

# COLLECTIVE FLOW AND MULTIPARTICLE AZIMUTHAL CORRELATIONS

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The measurement of azimuthal distributions in nucleus-nucleus collisions relies upon the assumption that azimuthal correlations between particles result solely from their correlation with the reaction plane (i.e. flow). We show that at SPS energies, the ansatz is no longer valid, and two-particle correlations due to momentum conservation, final state interactions or resonance decays become of the same order as those arising from flow. This leads us to introduce new methods to analyse collective flow, based on a cumulant expansion, which enable to extract smaller values than those accessible to the standard analysis.

In a collision between two heavy ions, the azimuthal distribution of outgoing particles with respect to the reaction plane, i.e. collective flow, is expected to reveal new insights on the central region of the collision: thermal equilibrium, equation of state, time evolution.<sup>1</sup> It is therefore important to have reliable flow values, and to be able to measure small values.

We first discuss in Sec. 1 the method usually used to analyze flow data, and we point out that it is based on an assumption which is not valid at ultrarelativistic energies. In the next two sections, we introduce a new method which allows flow measurements at such energies: the main ideas are presented in Sec. 2, while the practical implementation in terms of the event flow vector, including acceptance corrections, is given in Sec. 3. Finally, we briefly summarize our results in Sec. 4.

## 1 Flow and two-particle correlations

The standard method for analyzing the experimental data relies on a study of two-particle azimuthal correlations.<sup>2</sup> The one-particle azimuthal distribution is first expanded into a Fourier series, whose coefficients  $v_n$  characterize the flow.<sup>3</sup>

$$v_n = \langle e^{in\phi} \rangle, \quad (1)$$

where  $\phi$  denotes the azimuth with respect to the reaction plane, while the average is performed over a large number of events.

Since the actual orientation of the reaction plane is not known, only differences between particle azimuths can be measured. Thus, flow is extracted from the measured two-particle correlations  $\langle e^{in(\phi_1-\phi_2)} \rangle$ , under the assumption that they are only due to flow, or at least that the contribution  $\langle e^{in(\phi_1-\phi_2)} \rangle_c$  of direct, nonflow correlations is negligible. In other words, it is assumed that in the decomposition

$$\langle e^{in(\phi_1-\phi_2)} \rangle = v_n^2 + \langle e^{in(\phi_1-\phi_2)} \rangle_c \quad (2)$$

the second term in the right-hand side is much smaller than the first.

However, we have shown that this ansatz is not valid at SPS energies:<sup>4</sup> nonflow two-particle correlations, which can stem from various sources, are of the same magnitude as two-particle correlations due to flow. Thus, the NA49 values for pion and proton flow are significantly modified when two-particle azimuthal correlations arising from momentum conservation, HBT quantum effects, and resonance decays are subtracted from the measured correlations.

Two-particle correlations may not be easy to take into account in the standard analysis, even if the sources are well identified (as for example in the case of resonance decays). Moreover, the possibility that there are still unknown sources cannot be discarded. Thus, the safest approach consists in using a method which does not require a precise knowledge of these correlations.

## 2 Measuring flow using multiparticle azimuthal correlations

We recently proposed a new method for the analysis of flow, which does not depend on two-particle correlations since these are eliminated by means of a cumulant expansion of multiparticle azimuthal correlations.<sup>5</sup> In this section, we show how this method allows measurements of integrated flow (i.e. averaged over a phase space region, see 2.1) as well as differential flow for given transverse momentum and rapidity (2.2).

### 2.1 Integrated flow

The main idea underlying our method is the following. In a collision where  $N$  particles are emitted, *direct*  $k$ -particle correlations are typically of order  $1/N^{k-1}$ , so that they become smaller when  $k$  increases. In particular, their magnitude decreases faster than the flow contribution to the *measured*  $k$ -particle correlations: one can expect that the comparison between both terms allows measurements of smaller and smaller flow values.

Let us consider for instance the measured four-particle azimuthal correlations. These correlations can be expanded as the sum of products of direct one-, two- and three-particle correlations, where “one-particle correlation” means correlation with the reaction plane, that is flow. The order of magnitude of each term in the expansion can easily be estimated, keeping in mind that a flow term is of order  $v_n$  while direct  $k$ -particle correlations are  $O(1/N^{k-1})$ .<sup>5</sup> After calculation, the dominant terms in the expansion are:

$$\begin{aligned} \left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle &\simeq v_n^4 + \left\langle e^{in(\phi_1-\phi_3)} \right\rangle_c \left\langle e^{in(\phi_2-\phi_4)} \right\rangle_c \\ &+ \left\langle e^{in(\phi_1-\phi_4)} \right\rangle_c \left\langle e^{in(\phi_2-\phi_3)} \right\rangle_c + \left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle_c. \end{aligned} \quad (3)$$

[For simplicity, we have assumed that  $v_n$  is not much smaller than  $v_{2n}$ , and neglected a term  $O(v_{2n}^2/N^2)$ .] The first and the fourth terms in r.-h. s. are of magnitude  $v_n^4$  and  $1/N^3$  respectively: a direct comparison between them would allow flow measurements down to values  $v_n \gg 1/N^{3/4}$ , smaller than the  $v_n \gg 1/N^{1/2}$  limit of the standard analysis which can be deduced from Eq. (2). In order to compare these two terms, we have to get rid of the second and third terms, which are in fact equal, and correspond to two-particle correlations.

That can be done, taking the cumulant of the four-particle azimuthal correlation. The purpose of this cumulant is precisely to remove lower order (i.e. involving  $k'$  particles with  $k' < k = 4$ ) correlations. It is defined as the difference between measured correlations  $\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \rangle - 2\langle e^{in(\phi_1-\phi_3)} \rangle^2$ . Using Eqs. (2) and (3), one finds that the cumulant is:

$$\left\langle\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle\right\rangle = -v_n^4 + \left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle_c = -v_n^4 + O\left(\frac{1}{N^3}\right), \quad (4)$$

so that it gives the flow, provided this latter is larger than  $1/N^{3/4}$ . Measuring this cumulant is indeed sensitive to smaller flow values than the standard method. For instance, at SPS energies where about  $N = 2500$  particles are emitted, using the cumulant (4) allows measurements of  $v_n \gg 0.3\%$ , while the standard method is limited to  $v_n \gg 2\%$ ; these values are to be compared with the published values  $v_1 \simeq v_2 \simeq 3\%$  for pions.<sup>6</sup>

## 2.2 Differential flow

Let us assume that the average value  $v_n = \langle e^{in\phi} \rangle$  of flow is known for a given particle species, which we shall call “pions” to fix ideas. For any type of particles, let us say “protons”, although it can be anything—even “pions”—, it is possible to perform detailed measurements of the flow  $v'_n(p_T, y) = \langle e^{im\psi} \rangle$  using multiparticle correlations in order to go beyond the limitations of the

standard two-particle method.<sup>5</sup> To simplify the expressions, we shall assume  $n = m$ ; however, the ideas remain valid if  $m$  is a multiple of  $n$ .

Let us consider for example the four-particle azimuthal correlation between a proton and three pions  $\langle e^{in(\psi+\phi_1-\phi_2-\phi_3)} \rangle$ . As above, this measured correlation can be expanded in terms of lower order direct correlations, yielding a term  $v'_n v_n^3$ , a term of order  $1/N^3$  corresponding to direct four-particle correlations, and other terms. These latter, and in particular two-particle correlations, can be removed taking the cumulant [by analogy with Eq. (4)]

$$\begin{aligned} \langle\langle e^{in(\psi+\phi_1-\phi_2-\phi_3)} \rangle\rangle &\equiv \langle e^{in(\psi+\phi_1-\phi_2-\phi_3)} \rangle - 2 \langle e^{in(\psi-\phi_2)} \rangle \langle e^{in(\phi_1-\phi_3)} \rangle \\ &\simeq -v'_n v_n^3 + \langle e^{in(\psi+\phi_1-\phi_2-\phi_3)} \rangle_c = -v'_n v_n^3 + O\left(\frac{1}{N^3}\right). \end{aligned} \quad (5)$$

Inspection of this equation reveals that the measurement of the cumulant in the left-hand side gives access to  $v'_n$  as soon as it is larger than  $1/(Nv_n)^3$ , which is also the accuracy on  $v'_n$ .

### 3 Practical implementation using the event flow vector

#### 3.1 Integrated flow

An easy way to implement the method outlined in section 2 consists in using the event flow vector, which is defined as

$$Q_n = \frac{1}{\sqrt{M}} \sum_{j=1}^M e^{in\phi_j}, \quad (6)$$

where the sum runs over particles from the same event, and  $M$  should be chosen as large as possible. Averaging Eq. (6) over many events, one finds  $\langle Q_n \rangle = \sqrt{M} v_n$ : a nonvanishing  $\langle Q_n \rangle$  signals collective flow.

From definition (6), it is obvious that the powers of  $|Q_n|^2$  involve multiparticle azimuthal correlations, and only differences between angles (which require no knowledge of the reaction plane orientation). That explains why the method can naturally be expressed in terms of the flow vector: the average value  $\langle Q_n \rangle$  corresponds to  $v_n$ , i.e. what we want to extract from the data, while the measured quantity  $\langle |Q_n|^k \rangle$  corresponds to measured  $k$ -particle azimuthal correlations.

Let us turn once more to four-particle correlations. By analogy with what was found in section 2, the expansion of the moment  $\langle |Q_n|^4 \rangle$  involves various terms:  $\langle Q_n \rangle^4$ , a term  $\langle |Q_n|^4 \rangle_c$  corresponding to direct four-particle correlations, and other terms which we wish to get rid of. These latter can

indeed be removed, using the cumulant of the  $Q_n$  distribution, which at order  $k = 4$  is defined as

$$\langle\langle |Q_n|^4 \rangle\rangle \equiv \langle |Q_n|^4 \rangle - 2\langle |Q_n|^2 \rangle^2 = -\langle Q_n \rangle^4 + O\left(\frac{1}{M}\right). \quad (7)$$

Since  $\langle Q_n \rangle = \sqrt{M}v_n$ , this equation is strictly equivalent to Eq. (4), provided  $M$  and  $N$  are of the same magnitude. Therefore, the measurement of the cumulant (7) provides a way to extract smaller  $v_n$  values than what can be obtained within the standard two-particle analysis.

### 3.2 Acceptance corrections

An important feature of the method is the easy implementation of acceptance corrections.<sup>5</sup> The only modification regards the definition of the cumulant: in Eq. (7), the cumulant involves only two moments of the  $Q_n$  distribution because the other terms vanish for a perfectly isotropic detector. If the detector is not isotropic, these terms should be reintroduced in the definition, and the flow vector  $\bar{Q}_n$  measured in the laboratory frame is not equivalent to the flow vector  $Q_n$  with respect to the reaction plane. Taking into account these changes, the cumulant is (dropping the  $n$  for brevity):

$$\begin{aligned} \langle\langle |\bar{Q}|^4 \rangle\rangle &\equiv \langle |\bar{Q}|^4 \rangle - 2\langle \bar{Q} \rangle \langle \bar{Q} \bar{Q}^{*2} \rangle - 2\langle \bar{Q}^* \rangle \langle \bar{Q}^* \bar{Q}^2 \rangle - 2\langle |\bar{Q}|^2 \rangle^2 - \langle \bar{Q}^2 \rangle \langle \bar{Q}^{*2} \rangle \\ &\quad + 8\langle \bar{Q} \rangle \langle \bar{Q}^* \rangle \langle |\bar{Q}|^2 \rangle + 4\langle \bar{Q} \rangle^2 \langle \bar{Q}^{*2} \rangle + 4\langle \bar{Q}^* \rangle^2 \langle \bar{Q}^2 \rangle - 6\langle \bar{Q} \rangle^2 \langle \bar{Q}^* \rangle^2 \\ &= -\langle Q \rangle^4 + O(1/M). \end{aligned} \quad (8)$$

This expression generalizes Eq. (7) to the case of real, nonperfect detectors, and should be used to extract flow from experimental data.

### 3.3 Differential flow

Let us readopt the same notations as in Sec. 2.1. Representing the pions' azimuths by the flow vector  $Q_n$  [Eq. (6)], the four-particle (1 proton, 3 pions) correlations considered in Sec. 2.1 appears in the moment  $\langle |Q_n|^2 Q_n^* e^{in\psi} \rangle$ . The idea is again to expand this moment, and to take the cumulant in order to remove unwanted terms. More precisely, the corresponding cumulant for the (1+3)-particle azimuthal correlation is:

$$\begin{aligned} \langle\langle |Q_n|^2 Q_n^* e^{in\psi} \rangle\rangle &\equiv \langle |Q_n|^2 Q_n^* e^{in\psi} \rangle - 2\langle Q_n^* e^{in\psi} \rangle \langle |Q_n|^2 \rangle \\ &= -\langle Q_n \rangle^3 v'_n + O\left(\frac{1}{M}\right). \end{aligned} \quad (9)$$

Once the pion flow  $\langle Q_n \rangle$  has been measured, this cumulant gives  $v'_n$  with accuracy  $O(1/M\langle Q_n \rangle^3) = O(1/(Mv_n)^3)$ , as did the cumulant (5).

## 4 Summary

We have shown that the standard flow analysis, which relies on two-particle azimuthal correlations, is in trouble when  $v_n$  becomes of order  $1/N^{1/2}$ . In opposition, the new method we advocate, based on a cumulant expansion of four-particle correlations, allows integrated flow measurements down to values  $v_n \gtrsim 1/N^{3/4}$ . It is even possible to extract smaller values, using higher order cumulants, as for example the cumulant  $\langle\langle |Q_n|^6 \rangle\rangle$ : we have derived a generating equation which gives the cumulant of the  $Q_n$  distribution at a given order, and relates it to the corresponding power of  $\langle Q_n \rangle$ .<sup>5</sup>

This new method also allows the measurement of differential flow, from the correlation between a particle azimuth and the event flow vector  $Q_n$ . Once again, the results of Sec. 3.3 can be generalized to arbitrary order, using a generating equation.<sup>5</sup> Finally, corrections for detector inefficiencies are naturally implemented in the method, using a redefinition of the cumulants.

We wish to emphasize that all the cumulants, either of multiparticle azimuthal correlations or of the  $Q_n$  vector, can be obtained from measured quantities, that is, the moments of the  $Q_n$  distribution: our method relies only on measurable quantities, and therefore can easily be implemented.

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