

THERMALIZATION AND ELLIPTIC FLOW AT RHIC

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We discuss signatures of thermalisation in heavy ion collisions based on elliptic flow. We then propose a new method to analyse elliptic flow, based on multiparticle azimuthal correlations. This method allows one to test quantitatively the collective behaviour of the interacting system.

1 Introduction

A highly debated issue in ultrarelativistic heavy ion collisions is whether or not the colliding system reaches local thermal equilibrium at some stage of its evolution. Equilibrium with respect to inelastic collisions strongly constrains the ratios of particle abundances¹ (“chemical” equilibrium), as well as phase-space densities, which are obtained by combining informations from momentum spectra and two-particle HBT correlations;² on the other hand, equilibrium with respect to elastic collisions constrains momentum distributions, and implies in particular that they are isotropic in the local rest frame. This is the “kinetic” equilibrium, on which we concentrate here.

Kinetic equilibrium itself has (at least) two facets. One is the equilibration between longitudinal and transverse degrees of freedom, i.e., the implication that in the local rest frame, longitudinal and transverse momenta are of the same order of magnitude. This aspect of thermalisation can be discussed from first principles at the partonic level,³ and there is now a vast literature on this subject.⁴ But experimental signatures deal in fact rather with equilibration among the two transverse degrees of freedom. Due to the high Lorentz contraction at ultrarelativistic energies, the typical transverse scale is much larger than the longitudinal scale, so that this “transverse equilibrium” is probably easier to achieve than “longitudinal-transverse equilibrium”.

In Sec. 2, we discuss some experimental signatures of transverse equilibrium, especially elliptic flow, and the relevance of hydrodynamical models in this context. In Sec. 3, we present a new method recently developed to obtain more reliable measurements of elliptic flow, which are required in order to draw definite conclusions on the issue of thermalisation.

2 Elliptic flow: a signature of transverse thermalisation

2.1 When are hydrodynamical models useful?

If the colliding system thermalises, its evolution follows the laws of hydrodynamics; it is therefore natural to use hydrodynamical models in order to define signatures of thermalisation. There are two ingredients in these models: i) an equation of state; ii) initial conditions, i.e., energy density, baryon density and fluid velocity on a space-like hypersurface (typically, at some initial time).

Which parameters are under control? The initial time t_0 , at which the system thermalises, is hopefully short but to a large extent unknown, so that hydrodynamics has little predictive power concerning observables which depend strongly on t_0 , such as thermal photon production.⁵ Similarly, there is no serious motivation for studying the very first stages of the collision within the framework of hydrodynamics, either within a one-fluid model^{6,7} or a multifluid model,⁸ and the relevance of signatures based on such parametrisations is questionable. Finally, there is no well-defined prescription concerning the initial longitudinal fluid velocity and density, for which a variety of parametrisations exist, of either “Landau”⁹ or “Bjorken”¹⁰ type.

The situation is much clearer concerning transverse degrees of freedom: the transverse collective velocity must be initially zero since each nucleon-nucleon collision populates the transverse momentum space randomly; the initial density profile in the transverse plane is strongly constrained by observed multiplicity distributions.¹¹ Therefore, reliable signatures of thermalisation should rather be sought for in observables associated with transverse degrees of freedom: p_T spectra,¹² transverse radii¹³ and azimuthal anisotropies, in particular elliptic flow, on which we concentrate here.

2.2 Why elliptic flow?

Elliptic flow is defined as a correlation between the azimuth ϕ of an outgoing particle and the azimuth Φ_R of impact parameter:¹⁴

$$v_2 = \left\langle e^{2i(\phi - \Phi_R)} \right\rangle, \quad (1)$$

where brackets denote a statistical average. At ultrarelativistic energies, v_2 is positive for noncentral collisions.^{15,16} In a hydrodynamical picture, it results from anisotropic pressure gradients in the transverse plane, due to the almond-shaped region of the overlap region between the two nuclei.¹⁷ Microscopically, v_2 is created by rescattering among the produced particles, which makes it a sensitive probe of final state interactions: if there are none, it vanishes, while other observables such as p_T spectra may still look “thermal”.¹⁸

Furthermore, predictions of hydrodynamical models for v_2 are very stable: since v_2 is created by transverse pressure gradients, it strongly depends on the initial density transverse profile, which is well controlled as discussed above; it also depends significantly on the equation of state, on which it may thus provide valuable information; on the other hand, v_2 depends only weakly on arbitrary parameters, such as initial time and longitudinal velocity.¹⁷ Quite remarkably, simple hydrodynamical parametrisations are able to reproduce simultaneously the measured p_T spectra, HBT radii and elliptic flow at RHIC.¹⁹

2.3 Predictions for v_2

The momentum anisotropy v_2 calculated in hydro models is roughly equal to the anisotropy of the almond-shaped overlap region.¹⁷ This purely geometrical effect dominates the centrality dependence of v_2 , which decreases linearly with the number of participants. Deviations from this behaviour can be used to signal a phase transition²⁰ or a departure from thermal equilibrium.²¹ The latter is expected to occur for the most peripheral collisions, where v_2 should be smaller than the hydro prediction. This is indeed observed in several specific transport models like UrQMD²² (which however predicts a much too small value of v_2), QGSM,²³ and AMPT.²⁴ With a more systematic study,²⁵ one could relate the observed centrality dependence of v_2 to the degree of thermalisation of the system. Experimental results vary significantly depending on the method used to analyse elliptic flow.^{26,27} We come back to this issue in Sec. 3.

Hydrodynamical calculations²⁸ were also able to predict the p_T dependence of v_2 for identified hadrons, in remarkable agreement with experimental results:²⁹ v_2 is almost linear in p_T for pions and significantly smaller for protons. However, these non-trivial features are also reproduced by transport models.^{22,23,24} In addition, the latter predict a saturation²⁵ at high p_T which is not seen in hydro, suggesting that many elastic collisions are necessary to build the flow at high p_T . This saturation, which is seen in the data, could also be related to hard physics.³⁰

3 Analysing elliptic flow with multiparticle correlations

3.1 Flow from azimuthal correlations

Since the reaction plane Φ_R is unknown, v_2 cannot be derived directly from (1). It must be inferred from azimuthal correlations. The standard flow analysis³¹ relies on the key assumption that all azimuthal correlations are due to flow, i.e., that angles relative to the reaction plane $\phi - \Phi_R$ are statistically

independent. This allows one to write the two-particle correlation as³²

$$\left\langle e^{2i(\phi_1 - \phi_2)} \right\rangle = \left\langle e^{2i(\phi_1 - \phi_R)} e^{2i(\phi_R - \phi_2)} \right\rangle = (v_2)^2. \quad (2)$$

where brackets denote an average over pairs of particles belonging to the same event. One could also use higher order correlations, such as the four-particle correlation, which would give

$$\left\langle e^{2i(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \right\rangle = (v_2)^4. \quad (3)$$

However, such equations are not quite correct, since they neglect various other sources of azimuthal correlations (“nonflow correlations”), which is no longer justified at ultrarelativistic energies.³³

3.2 Simple illustration of nonflow correlations

In order to illustrate nonflow correlations, we consider the following example: assume that in each event, $M/2$ pairs of particles are emitted, where both particles in a pair have collinear momenta, but pairs are emitted with random orientations. Since pairs are emitted randomly, there is no flow ($v_2 = 0$), but there are azimuthal correlations. In each event, there is a total of $M(M - 1)/2$ particle pairs, among which $M/2$ are correlated, hence the two-particle correlation

$$\left\langle e^{2i(\phi_1 - \phi_2)} \right\rangle = \frac{1}{M - 1}. \quad (4)$$

A similar reasoning yields the four-particle correlation:

$$\left\langle e^{2i(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \right\rangle = \frac{2M(M - 2)}{M(M - 1)(M - 2)(M - 3)} = \frac{2}{(M - 1)(M - 3)}. \quad (5)$$

Applying Eqs. (2) and (4), or (3) and (5), one would obtain $v_2 \sim 1/\sqrt{M}$, although there is no flow: this is the typical order at which nonflow correlations spoil the standard flow analysis.

3.3 Subtraction of nonflow correlations

The contribution of nonflow correlations can be greatly reduced by combining the informations from two- and four-particle correlations. In Eq. (5), a 4-uplet of particles gives a nonvanishing contribution if it consists of two correlated pairs, either (1,3) and (2,4) or (1,4) and (2,3). Subtracting the corresponding contributions, one obtains

$$\left\langle e^{2i(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \right\rangle - 2 \left\langle e^{2i(\phi_1 - \phi_2)} \right\rangle^2 = \frac{4}{(M - 1)^2(M - 3)}. \quad (6)$$

The l.-h. s. of this equation defines the *cumulant* of four-particle correlations, $c_2\{4\}$. The r.-h. s. is the contribution of nonflow correlations, of order $1/M^3$, i.e., much smaller than the corresponding contribution to the four-particle correlation (5), of order $1/M^2$. On the other hand, the contribution of flow remains of the same magnitude: from Eqs. (2), (3) and (6), one obtains $c_2\{4\} = -(v_2)^4$. The subtraction therefore reduces the relative contribution of nonflow effects, and yields a more accurate estimate of the flow.

This method was recently applied to STAR data.²⁷ The resulting value of v_2 are smaller than those obtained with the standard analysis,²⁶ in particular for the most peripheral collisions: this is precisely where nonflow effects are expected to give the largest contribution since the multiplicity M is smaller. The centrality dependence obtained with this method suggests that departures from thermalisation at RHIC may be larger than was previously thought.

This cumulant expansion can be worked out to arbitrary orders, and allows one to extract the genuine 4-, 6-particle correlations and beyond.³⁴ The practical implementation of the method is described in detail elsewhere.³⁵ Flow, which is essentially a collective phenomenon, contributes to all orders, while the relative contribution of nonflow correlations decreases as the order increases. Higher order cumulants therefore provide a unique possibility to check quantitatively that azimuthal correlations are indeed of collective origin.

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