)\,, \quad \Psi^{\bar{1}}_{AS} = \tfrac{1}{2}(\Psi^1 - i\Psi^2)\,, \tag{A.17}$$

and so on for the other fermions. In this basis for the gamma matrices, SO(10) spinors are represented as a five-fold tensor product of SO(2) spinors,

$$S_a = S_\pm \otimes S_\pm \otimes S_\pm \otimes S_\pm \otimes S_\pm\,. \tag{A.18}$$

The correlators for an arbitrary number of spin fields $S_\pm$ and complex fermions (A.17) are known or can be derived through bosonisation. The ones we need can be found in Atick and Sen [13].

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