Closed Strings from SO(8) Yang-Mills Instantons

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When eight-dimensional instantons, satisfying $F \wedge F = \pm \star_8 (F \wedge F)$, shrink to zero size, we find stringy objects in higher order ten-dimensional Yang-Mills (viewed as a low-energy limit of open string theory). The associated F^4 action is a combination of two independent parts having a single-trace and a double-trace structure. As a result we get a D-string from the single-trace term and a fundamental string from the double-trace. The latter has (8,0) supersymmetry on the world-sheet and couplings to the background gauge fields of a heterotic string. A correlation between the conformal factor of the instanton and the tachyon field is conjectured.

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1. Introduction

Recent developments in string theory have given some hope for answering important questions which require off-shell formulation of a second quantized string theory. In the case of open string field theory, several off-shell formulations are currently known [1,2,3]. The situation with the closed string field theory is more complicated. One might consider the possibility that in fact the complete and consistent off-shell string theory can originate from properly defined open string field theory alone, with closed string being understood in terms of classical open string field theory (for motivations and references see [4,5]).

The possibility that a closed string world-sheet action arises from classical solutions in open string field theory will be the main theme of this paper. Although a complete picture might require incorporating the whole field theory of open strings, one might hope to get important insights by looking at low energy sector only. This is what we will attempt here. We start from a non-abelian super Yang–Mills theory including the higher-derivative F^4 terms in ten dimensions in the background of a SO(8) instanton and find stringy objects.¹ The presence of higher-derivative terms in the effective action is crucial for our analysis since not only the lowest order F^2 term cannot lead to stringy objects with a finite tension but, as we will argue, does not play any role. In zero size limit of the instanton, a Dstring and a fundamental heterotic string are found. The heterotic string solution will be supersymmetric only under a linear combination of the standard supersymmetry of the Yang–Mills theory together with an extra supersymmetry transformation. It is tempting to interpret this extra supersymmetry transformation as originating from the non-linear supersymmetry of a non-BPS D-brane sitting in our vacuum.

The consistency of the heterotic string background is related to the original symmetries of the Yang–Mills background; the chirality of the background will be reflected in the chirality of the string world-sheet theory. In particular, starting from an "anomaly free" open string theory (a correct choice of the gauge group for which the top term in the anomaly polynomial vanishes is what is meant here) will surely result in a closed heterotic string theory in a consistent gravitational and Yang–Mills background.

The D-string appears from the presence of $\operatorname{tr} F^4$ in the effective action, and its tension is quantized according the value of $\Pi_7(G) = \mathbb{Z}$ for G allowing non-trivial embedding of

¹ The structure and the supersymmetry of F^4 terms will be discussed in details in section 2. For now we just remark that due to different ways of contracting the gauge indices they can be of two types—single-trace terms tr F^4 and double-trace $(trF^2)^2$ terms.

SO(8) groups [6,7]. For obvious reasons, we chose G = SO(32). The novelty of our study resides in the origin of a fundamental heterotic string from the double-trace structure $(\text{tr}F^2)^2$ and its supersymmetric completion in the effective action.

In the next section, the precise structure of these terms will be shown from the supersymmetry analysis but we would like to make some qualitative comments here from a different point of view. Some relevant ideas come just from looking at the (bosonic part of) open string action

$$S = S_{\text{open}}(\phi, B, T) - \int d^{10}x \sqrt{\det(G+F)} f(T)$$
(1.1)

written here for the constant tachyon field T (the precise analog of this action for nonabelian gauge groups is not known yet but can be conceptually derived in any given string field theory). According to the well-known conjectures of Sen [8], f(T) has zero at the minimum of the tachyon potential where the closed string vacuum is expected. It can be calculated using the formalism of [2]; for bosonic string field theory it is given by $f(T) = e^{-T}$ while the tachyon potential is $V_B = (1 + T)e^{-T}$ and for the superstring, f(T) coincides with the tachyon potential $V_F(T) = e^{-T^2}$ [9]. Since our analysis relies on supersymmetry, from now on we will replace f by V.

In a general string field theory the lagrangian is defined up to field redefinitions and a correct choice of fields is dictated by the principle that equations of motion are proportional to world-sheet β -function with some Zamolodchikov metric G: $\partial_i S = G_{ij}\beta^j$. Here G is the metric on the space of fields and has to be non-degenerate. One can note that the perfect square part of double-trace terms, $(\text{tr}F_{\mu\nu}F^{\mu\nu})^2$, in a classical open string field theory lagrangian can be generated by a field redefinition $T \to T + trF^2$. The compatibility of the supersymmetry and the above principle should fix these terms unambiguously as well as complete the needed $t_{(8)} (\text{tr}F^2)^2$.

We conclude this introduction by a comment on the world-sheet topology. We already mentioned that by expanding around the solution we obtain couplings for collective modes which turn out to reproduce those of the heterotic string, and this should already indicate that the string under the consideration is closed. Much of this structure is directly inherited form the SO(8) instanton construction. It is yet another feature of the construction that the solution has in fact SO(9) symmetry and the fact that we are using only the SO(8)part leads to the periodicity of one of the string world-sheet coordinates. A brief outline of the paper is the following. In section 2, we present the Yang-Mills action and the supersymmetry transformations. In section 3, we look for BPS configurations associated with eight-dimensional instanton configurations having SO(8) symmetry. We consider the supersymmetric excitations in the section 4 and show the emergence of the two-dimensional action with couplings of the heterotic string with a particular embedding of the gauge group. Finally the section 5 contains a general discussion of our results. We have collected various definitions and identities needed in the main text in two appendices.

2. The effective action

Since we need to consider non-abelian and supersymmetric extensions of the effective theory (1.1), we start by recalling some facts about supersymmetrisation of open string effective actions.

2.1. The linear supersymmetry

The effective action can be expanded in power series of α' , and at each order supersymmetric action can be obtained by Noether procedure. This construction is valid for any gauge group. This is done in the context of *linearly realized* $\mathcal{N}_{10} = 1$ supersymmetry transformations. The lagrangian for the non-abelian degrees of freedom constructed by this procedure up to and including $(\alpha')^2 F^4$ terms is [10]:

$$\mathcal{L} = -\frac{1}{4} F^{A}{}_{\mu\nu} F^{A}{}_{\mu\nu} - 8\bar{\chi}^{A} \not D\chi^{A} - \frac{(\pi\alpha')^{2}}{32} M_{ABCD} \left[(F^{A}{}_{\mu\nu} F^{B}{}_{\nu\mu}) (F^{C}{}_{\rho\sigma} F^{D}{}_{\sigma\rho}) - 4F^{A}{}_{\mu\nu} F^{B}{}_{\nu\rho} F^{C}{}_{\rho\lambda} F^{D}{}_{\lambda\mu} \right] + (\pi\alpha')^{2} M_{ABCD} \left[4F^{A}{}_{\mu\lambda} F^{B}{}_{\lambda\nu} (\bar{\chi}^{C} \Gamma_{\mu} (D_{\nu} \chi)^{D}) + 2F^{A}{}_{\mu\nu} F^{B}{}_{\rho\lambda} (\bar{\chi}^{C} \Gamma_{\mu\nu\rho} (D_{\lambda} \chi)^{D}) \right] + quartic fermions.$$

$$(2.1)$$

This action (2.1) is invariant under the linear supersymmetry transformations (by a straightforward generalization of the results of [11])

$$\delta_{\epsilon}A^{A}_{\mu} = -4\bar{\epsilon}\Gamma_{\mu}\chi^{A} -\frac{(\pi\alpha')^{2}}{8}M^{A}_{BCD}\left[2(\bar{\epsilon}\Gamma_{\mu}\chi^{B})F^{C}_{\nu\rho}F^{D}_{\rho\nu} - 8(\bar{\epsilon}\Gamma^{\nu}\chi^{B})F^{C}_{\nu\lambda}F^{D}_{\lambda\mu} - (\bar{\epsilon}\Gamma^{\nu_{1}\cdots\nu_{4}}_{\mu}\chi^{B})F^{C}_{\nu_{1}\nu_{2}}F^{D}_{\nu_{3}\nu_{4}}\right]$$

$$(2.2)$$

$$\delta_{\epsilon}\chi^{A} = \frac{1}{8}\Gamma^{\mu\nu}\epsilon F^{A}_{\mu\nu} + \frac{(\pi\alpha')^{2}}{2^{5}\cdot 4!}M^{A}_{BCD} \Big[\big(t^{(\nu)}_{(8)}\Gamma_{\nu_{7}\nu_{8}}\epsilon - \Gamma^{\nu_{1}\cdots\nu_{6}}\epsilon\big)F^{B}_{\nu_{1}\nu_{2}}F^{C}_{\nu_{3}\nu_{4}}F^{D}_{\nu_{5}\nu_{6}} \Big]$$
(2.3)

 Γ^{μ} are the SO(1,9) Γ -matrices, as well as all the spinors in this expression; the tendimensional space-time indices are labeled by greek letters from the middle of the alphabet $\mu, \nu = 0, 1, ..., 9.$

All the fields are in the adjoint of the gauge group G = SO(32), labeled by a capital letter from the beginning of latin alphabet. The quadratic expressions in the field are contracted with the metric $\delta_{AB} = \text{tr}_{fund}(T_A T_B)$ and quartic expressions with the completely symmetric four rank tensor M_{ABCD} [10,12]. For any group, there are two independent fourrank symmetric tensors given by the symmetrized trace in the fundamental representation $\text{tr}_{fund}(T_{(A}T_BT_CT_D)) = \text{Str}(\cdots)$ and the double-trace $\delta_{(AB}\delta_{CD)} \sim \text{tr}_{fund}(\cdots)\text{tr}_{fund}(\cdots)$ in the fundamental representation

$$M_{ABCD} = m_1 \operatorname{tr}_{fund} (T_{(A}T_BT_CT_{D)}) + m_2 \,\delta_{(AB}\delta_{CD)} \,. \tag{2.4}$$

The analysis of [10,12] was purely in the framework of $\mathcal{N}_{10} = 1$ super-Yang-Mills theory without any reference to a particular string vacuum. Linear supersymmetry fixes the independent tensorials structures, or superinvariants, at each order in the derivative expansion, up to a number of unfixed coefficients (not affected by field redefinitions). We emphasize once more that in principle such structure can also be fixed from the requirement that the equations of motion are proportional to the world-sheet β -function.

When specifying the string vacuum of reference the coefficients multiplying the group tensors become field-dependent functions. In particular these coefficients will have a dependence on the dilaton. The open string effective action is normalized as $S = (\pi \alpha')^2 \int d^{10}x \mathcal{L}/(g_s g_o^2)$ with the standard normalisation value for the coupling constant $g_o^4 = 2^{10} \pi^7 (\alpha')^2 \kappa_{(10)}^2 = 2^{16} \pi^{14} (\alpha')^6$ [13]. The first term in (2.4) receives contributions starting from the tree-level (disc) open string diagram: $m_1(\varphi)/g_s = 1/g_s + 1/2 + o(g_s)$; the second starts from one-loop open string diagrams: $m_2(\varphi)/g_s = 1/16 + o(g_s)$. A dependence on the open string tachyon field can appear too. As we will see this difference will lead eventually to having D-string and a fundamental string tensions for solitons corresponding to the single- and the double-trace parts respectively. Looking at this theory as the low-energy limit of the SO(32) heterotic string, the coefficients m_1 and m_2 will have different dilaton dependence. These two different functional dependences on the dilaton

have been shown in [14] to be compatible with the strong coupling duality between type I and heterotic string. Bearing this in mind, we keep the field dependence of the coefficients unspecified, relying only on their independence under supersymmetrisation.

The presence of the two independent terms in (2.4) will be fundamental for our analysis. The single-trace term will carry the charge of the instanton introduced in section 3 and the double-trace term will be at the origin of the interactions of the fluctuations around the instanton.

2.2. The non-linear supersymmetry

Looking at the effective action as related to D-brane configurations in a type II vacuum, it is natural to look for the possibility of non-linear supersymmetry resulting from the breaking of some of the original supersymmetries of the vacuum. There are various ways to introduce non-linear supersymmetry depending on which fields Φ are coupled to the supersymmetry parameter [10,15] in $\delta_{\eta}\chi = \eta + \mathcal{O}(\Phi)\eta$.

For the case of open string actions with tachyon field and abelian gauge field, the quadratic effective action takes the form (abelian field have tilde)

$$\mathcal{L} = \frac{V(T)}{g_s} \left\{ 1 - \frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}_{\mu\nu} + \bar{\tilde{\chi}} \, \mathcal{D} \tilde{\chi} + \frac{1}{2} \partial^{\mu} T \partial_{\mu} T + \cdots \right\}$$
(2.5)

and is invariant under the non-linear supersymmetry transformations

$$\delta_{\eta}\tilde{\chi} = \left(1 + \frac{1}{\sqrt{2}}\partial^{\mu}T\Gamma_{\mu}\right)\eta$$

$$\delta_{\eta}T = \sqrt{2}\bar{\eta}\tilde{\chi} - 2\partial^{\mu}T\bar{\eta}\Gamma_{\mu}\tilde{\chi} - \sqrt{2}\partial^{\mu}T\partial_{\mu}T\bar{\eta}\tilde{\chi}$$

$$\delta_{\eta}\tilde{A}_{\mu} = \bar{\eta}\Gamma_{\mu}\tilde{\chi}.$$
(2.6)

For later reference we notice that the supersymmetry transformation on abelian gaugino take the form $\delta_{\eta}\tilde{\chi} = g(T)\eta$ where g(T) is function of the tachyon field only, taken its values in the spinorial representation of SO(1,9).

In a way, the non-linear supersymmetry involving the tachyon field is forced on us by the desire of working with pure Yang-Mills without the presence of the supergravity multiplet. The open string vacuum, where this second symmetry appears, will be important for us since as we will see in section 4 a linear combination of the linear and non-linear supersymmetries guarantees the supersymmetry of the fluctuations around the instanton.

3. Introducing the SO(8) Instanton

The main point of this paper is to search for closed strings inside the field theory of open strings. From the point of view of low-energy effective lagrangian this means that we shall look for classical solutions in the sector of theory containing the gauge fields and tachyon. Because we are looking for a string-like solitonic objects it is natural to investigate eight-dimensional gauge configurations. It is then not hard to see that due to the gauge configuration having codimension two, an expansion around such a soliton will result in a string. Understanding the nature of this string will be our task.

On a more technical level, the presence in the supersymmetry transfomations of the gauginos of terms linear and cubic in the the gauge field strenghts is also indicative of a special role of eight-dimensional gauge configurations. Luckily one such configuration, namely the eight-dimensional SO(8) instanton, is well known [16,17], and has been used for constructing string solitons previously in [6,18]. We start from examining the $(\alpha')^2$ corrections to $\delta_{\epsilon}\chi$ and observe that indeed it significantly simplifies for the gauge field given by [16,17]

$$F_{mn}^{ab}(x) = 4 \left(\Gamma_{mn} \mathcal{P}_{+}\right)^{ab} \frac{\rho^{2}}{(x^{2} + \rho^{2})^{2}},$$
(3.1)

where ρ is the size of the instanton, $m, n = 1, \dots, 8$, and a, b are SO(8) indices of positive chirality.² Hereafter the form factor will be abbreviated as $f(\rho, x) = \rho^2/(x^2 + \rho^2)^2$. We introduce the SO(8) chirality projector

$$\mathcal{P}_{\pm} = \frac{1}{2} (1 \pm \gamma_{(8)}) \,.$$

The SO(1,9) fermionic indices in (2.2) and (2.3) are split according to SO(8) indices as $\epsilon_{\alpha} = (\epsilon_a, \epsilon_{\tilde{a}})$ (note all the ten-dimensional spinors are Majorana-Weyl). Most importantly for our analysis the antisymmetric product of two SO(1,9) Γ -matrices decompose under SO(8) γ -matrices as

$$\Gamma_{mn} = \gamma_{mn} \mathcal{P}_+ + \tilde{\gamma}_{mn} \mathcal{P}_- \,.$$

As shown in appendix A, the eight-dimensional solution (3.1) satisfies a very important identity

$$M_{ABCD}\left(t_{(8)}^{r_1\cdots r_8} + \frac{1}{2}\varepsilon_{(8)}^{r_1\cdots r_8}\right)F_{r_1r_2}^AF_{r_3r_4}^BF_{r_5r_6}^C = 0,$$
(3.2)

² We refer to appendix A for details about SO(8) instantons, in particular the chirality discussion and their generalisations to multi-instantons solutions.

with the relative positive sign for our choice of the SO(8) representation for the instanton. (For the single-trace part this has been noticed in [6].)

For the solution (3.1), using (3.2) and the previous conventions, the $(\alpha')^2$ correction to the gaugino supersymmetry transformation (2.3) takes the form

$$\delta_{\epsilon}^{(2)}\chi^{A} = -\frac{(\pi\alpha')^{2}}{2^{4} \cdot 4!} M^{A}{}_{BCD} \left(\Gamma^{m_{1}\cdots m_{6}}\mathcal{P}_{-}\epsilon\right) F^{B}_{m_{1}m_{2}} F^{C}_{m_{3}m_{4}} F^{D}_{m_{5}m_{6}}, \qquad (3.3)$$

the appearance of the projector \mathcal{P}_{-} is related to the choice of the SO(8) representation used to define the instanton.

Since we have already restricted ourselves to eight-dimensional configurations, it is easy to see that there may be further relations between $\delta_{\epsilon}^{(0)}\chi$ and $\delta_{\epsilon}^{(2)}\chi$ by virtue of Hodge duality. But in order to see these we need some facts about eight-dimensional self-duality equations and their solutions [16,17], discussed in more details in appendix A. The gauge field (3.1) is a solution of the following self-duality equation in flat \mathbb{R}^8 [16,17]

$$M_{ABCD}\left(F^C \wedge F^D \mp \star_8(F^C \wedge F^D)\right) = 0.$$
(3.4)

This equation is conformal, and the self-duality is guaranteed by the properties of the $SO(8) \gamma$ -matrices.

One can check than the solution (3.1) also satisfies a relation

$$\frac{1}{2}m_1 \left(4f(\rho, x)\right)^2 \star_8 F^A = -M^A{}_{BCD}F^B \wedge F^C \wedge F^D \tag{3.5}$$

Note that we are not using the curved (non-conformal) counterpart of (3.4), but stay in the flat space and simply use another property of the solution. The equation (3.5) allows to establish a connection between the quadratic and quartic term in the gauge field in (2.1) and $\delta_{\epsilon}^{(0)}\chi$ and $\delta_{\epsilon}^{(2)}\chi$. Notice that in the lhs of (3.5) the dependence on m_2 has dropped out by virtue of the first Pontryagin class $p_1(F)$ vanishing on the solution. We finally rewrite the full resulting supersymmetry as

$$\delta_{\epsilon} \chi^{A} = \frac{1}{8} \Gamma^{rs} F^{A}_{rs} \left(1 + \frac{4}{3} (\pi \alpha')^{2} m_{1} (f(\rho, x))^{2} \mathcal{P}_{-} \right) \epsilon \quad .$$
(3.6)

Using (B.7) to work out the contraction between the spacetime Γ -matrices and the SO(8) γ -matrices used to construct the instanton we see that the supersymmetry transformation of the SO(8) part of the gaugino χ^{ab} takes the form

$$\delta_{\epsilon} \chi^{ab}_{\alpha} = -7 f(\rho, x) \left(\mathcal{P}_{+}\right)^{ab}_{\alpha\beta} \left(1 + \frac{4}{3} (\pi \alpha')^2 m_1 (f(\rho, x))^2 \mathcal{P}_{-}\right)_{\beta\gamma} \epsilon_{\gamma} , \qquad (3.7)$$

that may be further reduced to^3

$$\delta_{\epsilon} \chi^{ab}_{\alpha} = -7 f(\rho, x) \left(\mathcal{P}_{+} \right)^{ab}_{\alpha\beta} \epsilon_{\beta} , \qquad (3.8)$$

by remarking that the operator $(\mathcal{P}_{-})_{\alpha\beta}$ projects on the chirality space opposite to the one used to construct the instanton

$$(\mathcal{P}_{+})^{ab}_{\alpha\beta}\mathcal{P}_{-\beta\gamma} = 0$$

Therefore the $(\alpha')^2$ contribution to the supersymmetry transformation $\delta_{\epsilon}^{(2)}\chi$ vanishes identically. Since there is no background for the fermions $\delta_{\epsilon}^{(2)}A_{\mu} = 0$ for the instanton (3.1) and the $(\alpha')^2$ piece of the lagrangian (2.1), in the SO(8) bosonic background (3.1), is invariant under the lowest-order linear supersymmetry transformations

$$\delta_{\epsilon}^{(0)} \mathcal{L}^{(2)} \Big|_{\text{for (3.1)}} = 0.$$

$$(3.9)$$

From now on we will concentrate on this part of the Yang-Mills action.

The supersymmetry transformation (3.8) is reminiscent of the non-linear supersymmetry transformations (2.6) $\delta_{\eta}\tilde{\chi} = \eta$ up to the dependence on the profile of the instanton. That the instanton (3.1) realizes an identification between the spacetime SO(8) Lorentz group and the SO(8) sub-group of the gauge group, will be central, section 4, for getting a supersymmetric sigma-model from the fluctuations around this instanton.

The effects of the instanton are all controlled by the behavior of the form factor $f(\rho, x) = \rho^2/(\rho^2 + x^2)^2$. Running a little bit ahead of the time one can already note that for $\rho \to \infty$ or for $\rho \to 0$ and not sitting on the instanton $x \neq 0$ the form factor goes to zero, and the limit corresponds to the usual open string vacuum

Open string vacuum :
$$\lim f(\rho, x) = 0$$
 for $\begin{cases} \rho \to \infty & \text{for all } x \\ \rho \to 0 & \text{and } x \neq 0 \end{cases}$. (3.10)

In this regime the linear supersymmetry (2.3) is not broken and the non-linear supersymmetry (2.6) is present. There is no relation between these two supersymmetries.

When sitting on the instanton x = 0, the zero-size limit behavior is different as the form factor blows up to infinity: this is the closed string regime

Closed string vacuum :
$$\lim_{\rho \to 0} f(\rho, 0) \sim \rho^{-2} = \infty$$
. (3.11)

In this regime the instanton dissolves and the non-linear supersymmetry get promoted to linear supersymmetry and a linear combination of the two supersymmetries (3.8) and (2.6) is expected to survive.

³ The notation $(\mathcal{P}_+)^{ab}_{\alpha\beta}$ makes explicit that this operator projects on the positive SO(8) chirality space labeled by the indices a and b.

4. The string soliton

Since for the instanton (3.1), the quadratic $\mathcal{L}^{(0)}$ and quartic $\mathcal{L}^{(2)}$ pieces of the effective action (2.1) are decoupled (see (3.9)), and we can concentrate on the quartic piece of the action showing how the sigma-model for an heterotic string comes out

$$\mathcal{S}^{(2)} = -\frac{(\pi \alpha')^2}{2^9 \pi^7 (\alpha')^3 g_s} \int d^{10} x \,\mathcal{L}^{(2)} \,. \tag{4.1}$$

We shall now move to discussion of the string world-sheet and apply the standard technique of expansion around the soliton. We take $x^m = x^m(\sigma^{\alpha})$, where σ^{α} ($\alpha = 0, 1$) are two bosonic zero modes to be identified momentarily with the world-sheet coordinates, and at lowest order in $z^m = x^m - x_0^m$,

$$\hat{F}_{\alpha m} = \partial_{\alpha} A_m = -4i\gamma_{mn} \left(\partial_{\alpha} z^n f(\rho, x_0) + x^n (\partial_{\alpha} z \cdot x) f'(\rho, x_0) + \cdots\right).$$
(4.2)

As the size of the instanton goes to zero, $\lim_{\rho\to 0} (f(\rho, x_0))^4 = \delta^{(8)}(x_0)$ and this leads to emergence of two-dimensional string world-sheet action from the corresponding Yang-Mills action. Note that while the instanton solution enjoys a full SO(9) [17], only the SO(8)subgroup is left explicit by the identification of \mathbb{R}^8 as the complement of the point $r_9 = -\rho$. Any other point would have been as good. This remaining rotational invariance reflects itself in the periodicity of the world-sheet coordinate σ_1 . The construction puts no restriction on the world-sheet coordinate σ_0 . As far as the world-sheet action is concerned, at the leading order only the first term matters and realizes the pull-back from the spacetime to the world-volume theory. Using the decomposition of the antisymmetric product of two SO(8) gamma-matrices as $(\gamma_{mn})^{ab} = \delta_{mn}^{ab} + c_{mn}^{ab}$, where c_{mnab} is the completely antisymmetric SO(8) octonionic tensor, and covariantizing the result one gets from the F^4 terms in (4.1) the expected

$$\mathcal{S}^{(2)} = \frac{1}{2\pi\alpha'} \int d^2\sigma \left(2\frac{m_1}{g_s} + \left(\frac{17}{1920}\frac{m_1}{g_s} + \frac{1}{60}\frac{m_2}{g_s}\right) \partial_\alpha z_m \partial^\alpha z^m + \cdots \right)$$
(4.3)

We would like to take this a bit further and in particular study the coupling to background gauge fields.

Before we go on we comment on the coefficients in (4.3). The first constant term is the energy of the instanton, and thanks to the identities of section A.2, it receives contributions from the single-trace term only. It is therefore proportional to m_1 . The second term from the fluctuations around the instanton in the SO(8) part of the gauge group receives contributions from both the single- and the double-trace part, and depends on m_1 and m_2 . We observe here that this leads to emergence of two different string tensions here - the single-trace part $m_1/(2\pi\alpha' g_s) \sim 1/(2\pi\alpha' g_s)$ has a tension of a D-string while the double-trace part $m_2/(2\pi\alpha' g_s) \sim 1/(2\pi\alpha')$ has a fundamental string tension.

A more interesting test comes from considering the next order perturbation on the space of connections $A_m = A_m^{(0)} + A_m^{(1)}$ $(m = 1, \dots, 8)$ around the instantonic solution (3.1) $A_m^{(0)}$. The fluctuation $A_m^{(1)} = a_m$ lives in the SO(24) part of the gauge group. Since we choose a purely bosonic instanton background, the fermions are all of the first order type $\chi = \chi^{(1)}$. Fermions, respectively in SO(8) and SO(24), are associated with the fluctuations $\hat{F}_{\alpha m}$ and $A^{(1)}$. At this point another crucial difference between the single- and double-trace parts emerges. The former does not allow any mixing of the SO(8) and SO(24) parts and an action (4.3) with half of the original supersymmetries preserved is all one gets. The double-trace term however contains the supersymmetric extension

$$\Delta \mathcal{L}^{(2)} \sim \left[4m_2 \operatorname{tr}_{\mathrm{SO}(8)}(\bar{\chi}\Gamma_{\mu}D_{\nu}\chi) \operatorname{tr}_{\mathrm{SO}(8)}(\hat{F}_{\mu\lambda}\hat{F}_{\lambda\nu}) + 2m_2 \operatorname{tr}_{\mathrm{SO}(8)}(\bar{\chi}\Gamma_{\mu\nu\rho}D_{\lambda}\chi) \operatorname{tr}_{\mathrm{SO}(8)}(\hat{F}_{\mu\nu}\hat{F}_{\rho\lambda}) \right],$$

$$(4.4)$$

whose supersymmetry variation under (3.8) is

$$\delta_{\epsilon}(\Delta \mathcal{L}^{(2)}) \sim m_2 \left(f(\rho, x) \right)^3 \times \left(\bar{\epsilon} \Gamma \mathcal{P}_+ D \chi \right).$$
(4.5)

Contrary to the action evaluated for the classical background, the action for the fluctuations is not invariant by itself under the supersymmetry generated by ϵ . We have observed already the similarity between the linear and non-linear supersymmetries, and it looks natural to try to use the latter in order to compensate for $\delta_{\epsilon}(\Delta \mathcal{L}^{(2)})$. As noticed below (2.6) the geometry of the non-linear supersymmetry transformation is controlled by the profile of the tachyon field. Moreover, beyond the group theory identifications of the gauge SO(8)and the Lorentz SO(8), a tachyon field dependence can occur in the map between the two gaugini χ and $\tilde{\chi}$. We confine this freedom to a function W(T) whose relation to the potential is yet unclear. Acting with the non-linear supersymmetry (2.6) on (4.4) we get

$$\delta_{\eta}(\Delta \mathcal{L}^{(2)}) \sim m_2 W(T) \times (\bar{\eta} \Gamma D \chi), \qquad (4.6)$$

The fluctuations around the instanton (3.1) can be made supersymmetric under a linear combination of the two supersymmetry transformations δ_{ϵ} and δ_{η} provided the following identification between the supersymmetry parameters

$$\eta = \frac{(f(\rho, x))^3}{W(T)} \mathcal{P}_+ \epsilon \,. \tag{4.7}$$

It follows that the action (4.4) is supersymmetric granted

$$\mathcal{P}_{-}\eta = 0 \tag{4.8}$$

is satisfied and the positive-chirality component of $\tilde{\chi}_+$ survives. Notice that this chirality is the one of the SO(8) representation used to construct the instanton. The remaining supersymmetry, generated by η_+ , will give rise to light-cone Green-Schwarz fermions for an heterotic string.

As we will see in the next subsection, these fermionic zero modes will give rise to both the fermions of a chiral (8,0) supersymmetric sigma-model and the gauge degrees of freedom for the word-sheet theory of an heterotic string obtained from the fluctuations around the instantons (3.1).

4.1. Gauge coupling

The coupling between $A_m^{(1)} = a_m$ to the world-sheet fermions χ , both in SO(24), is essential for understanding the nature of the string. The effective action (2.1) has only two terms with ten-dimensional fermion bilinears, of the form $\chi\Gamma_{[1,3]}D\chi F^2$. Once again, the gauge indices are contracted with the tensor M_{ABCD} (2.4) containing a single-trace and a double-trace structure. Since the single-trace part gives a supersymmetric D-string, for which the SO(24) is totally decoupled, we concentrate here on the double-trace part only.

We proceed with decomposing the fermions into a SO(8) part, ξ , and SO(1, 1) fermions

$$\chi^{A} = \begin{cases} \xi(x) \otimes S(\sigma) & \text{for } A \in \operatorname{Adj}(\operatorname{SO}(8)) \\ \xi(x) \otimes \lambda^{A}(\sigma) & \text{for } A \in \operatorname{Adj}(\operatorname{SO}(24)) \end{cases}$$
(4.9)

 S^a is a eight-dimensional Majorana-Weyl fermion defined from the SO(8) component of χ by $\chi^{ab}_{\alpha} = S_{\beta} (\mathcal{P}_{+})^{ab}_{\alpha\beta}$. Its SO(8) chirality is the same as the one used to construct the instanton (3.1). The kinetic terms for the fermions S will come from the SO(8) - SO(8) part of the action. Since the fermions λ are the fluctuations in the complement of the SO(8), used to construct the instanton, in SO(32) the kinetic and gauge couplings for the these come from the SO(8) - SO(24) cross terms.

From the fermion bilinear terms in (2.1)

$$m_2 \int d^{10}x \operatorname{tr}_{\mathrm{SO}(8)} \left(\bar{\chi} \Gamma_{\alpha} \partial_{\beta} \chi \right) \operatorname{tr}_{\mathrm{SO}(8)} (F_{\alpha n} F_{n\beta})$$

we readily derive the kinetic terms for the light-cone fermions

$$m_2 \int d^2 \sigma \, \bar{S} \gamma_\alpha \partial_\beta S \, h_{\alpha\beta} \,, \tag{4.10}$$

with induced two-dimensional metric

$$h_{\alpha\beta} := \partial_{\alpha} z^n \partial_{\beta} z^n \times \lim_{\rho \to 0} \int d^8 x \ (f(\rho, x))^2 \ \bar{\xi} \xi(x) \,.$$

Notice that (3.8) implies that $\xi(x)$ scales as $f(\rho, x)$ and the previous integral has a finite non-zero limit when ρ goes to zero. The other fermion bilinear in (2.1) involving $\Gamma_{\mu\nu\rho}$ does not contribute to the kinetic term for the light-cone fermions S.

From the interactions between the SO(8) deformation (4.2) and the SO(24) fermions

$$m_2 \int d^{10} x \operatorname{tr}_{\mathrm{SO}(24)} \left(\bar{\chi} \Gamma_{\alpha} \partial_{\beta} \chi \right) \operatorname{tr}_{\mathrm{SO}(8)} (F_{\alpha n} F_{n\beta})$$

we derive the kinetic term for the twenty-four fermions λ

$$m_2 \int d^2 \sigma \operatorname{tr}_{\mathrm{SO}(24)}(\bar{\lambda}\gamma_\alpha \partial_\beta \lambda) h_{\alpha\beta} , \qquad (4.11)$$

From the coupling

$$m_2 \int d^{10}x \operatorname{tr}_{\mathrm{SO}(24)} \left(\bar{\chi} \Gamma_{\alpha}[a_m, \chi] \right) \operatorname{tr}_{\mathrm{SO}(8)}(F_{\alpha n} F_{nm})$$

we derive the coupling between the fermions λ and the background SO(24) gauge field

$$m_2 \int d^2 \sigma \operatorname{tr}_{\mathrm{SO}(24)}(\bar{\lambda}\gamma_{\alpha}[a_m,\lambda]) e_{\alpha}{}^m, \qquad (4.12)$$

where the contractions of the indices are performed with the vielbein

$$e_{\alpha}{}^m := \partial_{\alpha} z^m \times \lim_{\rho \to 0} \int d^8 x \ (f(\rho, x))^2 \ \bar{\xi} \xi(x) \, .$$

Finally the $\Gamma_{\mu\nu\rho}$ term in (2.1) does not contribute.

Putting everything together, we arrive at the world-sheet theory for a heterotic string in the light-cone gauge with the particular embedding of the spin connection into the SO(8) part of the SO(32) gauge group.⁴ In particular, the kinetic term proportional to

⁴ Curiously enough, one of very few features of ordinary four-dimensional instantons that is replicated by the SO(8) instanton is the possibility of extending the solution to a curved space by setting the gauge connection equal to spin connection [19].

 m_2 in (4.3) and (4.10) gives the (8,0) supersymmetric sigma model invariant under the supersymmetries generated by η . The kinetic term for the SO(24) fermions λ is obtained in (4.11) which together with the coupling (4.12) to the external field, gives a covariant derivative acting on the fermions.

It is well-known that the heterotic string can appear as a soliton in type I theory, and it is natural to inquire about the difference of our mechanism from the S-duality correspondance between type I and heterotic string of [20]. The latter requires a background metric (with the Lorentz invariance broken to $SO(1, 1) \times SO(8)$) and a non vanishing vev for the dilaton, while we only have a Yang-Mills background and a flat metric.

5. Discussion

We would like to underline some of the important points and list some open questions. Starting from pure ten-dimensional Yang-Mills action we are able to produce a D-string and more surprisingly/controversially a heterotic string in the background of eight-dimensional SO(8) instanton. The former has a non-vanishing vacuum energy and supersymmetric (8,0) world-sheet. Its very plausible connections to non-commutative solitons and to D1-D9 (D0-D8) systems are not very clear to us at the moment and are beyond the scope of this paper. There may be some interesting questions to be asked in this context. As for the heterotic string to which we will turn now, the SO(9) symmetry of the instanton solution gets reflected in the periodicty of one of the string world-sheet coordinates.

Our main interest is in the appearance of the fundamental heterotic string. Technically speaking, its existence is due to the presence of the double-trace structures in the Yang-Mills F^4 terms responsible for mixing of SO(8) and SO(24). These terms were first seen from the supersymmetry analysis on the ten-dimensional Yang-Mills action, and as we tried to argue here may also be related to the field redefinition freedom (fixed by the requirement that the equations of motion are proportional to the world-sheet β -function and by supersymmetry) in the classical open string field theory lagrangian. As a result by expanding around the soliton we obtain a string action in the background where the SO(8) gauge connection is set to the spin connection. It might be interesting to notice that this double-trace structure also allows to generate kinetic terms for Yang-Mills fields in the bulk by giving classical values to tr F^2 . Indeed, in the SO(8) instanton background, $t_{(8)} (\operatorname{tr} F^2)^2 \to \operatorname{tr} F_{\mu\nu} F^{\mu\nu}$, and in a way the SO(8) part is used as the "center" of the gauge group.

Finally, a crucial point in the analysis was the mutual cancellation of linear and nonlinear supersymmetry. The latter is known for the abelian case but applicable to the SO(8) group by the magic of the SO(8) instantons, which realizes an identification of the Lorentz and gauge SO(8) groups. The resulting connection between linear and nonlinear supersymmetry parameters contains a dependence of the tachyon and the size of the instanton via $(f(\rho, x))^3/W(T)$. Of course it is natural to expect that this ratio is a constant and thus one gets a correlation between the tachyon and the size of the instanton (e.g. since $f(\rho, x)$ is singular when the instanton shrinks to the zero size, W(T) should behave accordingly). This correlation is not very explicit though, since it is not yet completely clear how W(T) is related to the tachyon potential. One could hope to understand better what are the respective regimes where the D-string and the heterotic string discussed above dominate. Such a possibility of at least partially recovering the tachyon potential from the low-energy analysis in our opinion deserves further study.

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Appendix A. Instantons in eight-dimensions

In this appendix we review some facts about SO(8) instantons, and introduce the notations used in the main text. We also briefly discuss different variants of the self-duality equations and their solutions.

A.1. Definitions

▷ The SO(8) instantons satisfying the self-duality condition in euclidean flat eightdimensional \mathbb{R}^8

$$F_{\pm} \wedge F_{\pm} = \pm \star_8 (F_{\pm} \wedge F_{\pm}) = \pm \frac{1}{4!} \varepsilon_{(8)} (F_{\pm} \wedge F_{\pm})$$
(A.1)

have been constructed in [16,17]. This solution is defined on the eight-sphere of radius ρ , $r_m r^m + r_9 r^9 = \rho^2$ in terms of projective coordinates (x_m) on \mathbb{R}^8 , identified with the complement of the point $r_9 = -\rho$

$$r_m = \rho^2 \frac{2x_m}{\rho^2 + x^2} \ (m = 1, \dots, 8), \qquad r_9 = \rho \frac{\rho^2 - x^2}{\rho^2 + x^2}.$$

Thanks to the SO(9) rotational invariance of the solution, we could have considered the complement of $r_9 = +\rho$, without any change in the discussion of the main text. The solution with no singularity at the origin is given by

$$(A_{\pm})_m = -4i\Gamma_{mn}\mathcal{P}_{\pm} \frac{x^n \rho^2}{(\rho^2 + x^2)^2}$$
(A.2)

and its curvature is

$$(F_{\pm})_{mn} = 4\Gamma_{mn}\mathcal{P}_{\pm} \frac{\rho^2}{(\rho^2 + x^2)^2}$$
 (A.3)

where $\Gamma_{mn}\mathcal{P}_{+} = \gamma_{mn}$ belong to the $\mathbf{8}_{s}$ representation of SO(8) and correspond to the + sign in (A.1). The other representation of opposite SO(8) chirality $\Gamma_{mn}\mathcal{P}_{-}$ will be abbreviated as $\tilde{\gamma}_{mn}$. The two representations correspond to the choice of sign in (A.1). We refer to appendix B for details on γ -matrix algebra. These γ_{mn} matrices are the SO(8) equivalent of 't Hooft eta-symbols and this solution generalizes the Jackiw-Rebbi instanton.

In the main text we make the choice of the positive SO(8) chirality solution F_+ , and drop the + subscript.

▷ There is another solution [17] related to the one above by a conformal transformation, with a singular behavior at the origin

$$(A_{\pm})_m = -4i\Gamma_{mn}\mathcal{P}_{\pm} \frac{x^n \rho^2}{x^2(\rho^2 + x^2)^2}$$
(A.4)

and its associated curvature is

$$(F_{\pm})_{mn} = 4\Gamma_{mn}\mathcal{P}_{\pm} \frac{\rho^6}{x^4(\rho^2 + x^2)^2}.$$
 (A.5)

 \triangleright The solution (A.3) solves

$$\star_8 F_{\pm} = \mp F_{\pm} \wedge F_{\pm} \wedge F_{\pm} , \qquad (A.6)$$

with a conformal metric

$$g_{mn} = 4f(\rho, x)\eta_{mn} \,. \tag{A.7}$$

(A.5) has a form factor singular at the origin and therefore does not qualify for such non-conformal generalisation.

 \triangleright The solution (A.3) satisfies a relation

$$8 \times (f(\rho, x))^2 \star_8 F_{\pm} = \mp F_{\pm} \wedge F_{\pm} \wedge F_{\pm} , \qquad (A.8)$$

with the *flat* metric $g_{mn} = \eta_{mn}$. Some of the discussion in the main text relies heavily on this relation.

▷ One can show that in 4d dimensions the following double-duality relation holds for any Einstein metric

$$(2d)!^{2}(\wedge^{d}R)^{m_{1}\cdots m_{2d}}{}_{\mu_{1}\cdots \mu_{2d}} = \epsilon^{m_{1}\cdots m_{2d}}{}_{n_{1}\cdots n_{2d}}(\wedge^{d}R)^{n_{1}\cdots n_{2d}}{}_{\nu_{1}\cdots \nu_{2d}}\epsilon^{\nu_{1}\cdots \nu_{2d}}{}_{\mu_{1}\cdots \mu_{2d}}$$

where $\wedge^d R$ is the completely antisymmetrized product of 4*d*-dimensional Riemann curvatures. Similarly to four-dimensional case [21], for d = 2 this double-duality implies a single duality relation (A.1) for a gauge field A [19], provided

$$A^{\pm}_{\mu} = -\frac{1}{2}\omega^{mn}_{\mu}\mathcal{P}_{\pm}\gamma_{mn} \; ; \quad F^{\pm}_{\mu\nu} = -\frac{1}{2}R^{mn}_{\mu\nu}\mathcal{P}_{\pm}\gamma_{mn} \; . \tag{A.9}$$

Thus the procedure of setting the gauge connection to spin connection enables one to extend the construction of the instanton to curved backgrounds.

 \triangleright This setup is generalized to multiple instanton solutions classified by maps of the eight-sphere S^8 onto itself [22]:

$$(F_{\pm})_{mn} = 2 \frac{\partial_n w_p \partial_m w_q}{(1+w_l w_l)^2} \Gamma_{pq} \mathcal{P}_{\pm}$$

$$w_l \Gamma_l = \left(x_8 \mathbb{I} + \sum_{m=1}^7 x_m \Gamma_m \right)^N .$$
(A.10)

Such instanton carries a topological charge classified by $\Pi_7(SO(8)) = \mathbb{Z} \oplus \mathbb{Z}$.

- ▷ Finally we remark that the SO(8) instantons can be embedded in any SO(n) group with $n \ge 8$ ($\Pi_7(SO(n)) = \mathbb{Z}$ for n > 8) but not in $E_8 \times E_8$ group, since there is no non-trivial embedding of SO(8) in E_8 . This fact is easily understood by noticing that E_8 has no independent quartic invariants $\operatorname{tr}(F \land F \land F \land F) \propto \operatorname{tr}(F \land F) \land \operatorname{tr}(F \land F)$, and that $\operatorname{tr}(F \land F) = 0$ for the both the solutions (A.3) and (A.5).
- ▷ Since all the F^4 action is expressed in term of the completely symmetric tensor [10,12] we define the topological charge carried by the instanton as

$$q_{\pm} := \frac{1}{4!} \int d^8 x M_{ABCD} F^A_{\pm} \wedge F^B_{\pm} \wedge F^C_{\pm} \wedge F^D_{\pm} ,$$

which can be rewritten using first and second Pontrjagin classes

$$q_{\pm} = \frac{1}{4!} \int d^8 x \left(-4m_1 \, p_2(F) + 2(m_1 + m_2) \, p_1(F) \wedge p_1(F) \right)$$

Since for the instantons (A.3), (A.5) and (A.10) $p_1(F) = 0$, the charge carried by the instanton is

$$q_{\pm} = \frac{m_1}{4!} \int d^8 x \, p_2(F) \,. \tag{A.11}$$

(A.3) and (A.5) have charge $q_{\pm} = \pm 2^8 \pi^4$, and (A.10) has charge $q_{\pm} = \pm 2^8 \pi^4 \times N$.

A.2. Identities

We show that the SO(8) instanton (A.3) satisfies the identity

$$M_{ABCD}\left(t_{(8)}^{r_1\cdots r_8} \pm \frac{1}{2}\varepsilon_{(8)}^{r_1\cdots r_8}\right)(F_{\pm})^A_{r_1r_2}(F_{\pm})^B_{r_3r_4}(F_{\pm})^C_{r_5r_6}N^D_{r_7r_8} = 0$$
(A.12)

for any matrix N in the adjoint of the group, generalising the identity used in [6]. The completely symmetric tensor M_{ABCD} of equation (2.4) (defined in [10]) has two parts—the symmetrized single-trace and the double-trace. It is therefore sufficient to check that

$$\operatorname{tr}\left[t_{(8)}^{r_1\cdots r_8}(F_{\pm})_{r_1r_2}(F_{\pm})_{r_3r_4}(F_{\pm})_{r_5r_6}N_{r_7r_8}\right] = 2^5(7d-11)(d-4)(f(\rho,x))^2\operatorname{tr}((F_{\pm})_{r_7r_8}N_{r_7r_8})$$

$$\operatorname{tr}\left[\varepsilon_{(8)}^{r_1\cdots r_8}(F_{\pm})_{r_1r_2}(F_{\pm})_{r_3r_4}(F_{\pm})_{r_5r_6}N_{r_7r_8}\right] = \mp 2^4 \cdot 6! (f(\rho,x))^2\operatorname{tr}((F_{\pm})_{r_7r_8}M_{r_7r_8})$$

therefore for d = 8 (A.12) is satisfied.

$$t_{(8)}^{r_1\cdots r_8} \operatorname{tr}\left[(F_{\pm})_{r_1r_2}(F_{\pm})_{r_3r_4}\right] \operatorname{tr}\left[(F_{\pm})_{r_5r_6}N_{r_7r_8}\right] = -2^8(d-1)(d-8)(f(\rho,x))^2 \operatorname{tr}((F_{\pm})_{r_7r_8}M_{r_7r_8})$$

$$\operatorname{tr}\left[\varepsilon_{(8)}^{r_1\cdots r_8}(F_{\pm})_{r_1r_2}^A(F_{\pm})_{r_3r_4}^B(F_{\pm})_{r_5r_6}N_{r_7r_8}\right] = 0$$

therefore for d = 8 (A.12) is satisfied.

A.3. An action for the Instanton

This solution has been shown [6,18] to be of importance for constructing the string as a soliton for the five-brane in ten-dimension. It was there shown that the solution satisfies the equation-of-motion [23]

$$\mathcal{E}^{(0)}(A)^{\mu} = \frac{1}{f(\rho, x)} D_{\nu} \left(f(\rho, x) F^{\mu\nu} \right) = 0, \qquad (A.13)$$

and

$$\mathcal{E}^{(2)}(A)^{\mu} = -\frac{(\pi\alpha')^2}{g_o^2} \frac{e^{-\varphi}}{3! \cdot 8g_s} M^A{}_{BCD} \left(D_{\nu}(F^B)^{\mu}{}_{\nu}F^C_{\rho\sigma}F^D_{\sigma\rho} - \frac{1}{4}D_{\nu}(F^BF^CF^D)^{\mu}{}_{\nu} \right)$$
(A.14)

derived from the effective action (2.1).

Thanks to the equality (A.12), $\mathcal{E}^{(2)}(A)^{\mu} = 0$ and the Bianchi identity, the instanton (A.1) is an extremum of the $(\alpha')^2$ part of the effective action, $\mathcal{L}^{(2)}$, but not a global extremum.

Appendix B. Spinology, t_8 -ology and Γ -gymnastic

 \triangleright For a non-abelian gauge field

$$t_{(8)} \operatorname{tr} F^{4} = 8 \operatorname{tr} (F_{\mu\nu} F_{\nu\rho} F_{\rho\lambda} F_{\lambda\mu}) + 16 \operatorname{tr} (F_{\mu\nu} F_{\rho\lambda} F_{\nu\rho} F_{\lambda\mu}) - 4 \operatorname{tr} (F_{\mu\nu} F_{\mu\nu} F_{\rho\sigma} F_{\rho\sigma}) - 2 \operatorname{tr} (F_{\mu\nu} F_{\rho\sigma} F_{\mu\nu} F_{\rho\sigma}) t_{(8)} \operatorname{tr} F^{2} \operatorname{tr} F^{2} = -2 \operatorname{tr} (F_{\mu\nu} F_{\nu\mu}) \operatorname{tr} (F_{\rho\sigma} F_{\sigma\rho}) + 16 \operatorname{tr} (F_{\mu\nu} F_{\nu\rho}) \operatorname{tr} (F_{\rho\sigma} F_{\sigma\mu}) - 4 \operatorname{tr} (F_{\mu\nu} F_{\rho\sigma}) \operatorname{tr} (F_{\nu\mu} F_{\sigma\rho}) + 8 \operatorname{tr} (F_{\mu\nu} F_{\rho\sigma}) \operatorname{tr} (F_{\nu\rho} F_{\sigma\mu})$$
(B.1)

which is easily seen to be given by the following symmetrized trace formula

$$trt_{(8)}F^{4} = 4!Str\left(F_{\mu\nu}F_{\nu\rho}F_{\rho\sigma}F_{\sigma\mu} - \frac{1}{4}(F_{\mu\nu}F_{\mu\nu})^{2}\right)$$
(B.2)

 \triangleright The SO(1,9) Gamma matrices Γ_{μ} are decomposed into SO(8) gamma matrices as

$$\Gamma_{\mu} = \begin{pmatrix} 0 & \gamma_{\mu} \\ \tilde{\gamma}_{\mu} & 0 \end{pmatrix} . \tag{B.3}$$

 γ_{μ} acts from the space of spinor of positive SO(8) chirality into the space of negative SO(8) chirality: $\gamma_{\mu} : S_{+} \to S_{-}$. $\tilde{\gamma}_{\mu}$ does the opposite. Antisymmetric products of even numbers of Gamma matrices respects the direct sum $S_{+} \oplus S_{-}$. In particular

$$\Gamma_{\mu\nu} = \begin{pmatrix} \gamma_{\mu\nu} & 0\\ 0 & \tilde{\gamma}_{\mu\nu} \end{pmatrix} = \gamma_{\mu\nu}\mathcal{P}_{+} + \tilde{\gamma}_{\mu\nu}\mathcal{P}_{-}$$
(B.4)

 \triangleright The antisymmetric product of two SO(8) matrices can be expressed in terms of the completely antisymmetric octonionic structure constants

$$(\gamma_{mn})^{ab} = \delta^{ab}_{mn} + c_{mn}{}^{ab} \,. \tag{B.5}$$

 \triangleright We need the following Γ matrices identities (all antisymmetrizations are with unit weight)

$$\{\Gamma_{\mu}, \Gamma_{\nu}\} = 2\eta_{\mu\nu} \tag{B.6}$$

$$\Gamma_{pq}\Gamma^{rs} = \Gamma_{pq}{}^{rs} + 4\Gamma_{[p}{}^{[s}\eta_{q]}{}^{r]} + 2\eta_{pq}^{sr}$$
(B.7)

$$\Gamma_{\mu\lambda}\Gamma_{\lambda\nu} = (d-2)\Gamma_{\mu\nu} + (d-1)\eta_{\mu\nu} \tag{B.8}$$

$$\operatorname{tr}\left(\Gamma_{r_1\cdots r_n}\Gamma^{s_m\cdots s_1}\right) = 2^4 \times n! \times \eta^{s_1\cdots s_n}_{r_1\cdots r_n} \tag{B.9}$$

$$\varepsilon_{(8)r_1\cdots r_8} = \gamma_{r_1\cdots r_8}\gamma_{(8)} \tag{B.10}$$

$$\varepsilon_{(8)}^{r_1 \cdots r_n} {}_{s_1 \cdots s_{8-n}} \gamma^{r_n \cdots r_1} = n! \times \gamma_{s_1 \cdots s_{8-n}} \gamma_{(8)}$$
(B.11)

References

- [1] E. Witten, Interacting Field Theory of Open Superstrings, Nucl. Phys. B276 (86) 291.
- [2] E. Witten, On background independent open string field theory, Phys. Rev. D 46, 5467 (1992), hep-th/9208027;
 E. Witten, Some computations in background independent off-shell string theory, Phys. Rev. D 47, 3405 (1993), hep-th/9210065;
 S. L. Shatashvili, Comment on the background independent open string theory, Phys. Lett. B 311, 83 (1993), hep-th/9303143;
 S. L. Shatashvili, On the problems with background independence in string theory, Algebra and Anal., 6 (1994) 215, hep-th/9311177.
- [3] N. Berkovits, A new approach to superstring field theory, Fortsch. Phys. 48 (2000) 31, hep-th/9912121.
- S. L. Shatashvili, On field theory of open strings, tachyon condensation and closed strings, Talk at Strings 2001, Mumbai, India, hep-th/0105076.
- [5] D. J. Gross and W. Taylor, Split string field theory II, hep-th/0106036.
- [6] M.J. Duff and J.X. Lu, Strings from five-branes, Phys. Rev. Lett. 66 (1991) 1402.
- [7] E. Witten, *D*-branes and *K*-theory, JHEP, **12** (1998) 019, hep-th/9810188.
- [8] A. Sen, Descent relations among bosonic D-branes, Int. J. Mod. Phys. A 14 (1999) 4061, hep-th/9902105;

A. Sen, Non-BPS states and branes in string theory, hep-th/9904207;

A. Sen, Universality of the tachyon potential, JHEP **9912** (1999) 027, hep-th/9911116.

 [9] A. A. Gerasimov and S. L. Shatashvili, On exact tachyon potential in open string field theory, JHEP 0010 (2000) 034, hep-th/0009103;

D. Kutasov, M. Mariño and G. Moore, Some exact results on tachyon condensation in string field theory, JHEP **0010** (2000) 045, hep-th/0009148;

D. Kutasov, M. Mariño and G. Moore, Remarks on tachyon condensation in superstring field theory, hep-th/0010108;

A. A. Gerasimov and S. L. Shatashvili, *Stringy Higgs mechanism and the fate of open strings*, JHEP **0101** (2001) 019, hep-th/0011009.

- [10] M. Cederwall, B. E. Nilsson and D. Tsimpis, D = 10 super Yang-Mills at $O(\alpha^2)$, hepth/0104236.
- [11] K. Peeters, P. Vanhove and A. Westerberg, Supersymmetric higher-derivative actions in ten and eleven dimensions, the associated superalgebras and their formulation in superspace, Class. Quant. Grav. 18 (2001) 843, hep-th/0010167.
- [12] M. Cederwall, B. E. Nilsson and D. Tsimpis, The structure of maximally supersymmetric Yang-Mills theory: Constraining higher-order corrections, hep-th/0102009.
- [13] N. Sakai and M. Abe, Coupling Constant Relations And Effective Lagrangian In The Type I Superstring, Prog. Theor. Phys. 80 (1988) 162.

- [14] C. Bachas, C. Fabre, E. Kiritsis, N. A. Obers and P. Vanhove, *Heterotic/type-I duality* and D-brane instantons, Nucl. Phys. B 509 (1998) 33, hep-th/9707126.
- [15] J. Bagger and A. Galperin, A new Goldstone multiplet for partially broken supersymmetry, Phys. Rev. D 55 (1997) 1091, hep-th/9608177;
 T. Hara and T. Yoneya, Nonlinear supersymmetry without the GSO projection and unstable D9-brane, Nucl. Phys. B 602, (2001) 499, hep-th/0010173;
 E. Dudas and J. Mourad, Consistent gravitino couplings in non-supersymmetric strings, hep-th/0012071;
 S. Terashima and T. Uesugi, On the supersymmetry of non-BPS D-brane, JHEP 0105 (2001) 054, hep-th/0104176.
- [16] D. Tchrakian, Spherically Symmetric Gauge Field Configurations With Finite Action In 4 P-Dimensions (P = Integer), Phys. Lett. B 150 (1985) 360.
- B. Grossman, T. Kephart and J. Stasheff, Solutions To Yang-Mills Field Equations In Eight-Dimensions And The Last Hopf Map, Commun. Math. Phys. 96 (1984) 431.
- [18] M.J. Duff and J.X. Lu, A Duality between strings and five-branes, Class. Quant. Grav. 9 (1992) 1.
- [19] G. O'Brien and D. Tchrakian, Spin Connection Generalized Yang-Mills Fields On Double Dual Generalized Einstein-Cartan Backgrounds, J.Math.Phys. 29 (1988) 1212.
- [20] J. Polchinski and E. Witten, Evidence for Heterotic Type I String Duality, Nucl. Phys. B 460, 525 (1996), hep-th/9510169.
- [21] J. M. Charap and M. J. Duff, Gravitational effects on Yang-Mills topology, Phys. Lett. B69 (1977) 445.
- [22] A. Dundarer, Multi-Instantons Solutions in Eight-Dimensional Curved Space, Mod. Phys. Lett. A6 (1991) 409.
- [23] B. Grossman, T. Kephart and J. Stasheff, Solutions To Gauge Field Equations In Eight-Dimensions: Conformal Invariance And The Last Hopf Map, Phys. Lett. B220 (1989) 431.