Higher-derivative corrections in M-theory toward a superspace analysis

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what?

Find superspace manifestations of higher-derivative corrections to low-energy field theories arising from superstring theory, e.g.

$$\int \mathrm{d}^{10}x\, e\, [R + (\alpha')^3 R^4 + \cdots]$$

or

$$\int \mathrm{d}^{11}x\, e\, [R + l_{11}^3 R^4 + \cdots]$$

or

$$\int \mathrm{d}^{p+1}\xi \sqrt{-\det(g+\mathcal{F})}\,[1 + (\alpha')^2 R^2 + \cdots]$$

why?

Understand symmetry constraints and quantum consistency:

M-theory effective action:

UV behavior, symmetries, . . .

M/D-brane quantum physics

how?

classical supersymmetry → classical superspace

stringy supersymmetry → stringy superspace

Higher-derivative field theory from strings

- QFT/String scattering amplitudes upto two loops

[Bern, Dixon, Dunbar, Perelstein, Rozowski],[many authors]

$$\text{four gravitons} \rightsquigarrow \int \mathrm{d}^{10}x\, e\, f_k\, \Box^k R^4$$

- Consistency of sigma models in backgrounds [Grisaru, van de Ven, Zanon]

[Bachas, Bain, Green], [Shmakova], [Green, Gutperle]

$$S = T_p \int \mathrm{d}^{p+1}\sigma \sqrt{\det\left(G_{ab}(X) + 2\pi\alpha' \mathcal{F}_{ab}(X)\right)}$$

  - Higher loops $\rightsquigarrow$ higher-derivative constraints on background.
  - Loop order fixed by $T_p \sim 1/(\alpha')^{(p+1)/2}$.

- Anomaly considerations

[Vafa, Witten], [Duff, Liu, Minasian], [Green, Harvey, Moore]

$$S_{10}^{IIA} \ni \int B^{NS} \wedge R^4, \quad \text{or } S_{11} \ni \int C \wedge R^4$$

$$S_{\mathrm{D}p} \ni \int C \wedge e^{\mathcal{F}} \wedge \sqrt{\hat{A}(R)}$$

- Supersymmetry requirements

[de Roo, Suelmann, Wiedemann], [Green, Sethi]

$$S = \int \mathrm{d}^{10}x\, e\, [R^4 + BR^4 + \psi^2 (R^3 + R^2\, DR) + \cdots]$$

Supersymmetry not manifest!

Stringy superspace

- Very useful: manifestly susy construction of complicated actions

  Example: Linearized type IIB supergravity [Howe,West]

$$\Phi(x,\theta) = \phi(x) + \cdots + (\theta^T \mathcal{C}\Gamma^{abe}\theta)(\theta^T \mathcal{C}\Gamma_e{}^{cd}\theta) R_{abcd} + \cdots \, ,$$

$$\int \mathrm{d}^{10}x \, \mathrm{d}^{16}\theta^* \, \Phi^4(x,\theta) = \int \mathrm{d}^{10}x \, e \, R^4 + \text{other terms}$$

  1/2-superspace integration $\Rightarrow$ non-renormalization theorems.

  [Berkovits], [Pioline]

  Equivalent expression for higher-order $D^4 R^4$ will use 1/4-superspace integration. Proof of supersymmetry necessitates to use the constraints. Non-renormalization thms?

  [Howe, Stelle, Townsend]

- Essential: D-brane actions including gravitational curvature terms

  [Green, Harvey, Moore], [Bachas, Bain, Green], [Cheung, Yin]

$$-\int \mathrm{d}^{p+1}\xi \sqrt{-\det(g+\mathcal{F})}\,[1+(\alpha')^2 R^2+\cdots] + \int C \wedge e^{\mathcal{F}} \wedge \sqrt{\frac{\hat{A}(\alpha' R_{\mathrm{T}})}{\hat{A}(\alpha' R_{\mathrm{N}})}}$$

require quantum kappa symmetry

Kappa symmetry

κ-symmetry is *essential* for extended objects to be supersymmetric, by allowing a match between the dof for bosons and fermions.

$$\delta_\kappa Z^M = \kappa^\alpha E_\alpha{}^M \,,$$
$$Z^M(\xi) \equiv (X^m(\xi), \theta^\mu(\xi)) \,, \quad \kappa^\alpha(\xi) = P_+ \zeta$$

Here $P_\pm = \frac{1}{2}(1 \pm \Gamma_*)$ are projection operators of half-maximal rank ($\Leftrightarrow \Gamma_*^2 = 1$ and $\mathrm{tr}\, \Gamma_* = 0$) $\rightsquigarrow$ half of $\theta^\mu(\xi)$ projected out

[Bergshoeff, Sezgin, Townsend], [Cederwall, von Gussich, Nilsson, Sundell, Westerberg]

Invariance only for specific backgrounds $(E_M{}^A, C_{M_1 \dots M_k}, \cdots)$ for which the on-shell constraints are satisfied, e.g.

$$T_{\alpha\beta}{}^c = (\Gamma^c)_{\alpha\beta} \,, \quad T_{\alpha b}{}^c = 0 \,, \quad H_{\alpha\beta cd} = (\Gamma_{cd})_{\alpha\beta} \,.$$

(Bianchi identities $\rightsquigarrow$ further constraints + eqs of motion)

Classically, the kappa-symmetry constraints coincide with the supergravity constraints. At the quantum level, anomalies may spoil this.

Kappa anomalies and stringy constraints

Example: the heterotic string in a gauge and gravitational background

[Tonin], [Candiello, Lechner, Tonin]

$$S = \int \mathrm{d}^2\sigma \left[\sqrt{h} h^{ij} E_i{}^a E_{ja} + \epsilon^{ij} E_j{}^B E_i{}^A B_{AB} + \sqrt{h}\epsilon_-{}^i \bar{\psi}(\partial_i - E_i{}^A A_A)\psi\right]$$

Induced kappa transformations for background fields include:

$$\delta_\kappa A_i = \mathcal{D}_i\big(\kappa^\alpha A_\alpha\big) + \cdots ,$$

$$\delta_\kappa \Omega_{ia}{}^b = D_i\big(\kappa^\alpha \Omega_{\alpha a}{}^b\big) + \cdots .$$

like gauge symmetries

Quantum kappa symmetry requires:

$$(\mathrm{d}B)_{\alpha\beta c} = (\Gamma_c)_{\alpha\beta} + \alpha'(\omega_{3\,\mathrm{YM}} - \omega_{3\,\mathrm{L}})_{\alpha\beta c} + \mathcal{O}(\alpha'^2) .$$

anomaly-cancellation terms reproduced

Bianchi identities on $\mathrm{d}^2 B = 0$ *are satisfied* for this model.

Where are the modified constraints?

In $D = 11$ the corrections to the constraints are possible, e.g.:

$$T_{\alpha\beta}{}^{a} = (\mathcal{C}\Gamma^{a})_{\alpha\beta} + (\mathcal{C}\Gamma^{bc})_{\alpha\beta} X^{a}{}_{bc} + (\mathcal{C}\Gamma^{bcdef})_{\alpha\beta} X^{a}{}_{bcdef}$$

5808 **1** **429**+165+11 **4290**+462+330

Only the diagrams **429** and **4290** are independent of the choice of the bundle (i.e. of super-vielbein redefinitions). [Cederwall]

- impose only $T_{\alpha\beta}{}^{c} = (\Gamma^{c})_{\alpha\beta}$ $\Rightarrow$ standard $D{=}11$ sugra eom on-shell [Howe]

- Quantum corrections will modify

$$S_{11} = \int \mathrm{d}^{11}x\, e[R + \alpha^{3} R^{4} + \cdots] \Rightarrow T_{\alpha\beta}{}^{a} = (\Gamma^{a})_{\alpha\beta} + \alpha^{3} X^{a}_{\alpha\beta}$$

- Fit component results in superspace $\rightsquigarrow$ automatically leads to fields that satisfy constraints. [Peeters,Vanhove,Westerberg]

Complementary approaches

[Cederwall,Gran,Nielsen,Nilsson]

- starts from superspace

modified component action

$$\int R + (\alpha')^3 (R^4 + \psi^2 R^3 + \cdots)$$

use string input here

modified transformation rules

$$\delta e_m{}^a = \bar{\epsilon}\Gamma^a \psi_m + (\alpha')^3 (\ldots)$$

absolutely needs string input

modified superalgebra

$$[\delta_1, \delta_2] e_m{}^a = (\ldots) + (\alpha')^3 (\ldots)$$

modified supertorsion constraints

$$T_{\alpha\beta}{}^a = (\Gamma^a)_{\alpha\beta} + (\alpha')^3 (\ldots)$$

- interpretation straightforward
- systematic (but complicated!)

[Peeters,Vanhove,Westerberg]

in $d = 11$

From superalgebra to constraints

Higher-derivative action $S = S^{(0)} + (\alpha')^3 S^{(3)} \cdots$ is susy only up to the equations of motion for classical action $S^{(0)}$; invariance to order $(\alpha')^3$ achieved by modifying the susy transformation laws: [Green,Sethi]

$$\delta_\epsilon^{(3)} e = \epsilon\psi + (\alpha')^3 f(W)\epsilon\psi \,, \cdots$$

field-dependent coefficients

The susy algebra on ${E_m}^a$ takes the form

$$[\delta_{1,\mathrm{sg}}, \delta_{2,\mathrm{sg}}] = \delta^{\mathrm{translation}}(\xi^m) + \delta^{\mathrm{susy}}(-\xi^m\psi_m) + \delta^{\mathrm{Lorentz}}(\xi^m {\hat\omega_m}{}^a{}_b + 4\,\bar\epsilon_2 \mathcal{S}^{a\;mnpq}{}_b \epsilon_1 \hat F_{mnpq}) + (\alpha')^3(\cdots)\,,$$

The corresponding transformations of the superfield

$$\delta {E_M}^A = \Xi^N \partial_N {E_M}^A + \partial_M \Xi^M {E_N}^A + \tfrac{1}{2}(\Lambda^{cd}X_{cd})^A{}_B {E_M}^B$$

obey the algebra

$$\Xi^M(\epsilon_1,\epsilon_2) = \Xi^N(\epsilon_2)\partial_N \Xi^M(\epsilon_1) + \delta_{1,\mathrm{sg}}\Xi^M(\epsilon_2) - (1 \leftrightarrow 2)$$

The superfield is invariant under the combined transformation

$$\delta_{\mathrm{tot}} {E_M}^A \equiv \delta {E_M}^A - \delta_{\mathrm{sg}} {E_M}^A = 0$$

Choosing the gauge

$$E_\alpha{}^m = \mathcal{O}(\theta)\,, \quad E_\alpha{}^\mu = \delta_\alpha{}^\mu\,, \quad E_\mu{}^a = \mathcal{O}(\theta)\,,$$
$$\Xi^m(x,\theta=0) = \xi^m\,, \quad \Xi^\mu(x,\theta=0) = \epsilon^\mu\,,$$

and using the definition $T^C = DE^C$ then allows certain constraints to be determined, e.g.

$$T_{\alpha\beta}{}^a = (\Gamma^a)_{\alpha\beta} + (\alpha')^3 X^a{}_{\alpha\beta}\,.$$

Supersymmetry building blocks

- Field-strength terms can easily be included using the Holonomy tensor: Define the super-covariant derivative

$$\delta_\epsilon \psi_\mu = \nabla_\mu \epsilon = \mathcal{D}_\mu \epsilon + \beta\, T_\mu \cdot \hat{F}$$

  Vary the gravitino curvature

$$\delta_\epsilon \nabla_{[\mu} \psi_{\nu]} = \mathcal{H}_{\mu\nu}{}^{ab} \gamma^{ab} \epsilon + \beta \mathcal{D}_{[\mu} (T_{\nu]} \cdot \hat{F}) + \text{Ricci terms} + \cdots$$

  appears in the invariants

- In $d = 11$ $\mathcal{H}_{\mu\nu}{}^{ab} = W_{\mu\nu}{}^{ab} - \frac{1}{12} \hat{F}_{\mu\nu} \cdot \hat{F}^{ab}$ and $\mathcal{D}\hat{F}_{(4)}$ independent building block as required by Cremmer/Ferrara's superfield.
- In $d = 10a$ $\mathcal{H}_{\mu\nu}{}^{ab} = W_{\mu\nu}{}^{ab} + \cdots$ and $\mathcal{D}\hat{F}_{(5)}$ independent building blocks. $\mathcal{H}$ contains Ramond fields.
- In $d = 10b$ $\mathcal{H}_{\mu\nu}{}^{ab} = W_{\mu\nu}{}^{ab}$ and $\mathcal{D}\hat{F}_{(5)}$ independent building blocks as required by Howe/West's *linearized* superfield.

Superstring Theory Input

- Five-point function are needed by local supersymmetry because of super-Ward identities:

$$\mathrm{Res}\left\langle \left(V_g^{(0)}\right)^2 V_\psi^{(\frac{1}{2})} V_\psi^{(-\frac{1}{2})}(z_4) V_g^{(0)}(z_5)\right\rangle\Big|_{z_4=z_5} = \left\langle \left(V_g^{(0)}\right)^2 V_\psi^{(\frac{1}{2})} V_\psi^{(-\frac{1}{2})}(z_4)\right\rangle$$

  recall $Q_\alpha = \oint \frac{dw}{2i\pi} V_\alpha^{(-\frac{1}{2})}(k=0)$ and $[\epsilon Q, V_g^{(0,0)}] = V_{\delta\psi}^{(-\frac{1}{2},0)}$.
- Superstring organizes the various tensorial structures: useful for $(\mathrm{fermi})^2$ terms.

One-loop four-point amplitudes

- 4 gravitons even-even spin structure

$$t_8^{(r)} t_8^{(s)} W^4$$

- 2 gravitini/2 gravitons even-even spin structure

$$t_8^{(r)} t_8^{(s)} (\psi_{(2)} \left[\Gamma_{r_1 r_2}, \Gamma_{r_3}\right] D_{r_4} \psi_{(2)} W^2)$$

$$t_8^{(s)} (\psi_{s_1 s_2} \Gamma_{r_1} D_{r_2} \psi_{s_3 s_4} W_{r_1 m s_5 s_6} W_{m r_2 s_7 s_8}$$

- 2 gravitini/2 gravitons odd-even spin structure

$$\varepsilon_{(10)}^{(r)} t_8^{(s)} (\psi_{(2)} \Gamma_{r_1 \cdots r_5} D_{r_6} \psi_{(2)} W^2)$$

One-loop five-point amplitudes

- 5 gravitons odd-odd spin structure

$$\varepsilon_{(10)}^{(r)} \varepsilon_{(10)}^{(s)} W^4$$

- 2 gravitini/3 gravitons odd-even spin structure

$$\varepsilon_{(10)}^{(r)} t_8^{(s)} (\psi_{r_1} \Gamma_{r_2 \cdots r_4} \psi_{(2)} W^3)$$

- B-field/4 gravitons odd-even spin structure

$$\varepsilon_{(10)}^{(r)} \, B \, t_8^{(s)} W^4$$

$$t_8(M_1 M_2 M_3 M_4) = -\, 2\mathrm{tr}(M_1 M_2)\mathrm{tr}(M_3 M_4) + 8\mathrm{tr}(M_1 M_2 M_3 M_4) + \text{cycl.(2,3,4)}$$

$$\varepsilon_{(10)}^{\mu\nu\, r_1 \cdots r_8} \varepsilon_{(10)}^{\mu\nu\, s_1 \cdots s_8} \propto \delta_{s_1 \cdots s_8}^{r_1 \cdots r_8}$$

$N=2\ D=10$ type IIa 'R^4' invariants

$$\begin{aligned}
\mathcal{L}_X = +\quad & \alpha_0^{(1)}((W^2)^2 - 4W^4) + \alpha_0^{(2)}\varepsilon_{(11)}BW^4 \\
& + \alpha_1\, t_8^{(s)}(\bar\psi_{s_1s_2}\Gamma_{r_1}D_{r_2}\psi_{s_3s_4})W_{r_1r_3s_5s_6}W_{r_3r_2s_7s_8} \\
& + \alpha_2\, t_8^{(s)}(\bar\psi_{r_1}\Gamma_{r_2}\psi_{s_7s_8})W^3 \\
& + \alpha_3\, t_8^{(s)}(\bar\psi_{s_5s_6}\Gamma_{r_1r_2r_3}D_{r_4}\psi_{s_7s_8})W_{r_1r_2s_1s_2}W_{r_3r_4s_3s_4} \\
& + \alpha_4\, t_8^{(s)}(\bar\psi_m\Gamma^{mr_1r_2}\psi_{s_7s_8})W^3 \\
& + \alpha_5\, t_8^{(s)}(\bar\psi_m\Gamma^{r_1r_2r_3}\psi_{s_7s_8})W_{r_1r_2s_1s_2}W_{mns_3s_4}W_{nr_3s_5s_6} \\
& + \alpha_6\, t_8^{(s)}(\bar\psi_m\Gamma_{r_1\cdots r_5}\psi_{s_7s_8})W_{mr_1s_1s_2}W_{r_2r_3s_2s_3}W_{r_4r_5s_4s_5} \\
& + \alpha_7\, t_8^{(s)}(\bar\psi_m\Gamma_{mr_1\cdots r_6}\psi_{s_7s_8})W_{r_1r_2s_1s_2}W_{r_3r_4s_3s_4}W_{r_5r_6s_5s_6} \\
& + \mathcal{O}(\psi^4)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{X-Z} = +& \beta_0^{(1)}((W^2)^2 - 4W^4) + \beta_0^{(2)}\varepsilon_{(10)}\varepsilon_{(10)}W^4 \\
& + \beta_1\, t_8^{(s)}(\bar\psi_{s_1s_2}\Gamma_{r_1}D_{r_2}\psi_{s_3s_4})W_{r_1r_3s_5s_6}W_{r_3r_2s_7s_8} \\
& + \beta_2\, t_8^{(s)}(\bar\psi_{r_1}\Gamma_{r_2}\psi_{s_7s_8})W^3 \\
& + \beta_3\, t_8^{(s)}(\bar\psi_{s_5s_6}\Gamma_{r_1r_2r_3}D_{r_4}\psi_{s_7s_8})W_{r_1r_2s_1s_2}W_{r_3r_4s_3s_4} \\
& + \beta_4\, t_8^{(s)}(\bar\psi_m\Gamma^{mr_1r_2}\psi_{s_7s_8})W^3 \\
& + \beta_5\, t_8^{(s)}(\bar\psi_m\Gamma^{r_1r_2r_3}\psi_{s_7s_8})W_{r_1r_2s_1s_2}W_{mns_3s_4}W_{nr_3s_5s_6} \\
& + \beta_6\, t_8^{(r)}(\bar\psi_m\Gamma^{s_1\cdots s_5}\psi_{r_7r_8})W^3 + \mathcal{O}(\psi^4)
\end{aligned}$$

- Necessary to expand one $t_8^{(r)}$ to make susy manifest and to lift the result to $d=11$.
- Form-field included in the holonomy tensor.

Summary and open questions

Open questions:

- Precise tensorial structure in $D = 11$: off-shell $t_{16} \neq t_8^{(r)} t_8^{(s)} \pm \varepsilon_{(11)} \varepsilon_{(11)}$ (No Weyl condition in $D = 11$)
- Discrepancy with Matrix Model computation

  [Green, Gutperle, Kwon], [Nicolai, Plefka],[Dasgupta, Nicolai, Plefka]

In progress/to do:

- Construct the torsion constraints
- Solve these constraints
- Relation to M/D-brane world-volume QFT and anomalies
- Membrane analysis of various tensorial structures appearing in the invariants.