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SUPERSYMMETRIC R^4 ACTIONS AND QUANTUM CORRECTIONS TO SUPERSPACE TORSION CONSTRAINTS

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We present the supersymmetrisation of the anomaly-related R^4 term in eleven dimensions and show that it induces no non-trivial modifications to the on-shell supertranslation algebra and the superspace torsion constraints before inclusion of gauge-field terms.[1]

1 Higher-derivative corrections and supersymmetry

The low-energy supergravity limits of superstring theory as well as D-brane effective actions receive infinite sets of correction terms, proportional to increasing powers of $\alpha' = l_s^2$ and induced by superstring theory massless and massive modes. At present, eleven-dimensional supergravity lacks a corresponding microscopic underpinning that could similarly justify the presence of higher-derivative corrections to the classical Cremmer-Julia-Scherk action [1]. Nevertheless, some corrections of this kind are calculable from unitarity arguments and super-Ward identities in the massless sector of the theory [2] or by anomaly cancellation arguments [3, 4].

Supersymmetry puts severe constraints on higher-derivative corrections. For example, it forbids the appearance of certain corrections (like, e.g, R^3 corrections to supergravity effective actions [5]), and groups terms into various invariants [6–9]. The structure of the invariants that contain anomaly-cancelling terms is of great importance due to the quantum nature of the

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anomaly-cancellation mechanism and is the main concern of this note.

Higher-derivative additions to the supergravity actions are in general compatible with supersymmetry only if the transformation rules for the fields also receive higher-derivative corrections:

$$\left(\delta_0 + \sum_n (\alpha')^n \delta_n\right)\left(S_0 + \sum_n (\alpha')^n S_n\right) = 0\,. \tag{1}$$

As a consequence, the field-dependent structure coefficients on the right-hand side of the supersymmetry algebra,

$$[\delta_1^{\rm susy}, \delta_2^{\rm susy}] = \delta^{\rm translation} + \delta^{\rm susy} + \delta^{\rm gauge} + \delta^{\rm Lorentz}\,, \tag{2}$$

will be modified as well. When the theory is formulated in superspace the structure of the algebra is related to the structure of the tangent bundle, the link being provided by the constraints on the superspace torsion. In particular, corrections to the parameters modify the superspace constraints. However, since some corrections are reabsorbable by suitable rotations of the tangent bundle basis, not all corrections are physical.

We report here on the supersymmetrisation of the anomaly-related terms $(\alpha')^2 B \wedge F^4$ for super-Maxwell theory coupled to N=1 supergravity in ten dimensions and $(\alpha'_M)^3 C \wedge t_8 R^4$ (where $(\alpha'_M)^3 = 4\pi\,(l_P)^6$) in eleven dimensions performed in [10]. While the former does not require any corrections to the superspace constraints, there are strong indications based on a previous superspace analysis [20] that the latter does induce such modifications. However, we show that there are no such corrections which are proportional to (powers of) the Weyl tensor only. We present here only the more salients aspect of the analysis and refer to the article [10] for computational and bibliographical details.

Our main motivation to look for non-trivial corrections to superspace constraints comes from the link between these constraints and the kappa symmetry of M-branes [11–13] and D-branes [14–16]. Classical kappa invariance of the M- and D-brane world-volume actions — a key requirement for these objects to be supersymmetric — imposes the on-shell constraints on the background superspace supergravity fields, among them the superspace torsion. For this reason, any non-trivial modification to the constraints is expected to require new terms in the world-volume actions for the branes in order for kappa symmetry to be preserved.

2 Construction of an abelian F^4 superinvariant in D=10

As a first step in our analysis of the implications of higher-derivative corrections to the supersymmetry algebra, we discuss the construction of the abelian $(\alpha')^2(t_8F^4 - B\wedge F^4)$ for N=1 super-Maxwell theory coupled to gravity in ten dimensions.

The field content of the on-shell super-Maxwell theory comprises an abelian vector A_μ and a negative-chirality Majorana-Weyl spinor χ. Since we are interested in local supersymmetry invariance we have to take into account also the interactions with the zehnbein $e_\mu{}^r$, the negative-chirality Majorana-Weyl gravitino ψ_μ and the two-form $B_{\mu\nu}$ from the supergravity multiplet. The classical action (leaving out the gravitational sector)

$$S_{F^2} = \int \mathrm{d}^{10}x\, e\big[-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - 8\,\bar\chi\, \not{D}(\omega)\chi + 2\,\bar\chi\Gamma^\mu\Gamma^{\nu\rho}\psi_\mu\, F_{\nu\rho}\big] \tag{3}$$

is invariant under the local supersymmetry transformations

$$\delta A_\mu = -4\,\bar\epsilon\Gamma_\mu\chi\,, \qquad \delta\chi = \tfrac{1}{8}\Gamma^{\mu\nu}\epsilon\, F_{\mu\nu}\,. \tag{4}$$

For local supersymmetry we have to consider the transformations of the supergravity multiplet fields as well (neglecting terms proportional to the two-form $B_{\mu\nu}$ and the corresponding field strength, $H_{\mu\nu\rho}$):

$$\delta e_\mu{}^r = 2\,\bar\epsilon\Gamma^r\psi_\mu\,, \quad \delta\psi_\mu = D_\mu(\omega)\epsilon + \cdots\,, \quad \delta B_{\mu\nu} = 2\,\bar\epsilon\Gamma_{[\mu}\psi_{\nu]}\,. \tag{5}$$

The F^4 action invariant under the local supersymmetry transformations listed above has been determined previously [7, 8] and extends the globally supersymmetric action of [17, 18]. Using additional string input, we have managed to group all terms (including the fermionic bilinears) in a very compact way using the well-known t_8-tensor [10]. The result is

$$\begin{aligned} S_{F^4} = \frac{(\alpha')^2}{32}\int \mathrm{d}^{10}x\Big[\, & \tfrac{1}{6}e\, t_8^{(r)}F_{r_1r_2}\cdots F_{r_7r_8} + \tfrac{1}{12}\varepsilon_{10}^{(r)}B_{r_1r_2}F_{r_3r_4}\cdots F_{r_9r_{10}} \\ & - \tfrac{32}{5}e\, t_8^{(r)}\eta_{r_2r_3}(\bar\chi\Gamma_{r_1}D_{r_4}(\omega)\chi)F_{r_5r_6}F_{r_7r_8} + \tfrac{12\cdot 32}{5}e\,(\bar\chi\Gamma_{r_1}D_{r_2}(\omega)\chi)F^{r_1m}F_m{}^{r_2} \\ & - \tfrac{16}{5!}\varepsilon_{10}^{(r)}(\bar\chi\Gamma_{r_1\cdots r_5}D_{r_6}(\omega)\chi)F_{r_7r_8}F_{r_9r_{10}} + \tfrac{16}{3}e\, t_8^{(r)}(\bar\psi_{r_1}\Gamma_{r_2}\chi)F_{r_3r_4}F_{r_5r_6}F_{r_7r_8} \\ & + \tfrac{8}{3}e\,(\bar\psi_m\Gamma^{mr_1\cdots r_6}\chi)F_{r_1r_2}\cdots F_{r_5r_6}\Big]\,. \end{aligned} \tag{6}$$

The local supersymmetry invariance of the combined action $S_{F^2}+S_{F^4}$ requires that the supersymmetry transformations be modified according to ($F^2 := F^{mn}F_{nm}$)

$$\delta A_\mu = -4\,\bar\epsilon\Gamma_\mu\chi - (\alpha')^2\Big[\tfrac{1}{4}(\bar\epsilon\Gamma_\mu\chi)F^2 - (\bar\epsilon\Gamma^m\chi)F^2_{m\mu} - \tfrac{1}{8}(\bar\epsilon\Gamma^{r_1\cdots r_4}{}_\mu\chi)F_{r_1r_2}F_{r_3r_4}\Big]\,,$$
$$\delta\chi = \tfrac{1}{8}\Gamma^{\mu\nu}\epsilon\,F_{\mu\nu} + \tfrac{1}{768}(\alpha')^2\Big[t_8^{(r)}\Gamma_{r_7r_8}\epsilon - \Gamma^{r_1\cdots r_6}\epsilon\Big]F_{r_1r_2}F_{r_3r_4}F_{r_5r_6}\,. \tag{7}$$

It can be verified that the structure of the supersymmetry algebra is not modified by the order-$(\alpha')^2$ corrections [10,17,18]:

$$\big[\delta^{(\alpha')^0}_{\epsilon_1}+\delta^{(\alpha')^2}_{\epsilon_1},\delta^{(\alpha')^0}_{\epsilon_2}+\delta^{(\alpha')^2}_{\epsilon_2}\big]A_\mu = \big[\delta^{(\alpha')^0}_{\epsilon_1},\delta^{(\alpha')^0}_{\epsilon_2}\big]A_\mu + \mathcal{O}\big((\alpha')^4\big)\,. \tag{8}$$

Consequently, the structure of the superspace torsion constraints will be the same as for the classical theory to this order. This observation is related to the fact that it is possible to supersymmetrise the Dirac-Born-Infeld actions while imposing only the classical constraints [19].

3 Construction of the $C\wedge R^4$ superinvariant in D=11

Noticing the close parallel between the classical supersymmetry transformations for the super-Maxwell and the supergravity fields

$$\begin{aligned}
\delta\chi &= \tfrac{1}{8}\Gamma^{\mu\nu}\epsilon\,F_{\mu\nu}\,, & \delta\psi_{rs} &= \tfrac{1}{8}\Gamma^{\mu\nu}\epsilon\,R_{\mu\nu rs}+\cdots\,,\\
\delta F_{\mu\nu} &= -8D_{[\mu}(\bar\epsilon\Gamma_{\nu]}\chi)\,, & \delta R_{\mu\nu}{}^{rs} &= -8D_{[\mu}(\bar\epsilon\Gamma_{\nu]}\psi^{rs})\\
& & &\quad + 4D_{[\mu}(\bar\epsilon\Gamma_{\nu]}\psi^{rs}+2\,\bar\epsilon\Gamma^{[r}\psi^{s]}{}_{\nu]})+\cdots\,,
\end{aligned} \tag{9}$$

it is tempting to make the following substitution in the super-Maxwell action:

$$F_{r_1r_2}\to R_{r_1r_2s_1s_2}\,,\quad \chi\to\psi_{s_1s_2}\,,\quad D_r\chi\to D_r\psi_{s_1s_2}\,, \tag{10}$$

Unfortunately, the difference in structure between the equations of motion for the gauge potential and the spin connection implies that the previous mapping does not commute with supersymmetry, as can be seen by the presence of the second line in the supersymmetry transformation of the Riemann tensor above. Another crucial difference between the super-Maxwell and supergravity cases is that, when subtracting all the lowest-order equations of motions, it is necessary to make the following substitution for the Riemann tensor:

$$R_{mn}{}^{pq}\to W_{mn}{}^{pq} - \frac{16}{d-2}\delta_{[m}{}^{[p}(\bar\psi_{|r|}\Gamma^{|r|}\psi_{n]}{}^{q]} - \bar\psi^{|r|}\Gamma^{q]}\psi_{n]r})\,. \tag{11}$$

Taking all these facts into account, as well as the information from string-amplitude analysis that the extra s-type indices in (10) should be contracted with an additional $t_8^{(s)}$ tensor, we arrive at the following M-theory $C \wedge R^4$ invariant after lifting to eleven dimensions [10]:

$$
\begin{aligned}
(\alpha'_M)^{-3}\mathcal{L}_{\Gamma^{[0]}} &= +\tfrac{1}{192} e\, t_8^{(r)} t_8^{(s)}\, W_{r_1r_2s_1s_2} \cdots W_{r_7r_8s_7s_8} \\
&\quad + \quad \tfrac{1}{(48)^2} \varepsilon^{t_1t_2t_3r_1\cdots r_8}\, t_8^{(s)} C_{t_1t_2t_3} W_{r_1r_2s_1s_2} \cdots W_{r_7r_8s_7s_8}\,, \\
(\alpha'_M)^{-3}\mathcal{L}_{\Gamma^{[1]}} &= -\ 4\, e\, t_8^{(s)} (\bar\psi_{s_1s_2}\Gamma_{r_1} D_{r_2}\psi_{s_3s_4}) W_{r_1r_3s_5s_6} W_{r_3r_2s_7s_8} \\
&\quad - \ \tfrac{1}{4} e\, t_8^{(s)} (\bar\psi_{r_1}\Gamma_{r_2}\psi_{s_7s_8}) W_{r_1r_2s_1s_2} W_{mns_3s_4} W_{nms_5s_6} \\
&\quad - \quad e\, t_8^{(s)} (\bar\psi_{r_1}\Gamma_{r_2}\psi_{s_7s_8}) W_{r_1ms_1s_2} W_{mns_3s_4} W_{nr_2s_5s_6} \\
&\quad + \quad e\, t_8^{(s)} (\bar\psi_{r_1}\Gamma_{s_7}\psi_{r_2s_8}) W_{r_1r_2s_1s_2} W_{mns_3s_4} W_{nms_5s_6} \\
&\quad - \ 4\, e\, t_8^{(s)} (\bar\psi_{r_1}\Gamma_{s_7}\psi_{r_2s_8}) W_{r_1ms_1s_2} W_{mns_3s_4} W_{nr_2s_5s_6} \\
&\quad + \ \tfrac{2}{9} e\, t_8^{(s)} (\bar\psi_{m}\Gamma_{n}\psi_{ms_8}) W_{pqs_1s_2} W_{qps_3s_4} W_{ns_7s_5s_6} \\
&\quad - \ \tfrac{8}{9} e\, t_8^{(s)} (\bar\psi_{m}\Gamma_{n}\psi_{ms_8}) W_{nps_1s_2} W_{pqs_3s_4} W_{qs_7s_5s_6}\,, \\
(\alpha'_M)^{-3}\mathcal{L}_{\Gamma^{[3]}} &= +\ 2\, e\, t_8^{(s)} (\bar\psi_{s_5s_6}\Gamma_{r_1r_2r_3} D_{r_4}\psi_{s_7s_8}) W_{r_1r_2s_1s_2} W_{r_3r_4s_3s_4} \\
&\quad - \ \tfrac{1}{8} e\, t_8^{(s)} (\bar\psi_{m}\Gamma^{mr_1r_2}\psi_{s_7s_8}) W_{r_1r_2s_1s_2} W_{pns_3s_4} W_{nps_5s_6} \\
&\quad + \ \tfrac{1}{2} e\, t_8^{(s)} (\bar\psi_{m}\Gamma^{mr_1r_2}\psi_{s_7s_8}) W_{r_1ps_1s_2} W_{pns_3s_4} W_{nr_2s_5s_6} \\
&\quad + \quad e\, t_8^{(s)} (\bar\psi_{m}\Gamma^{r_1r_2r_3}\psi_{s_7s_8}) W_{r_1r_2s_1s_2} W_{mns_3s_4} W_{nr_3s_5s_6}\,, \\
(\alpha'_M)^{-3}\mathcal{L}_{\Gamma^{[5]}} &= +\ \tfrac{1}{8} e\, t_8^{(s)} (\bar\psi^{r_6}\Gamma^{r_1\cdots r_5}\psi_{s_7s_8}) W_{r_1r_2s_1s_2} W_{r_3r_4s_3s_4} W_{r_5r_6s_5s_6}\,, \\
(\alpha'_M)^{-3}\mathcal{L}_{\Gamma^{[7]}} &= +\ \tfrac{1}{48} e\, t_8^{(s)} (\bar\psi_{m}\Gamma_{mr_1\cdots r_6}\psi_{s_7s_8}) W_{r_1r_2s_1s_2} W_{r_3r_4s_3s_4} W_{r_5r_6s_5s_6}\,.
\end{aligned}
\tag{12}
$$

Even though the elfbein supersymmetry transformation rule receives $(\alpha'_M)^3$ modifications, by computing the closure of the supersymmetry algebra (2), we find [10] that the translation parameter does *not* receive corrections that cannot be absorbed by field redefinitions.

4 Superspace approach

It can be argued that in the completely general ansatz for the dimension-zero torsion constraint

$$T_{ab}{}^{r} = 2\Big((\mathcal{C}\Gamma^{r_1})_{ab}\, X^{r}{}_{r_1} + \tfrac{1}{2!}\,(\mathcal{C}\Gamma^{r_1 r_2})_{ab}\; X^{r}{}_{r_1 r_2} + \tfrac{1}{5!}\,(\mathcal{C}\Gamma^{r_1\cdots r_5})_{ab}\; X^{r}{}_{r_1\cdots r_5}\Big)\,, \quad (13)$$

the coefficient $X^{r}{}_{r_1}$ can be set equal to $\delta^{r}{}_{r_1}$ and all fully antisymmetric tensors contained in $X^{r}{}_{r_1 r_2}$ and $X^{r}{}_{r_1\cdots r_5}$ can be set to zero by a choice of tangent bundle basis (see, e.g., [22]). This leaves as the only candidates for non-trivial M-theory corrections to the standard dimension-zero constraint

$$T_{ab}{}^{r} = 2\,(\mathcal{C}\Gamma^{r})_{ab} \quad (14)$$

the SO(1,10) representations **429** and **4290** of the $\Gamma_{[2]}$ and $\Gamma_{[5]}$ coefficients, respectively. Therefore, from the component-field analysis of the previous section we conclude that the higher-order invariant (12) does not induce any modifications to the torsion constraint (13).

Howe has shown in [20] that by imposing *only* the constraint (14), the full classical, on-shell, eleven-dimensional supergravity theory of [1] follows. Hence, we conclude from the absence of corrections to (13) induced by the R^4 invariant (12), that any non-trivial M-theory corrections to the classical supergravity theory requires the inclusion of the four-form field strength. In addition, one may get such corrections from the inclusion of the $\epsilon\epsilon R^4$ interaction, which at the level of our analysis is part of a separate superinvariant and has therefore not been taken into account yet.

One can argue, on the basis of lifting of the type IIA action [21], that this $\epsilon\epsilon R^4$ term should be present in the eleven-dimensional theory as well, but it is interesting to note that our results give another strong indication that this term should be present. Otherwise our analysis, when combined with Howe's, would imply that the dynamics encoded in the action (12) does not correspond to any non-trivial corrections to the classical supergravity theory of [1] for configurations with vanishing four-form field strength.

In this context, let us also mention that in parallel with our component-field based approach to uncover the superspace underlying M-theory, a complementary line of attack based on an analysis of the superspace Bianchi identities has been initiated by Cederwall et al. in [22].

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